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INTEGRABILITY OF THE MAJORANT OF THE FOURIER SERIES PARTIAL SUMS WITH RESPECT TO BASES

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Abstract. The majorant of Fourier series partial sums with respect to the system of functions formed by the product of $L([0,1])$ space bases is considered. It is proved that in any Orlicz space wider than $L(\log^+ L)^d([0,1]^d)$, $d \geq 1$, the set of functions with such a majorant is integrable on $[0,1]^d$ and has the first Baire category.

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In investigations of the convergence of Fourier series with respect to various systems of functions the majorant of partial sums plays an important role (see [1], [2]). P. L. Ul'janov ([3], p. 928) constructed a function $f \in \bigcup_{0\leq r<1} L(\log^+ L)^r(I)$, $I=[0,1]$, such that its majorant of partial Fourier–Haar sums is not integrable. A. M. Olevskii ([1], p. 75) extended this statement to arbitrary bases of $L(I)$ and arbitrary spaces of functions wider than $L\log^+ L$. O. Kovachik [4] proved that, for an arbitrary convex function $Q:[0,\infty[\to[0,\infty[$, under the condition $Q(u)=o(u\log^2 u)$ for $u\to\infty$ there exists a function $f(x_1,x_2)$ such that $\int_{I^2} Q(f)<\infty$ and its majorant of partial sums is not integrable on I^2. Similar problems for martingales are considered in [5], [6], see also [7].

The aim of the present paper is to establish a condition for the integrability of the majorant of partial sums with respect to the system formed by the product of bases.

For an integer $d \geq 1$ let I^d be the d-dimensional unit cube and let Ψ_d be the system defined by

$$\Psi_d=\Big\{\prod_{j=1}^{d}\psi_{i_j}^{(j)}(x_j),\quad x_j\in I,\quad i_j\in\mathbb{N},\quad j=1,\dots,d\Big\}, \tag{1}$$

where $\{\psi_{i_j}^{(j)}(x_j)\}_{i_j=1}^{\infty}$, $x_j\in I$, $j=1,\dots,d$, are the bases of the space $L(I)$ and let $\{\overline{\psi}_{i_j}^{(j)}\}_{i_j=1}^{\infty}$, $j=1,\dots,d$, be the system of functionals dual to them. Let the partial Fourier sums of the function $f\in L(I^d)$ with respect to the system Ψ_d be defined by

$$S_{k_1,\dots,k_d}(f,\Psi_d)=\sum_{i_1=1}^{k_1}\cdots\sum_{i_d=1}^{k_d}c_{i_1,\dots,i_d}(f,\Psi_d)\prod_{j=1}^{d}\psi_{i_j}^{(j)}(x_{i_j}), \tag{2}$$

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where $c_{i_1,\dots,i_d}(f,\Psi_d)$ is the Fourier coefficient of the function f with respect to the system Ψ_d. We use below the following majorants of partial sums:

$$S^*_{M_1,\dots,M_d}(f,\Psi_d) = \max\big\{|S_{k_1,\dots,k_d}(f,\Psi_d)| : \quad 1 \le k_j \le M_j, \quad 1 \le j \le d\big\}, \qquad (3)$$

$$S^*(f,\Psi_d) = \max\big\{|S_{k_1,\dots,k_d}(f,\Psi_d)| : \quad 1 \le k_j < \infty, \quad 1 \le j \le d\big\}. \qquad (4)$$

Let $H_1 = \{\mathcal{X}_{i_1}(x_1)\}_{i_1=1}^{\infty}$ be the Haar system on the interval $I^1 = I$ (see [1], p. 77), and for $d \ge 2$, let $H_d = \{\prod_{j=1}^{d} \mathcal{X}_{i_j}^{(j)}(x_j),\ x_j \in I,\ i_j \in \mathbb{N},\ j = 1,\dots,d\}$ be the d-multiple Haar system defined on I^d.

Throughout the paper C_r, $r = 0,\dots,9$, are positive absolute constants and $\log x = \log_2 x$, $\log^+ u = \max\{0, \log u\}$. In the sequel $L_Q(I^d)$ denotes the Orlicz space of functions defined on the cube I^d which is generated by the Young function Q and equipped with the norm $\|\cdot\|_{L_Q(I^d)}$.

In addition, $L(\log^+ L)^r(I^d)$, $r \ge 0$, denotes the Orlicz space generated by the Young function which is equal to $u \log^r u$ for $u \ge 2$ ([8], p. 28) and equipped with the norm $\|\cdot\|_{L(\log^+ L)^r(I^d)}$.

We shall prove

Theorem 1. *Let $L_Q(I^d)$ be the Orlicz space satisfying the condition*

$$\liminf_{u\to\infty} Q(u)u^{-1}\log^{-d} u = 0 \qquad (5)$$

and let Ψ_d be a system of form (1) *generated by arbitrary bases of the space $L(I)$. Then the set*

$$E = \big\{f \in L_Q(I^d) : S^*(f,\Psi_d) \in L(I^d)\big\} \qquad (6)$$

has the first Baire category in the space $L_Q(I^d)$.

First we prove the following

Lemma 1. *Let $L_Q(I^d)$ be an Orlicz space satisfying condition* (5)*. Then for any $R \in \mathbb{N}$ there exists a polynomial $g^{(d)}_{R,Q}$ with respect to the system H_d*

$$g^{(d)}_{R,Q} = g^{(d)}_{R,Q}(x_1,\dots,x_d) = \sum_{i_1=1}^{M} \cdots \sum_{i_d=1}^{M} a_{i_1,\dots,i_d}\, \mathcal{X}_{i_1}(x_1)\dots\mathcal{X}_{i_d}(x_d) \qquad (7)$$

such that the following estimates hold:

$$\big\|g^{(d)}_{R,Q}\big\|_{L_Q(I^d)} \le 2, \qquad (8)$$

$$\int_{I^d} \big|g^{(d)}_{R,Q}\big|\big(log^+\big|g^{(d)}_{R,Q}\big|\big)^{d-1} \le 1, \qquad (9)$$

$$\big\|S^*_{M,\dots,M}(g^{(d)}_{R,Q}, H_d)\big\|_{L(I^d)} > R8^{-d}d^{-d}. \qquad (10)$$

Proof. Let R be an arbitrary positive integer. By virtue of (5) there exists $u_{R,Q} > 2^R$ such that

$$Q(u_{R,Q})u^{-1}_{R,Q}\log^{-1} u_{R,Q} < 2^{-d}R^{-1} \qquad (11)$$

and
$$2^{dp_R} \le u_{R,Q} < 2^{d(p_R+1)}, \tag{12}$$
where $p_R = p_R(Q,d) = [\frac{1}{d}\log u_{R,Q}]$; here $[a]$ denotes the integer part of a. We put
$$g^{(d)}_{R,Q} = g^{(d)}_{R,Q}(x_1,\dots,x_d) = \begin{cases} Rd^{-d}(p_R+1)^{-d}2^{dp_R} & \text{for} \quad 0\le x\le 2^{-p_R}, \\ & \qquad 1\le j\le d, \\ 0, \quad \text{in other points on } I^d. \end{cases} \tag{13}$$
In view of the convexity of the function Q and the property of an Orlicz space norm (see [8], p. 89), from (11) and (12) we have
$$\begin{aligned}\|g^{(d)}_{R,Q}\|_{L_Q(I^d)} &\le 1 + \int\limits_{I^d} Q(g^{(d)}_{R,Q}) \le 1 + Q(2^{dp_R})2^{-dp_R}Rd^{-d}(p_R+1)^{-d} \\ &\le 1 + Q(u_{R,Q})R\,2^d 2^{-d(p_R+1)}d^{-d}(p_R+1)^{-d} \\ &\le 1 + Q(u_{R,Q})R\,2^d u^{-1}_{R,Q}\log^{-1}u_{R,Q} \le 2.\end{aligned}$$
Estimate (9) follows directly from (13):
$$\int\limits_{I^d} |g^{(d)}_{R,Q}|\big(\log^+ g^{(d)}_{R,Q}|\big)^{d-1} \le Rd^{-d}(p_R+1)^{-d}d^{d-1}p_R^{d-1} \le 1.$$
By (13) the function g can be represented as polynomial (7) with respect to the system H_d, where $M = M(R,Q,d) = 2^{p_R+1}$. From the estimate of [1], p. 96, follows (10):
$$\big\|S^*_{M,\dots,M}(g^{(d)}_{R,Q},H_d)\big\|_{L(I^d)} \ge Rd^{-d}(p_R+1)^{-d}2^{-2d}p_R^d \ge R8^{-d}d^{-d}. \qquad \square$$

Proof of Theorem 1. We consider the case $d=2$ as a typical one, since in the general case the proof is similar but cumbersome. Let $\{\psi^{(j)}_{i_j}(x_j)\}^\infty_{i_j=1}$, $x_j\in I$, $j=1,2$, be the bases of the space $L(I)$ and Ψ_2 be the system defined according to (1). Using A. M. Olevskii's theorem ([2], p. 62) we find, for $j=1,2$, systems $\{u^{(j)}_{i_j}(x_j)\}^\infty_{i_j=1}$ and sequences $\{n^{(j)}_{p_j}\}^\infty_{p_j=1}$, $0<n^{(j)}_1<n^{(j)}_2<\cdots$, such that the following conditions hold:

a) for any integer N_j, $j=1,2$, there exist measure-preserving bijections $\lambda^{(j)}_{N_j}; I\to I$ such that
$$u_{i_j}(x_j) = \mathcal{X}_{i_j}\big(\lambda^{(j)}_{N_j}(x_j)\big) \quad \text{for} \quad i_j=1,\dots,N_j, \quad j=1,2; \tag{14}$$

b)
$$\big\|A^{i_j}_{p_j,j}\big\|_{L(I)} \le 2^{-p_j-i_j}, \quad p_j\ne i_j, \quad p_j,i_j=1,2,\dots, \tag{15}$$
where
$$A^{i_j}_{p_j,j}(x_j) = \sum_{n_j\in\Gamma^{(j)}(p_j)} \big(u^{(j)}_{i_j},\overline{\psi}^{(j)}_{n_j}\big)\psi^{(j)}_{n_j} \tag{16}$$
and
$$\Gamma^{(j)}(p_j) = \big\{n^{(j)}_{p_j}+1,\dots,n^{(j)}_{p_j+1}\big\}. \tag{17}$$

Let arbitrary $R \in \mathbb{N}$ be given and let the function $f_{R,Q}$ be defined by

$$f_{R,Q} = f_{R,Q}(x_1, x_2) = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} a_{i_1,i_2} u_{i_1}^{(1)}(x_1) u_{i_2}^{(2)}(x_2), \tag{18}$$

where M and a_{i_1,i_2} are defined according Lemma 1 (see (7)–(10)).

Then by (14), (8) and (18) we have

$$\|f_{R,Q}\|_{L_Q(I^2)} \le 2. \tag{19}$$

Now we shall prove that for the system Ψ_2 the following inequality holds (see (3)):

$$\left\| S^*_{r_1(M+1), r_2(M+1)}(f_{R,Q}, \Psi_2) \right\|_{L(I^2)} > 2^{-8} R - C^*, \tag{20}$$

where C^* is the absolute positive constant defined below and $r_j(p) = n_p^{(j)}$, $j = 1, 2$.

By virtue of (16), (17), (2), for $k_j \le M$, $j = 1, 2$, we have

$$\begin{aligned}
&S_{r_1(k_1+1), r_2(k_2+1)}(f_{R,Q}, \Psi_2) \\
&\quad = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} a_{i_1,i_2} \sum_{p_1=1}^{k_1} \sum_{n_1 \in \Gamma^{(1)}(p_1)} \big(u_{i_1}^{(1)}, \overline{\psi}_{n_1}^{(1)}\big) \psi_{n_1}^{(1)} \sum_{p_2=1}^{k_2} \sum_{n_2 \in \Gamma^{(2)}(p_2)} \big(u_{i_2}^{(2)}, \overline{\psi}_{n_2}^{(2)}\big) \psi_{n_2}^{(2)} \\
&\quad = \sum_{p_1=1}^{k_1} \sum_{p_2=1}^{k_2} a_{p_1,p_2} A_{p_1,1}^{p_1}(x_1) A_{p_2,2}^{p_2}(x_2) \\
&\qquad + \sum_{p_1=1}^{k_1} \sum_{i_2=1}^{M} a_{p_1,i_2} A_{p_1,1}^{p_1}(x_1) \sum_{p_2=1, p_2 \ne i_2}^{k_2} A_{p_2,2}^{p_2}(x_2) \\
&\qquad + \sum_{p_2=1}^{k_2} \sum_{i_1=1}^{M} a_{i_1,p_2} A_{p_2,2}^{p_2}(x_2) \sum_{p_1=1, p_1 \ne i_1}^{k_1} A_{p_1,1}^{p_1}(x_1) \\
&\qquad + \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} a_{i_1,i_2} \sum_{p_1=1, p_1 \ne i_1}^{k_1} A_{p_1,1}^{p_1}(x_1) \sum_{p_2=1, p_2 \ne i_2}^{k_2} A_{p_2,2}^{p_2}(x_2) \\
&\quad = \sum_{i_1=1}^{k_1} \sum_{i_2=1}^{k_2} a_{i_1,i_2} u_{i_1}^{(1)}(x_1) u_{i_2}^{(2)}(x_2) \\
&\qquad + \sum_{i_1=1}^{k_1} \sum_{i_2=1}^{k_2} a_{i_1,i_2} u_{i_1}^{(1)}(x_1) \big[A_{i_2,2}^{i_2}(x_2) - u_{i_2}^{(2)}(x_2)\big] \\
&\qquad + \sum_{i_1=1}^{k_1} \sum_{i_2=1}^{k_2} a_{i_1,i_2} u_{i_2}^{(2)}(x_2) \big[A_{i_1,1}^{i_1}(x_1) - u_{i_1}^{(1)}(x_1)\big] \\
&\qquad + \sum_{i_1=1}^{k_1} \sum_{i_2=1}^{k_2} a_{i_1,i_2} \big[A_{i_1,1}^{i_1}(x_1) - u_{i_1}^{(1)}(x_1)\big] \big[A_{i_2,2}^{i_2}(x_2) - u_{i_2}^{(2)}(x_2)\big]
\end{aligned}$$

$$+\sum_{i_1=1}^{k_1}\sum_{i_2=1}^{M} a_{i_1,i_2} u_{i_1}^{(1)}(x_1) \sum_{p_2=1,p_2\neq i_2}^{k_2} A_{i_2,2}^{p_2}(x_2)$$

$$+\sum_{i_1=1}^{k_1}\sum_{i_2=1}^{M} a_{i_1,i_2} \left[A_{i_1,1}^{i_1}(x_1) - u_{i_1}^{(1)}(x_1)\right] \sum_{p_2=1,p_2\neq i_2}^{k_2} A_{i_2,2}^{p_2}(x_2)$$

$$+\sum_{i_1=1}^{M}\sum_{i_2=1}^{k_2} a_{i_1,i_2} u_{i_2}^{(2)}(x_2) \sum_{p_1=1,p_1\neq i_1}^{k_1} A_{i_1,1}^{p_1}(x_1)$$

$$+\sum_{i_1=1}^{M}\sum_{i_2=1}^{k_2} a_{i_1,i_2} \left[A_{i_2,2}^{i_2}(x_2) - u_{i_2}^{(2)}(x_2)\right] \sum_{p_1=1,p_1\neq i_1}^{k_1} A_{i_1,2}^{p_1}(x_1)$$

$$+\sum_{i_1=1}^{M}\sum_{i_2=1}^{M} a_{i_1,i_2} \sum_{p_1=1,p_1\neq i_1}^{k_1} A_{p_1,1}^{i_1}(x_1) \sum_{p_1=1,p_1\neq i_1}^{k_1} A_{p_2,2}^{i_2}(x_2)$$

$$=\sum_{r=1}^{9} L_r(x_1,x_2). \tag{21}$$

It follows from (10) and (14) that

$$\left\| \max\{|L_1| : 1 \le k_j \le M,\;\; j=1,2\}\right\|_{L(I^2)} > 2^{-8}R. \tag{22}$$

It is well known that the Hardy–Littlewood maximal operator is bounded from the space $L\log^+ L(I)$ into $L(I)$(see [7], pp. 59 and 279).

Therefore taking into account the fact that for any $i_2 \in \mathbb{N}$ the function

$$h(t_1) = \int_I g_{R,Q}^{(2)}(t_1,t_2)\mathcal{X}_{i_2}(t_2)dt_2$$

belongs to the space $L\log^+ L(I)$ and using the well-known estimates of the majorant of partial sums of a Fourier–Haar series by the maximal function (see [2], p. 74) and the Haar function estimate, we obtain

$$\left\| \max_{1\le k_1\le M} \left| \sum_{i_1=1}^{k_1} a_{i_1,i_2}\mathcal{X}_{i_1}\right|\right\|_{L(I)} = \left\|S_M^*(h,H_1)\right\|_{L(I)}$$

$$\le C_0\|h\|_{L\log^+ L(I)} \le C_0\sqrt{i_2}\left\| \int_I \left|g_{R,Q}^{(2)}(\cdot,t_2)\right|dt_2\right\|_{L\log^+ L(I)}. \tag{23}$$

Since the system $\{\psi_{i_2}^{(2)}\}_{i_2=1}^{\infty}$ forms the basis in the space $L(I)$, from (14)–(17) follows

$$\left\|u_{i_2}^{(2)} - A_{i_2,2}^{i_2}\right\|_{L(I)} \le \sum_{p_2\neq i_2} \left\|A_{i_2,2}^{i_2}\right\|_{L(I)} \le 2^{i_2}, \quad i_2=1,2,\ldots. \tag{24}$$

Now taking into account (14), (9) and the Jenssen inequality, from (23) and (24) we obtain

$$\Big\|\max\big\{|L_2| : 1\le k_j\le M,\quad j=1,2\big\}\Big\|_{L(I^2)}$$

$$\le\sum_{i_2=1}^{M}\big\|A^{i_2}_{i_2,2}-u^{(2)}_{i_2}\big\|_{L(I)}\Big\|\max_{1\le k_j\le M}\Big|\sum_{i_1=1}^{k_1}a_{i_1,i_2}u^{(1)}_{i_1}\Big|\Big\|_{L(I)}$$

$$\le C_1\bigg(1+\int\limits_I\bigg[\int\limits_I\big|g^{(2)}_{R,Q}(t_1,t_2)\big|dt_1\log^+\int\limits_I\big|g^{(2)}_{R,Q}(t_1,t_2)\big|dt_1\bigg]dt_2\bigg)\sum_{i_2=1}^{\infty}\sqrt{i_2}2^{-i_2}$$

$$\le C_1'\bigg(1+\int\limits_{I^2}\big|g^{(2)}_{R,Q}\big|\log^+\big|g^{(2)}_{R,Q}\big|\bigg)=2C_1'=C_2. \tag{25}$$

Similarly, we can prove for $r=3,\dots,9$ the estimates

$$\big\|\max\{|L_r| : 1\le k_j\le M,\ \ j=1,2\}\big\|_{L(I^2)}\le C_r.$$

Setting $C^*=\sum_{r=2}^{9}C_r$, from (22), (25) and (21) we have (20). Finally, we shall prove that the set E (see (6)) has, in $L_Q(I^2)$, the first Baire category. We use the representation $E=\bigcup_{k=1}^{\infty}E_k$, where (see (3), (4))

$$E_k=\bigcap_{N_1=1}^{\infty}\bigcap_{N_2=1}^{\infty}\big\{f\in L_Q(I^2) : \big\|S^{(*)}_{N_1,N_2}(f,\Psi_2)\big\|_{L(I^2)}\le k\big\},$$

and assume that even if one of the sets E_k contains a sphere, i.e., there exist a function $F_0\in L_Q(I^2)$, a positive number ε_0 and an integer k_0 such that for any integers N_1 and N_2 and for any function $F\in L_Q(I^2)$, under the condition

$$\big\|F-F_0\big\|_{L_Q(I^2)}\le\varepsilon_0, \tag{26}$$

we have

$$\big\|S^{(*)}_{N_1,N_2}(f,\Psi_2)\big\|_{L(I^2)}\le k_0. \tag{27}$$

On the other hand, by virtue of the above reasoning (see (18)–(20)), for the number $R=2^8(5k_0\varepsilon_0^{-1}+C^*)$ one can find a function $f_{R,Q}(x_1,x_2)$ such that

$$\big\|f_{R,Q}\big\|_{L(I^2)}\le 2$$

and

$$\big\|S^{*}_{r_1(M+1),r_2(M+1)}(f_{R,Q},\Psi_2)\big\|_{L(I^2)}>5k_0\varepsilon_0^{-1}. \tag{28}$$

Let now

$$F(x_1,x_2)=\frac{\varepsilon_0}{2}f_{R,Q}(x_1,x_2)+F_0(x_1,x_2).$$

Then

$$\big\|F-F_0\big\|_{L_Q(I^2)}\le\varepsilon_0 \tag{29}$$

and, by virtue of (26) and (27), we have

$$\big\|S^{*}_{r_1(M+1),r_2(M+1)}(F,\Psi_2)\big\|_{L(I^2)}\ge\frac{\varepsilon_0}{2}\big\|S^{*}_{r_1(M+1),r_2(M+1)}(f_{R,Q},\Psi_2)\big\|_{L(I^2)}$$

$$-\left\|S^*_{r_1(M+1),r_2(M+1)}(F_0,\Psi_2)\right\|_{L(I^2)} > k_0. \tag{30}$$

The contradiction between (30) and (27) (see also (29) and (26)) shows that the above assumption does not hold.

Since condition (5) is equivalent to the condition $L_Q(I^d)\setminus L(\log^+ L)^d(I^d) \neq \varnothing$, the following theorem is valid.

Theorem 2. *Let Ψ_d be an arbitrary system of form* (1) *and*

$$L_Q(I^d) \setminus L(\log^+ L)^d(I^d) \neq \varnothing.$$

Then there exists a function $f_0 \in L_Q(I^d)$ such that $S^*(f_0,\Psi_d) \notin L(I^d)$.

Remark 1. The example of the Haar system shows that condition (5) is necessary for Theorem 1 to hold. For $d = 1$ it is proved in [3], p. 928, with the help of the Hardy–Littlewood maximal function; for $d > 1$ one can apply similar arguments (see, for example, [11], Chapter II).

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