Abstract. In this paper, we show that every conjugacy class of the imprimitive complex reflection group $G(m, 1, n)$ can be represented by an admissible diagram. For this, we introduce a length function for elements of $G(m, 1, n)$ and study its properties. This then allows us to establish the admissible diagram for each conjugacy class of $G(m, 1, n)$. The corresponding results for Weyl groups and their conjugacy classes are well known.

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1. Introduction

The main objective of this paper is to show that there is a one-to-one correspondence between conjugacy classes in the imprimitive complex reflection group $G(m, 1, n)$ and admissible diagrams whose connected components all have type $A$ or $B$. The group $G(m, 1, n)$ can be viewed as the generalized symmetric group and its conjugacy classes can be found in Kerber [8]. To establish an admissible diagram for every conjugacy class of $G(m, 1, n)$, in the second section we define a length function for $G(m, 1, n)$ and study its properties. The corresponding results for Weyl groups were studied by Carter [5].

We first give the basic notation and state some results which are required later. We refer the reader to [3] and [6] for most of the undefined terminology and quoted results. As a convention, throughout this paper, we assume that $\xi$ is a primitive $m$-th root of unity.

Let $V = \mathbb{C}^n$ be the complex vector space of dimension $n$ with standard unitary inner product $(\cdot, \cdot)$ and the standard basis $\{e_1, e_2, \ldots, e_n\}$. A reflection in $V$ is a linear transformation of $V$ of finite order with exactly $n-1$ eigenvalues equal to 1. A reflection group $G$ in $V$ is a finite group generated by reflections in $V$.

For each non-zero vector $\alpha \in V$, let $w_\alpha$ be a reflection in $V$ of order $m > 1$. Then there is a primitive $m$-th root of unity $\xi$ such that

$$w_\alpha(v) = v - (1 - \xi) \frac{(v, \alpha)}{(\alpha, \alpha)} \alpha$$

for all $v \in V$. Thus $w_\alpha(\alpha) = \xi \alpha$ and $w_\alpha(v) = v$ if $v \in \langle \alpha \rangle^\perp$, where $\langle \alpha \rangle^\perp$ is the orthogonal complement of $\langle \alpha \rangle$ with respect to the given unitary inner product. Define $o_G : V \to \mathbb{N}$ by $o_G(v) = |G_{\langle v \rangle^\perp}|$ ($v \in V$). Then $o_G(v) > 1$ if and only if $v$ is a root of $G$. In this case, $o_G(v)$ is the order of the cyclic group generated...
by the reflections in \( G \) with root \( v \). If \( \alpha \) is a root of \( G \), then the number \( \alpha_G(\alpha) \) is called the order of \( \alpha \).

Let \( G(m,1,n) \) be the \textit{imprimitive complex reflection group} in \( V \) generated by reflections of order 2 with roots \( e_1 - e_2, e_2 - e_3, \ldots, e_{n-1} - e_n \) and the reflection of order \( m \) with root \( e_n \). A root system for \( G(m,1,n) \) may be defined as follows (see [6]). Let \( \mu_m = \{ \xi^l \mid l \in \mathbb{N}, \xi \text{ be a primitive } m-\text{th root of unity} \} \). Put

\[
R(m,1,n) = \mu_m \{ \pm (e_i - \xi^l e_j), e_k \mid i, j, k, l \in \mathbb{N}, i \neq j, 1 \leq i, j, k \leq n \}
\]

and let \( f_{m,1,n} : R(m,1,n) \to \mathbb{N}\backslash\{1\} \) be defined by

\[
f_{m,1,n}(\alpha) = \begin{cases} m & \text{if } \alpha \in \mu_m \{ e_k \mid 1 \leq k \leq n \}, \\ 2 & \text{otherwise}; \end{cases}
\]

then we have that \( \Phi = \Phi(m,1,n) = (R(m,1,n), f_{m,1,n}) \) is a root system with \( W(\Phi) = G(m,1,n) \). The group \( G(m,1,n) \) has the following presentation (see [7]):

\[
G(m,1,n) = \langle r_1, \ldots, r_{n-1}, w_1, \ldots, w_n \mid r_i^2 = (r_ir_{i+1})^3 = (r_i r_j)^2 = 1, |i-j| \geq 2, w_i^m = 1, w_iw_j = w_jw_i, r_i w_i = w_i r_i, r_i w_j = w_j r_i, j \neq i, i + 1 \rangle.
\]

2. The Length Function

Let \( W = G(m,1,n) \) denote the imprimitive reflection group corresponding to \( \Phi = \Phi(m,1,n) \). In this section we introduce a length function for \( W \) and study its properties.

Now each element \( w \) in \( W \) can be expressed as a product of reflections \( w = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k} \), where \( a_i \in \Phi \) and \( s_i \in \{1,\ldots, m-1\} \). The length of \( w \), denoted by \( l(w) \) is the smallest value of \( \sum_{i=1}^{k} s_i \) in any such expression for \( w \). (Here if \( o_W(a_i) = 2 \) then \( s_i = 1 \), and if \( o_W(a_i) = m \) then \( s_i \in \{1,\ldots, m-1\} \).) By convention, \( l(1) = 0 \). Clearly, \( l(w) = 1 \) if and only if \( w = w_a \) where \( a \in \Phi \). It is also clear that if \( w = w_s^a \) with \( o_W(a) = m \) and \( s \in \{1,\ldots, m-1\} \), then \( l(w) = s \). We say that \( w \) is a product of \( k \) reflections if \( l(w) = \sum_{i=1}^{k} s_i \).

Any element \( \sigma \in W \) can be written uniquely (up to reordering) as the product of disjoint cycles \( \sigma = \theta_1 \cdots \theta_t \), where

\[
\theta_i = \left( \begin{array}{cccc} b_{i1} & b_{i2} & \cdots & b_{ik_i} \\ \xi^{s_{i1}}b_{i2} & \xi^{s_{i2}}b_{i3} & \cdots & \xi^{s_{ik_i}}b_{i1} \end{array} \right),
\]

\( b_{ij} \in \{1,\ldots,n\} \), \( s_{ij} \in \{1,\ldots,m\} \), \( k_i \) is the length of the cycle \( \theta_i, i = 1,\ldots,t \). Let \( f(\theta_i) = \sum_{j=1}^{k_i} s_{ij} \), and put \( f(\sigma) = \sum_{i=1}^{t} f(\theta_i) \).

Now, define \( a_{pq}(\sigma) \) to be the number of cycles \( \theta_i \) of \( \sigma \) of length \( q \) such that \( f(\theta_i) \equiv p \pmod{m} \) for \( 1 \leq p \leq m, 1 \leq q \leq n \). The \( m \times n \) matrix \( (a_{pq}(\sigma)) \) is called the \textit{type} of \( \sigma \), denoted by \( Ty(\sigma) \) (see [9]). Then \( \sigma, \pi \in W \) are conjugate in \( W \) if and only if \( Ty(\sigma) = Ty(\pi) \) (see [8]).
Lemma 2.1. If σ, π ∈ W are conjugate in W, then l(σ) = l(π).

Proof. Let σ = w_{a_1}^{s_1} · · · w_{a_k}^{s_k}, where a_i ∈ Φ and s_i ∈ {1, . . . , m − 1}. Since σ is conjugate in W to π, π = wσw^{−1} for some w ∈ W. But wσw^{−1} = w_{b_1}^{s_1} . . . w_{b_k}^{s_k}
where b_i = w(a_i) implies that l(σ) = \sum_{i=1}^{k} s_i = l(π). □

The above lemma says that two conjugate elements in W have the same length and each of them is also the product of the same number of reflections.

The lemma below is a well-known property of reflection groups (see [10]).

Lemma 2.2. Let G be a reflection group in an n-dimensional complex vector space V. If g ∈ G and U is the subspace of V composed of all vectors fixed by g, then g is a product of the reflections corresponding to the roots in the orthogonal complement U⊥ of U.

Lemma 2.3. Let w ∈ W. Then l(w) is a sum of the powers of eigenvalues of w on V which are not equal to 1.

Proof. Suppose that l(w) = \sum_{i=1}^{k} s_i. Then w is a product of k reflections and has an expression of the form w = w_{a_1}^{s_1} . . . w_{a_k}^{s_k}, where a_i ∈ Φ and s_i ∈ {1, . . . , m − 1}. Now, let H_{a_i} be the reflecting hyperplane of a_i in V and let

\[ H = H_{a_1} \cap H_{a_2} \cap \cdots \cap H_{a_k}. \]

Then w acts trivially on H and dim H ≥ n − k. Thus w has at least n − k eigenvalues equal to 1, and so at most k eigenvalues \(ξ^{s_1}, ξ^{s_2}, \ldots, ξ^{s_k}\) which are not equal to 1, by the definition of a reflection. Therefore, the sum of the powers of these eigenvalues is not more than l(w).

Conversely, suppose w has k eigenvalues \(ξ^{s_1}, ξ^{s_2}, \ldots, ξ^{s_k}\) which are not equal to 1, where s_i ∈ {1, . . . , m − 1}. Let U be the subspace of V composed of all vectors fixed by w, and U⊥ be the orthogonal subspace. Then at once dim U = n − k and dim U⊥ = k, and by Lemma 2.2 w is a product of the reflections corresponding to the roots in U⊥.

Suppose that w fixes some vector in V. Then k < n and so dim U⊥ < dim V. The roots in U⊥ form a root system in the subspace they generate which has dimension less than n, and w is an element of the reflection group associated with this root system. Thus, by induction, w is a product of at most k reflections, i.e., \(w = w_{a_1}^{s_1} . . . w_{a_k}^{s_k}\), and so l(w) ≤ \sum_{i=1}^{k} s_i. □

An expression \(w_{a_1}^{s_1} . . . w_{a_k}^{s_k}\) ∈ W is called reduced if \(l(w_{a_1}^{s_1} . . . w_{a_k}^{s_k}) = \sum_{i=1}^{k} s_i\).

Lemma 2.4. Let a_1, . . . , a_k ∈ Φ and s_i ∈ {1, . . . , m − 1} for i = 1, . . . , k. Then \(w_{a_1}^{s_1} . . . w_{a_k}^{s_k}\) is reduced if and only if a_1, . . . , a_k are linearly independent.
Proof. Let $w = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k}$. Suppose that the expression is reduced. Then by Lemma 2.3, $w$ has $k$ eigenvalues not equal to 1, and so
\[
\dim(H_{a_1} \cap H_{a_2} \cap \cdots \cap H_{a_k}) = n - k.
\]
(Here, the dimension cannot be larger, since $w$ acts as the identity on this subspace.) Thus it follows that the roots $a_1, \ldots, a_k$ are linearly independent.

Conversely, suppose that the roots $a_1, \ldots, a_k$ are linearly independent. Now, consider the subspace $\text{im}(w - 1)$, and choose a vector $v_1 \in V$ such that
\[
v_1 \in H_{a_2} \cap \cdots \cap H_{a_k} \text{ but } v_1 \not\in H_{a_1}.
\]
Then $w(v_1) - v_1$ is a non-zero multiple of $a_1$. Thus $a_1 \in \text{im}(w - 1)$. Now, select once again a vector $v_2 \in V$ with
\[
v_2 \in H_{a_3} \cap \cdots \cap H_{a_k} \text{ but } v_2 \not\in H_{a_2}.
\]
Then $w(v_2) - v_2 = \alpha a_1 + \beta a_2$, where $\alpha, \beta \in \mathbb{C}$ and $\alpha \neq 0$. Hence $a_2 \in \text{im}(w - 1)$. Repeating this argument will eventually show that $a_1, \ldots, a_k \in \text{im}(w - 1)$, and so $\dim \text{im}(w - 1) = k$.

Then $w$ is reduced, for if $w$ has a shorter expression $w = w_{b_1}^{r_1} \cdots w_{b_l}^{r_l}$ with $l < k$ and $r_i \in \{1, \ldots, m - 1\}$, then every element of $\text{im}(w - 1)$ can be written as a linear combination of $b_1, \ldots, b_l$ and so $\dim \text{im}(w - 1) < k$, which is a contradiction. Furthermore, if $w$ has an expression $w = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k}$ with $r_i \leq s_i$, then $w_{a_1}^{s_1} \cdots w_{a_k}^{s_k} w = 1$ and $w_{a_k}^{s_k} \cdots w_{a_1}^{s_1} w_{a_1}^{r_1} \cdots w_{a_k}^{r_k} \neq 1$ where $\rho_i = \alpha w(a_i) - s_i (1 \leq i \leq k)$, a contradiction. \hfill $\Box$

3. Admissible Diagrams

Let $\Phi(m, p, n) (p = 1, \ m)$ be an imprimitive root system with simple system $\pi(m, p, n) = (B, \theta)$, where
\[
B = \begin{cases} 
\{\alpha_i = e_i - e_{i+1} \ (i = 1, \ldots, n - 1), \ \alpha_n = e_n\} & \text{if } p = 1, \\
\{\beta_i = e_i - e_{i+1} \ (i = 1, \ldots, n - 1), \ \beta_n = e_{n-1} - \xi e_n\} & \text{if } p = m.
\end{cases}
\]

Then the Cohen diagrams for $\Phi(m, 1, n)$ and $\Phi(m, m, n)$ are respectively

\[B_n^m: \quad \begin{array}{c}
\circ \quad \circ \quad \cdots \quad \circ \quad \circ \quad \circ \\
1 \quad 2 \quad \cdots \quad n-1 \quad n
\end{array}
\]

where the node corresponding to $\alpha_i \ (i = 1, \ldots, n)$ is denoted by $i$ and

\[D_n^m: \quad \begin{array}{c}
\circ \quad \circ \quad \cdots \quad \circ \quad \circ \\
1 \quad 2 \quad \cdots \quad n-2 \quad n-1 \quad \frac{1+\xi}{2} \quad \frac{1-\xi}{2} \\
\end{array}
\]

where the node corresponding to $\beta_i \ (i = 1, \ldots, n)$ is denoted by $i$. 
A web is a graph of the form $\begin{array}{c} \text{node} \\ \text{node} \end{array}$, where $s \in \{1, \ldots, m - 1\}$.

Any element $w = w_{a_1}^{s_1} \cdots w_{a_k}^{s_k} \in W$ with $l(w) = \sum_{i=1}^{k} s_i$ can be decomposed as follows (see [1] or [2]):

$$w = \tau w_{a_1+1}^{s_1+1} \cdots w_{a_k}^{s_k}, \quad \text{where} \quad \tau = w_{a_1} \cdots w_{a_i} \in W(A_{n-1}).$$

For each such decomposition of $w$, we define the graph $\Gamma$ as having $k$ nodes, one corresponding to each of the roots $a_1, \ldots, a_k$ with the value $o_W(a_i)$. The nodes corresponding to distinct roots $a_i, a_j$ are joined by a bond of weight $(a_i, a_j)$. If $o_W(a_i) = 2$, then the number 2 in the node corresponding to the root $a_i$ is omitted, as in Cohen [6].

If $w \in W$ has a decomposition with graph $\Gamma$, then any conjugate of $w$ also has a decomposition with graph $\Gamma$. For if $w = w_{a_1} \cdots w_{a_i} w_{a_i+1}^{s_i+1} \cdots w_{a_k}^{s_k}$, where $w_{a_1} \cdots w_{a_i} \in W(A_{n-1})$, then we have $w' w w'^{-1} = w_{b_1} \cdots w_{b_i} w_{b_i+1}^{s_i+1} \cdots w_{b_k}$, where $b_j = w'(a_j)$ for $j = 1, \ldots, k$.

Therefore we say that the graph $\Gamma$ is associated with this conjugacy class. (Here, we assume that the conjugacy class containing the identity element is represented by the empty graph.) By Lemma 2.4 the nodes of $\Gamma$ correspond to a set of linearly independent roots.

Now we can give our basic definition.

**Definition 3.1.** Let $\Gamma$ be a graph, then $\Gamma$ is called an **admissible diagram** if

(i) the nodes of $\Gamma$ correspond to a set of linearly independent roots of $\Phi$,

(ii) each subgraph of $\Gamma$ which is a cycle is equivalent to a web.

(A subgraph of $\Gamma$ in this context is a subset of the nodes, together with the bonds joining the nodes in the subset. A cycle is a graph in which each node is connected only to two other nodes.)

**Lemma 3.2.** Every admissible diagram associated with a conjugacy class of $W$ is the Cohen (Dynkin) diagram of some reflection subgroup of $W$.

**Proof.** Let $\Gamma$ be such a graph. Let $J$ be a set of the roots corresponding to the nodes of $\Gamma$. Denote by $W(J)$ the group generated by all reflections $w_{a, o_W(a)}$ with $a \in J$, then $W(J)$ is a subgroup of $W$ and so is a finite reflection group. Furthermore, $J$ is linearly independent by the definition of an admissible diagram. Thus, by (4.2) of Cohen [6], $\Gamma$ is a root graph.

Now, put $S = W(J)J$, define a map $g : S \to \mathbb{N}\{1\}$ by $g(a) = o_W(J)(a)$ for all $a \in S$, then the pair $\Psi = (S, g)$ is the pre-root system corresponding to $J$ with $W(\Psi) = W(J)$ by 1.2 (ii) of Can [3]. By 1.2 (iii) of Can [3], the pair $\Psi = (S, g)$ is a root system and so is a subsystem of $\Phi$. Thus, $W(\Psi)$ is the reflection group of $\Psi$, and so $\Gamma$ is the Cohen (Dynkin) diagram of the reflection subgroup $W(\Psi)$ of $W$, as desired. \[\square\]
Now, recall that $\Gamma$ may be a union of disconnected graphs $\Gamma_i$, which, say, satisfy the following: if $\Gamma_i$ contains no web, then $\Gamma_i$ is either of type $A$ or $B$, and if $\Gamma_i$ does contain a web, then $\Gamma_i$ must be of type $D$.

The present author [3] has presented an algorithm for obtaining graphs which are Cohen (Dynkin) diagrams of reflection subgroups of $W$ without any reference to extended diagrams. In [4], we interpreted this algorithm as a computer program written by using the Maple symbolic computation system.

We now show that admissible diagrams can be used to parametrize the conjugacy classes of $W$.

**Theorem 3.3.** Let $W = G(m, 1, n)$. There is a one-to-one correspondence between conjugacy classes in $W$ and admissible diagrams of the form

$$\sum_{p=1}^{m} \left( B^{m_p}_{\lambda_1^{(p)}} + B^{m_p}_{\lambda_2^{(p)}} + \cdots + B^{m_p}_{\lambda_s^{(p)}} \right),$$

where

$$\sum_{p=1}^{m-1} \sum_{q=1}^{s_p} \lambda_1^{(p)} + \sum_{q=1}^{s_m} (\lambda_1^{(m)} + 1) = n \quad \text{and} \quad m_p = \frac{m}{(m, p)},$$

where $(m, p)$ is the greatest common divisor of $m$ and $p$.

**Proof.** The elements of $W$ operate on the orthonormal basis $e_1, \ldots, e_n$ of $V$ by permuting the basis vectors and multiplying arbitrary subsets of them by a power of $\xi$. Ignoring these multiples, each element $w$ of $W$ determines a permutation of $\{1, \ldots, n\}$ which can be expressed in the usual way as a product of disjoint cycles. Let $(k_1 k_2 \cdots k_r)$ be such a cycle written as

$$e_{k_1} \rightarrow \xi^{p_1} e_{k_2} \rightarrow \xi^{p_1+p_2} e_{k_3} \rightarrow \cdots \rightarrow \xi^{p_1+\cdots+p_{r-1}} e_{k_r} \rightarrow \xi^{p_1+\cdots+p_r} e_{k_1},$$

where $p_i \in \{1, \ldots, m\}$. The cycle $(k_1 k_2 \cdots k_r)$ is said to be a $(\xi^p, r)$-cycle, denoted by $[r^p]$, if $w^r(e_{k_1}) = \xi^p e_{k_1}$, where $\sum_{i=1}^{r} p_i \equiv p \pmod{m}$. Then the lengths of the cycles together with their values $\sum p_i$ determine the type of $w$, and two elements of $W$ are conjugate if and only if they have the same type, as in Kerber [8]. Thus there is a one-to-one correspondence between conjugacy classes and types. Now, consider the $(\xi^p, r)$-cycle

$$e_1 \rightarrow e_2 \rightarrow \cdots \rightarrow e_{r-1} \rightarrow e_r \rightarrow \xi^p e_1,$$

where $p \in \{1, \ldots, m\}$. If $p = m$, then this can be expressed as the product of elements $(12)(23) \cdots (r-1 \; r)$. These factors form a complete set of simple reflections of the Weyl subgroup of type $A_{r-1}$, and so this $(1, r)$-cycle, denoted by $[r]$, is represented by an admissible diagram $A_{r-1}$, as in type $A_n$ (see Carter [5]). If $p \in \{1, \ldots, m-1\}$, then this can be expressed as the product of elements $(12)(23) \cdots (r-1 \; r)w_p^k$, where $w_p^k$ changes $e_r$ into $\xi^p e_r$ and fixes all other $e_i$. Thus these factors form a complete set of simple reflections of the reflection subgroup of type $B_r^{m_p}$, where $m_p = \frac{m}{(m, p)}$ where $(m, p)$ is the g.c.d. of $m$ and $p$, and so this $(\xi^p, r)$-cycle is represented by an admissible diagram
(This is a natural choice for the following reason. If for the group $G(m, 1, n)$ we take $n = 1$, then we have $C_m = G(m, 1, 1) = \langle w \mid w^m = 1 \rangle$ which is the cyclic group of order $m$. We attach to each non-identity class (element) $w^p$ ($1 \leq p \leq m - 1$) in $C_m$ an admissible diagram

\[
\begin{array}{c}
\begin{array}{c}
B_{mp}^r \text{ (} r \text{ nodes)} : \quad \bullet \cdots \bullet
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
w^p
\end{array}
\end{array}
\]

i.e., the admissible diagram depends on the order of the element.)

Now consider an arbitrary element of $W$, expressed as a product of disjoint $(\xi^p, r)$-cycles. Since disjoint cycles operate on orthogonal subspaces of $V$, the admissible diagram splits into connected components corresponding to the cycle decomposition, and so takes form

\[
\sum_{p=1}^{m} \left( B_{mp}^{\lambda(p)} + B_{mp}^{\lambda(p)} + \cdots + B_{mp}^{\lambda(p)} \right),
\]

where

\[
\sum_{p=1}^{m-1} \sum_{q=1}^{s_p} \lambda_q^{(p)} + \sum_{q=1}^{s_m} \lambda_q^{(m)} + 1 = n \quad \text{and} \quad m_p = \frac{m}{(m, p)},
\]

where $(m, p)$ is the g.c.d. of $m$ and $p$, as desired. \qed

Remark 3.4. Now, define $m$ partitions $\lambda^{(1)}, \ldots, \lambda^{(m)}$ by

$\lambda^{(p)} = (\lambda_1^{(p)}, \ldots, \lambda_{s_p}^{(p)}) \quad (p = 1, \ldots, m - 1), \quad \lambda^{(m)} = (\lambda_1^{(m)} + 1, \ldots, \lambda_{s_m}^{(m)} + 1),$

then there is a one-to-one correspondence between conjugacy classes in $W$ and $m$-sets of partitions $\lambda^{(1)}, \ldots, \lambda^{(m)}$ of $n$ with

\[
\sum_{p=1}^{m-1} \sum_{q=1}^{s_p} \lambda_q^{(p)} + \sum_{q=1}^{s_m} \lambda_q^{(m)} + 1 = n \quad \text{(see [8]).}
\]

If $m = 1$, then $W = W(A_{n-1})$ (a Weyl group of type $A_{n-1}$), and if $m = 2$, then $W = W(C_n)$ (a Weyl group of type $C_n$), and so by putting $m = 1, 2$ in Theorem 3.3, we recover the results of Carter [5].

The admissible diagrams given in Theorem 3.3 are not the only ones which can be taken. We know that $W$ contains a reflection subgroup $G(m, m, n) = W(D_n^m)$ (see [6]), and so $D_n^m$ is an admissible diagram for $W$. However since the admissible diagrams given in Theorem 3.3 are in one-to-one correspondence with the conjugacy classes of $W$, we do not need the remaining part of the proof.
As an application of Theorem 3.3, we now give the following example.

**Example 3.5.** Consider the group $G(3, 1, 3)$. Then we have

<table>
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<th>Cycle type</th>
<th>Conjugacy class</th>
<th>Admissible diagram</th>
</tr>
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<td>(3) w</td>
</tr>
<tr>
<td>$[2^21]$</td>
<td>(12)$w_3^2$</td>
<td>(3) w</td>
</tr>
<tr>
<td>$[3]$</td>
<td>(12)(23)</td>
<td>$w$</td>
</tr>
<tr>
<td>$[3^2]$</td>
<td>(12)(23)$w_3$</td>
<td>$w$</td>
</tr>
<tr>
<td>$[3^2]$</td>
<td>(12)(23)$w_3^2$</td>
<td>$w$</td>
</tr>
</tbody>
</table>

The elements in column 2 are representatives of the conjugacy classes of $G(3, 1, 3)$.

**References**


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