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ON THE UNIQUENESS OF MEASURE AND CATEGORY σ-IDEALS ON 2^ω

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Abstract. We prove that if a pair $\langle I, J\rangle$ of ccc, translation invariant σ-ideals on 2^ω has the Fubini Property, then $I = J$. This leads to a slightly improved exposition of a part of the Farah-Zapletal proof of an invariant version of their theorem which characterizes the measure and category σ-ideals on 2^ω as essentially the only ccc definable σ-ideals with Fubini Property.

1. Introduction

A *σ-ideal* on an uncountable Polish space X is a family $I \subseteq \mathcal{P}(X)$ which is closed under taking subsets and countable unions. Throughout the paper we assume that I is *proper*, i.e., $X \notin I$, contains all singletons and has a basis consisting of Borel sets, i.e., every set from I is covered by a Borel set from I (we will sometimes abuse the notation by identifying I with $I \cap \mathbf{B}(X)$, where $\mathbf{B}(X)$ is the family of all Borel subsets of X).

Given σ-ideals I and J on Polish spaces X and Y, respectively, we say that the pair $\langle I, J\rangle$ has the *Fubini Property* (FP) if for every Borel set $B \subseteq X \times Y$, if all its vertical sections $B_x = \{y \colon \langle x, y\rangle \in B\}$ are in J, then

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its horizontal sections $B^y = \{x\colon \langle x,y\rangle \in B\}$ are in I, for every y outside a set from J. If the pair $\langle I,I\rangle$ has the FP, then we simply say that I has the Fubini Property. In view of the classical Fubini and Kuratowski–Ulam theorems (see [5, Theorem 8.41]), the pairs $\langle \mathcal{N}_\mu, \mathcal{N}_\nu\rangle$ and $\langle \mathcal{M}(X), \mathcal{M}(Y)\rangle$ have the FP. Here $\mathcal{N}_\mu$ is the collection of all subsets of X, having outer measure zero with respect to a Borel σ-finite continuous measure μ on X and $\mathcal{M}(X)$ is the family of all meager subsets of X (we tacitly assume that X has no isolated points, so that the σ-ideal $\mathcal{M}(X)$ contains all singletons). In particular, if $X = 2^\omega$ is the Cantor group equipped with the Haar measure, then the respective σ-ideals will be denoted by $\mathcal{N}$ and $\mathcal{M}$ and referred to as the measure and the category σ-ideals.

We say that σ-ideals I and J on spaces X and Y, respectively, are *Borel isomorphic* ($I \equiv_B J$) if there exists a Borel isomorphism $f\colon X \to Y$ between X and Y such that $A \in I \iff f[A] \in J$ for every $A \in \mathbf{B}(X)$. It is well-known that all σ-ideals of the form $\mathcal{N}_\mu$ and, respectively, of the form $\mathcal{M}(X)$, are Borel isomorphic.

Thus the Fubini and Kuratowski–Ulam theorems show just two, up to a Borel isomorphism, examples of σ-ideals with the FP: the measure and the category one.

We say that a σ-ideal I on X is *ccc* if there is no uncountable family of disjoint Borel subsets of X outside I. Note that the measure and category σ-ideals are ccc.

A σ-ideal I on 2^ω is *translation invariant* (shortly: *invariant*), if

$$\forall x \in 2^\omega \ \forall A \subseteq 2^\omega \ (A \in I \ \Rightarrow \ x + A \in I).$$

The main result of the paper is the following theorem, the proof of which will be presented in Section 2.

Theorem 1.1. *If a pair $\langle I,J\rangle$ of ccc invariant σ-ideals on 2^ω has the FP, then $I = J$.*

The classical examples of ccc invariant σ-ideals on 2^ω are $\mathcal{M}$ and $\mathcal{N}$ but by the work of Rosłanowski and Shelah [10] plenty of other examples exist. On the other hand, none of them has the FP and a version of a problem of Kunen (see [7]) is whether $\mathcal{M}$ and $\mathcal{N}$ are the only ccc invariant σ-ideals on 2^ω with the Fubini Property. Recently, Farah and Zapletal [4] obtained the positive answer under the additional assumption that the σ-ideals in question are definable (in the sense to be explained in Section 3).

More precisely, the (ZFC version of) Farah-Zapletal theorem states the following.

Theorem 1.2 (Farah, Zapletal)**.** *If a pair $\langle I,J\rangle$ of ccc, definable σ-ideals on 2^ω has the FP, then either $I \equiv_B J \equiv_B \mathcal{N}$ or $I \equiv_B J \equiv_B \mathcal{M}$.*

In Section 3 we show how Theorem 1.1 can be applied in an exposition of a part of the Farah-Zapletal proof of the following invariant version of their theorem.

Theorem 1.3. *If a pair $\langle I, J\rangle$ of invariant, ccc, definable σ-ideals on 2^ω has the FP, then either $I = J = \mathcal{N}$ or $I = J = \mathcal{M}$.*

2. Fubini Property for invariant σ-ideals on 2^ω

This section is devoted to the proof of Theorem 1.1. Our notation is standard. The complement of a set A in the space X will be denoted by A^c.

If J is a σ-ideal on a Polish space X and $A \in \mathbf{B}(X) \setminus J$ then we define $J|A = \{C \subseteq X : C \cap A \in J\}$. Clearly, $J|A$ is also a σ-ideal on X and $J \subseteq J|A$. Moreover, if I and J are ccc σ-ideals on X and $J \subseteq I$, then there is a set $A \in \mathbf{B}(X)$ with $A^c \in I$ (in particular: $A \notin J$) such that $I = J|A$.

Proof of Theorem 1.1. The following lemma takes care of one of the two inclusions we are going to prove.

Lemma 2.1. *If a pair $\langle I, J\rangle$ of ccc invariant σ-ideals on 2^ω has the FP, then $J \subseteq I$.*

Proof. Take $A \in J \cap \mathbf{B}(2^\omega)$ and let $B = \{\langle x, y\rangle : x + y \in A\}$. Then $B \in \mathbf{B}(2^\omega \times 2^\omega)$ and $B_x = x + A$ for every $x \in 2^\omega$. Since J is invariant it follows that $B_x \in J$ for every $x \in 2^\omega$. Hence, by the FP, $\{y \in 2^\omega : B^y \notin I\} \in J$ and, in particular, there exists $y \in 2^\omega$ such that $B^y \in I$ (recall that all σ-ideals under consideration are proper, so $J \neq \mathcal{P}(2^\omega)$). But $B^y = y + A$, so in view of the invariance of I we conclude that $A \in I$. $\square$

Now let I and J be ccc invariant σ-ideals on 2^ω and assume that $\langle I, J\rangle$ has the FP. Using remarks preceding the proof fix an $A \in \mathbf{B}(X)$ such that $A^c \in I$ and $I = J|A$.

Suppose, towards a contradiction, that $J \neq I$. Then $A^c \notin J$ and $J = I \cap J|A^c$.

Lemma 2.2. *The σ-ideal $J|A^c$ is invariant.*

Proof. Take arbitrary $B \in J|A^c$ and $t \in 2^\omega$. Hence we assume that $B \cap A^c \in J$ and we want to show that $(t + B) \cap A^c \in J$. Since $(t + B) \cap (t + A^c) = t + (B \cap A^c) \in J$, it is enough to prove that

$$(t + B) \cap \big(A^c \setminus (t + A^c)\big) \in J$$

or, equivalently, translating by t and using the invariance of J, that

$$B \cap \big((t + A^c) \cap A)\big) \in J \tag{$*$}$$

Let $C = B \cap \big((t + A^c) \cap A)\big)$. Note that since $t + C \subseteq A^c$ we have $t + C \in I$. Hence, in view of the invariance of I, also $C \in I$. But taking into account that $C \subseteq A$ and $I = J|A$ we conclude that $C \in J$ which gives $(*)$ and completes the proof of Lemma 2.2. □

Finally, since the FP for $\langle I, J\rangle$ easily implies the FP for $\langle I, J|A^c\rangle$ it follows from Lemma 2.1 that $J|A^c \subseteq I$.

But $J = I \cap J|A^c$ hence $J = J|A^c$, so $A \in J$ which contradicts the choice of A, completing the proof of Theorem 1.1. □

3. The uniqueness of measure and category σ-ideals on 2^ω

Actually, Theorem 1.2 referred to in this paper as the Farah-Zapletal theorem, relies on a fundamental dichotomy resulting from an earlier theorem of Shelah [11] and the work of the two authors concerning von Neumann's problem on the existence of strictly positive continuous submeasures on weakly distributive Boolean algebras (see [4]; compare also [2] and [1] where a much more general approach to von Neumann's problem is presented). In order to state the ZFC version of this dichotomy, let us first explain what is meant by definable σ-ideals in the statement of Theorems 1.2 and 1.3.

We say that a σ-ideal I on 2^ω is

- *analytic on G_δ* if for every G_δ subset G of $2^\omega \times 2^\omega$ the set $\{x\colon G_x \in I\}$ is an analytic subset of 2^ω,
- *Souslin* if there is a Souslin poset P such that the quotient Boolean algebra $\mathbf{B}(2^\omega)/I$ is isomorphic to the completion of P (a poset P is Souslin if its domain is an analytic subset of an uncountable Polish space and both the order and the incompatibility relation of P are analytic, see [3]),
- *definable* if it is analytic on G_δ and Souslin.

It is well-known that the σ-ideals $\mathcal{N}$ and $\mathcal{M}$ are definable.

If I_1 and I_2 are σ-ideals on Polish spaces X_1 and X_2, respectively, we write $I_2 \leq_B I_1$ if there exists a Borel function $\varphi\colon X_1 \to X_2$ such that

$$C \in I_2 \quad \Leftrightarrow \quad \varphi^{-1}[C] \in I_1 \qquad \text{for every } C \in \mathbf{B}(X_2).$$

Note that if $I_2 \leq_B I_1$ and $\langle I_1, J\rangle$ has the FP then $\langle I_2, J\rangle$ has the FP as well (see [14, Proposition 2.3]).

Now the dichotomy mentioned above can be summarized in the following two theorems (see comments in Section 4). Let I be a ccc σ-ideal on 2^ω.

Theorem 3.1 (Shelah, [11])**.** *If I is Souslin and the quotient Boolean algebra $\mathbf{B}(2^\omega)/I$ is not weakly σ-distributive, then $\mathcal{M} \leq_B I|A$ for a certain $A \in \mathbf{B}(X) \setminus I$.*

Theorem 3.2 (Farah-Zapletal, [4])**.** *If I is analytic on G_δ and the quotient Boolean algebra $\mathbf{B}(2^\omega)/I$ is weakly σ-distributive, then there exists a continuous diffused Borel submeasure Φ such that $I = Null(\Phi)$, the collection of its null sets. Moreover, if additionally I has the FP, then $Null(\Phi) \equiv_B \mathcal{N}$.*

Taking the above for granted, a proof of the invariant version of the Farah-Zapletal Theorem (Theorem 1.3) can now be completed with the help of Theorem 1.1 as follows.

Proof of Theorem 1.3. Let I, J be invariant, ccc, definable σ-ideals on 2^ω and assume that the pair $\langle I, J\rangle$ has the FP. We want to prove that either $I = J = \mathcal{N}$ or $I = J = \mathcal{M}$.

Since I and J are invariant and ccc we readily have from Theorem 1.1 that $I = J$. Since I and J are, moreover, definable, by the dichotomy above it is enough to consider two cases.

- *Case 1.* There is a set $A \in \mathbf{B}(2^\omega) \setminus I$ such that $\mathcal{M} \leq_B I|A$. Then the FP for $\langle I, I\rangle$ immediately gives the FP for $\langle I|A, I\rangle$ which in turn implies the FP for $\langle \mathcal{M}, I\rangle$. Finally, Theorem 1.1 gives $I = \mathcal{M}$.
- *Case 2.* There is a continuous diffused Borel submeasure Φ such that $I = Null(\Phi)$ and $Null(\Phi) \equiv_B \mathcal{N}$. Then the FP for $\langle I, I\rangle$ implies the FP for $\langle \mathcal{N}, I\rangle$, which again by Theorem 1.1 gives $I = \mathcal{N}$, completing the proof.

□

4. Additional remarks

4.1. Translation invariant ccc ideals on 2^ω with the FP. Although the family of ccc invariant σ-ideals on 2^ω with the FP has just two members ($\mathcal{N}$ and M) that have been identified so far (and, by Theorem 1.3, no more definable members) the following consequence of Theorem 1.1 seems to be of some interest. Let us say that σ-ideals I and J on a Polish space X are *orthogonal* (the fact denoted in this paper by $I \perp J$) if there exist a Borel set $B \subseteq X$ such that $B^c \in I$ and $B \in J$. Equivalently, $I \perp J$ iff there is no (proper!) σ-ideal on X extending both I and J. Note that $\mathcal{N} \perp \mathcal{M}$.

Proposition 4.1.

1. *If I is a ccc invariant σ-ideal on 2^ω with the FP, then I is a maximal invariant σ-ideal on 2^ω. In particular, for every Borel set B not in I there are countably many elements $t_n \in 2^\omega$ such that $2^\omega \setminus \bigcup_{n<\omega} (t_n + B) \in I$.*
2. *If I_1 and I_2 are ccc invariant σ-ideals on 2^ω with the FP, then either $I_1 = I_2$ or $I_1 \perp I_2$.*

Proof. To prove part (1) assume that $\bar{I}$ is an invariant σ-ideal on 2^ω with $I \subseteq \bar{I}$ (clearly, $\bar{I}$ is ccc as well). Then the FP for $\langle I, I\rangle$ implies the FP for $\langle I, \bar{I}\rangle$ which by Theorem 1.1 gives $I = \bar{I}$.

To prove part (2) assume that I_1 and I_2 are not orthogonal and let J be the σ-ideal generated by $I_1 \cup I_2$. Since, clearly, J is invariant, it follows from part (1) that $J = I_1 = I_2$. $\square$

4.2. Shelah's Theorem 3.1 and forcing. The conclusion of Shelah's theorem (3.1) is usually formulated in forcing terms as "forcing with $\mathbf{B}(2^\omega)/I$ adds a Cohen real". This in turn in Boolean algebraic terms means that there exists a set $A \in \mathbf{B}(X) \setminus I$ such that the Cohen algebra $\mathbf{B}(2^\omega)/\mathcal{M}$ is a complete subalgebra of the quotient algebra $\mathbf{B}(2^\omega)/(I|A)$. However, by Sikorski's theorem on inducing homomorphisms of σ-algebras by point maps (see [5, 15.9]) the latter condition is equivalent to $\mathcal{M} \leq_B I|A$ which is just the way we stated it.

The following result and its corollary may perhaps also shed more light on the meaning of the notion of reducing one σ-ideal to another.

Proposition 4.2. *Let I_1 and I_2 be σ-ideals on Polish spaces X_1 and X_2, respectively. For a Borel function $\varphi\colon X_1 \to X_2$ the following conditions are equivalent:*

1. $\forall B \in \mathbf{B}(X_1)$ $(X_1 \setminus B \in I_1 \ \Rightarrow\ \varphi[B] \notin I_2)$,
2. $\forall B \in \mathbf{B}(X_2)$ $(B \in I_2 \ \Rightarrow\ X_1 \setminus \varphi^{-1}[B] \notin I_1)$,
3. *φ witnesses that $I_2|C \leq_B I_1|A$ for certain sets $A \in \mathbf{B}(X_1) \setminus I_1$ and $C \in \mathbf{B}(X_2) \setminus I_2$ where, moreover, $A = \varphi^{-1}[C]$.*

Proof. The equivalence of conditions (1) and (2) is almost obvious.

To prove that (3) $\Rightarrow$ (2) take a $B \in I_2 \cap \mathbf{B}(X_2)$. Then $B \in I_2|C$ so $\varphi^{-1}[B] \in I_1|A$, i.e., $\varphi^{-1}[B] \cap A \in I_1$. But since $A \notin I_1$ the latter implies that $X_1 \setminus \varphi^{-1}[B] \notin I_1$.

To prove that (2) $\Rightarrow$ (3) let

$$I_3 = \{B \in \mathbf{B}(X_2)\colon \varphi^{-1}[B] \in I_1\}.$$

Note that I_3 is a ccc σ-ideal on X_2 and $I_3 \not\leq I_2$. Indeed, if $B \subseteq X_2$ is a Borel set such that $X_2 \setminus B \in I_3$, then $X_1 \setminus \varphi^{-1}[B] \in I_1$ which by (2) implies that $B \notin I_2$.

So let J be a σ-ideal on X_2 such that $I_2 \cup I_3 \subseteq J$. Then there are Borel sets $C_2,\ C_3 \subseteq X_2$ with ${C_2}^c, {C_3}^c \in J$ such that $J = I_2|C_2$ and $J = I_3|C_3$. Let $C = C_2 \cap C_3$. Then $C \notin J$ and $I_2|C = I_3|C$. Let $A = \varphi^{-1}[C]$. Then $A \notin I_1$ and we claim that φ witnesses $I_2|C \leq_B I_1|A$. Indeed, take $B \in \mathbf{B}(X_2)$ and to complete the proof examine the following sequence of conditions: $B \in I_2|C$ iff $B \cap C \in I_2$ iff $B \cap C \in I_3$ iff $\varphi^{-1}[B \cap C] \in I_1$ iff $\varphi^{-1}[B] \cap A \in I_1$ iff $\varphi^{-1}[B] \in I_1|A$. $\square$

Corollary 4.3. *Let I be a ccc σ-ideal on a Polish space X. Then the following conditions are equivalent:*

1. $\mathcal{M} \leq_B I|A$ *for a certain* $A \in \mathbf{B}(X) \setminus I$.
2. *There is a Borel function* $\psi\colon X \to 2^\omega$ *such that*

$$\forall B \in \mathbf{B}(X)\ (X \setminus B \in I \ \Rightarrow\ \psi[B] \notin \mathcal{M}).$$

Proof. This follows immediately from Proposition 4.2 and the fact that for any $C \in \mathbf{B}(2^\omega) \setminus \mathcal{M}$ we have $\mathcal{M}|C \equiv_B \mathcal{M}$. $\square$

4.3. The definability assumptions on *I*. Actually, the Farah-Zapletal theorem (1.2) is formulated in [4] with just one definability assumption on I and J, namely that both are analytic on G_δ, together with the remark that if I is analytic on G_δ, then it is easily Souslin (see [4]). However, the referee's comments on an earlier version of the present paper and the correspondence with the authors of [4] caused doubts if the latter is true in such a generality. Nevertheless, we have the following remark, due in its final form to J. Zapletal (private communication). It explains the situation under an additional condition, which covers all known cases and is easily satisfied by the σ-ideals $\mathcal{N}$ and $\mathcal{M}$.

Remark 4.4. Let I be a ccc analytic on G_δ σ-ideal on 2^ω and suppose that the (equivalence classes of) compact sets not in I are dense in the quotient Boolean algebra $\mathbf{B}(2^\omega)/I$. Then the σ-ideal I is Souslin.

Proof. Consider the poset $P = \{K \in K(2^\omega)\colon K \notin I\}$, ordered by inclusion; $K(2^\omega)$ is the hyperspace of all compact subsets of 2^ω equipped with Vietoris topology. Recall that the relation "$K \subseteq L$" is closed in $K(2^\omega)^2$ and the map $\langle K, L\rangle \mapsto K \cap L$ from $K(2^\omega)^2$ to $K(2^\omega)$ is Borel (see [5]). Note also that, I being analytic on G_δ, the set $I \cap K(2^\omega)$ is analytic and hence actually (by a theorem of Kechris, Louveau, and Woodin [6]) a G_δ subset of $K(X)$. It follows that P is analytic (actually F_σ) in $K(2^\omega)$ and sets $K,\ L \in P$ are incompatible iff $K \cap L \in I$, which in view of the preceding remarks is also a Borel relation.

When we additionally assume that I is invariant and has the FP, which is the special case dealt with in this note, the fact that P is analytic (actually Borel) in $K(2^\omega)$ can also be proved without resorting to the Kechris-Louveau-Woodin theorem. Namely, by Proposition 4.1, I is a maximal invariant σ-ideal on 2^ω. So for a set $K \in K(2^\omega)$ we have that $K \in P$ iff there is a sequence $\langle t_n : n < \omega\rangle$ of elements of 2^ω such that $2^\omega \setminus \bigcup_{n<\omega} (t_n + K) \in I$. This is an analytic statement provided the relation $2^\omega \setminus \bigcup_{n<\omega} (t_n + K) \in I$ is analytic. Since I is analytic on G_δ, to prove the latter it is enough to find a universal G_δ subset U of $2^\omega \times 2^\omega$ and a Borel function $S\colon (2^\omega)^\omega \times K(2^\omega) \to 2^\omega$ such that if $x = S(\langle t_n : n < \omega\rangle, K)$ then $2^\omega \setminus \bigcup_{n<\omega} (t_n + K) = U_x$, the vertical section of U at x. In fact, we shall identify 2^ω with $\mathcal{P}(2^{<\omega})$ and use a well-known universal G_δ-set defined by

$$U = \{\langle A, y\rangle \in \mathcal{P}(2^{<\omega}) \times 2^\omega : \text{for infinitely many } n,\ y \restriction n \in A\}.$$

(see e.g. [4] or [8]). Now, the proof given in [8] that for every G_δ-set $V \subseteq 2^\omega$ there is a set $A \subseteq 2^{<\omega}$ such that $U_A = V$, applied to $V = \bigcap_{n<\omega} V_n$ and $V_n = \bigcap_{i\le n}(t_i + K)$, easily gives a function with desired properties. Namely, define $S(\langle t_n : n < \omega\rangle, K) = A$, where

$$A = \{\sigma \in 2^{<\omega} : [\sigma] \subseteq V \text{ or } \exists n < \omega\ ([\sigma] \subseteq V_n \text{ and } [\sigma^*] \nsubseteq V_n)\}$$

(if σ is a finite non-empty binary sequence, σ^* denotes its initial segment of length exactly one less than σ).

It is easy to check that this is a Borel definition. □

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