Journal of Convex Analysis
Volume 9 (2002), No. 2, 535–542

Rank Condition and Controllability of Parametric Convex Processes

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Received December 5, 2000
Revised manuscript received October 16, 2001

This note is concerned with the controllability of differential inclusions whose right-hand sides are convex processes. More precisely, it relates the controllability of $\dot{x}(t) \in F(x(t))$ with the controllability of a perturbed version $\dot{x}(t) \in F_n(x(t))$. The reference (or nominal) convex process F is seen as the "limit" of a sequence $\{F_n\}_{n\in \boldsymbol{N}}$ of approximations.

Keywords: Convex process, differential inclusion, controllability, point spectrum, rank condition, Painlevé-Kuratowski convergence

1991 Mathematics Subject Classification: 93B05, 47H04, 34A60

1. Introduction

Throughout this paper X denotes a finite dimensional real Hilbert space with inner product $\langle \cdot\, ,\ \cdot \rangle$ and associated norm $\|\cdot\|$. A *convex process* on X is a multivalued operator $F : X \rightrightarrows X$ whose graph

$$Gr\ F := \{(s,v) \in X \times X \mid \quad v \in F(s)\}$$

is a convex cone containing the origin. Equivalently, a convex process $F : X \rightrightarrows X$ is characterized by the following three requirements :

(a) $0 \in F(0)$; (normalization)
(b) $F(\alpha s) = \alpha F(s) \quad \forall \alpha > 0\ ,\ s \in X$; (positive homogeneity)
(c) $F(s_1) + F(s_2) \subset F(s_1 + s_2) \quad \forall s_1, s_2 \in X$. (super-additivity)

A convex process is said to be *closed* if its graph is a closed set. For the sake of convenience, we shall write

$$Q(X) := \{F : X \rightrightarrows X \mid \quad F \text{ is a closed convex process}\}\ .$$

To each operator $F : X \rightrightarrows X$ and terminal time $T > 0$, one associates the differential inclusion

$$\dot{x}(t) \in F(x(t)) \quad \text{for } a.e. \quad t \in [0,T]\ , \tag{1}$$

ISSN 0944-6532 / \$ 2.50

whose solutions are sought in the class

$$W_X[0,T] := \{x : [0,T] \longrightarrow X \mid \quad x \text{ is absolutely continuous}\} .$$

A central issue of control theory is to know whether a given dynamical system is controllable or not. In the context of our work, the precise meaning of controllability is as follows :

Definition 1.1. The operator $F : X \rightrightarrows X$ is said to be controllable if for each state $\xi \in X$, there are a finite time $T > 0$ and a solution x to (1) such that $x(0) = 0$ and $x(T) = \xi$.

Necessary and sufficient controllability conditions for convex processes have been suggested by Aubin, Frankowska and Olech in their paper [4] of 1986. These authors proved that

$$F \text{ is controllable} \iff \begin{cases} F \text{ is nonempty-valued and reproducing,} \\ \text{and its adjoint } F^* \text{ has no eigenvalues .} \end{cases} \tag{2}$$

Reproducibility refers to a certain rank condition that is discussed in Section 3. The orientation of our work deviates from [4] in a fundamental way : F is no longer seen as a fixed element of $Q(X)$, but rather as a "limit" of a sequence $\{F_n\}_{n\in \mathbb{N}}$ of approximate or perturbed convex processes. The differential inclusion (1) is interpreted as the "limiting" version of

$$\dot{x}(t) \in F_n(x(t)) \quad \text{for} \quad a.e. \quad t \in [0,T] .$$

Under suitable "inner convergence" and "boundedness" assumptions on $\{F_n\}_{n\in \mathbb{N}} \subset Q(X)$, one can show that the controllability of F forces the controllability of F_n for each $n \in \mathbb{N}$ sufficiently large. A result of this type has been obtained recently by Naselli-Ricceri [9, Theorem 3.2]. The proof technique used in [9] relies on semicontinuity properties of solution-sets to parametric differential inclusions. As shown in this note, preservation of controllability can also be obtained from the equivalence (2). This alternative approach is more direct and does not involve heavy mathematical machinery.

2. Notation and preliminary results

The notation that we employ is for the most part standard :

$$\begin{array}{rl} B_X & := \{s \in X : ||s|| \leq 1\} , \\ \text{dist}\,[z;S] & := \text{ distance from } z \in X \text{ to the set } S \subset X, \\ K^+ & := \{w \in X \mid \langle w, s\rangle \geq 0 \quad \forall s \in K\} , \\ \text{dom}\, F & := \{s \in X \mid F(s) \neq \emptyset\} . \end{array}$$

Recall that the composition $G \circ F : X \rightrightarrows X$ of two operators $G, F : X \rightrightarrows X$ is defined by the rule

$$(G \circ F)(s) := G(F(s)) = \bigcup_{v\in F(s)} G(v) \qquad \forall s \in X .$$

The composition of $F : X \rightrightarrows X$ with itself can be repeated as many times as one wishes. In this way one gets the operators $F^p : X \rightrightarrows X$ defined recursively by

$$F^1 := F , \qquad F^{p+1} := F^p \circ F \qquad \forall p \geq 1 .$$

Inner and outer-limits will be understood in the sense of Painlevé-Kuratowski:

Definition 2.1. Let $\{C_n\}_{n\in \mathbb{N}}$ be a sequence of sets in a topological space Z. The outer-limit of $\{C_n\}_{n\in \mathbb{N}}$ is defined by

$$z \in \textit{ outlim } C_n \Longleftrightarrow \begin{cases} \textit{there are a sequence } \{z_n\}_{n\in \mathbb{N}} \longrightarrow z \textit{ and a strictly} \\ \textit{increasing function } \varphi : \mathbb{N} \longrightarrow \mathbb{N} \textit{ such that} \\ z_n \in C_{\varphi(n)} \textit{ for all } n \in \mathbb{N} \ . \end{cases}$$

The inner-limit of $\{C_n\}_{n\in \mathbb{N}}$ corresponds to the set given by

$$z \in \textit{ innlim } C_n \Longleftrightarrow \begin{cases} \textit{there is a sequence } \{z_n\}_{n\in \mathbb{N}} \longrightarrow z \textit{ such that} \\ z_n \in C_n \textit{ for all } n \in \mathbb{N} \textit{ large enough} \ . \end{cases}$$

The operators outlim $F_n : X \rightrightarrows X$ and innlim $F_n : X \rightrightarrows X$ are defined respectively by

$$\text{Gr [outlim } F_n] = \text{ outlim [Gr } F_n] \quad , \qquad \text{Gr [innlim } F_n] = \text{ innlim [Gr } F_n] \ .$$

If the sequence $\{F_n\}_{n\in \mathbb{N}}$ lies in $Q(X)$, then so does innlim F_n. However, the graph of outlim F_n may fail to be convex. See the book by Rockafellar and Wets (1998) for an elaborate discussion on Painlevé-Kuratowski limits.

One says that $F^* : X \rightrightarrows X$ is the adjoint (or transpose) of $F : X \rightrightarrows X$ if

$$Gr\ F^* := \{(q, w) \in X \times X \mid (-w, q) \in (Gr\ F)^+\} \ .$$

For an arbitrary sequence $\{F_n\}_{n\in \mathbb{N}}$ in $Q(X)$, one has always the inclusion

$$Gr[\text{outlim } F_n^*] \ \subset \ Gr[(\text{innlim } F_n)^*] \ . \tag{3}$$

To each convex process $F : X \rightrightarrows X$, one can associate the nonnegative number

$$N[F] := \sup_{s\in B_X \cap \text{ dom } F} \text{dist}[0; F(s)] \ .$$

Loosely speaking, $N[F]$ can be seen as the "magnitude" of F. There is a rich theory behind this concept, but we just need to retain the following two results.

Proposition 2.2. *Let $F \in Q(X)$ be nonempty-valued. Then,*

(a) $N[F]$ *is finite* ;

(b) $||w|| \le N[F]\, ||q|| \qquad \forall (q, w) \in Gr\ F^*$.

Proof. Part (a) is due to Robinson (1972). Part (b) can be found in [2, p. 71]. □

Proposition 2.3. *Suppose the sequence $\{F_n\}_{n\in \mathbb{N}} \subset Q(X)$ satisfies the boundedness criterion*

$$\forall s \in X, \exists r \in \mathbb{R}_+ \quad \text{such that } F_n(s) \cap rB_X \neq \emptyset \quad \forall n \in \mathbb{N} \ . \tag{4}$$

Then, $\sup\{N[F_n] \mid n \in \mathbb{N}\} < \infty$.

Proof. See Theorem 2.3.1 in [2], or the original source [3] . □

Observe that (4) forces each F_n to be nonempty-valued.

3. Rank condition

This section can be considered as the core of our work. The concept of reproducibility is studied here for its own sake. If $F : X \rightrightarrows X$ is a convex process, then $\{F^p(0)\}_{p\geq 1}$ is a sequence of convex cones arranged in a nondecreasing order :

$$\{0\} \subset F^1(0) \subset F^2(0) \subset F^3(0) \subset \cdots \subset X . \tag{5}$$

Definition 3.1. A convex process $F : X \rightrightarrows X$ is said to be reproducing if

$$\text{there is an integer } p \geq 1 \quad \text{such that} \qquad X = F^p(0) - F^p(0) . \tag{6}$$

Of course, $F^p(0) - F^p(0)$ corresponds to the linear space spanned by $F^p(0)$. The "rank condition" (6) was explored by Korobov (1980) in the particular case $F(x) := Ax + K$, with $A : X \longrightarrow X$ being a linear operator, and K a convex cone of controls. The set $F^p(0)$ takes then the particular form

$$F^p(0) = K + AK + \cdots + A^{p-1}K .$$

In the general case, the computation of $F^p(0)$ can be quite cumbersome. However, the following lemma helps us to understand the nature of this cone.

Lemma 3.2. *Let $F \in Q(X)$ be nonempty-valued. Then,*

$$[F^p(0)]^+ = \operatorname{dom}[(F^*)^p] \qquad \forall p \geq 1 . \tag{7}$$

If $F \in Q(X)$ fails to be nonempty-valued, then one still has the inclusion

$$[F^p(0)]^+ \supset \operatorname{dom}[(F^*)^p] \qquad \forall p \geq 1 .$$

Proof. Formula (7) appears in Phat [10, Proposition 2.3]. □

Next we establish an estimate for $F^p(0)$ when F is the inner-limit of a certain sequence $\{F_n\}_{n\in\mathbb{N}}$.

Proposition 3.3. *Assume that $\{F_n\}_{n\in\mathbb{N}} \subset Q(X)$ satisfies the boundedness criterion* (4)*. Then,*

$$(\textit{innlim } F_n)^p(0) \subset \operatorname{innlim}[F_n^p(0)] \quad \forall p \geq 1 . \tag{8}$$

Proof. As a preliminary step, we shall prove that

$$\operatorname{outlim}\big[\operatorname{dom}(G_n^p)\big] \subset \operatorname{dom}\big[(\operatorname{outlim} G_n)^p\big] \qquad \forall p \geq 1 , \tag{9}$$

where $G_n = F_n^*$. We proceed by induction. For $p = 1$, the above inclusion becomes

$$\operatorname{outlim}\big[\operatorname{dom} G_n\big] \subset \operatorname{dom}\big[\operatorname{outlim} G_n\big] .$$

Let q be in the outer-limit of $\{\mathit{dom}\, G_n\}_{n\in\mathbb{N}}$. One can find a strictly increasing function $\varphi : \mathbb{N} \longrightarrow \mathbb{N}$ and a sequence $\{q_n\}_{n\in\mathbb{N}} \longrightarrow q$ such that

$$q_n \in \operatorname{dom} G_{\varphi(n)} \qquad \forall n \in \mathbb{N} .$$

For each $n \in \mathbb{N}$, pick up any $w_n \in G_{\varphi(n)}(q_n)$. By Propositions 2.2 and 2.3, we know that

$$||w_n|| \leq N\big[F_{\varphi(n)}\big]\; ||q_n|| \leq M$$

for some constant $M < \infty$. Since $\{w_n\}_{n\in \mathbb{N}}$ is bounded, the limit $w := \lim w_{\psi(n)}$ exists for some strictly increasing $\psi : \mathbb{N} \longrightarrow \mathbb{N}$. Thus,

$$\big(q, w\big) = \lim \big(q_{\psi(n)}, w_{\psi(n)}\big) \quad \text{with} \quad \big(q_{\psi(n)}, w_{\psi(n)}\big) \in \; Gr\; G_{(\varphi\circ\psi)(n)} \;.$$

Hence, $(q, w) \in$ outlim [Gr G_n]. This proves that $q \in$ dom [outlim G_n]. So, the case $p = 1$ has been taken care of. Let us admit (9) for a given p. We need to show that (9) remains true for $p + 1$. Pick up any

$$q \in \text{ outlim } [\text{dom } (G_n^{p+1})] \;.$$

Then one can find a strictly increasing function $\varphi : \mathbb{N} \longrightarrow \mathbb{N}$ and a sequence $\{q_n\}_{n\in \mathbb{N}} \longrightarrow q$ such that

$$q_n \in \text{ dom}\big[G^{p+1}_{\varphi(n)}\big] \quad \forall n \in \mathbb{N} \;.$$

For each $n \in \mathbb{N}$, pick up any $w_n \in G^{p+1}_{\varphi(n)}(q_n)$. Thus, one can find a sequence $\{z_n\}_{n\in \mathbb{N}}$ such that

$$w_n \in G^p_{\varphi(n)}(z_n) \quad \textit{and} \quad z_n \in G_{\varphi(n)}(q_n) \;.$$

Propositions 2.2 and 2.3 ensure the boundedness of $\{z_n\}_{n\in \mathbb{N}}$. Thus, $z := \lim z_{\psi(n)}$ exists for some strictly increasing $\psi : \mathbb{N} \longrightarrow \mathbb{N}$. The relations

$$z_{\psi(n)} \in \text{ dom}[G^p_{(\varphi\circ\psi)(n)}] \;, \qquad z_{\psi(n)} \in G_{(\varphi\circ\psi)(n)}\; \big(q_{\psi(n)}\big) \qquad \forall n \in \mathbb{N} \;,$$

yield $z \in$ outlim [dom (G_n^p)] and $z \in$(outlim G_n)(q), respectively. By invoking the induction hypothesis, one arrives at $z \in$ dom [(outlim $G_n)^p$], that is to say, the set (outlim $G_n)^p(z)$ contains at least one element, say $w \in X$. From the definition of the composition operation, it is clear that

$$w \in \big[\big(\text{outlim } G_n\big)^p \circ (\text{outlim } G_n)\big](q) \;.$$

In this way one arrives at the desired conclusion, namely

$$q \in \text{ dom}\big[(\text{outlim } G_n)^{p+1}\big] \;.$$

Now we can prove the inclusion (8). Let $F :=$ innlim F_n. Fix any $p \geq 1$ and define $K_n := [\text{dom}((F_n^*)^p)]^+$. According to Lemma 3.2 , K_n coincides with the closure of $F_n^p(0)$. Thus

$$\text{innlim}[F_n^p(0)] = \text{ innlim } K_n = \; \{\text{outlim } K_n^+\}^+$$

$$= \{\text{outlim}[\text{dom}((F_n^*)^p)]\}^+ \supset \{\text{dom}[(\text{outlim } F_n^*)^p]\}^+ \;.$$

But, (3) says that

$$Gr[\text{outlim } F_n^*] \subset Gr[F^*] \;,$$

from where one obtains

$$\text{dom}[(\text{outlim } F_n^*)^p] \subset \text{ dom}[(F^*)^p] \;.$$

The conclusion is that

$$\text{innlim}[F_n^p(0)] \supset \{\text{dom}[(F^*)^p]\}^+ \supset F^p(0) \;. \qquad \square$$

Due to the monotonicity property (5), it is natural to introduce the expression

$$\mathrm{Ind}(F) := \inf\{p \geq 1 \quad | \quad X = F^p(0) - F^p(0)\ \} ,$$

which will be called the *index* of the convex processs $F : X \rightrightarrows X$. Observe that $\mathrm{Ind}(F) = \infty$ if the rank condition (6) fails. Thus, a convex process is reproducing if and only if, its index is finite. Some calculus rules for computing indices are presented below :

Proposition 3.4. *(Rule of the iterated composition). For a convex process $F : X \rightrightarrows X$, the following three conditions are equivalent :*

(a) F is reproducing ;
(b) there exists $m \geq 1$ such that F^m is reproducing ;
(c) $\forall m \geq 1$, F^m is reproducing.

More precisely,

$$\forall m \geq 1 \, , \begin{cases} \mathrm{Ind}(F^m) = \left\lfloor \frac{\mathrm{Ind}(F)-1}{m} \right\rfloor + 1 = \left\lceil \frac{\mathrm{Ind}(F)}{m} \right\rceil \\ \\ m\,\mathrm{Ind}(F^m) - m + 1 \leq \mathrm{Ind}(F) \leq m\,\mathrm{Ind}(F^m) \, , \end{cases} \tag{10}$$

where $\lfloor \gamma \rfloor$ and $\lceil \gamma \rceil$ denote, respectively, the lower and upper integer part of $\gamma \in I\!R$.

Proof. It is based on the relation

$$(F^m)^p(0) = F^{m\,p}(0) \qquad \forall m, p \geq 1 \, . \qquad \square$$

Proposition 3.5. *(Rule of the inner-limit). Let $F = \mathrm{innlim}\ F_n$, with $\{F_n\}_{n\in I\!N} \subset Q(X)$ satisfying the boundedness criterion (4). Then,*

$$\mathit{Ind}(F_n) \leq \mathit{Ind}(F) \quad \textit{for all } n \in I\!N \textit{ large enough} \, . \tag{11}$$

Proof. One can assume that F is reproducing ; otherwise $\mathrm{Ind}(F) = \infty$, and there is nothing to prove. Let p be the index of F. If the conclusion (11) was false, then one could find a strictly increasing function $\varphi : I\!N \longrightarrow I\!N$ such that

$$\mathrm{Ind}\big(F_{\varphi(n)}\big) > p \qquad \forall n \in I\!N \, .$$

The above inequality says that $F^p_{\varphi(n)}(0)$ does not span the whole space X. In other words, $F^p_{\varphi(n)}(0)$ lies in a set of the form

$$\mathrm{Ker}\ q_n := \{v \in X \mid \langle q_n, v\rangle = 0\} \, , \quad \text{with} \quad ||q_n|| = 1 \, .$$

For some strictly increasing $\psi : N \longrightarrow I\!N$, the sequence $\{q_{\psi(n)}\}$ converges to a nonzero vector $q \in X$. By applying Proposition 3.3, one gets

$$F^p(0) \subset \ \mathrm{innlim}\ [F^p_n(0)] \subset \ \mathrm{innlim}\ [F^p_{(\varphi\circ\psi)(n)}(0)] \subset \ \mathrm{innlim}[\mathrm{Ker}\ q_{\psi(n)}] \subset \ \mathrm{Ker}\ q \, .$$

The linear space spanned by $F^p(0)$ is thus contained in Ker q. The fact that $F^p(0)$ does not span X contradicts the very definition of p. $\square$

We state now the main result of this section.

Proposition 3.6. *Let $F = \mathrm{innlim}\ F_n$, with $\{F_n\}_{n\in I\!N} \subset Q(X)$ satisfying the boundedness criterion (4). Then, the following implication holds true :*

$$F \textit{ is reproducing} \quad \Longrightarrow \quad \textit{for all } n \in I\!N \textit{ large enough}, \quad F_n \textit{ is reproducing} \, . \tag{12}$$

Proof. Apply Proposition 3.5. $\square$

4. The role of eigenvalues

As in the case of linear differential systems, eigenvalues and eigenvectors play a fundamental role in the analysis of differential inclusions whose right-hand sides are convex processes.

Definition 4.1. The number $\lambda \in I\!R$ is said to be an eigenvalue of the multivalued operator $G : X \rightrightarrows X$ if $\lambda\xi \in G(\xi)$ for some nonzero vector $\xi \in X$. The point spectrum of G is the set

$$\sigma(G) := \{\lambda \in I\!R : \lambda\xi \in G(\xi) \quad \text{for some} \quad \xi \in X \setminus \{0\}\}$$

of all eigenvalues of G.

General spectral results for multivalued operators can be found in the papers by Seeger and collaborators [5, 6, 7, 13]. We shall borrow from them the formula (cf.[6, Corollary 3.3])

$$\text{outlim}\ \sigma(G_n) \subset \sigma(\text{outlim}\ G_n) \quad , \tag{13}$$

which applies to any sequence $\{G_n\}_{n\in\boldsymbol{N}}$ of positively homogeneous operators $G_n : X \rightrightarrows X$. Besides (13), we shall need also the following two auxiliary results.

Proposition 4.2. *Let $F \in Q(X)$ be nonempty-valued. Then,* $\sigma(F^*) \subset [-N(F), N(F)]$.

Proof. It follows from Proposition 2.2. □

Proposition 4.3. *Let $F = \text{innlim}\ F_n$, with $\{F_n\}_{n\in\boldsymbol{N}} \subset Q(X)$ satisfying the boundedness criterion* (4). *Then,*

$$\sigma(F^*) = \emptyset \quad \Longrightarrow \quad \textit{for all } n \in I\!N \textit{ large enough}, \quad \sigma(F_n^*) = \emptyset\ . \tag{14}$$

Proof. If the conclusion of (14) was false, then it would be possible to find a strictly increasing function $\varphi : I\!N \longrightarrow I\!N$ such that

$$\sigma(F^*_{\varphi(n)}) \neq \emptyset \qquad \forall n \in I\!N\ .$$

Built up any sequence $\{\lambda_n\}_{n\in\boldsymbol{N}}$ satisfying

$$\lambda_n \in \sigma(F^*_{\varphi(n)})\ , \quad \forall n \in I\!N\ .$$

Thanks to Propositions 2.3 and 4.2, $\{\lambda_n\}_{n\in\boldsymbol{N}}$ is necessarily bounded. Thus, for some increasing function $\psi : I\!N \longrightarrow I\!N$, the sequence $\{\lambda_{\psi(n)}\}_{n\in\boldsymbol{N}}$ converges to a certain $\lambda \in I\!R$. Due to (3) and (13), one can write

$$\lambda \in \text{ outlim}\ \sigma(F^*_{(\varphi\circ\psi)(n)}) \subset \text{ outlim}\ \sigma(F_n^*) \subset \sigma(\text{outlim}\ F_n^*) \subset \sigma(F^*)\ ,$$

contradicting in this way the fact that the point spectrum of F^* is empty. □

5. Preservation of controllability

The results of the preceding sections are used here to prove that the concept of controllability is stable with respect to a certain class of perturbations.

Theorem 5.1. *Let $F = \text{innlim}\ F_n$, with $\{F_n\}_{n\in\boldsymbol{N}} \subset Q(X)$ satisfying the boundedness criterion* (4). *Then, the following implication holds true :*

$$F \textit{ is controllable} \quad \Longrightarrow \quad \textit{for all } n \in I\!N \textit{ large enough}, \quad F_n \textit{ is controllable}\ . \tag{15}$$

Proof. Each F_n is nonempty-valued. If one relies on the equivalence (2), then it suffices to apply Proposition 3.6 and 4.3. □

Some final comments are in order :

(a) That each F_n is controllable does not guarantee the controllability of the limit process F. The converse of implication (15) does not hold, even in the context of linear control systems!

(b) The above remark applies also to the concept of reproducibility : the converse of implication (12) is not true in general.

(c) The boundedness criterion (4) has played a crucial role in Sections 3 and 4. This condition has secured not only the nonempty-valuedness of each F_n, but also a more substantial property :

$$\bigcup_{n\in\boldsymbol{N}} F_n^*(B) \text{ is bounded whenever } B \text{ is bounded .}$$

Dropping the latter requirement would invalidate Propositions 3.3, 3.5, 3.6, and 4.3 .

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