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Behavioral Economics, Competitive Equilibrium and Multiple Phase Dynamics in an Abstract, Adaptive Society: Remarks in Honor of Jean–Pierre Aubin*

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Dedicated to Jean-Pierre Aubin on the occasion of his 65th birthday.

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This paper gives a general characterization of satisficing, lexicographic agents, provides conditions for the existence of a competitive equilibrium and considers the viability and multiphase dynamic of an abstract adaptive society whose agents are governed by satisficing, lexicographic choice.

1. Introduction

If pure mathematics is the language and logic of symbolic relationships, then applied mathematics is the synthesis and analysis of models that symbolize empirical experience. The French school of mathematical economics beginning with Cournot and Walras created a kind of *pure* applied mathematics when Arrow and Debreu (1954, [1]) re-expressed general economic equilibrium theory in the language of Bourbaki, using set valued maps and more general concepts of regularity. A decade later a newly minted functional analyst set about mastering the foundations of this by then flourishing field. At the same time he began extending the functional analysis itself, subsequently turning to the more general domain of differential inclusions. Just as set valued maps greatly extend the applicable domain of static, general equilibrium analysis, differential inclusions encompass a more general way of characterizing change through time. The establishment of this new field of applied mathematics implied a new pure mathematics, the structure and properties of which our hero proceeded to develop and extend. I am speaking, of course, about Jean–Pierre Aubin, whose gift for constructive abstraction relevant for modeling varied phenomonological domains and whose genius for extending the pure mathematics these models imply, we have long admired. Equally admirable has been his knack for recognizing talent in others, inspiring and engaging them in a collaborative enterprise that now constitutes an identifiable school, many members of which are assembled here at this conference.

Just when Jean–Pierre was launching this remarkable enterprise, I met him at the University of Wisconsin's Mathematical Research Center some time around 1965. I visited him shortly thereafter at his newly founded CEREMAD (Centre de Recherche de

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Mathématiques de la Décision) where I presented an abstract reformulation of the recursive programming models that my students and I had constructed to explain production, investment, and technological change in specific industries and agricultural regions. I exploited discrete time difference inclusions to do so, which could make use of the already existing topology of set valued maps, to establish existence of compact orbits (Day and Kennedy (1970)) and contraction properties to establish some weak stability results (Cherene (1978)). Jean–Pierre, with Helene and other collaborators has used the alternative framework of differential inclusions to develop an economic viability theory far richer in mathematical results.

I had hoped on this occasion to establish a link between these alternative approaches by exploiting the recent work on impulse control and reset maps. After some intense effort, I have not succeeded so far. I have not given up but for now I will have to be content with an elaboration of a version of an abstract, adaptive society about which one can say a few things not wholly without economic and mathematical interest.

In particular, I show how two fundamental concepts of behavioral economics, satisficing, and sequential attention to goals (Simon (1957), [13]), March and Simon (1960), and Cyert and March (1963, [4]) fit into the standard theory of rational choice and equilibrium, and explain some interesting implications that arise in a dynamic context when cognitive and informational limitations are important. A general discussion with illustrations and references to the related literature is presented elsewhere (Day (1996), [6]).

2. Satisficing

As a preliminary, consider the abstract parametric optimal choice problem. Let x be a choice vector in a linear, locally convex, topological *choice space* $\mathcal{X}$ and let w be a datum or parameter in a topological *information space* $\mathcal{W}$. The *feasibility correspondence* is a nonempty, continuous multivalued map $\Gamma : \mathcal{W} \to 2^{\mathcal{X}}$, the *utility* or more generally, the *objective function* is a real single-valued continuous function $\varphi : \mathcal{X} \times \mathcal{W} \to \mathbb{R}$. The *indirect objective function* $\pi : \mathcal{W} \to \mathbb{R}$ is defined by

$$\pi(w) := \max_{x \in \Gamma(w)} \varphi(x, w). \tag{1}$$

The *choice correspondence* is a multi-valued map $\Psi : \mathcal{W} \to 2^{\mathcal{X}}$ defined by

$$\Psi(w) := \operatorname{Arg} \max_{x \in \Gamma(w)} \varphi(x, w). \tag{2}$$

Remark 2.1. According to the Maximum Theorem (Berge (1963, [3], pp. 115–116)), π is continuous and single-valued on $\mathcal{W}$ and Ψ is upper semi-continuous on $\mathcal{W}$. If for each w, $\varphi(x, w)$ is quasi-concave, then the upper contour sets of the preference ordering,

$$M(x) := \Big\{ x \mid \varphi(x, w) \geq \pi(w) \Big\}, \tag{3}$$

are convex. Therefore, if for each w, $\Gamma(w)$ is a convex set which is assumed henceforth, then $\Psi(w)$ is convex also.

Define a real single-valued, continuous *satisficing function*, $\sigma : \mathcal{W} \to \mathbb{R}$. For each w the set of satisficing choices is

$$\Gamma(x, w) \cap \Big\{x \mid \varphi(x, w) \geq \sigma(w)\Big\}. \tag{4}$$

Assume that if no satisficing choice is attainable, the problem reverts to that of finding the best feasible choice.

Now consider the utility function defined by

$$\varphi(x, w) = \min\Big[\mu(x, w),\ \sigma(w)\Big] \quad x \in \mathcal{X},\ w \in \mathcal{W}, \tag{5}$$

where $\mu(x, w)$ is a real continuous, quasi-convex, single-valued function on $\mathcal{X}, \mathcal{W}$.

Remark 2.2. Given Remark 2.1 and the assumption just listed, Ψ is a nonempty, convex-valued, *upper semi-continuous map.*

A stronger result, however, is needed for the lexicographic case.

I. Existence and Continuity of Satisficing Choices. (Robinson and Day (1974, [12])). *If μ is strictly quasi-concave in x for each fixed w and σ is continuous on $\mathcal{W}$, then the satisficing choice correspondence Ψ is nonempty, continuous and convex-valued on $\mathcal{W}$.*

3. Satisficing Lexicographic Choice

Let $N := (1, \dots, n)$. Let $(\varphi_i : \mathcal{X} \times \mathcal{W} \to \mathbb{R},\ i \in N)$ be a family of utility or preference functions arranged in a *priority order* given by the index i. Consider the lexicographic sequence of choice problems

$$\pi_i(w) := \max_{x \in \Psi_{i-1}(w)} \varphi_i(x, w),\ i \in N \tag{6}$$

where $\Psi_0(w) := \Gamma(w)$. Lexicographic choices are constructed recursively by the sequence

$$\Psi_i(w) := \Psi_{i-1}(w) \cap \Big\{x \mid \varphi_i(x, w) \geq \pi_i(w)\Big\},\ i \in N. \tag{7}$$

Let σ_1 be a nonnegative real number. Assume $\varphi_1(x, w) \equiv \sigma_1$ for all $w \in \mathcal{W}$. Define the *lexicographic choice correspondence*, $\Psi_l : W \to 2^{\mathcal{X}}$ by

$$\Psi_l(w) := \bigcap_{i \in N} \Psi_i(w),\ w \in \mathcal{W}. \tag{8}$$

For a given index $i > 1$ the decision maker is indifferent among all choices in $\Psi_{i-1}(w)$ according to the criteria $1, \dots, i-1$, but prefers a choice y to another choice x in $\Psi_{i-1}(w)$ whose utility is greater according to the i^{th} criterion. Clearly, $\Psi_i(w) \subset \Psi_{i-1}(w)$ for all $w \in \mathcal{W}$. Note that for $l = 1$, $\Psi_l(w) = \Gamma(w)$: the highest priority is to find a feasible solution. In practice this may not be an easy problem. Indeed, it may be the most difficult problem in the sequence and the most costly one to solve.

For the narrowing down process to be nontrivial, there must be room at least in the choice correspondence $\Psi_1(w)$ for further choice according to the second criterion φ_2. To define such situations, we adopt the hypothesis of satisficing in priority order using a family of satisficing functions ($\sigma_i : \mathcal{W} \to \mathbb{R}. i \in N$) and a family of utility functions ($\mu_i : \mathcal{X} \times \mathcal{W} \to \mathbb{R}$, $i \in N$). We define the L^* family of utility functions,

$$\varphi_i(x, w) := \min\Big[\mu_i(x, w), \sigma_i(w)\Big],\ x \in \mathcal{X},\ w \in \mathcal{W},\ i \in N, \tag{9}$$

where $\mu_1(x, w) \equiv \sigma_1(w) \equiv \sigma_1$ for all $w \in \mathcal{W}$. Given (9), the sequence (7) is called an L^* *decision sequence* and Ψ_l an L^* *choice correspondence.* A decision maker so described is an L^* *agent*, and a choice $x \in \Psi_l(w)$ is called an L^* decision.

Obviously, for any given problem i in an L^* decision sequence to have a solution and hence for each correspondence in the L^* decision sequence to be nonempty, the preceding problem in the sequence must have had a solution. For each one to be upper semi-continuous, its predecessor must be continuous.

II. Existence and Continuity of L^* Choices (Day and Robinson (1972)).

(i) *If $\mu_i : \mathcal{X} \times \mathcal{W} \to \mathbb{R}$ is continuous on $\mathcal{X} \times \mathcal{W}$, strictly quasi-concave in x for each $w \in \mathcal{W}$, for each $i = 2, \ldots, n$,*
then Ψ_l is a single-valued, continuous function. In this case, define $\psi_l \equiv \Psi_l$.

(ii) *If $N = \{1, \ldots, n\}$ for n finite and assumption (i) holds for each $i = 1, \ldots, n-1$ and μ_n is continuous but merely quasi-concave in x, then Ψ_l is a convex-valued, upper semi-continuous function.*

For lack of a better terminology, refer to an L^* ordering satisfying (i) as strong and one satisfying (ii) as week.

Remark 3.1. For each w there must exist an $l \in N$ depending on w such that $\Psi_i(w) = \Psi_{l(w)}(w)$ for all $i > l(w)$. Call $l(w)$ the *determining criterion governing choice.*

Remark 3.2. The continuity of L^* decisions suggests that L^* behavior can be rationalized by a single ordering. Indeed, a trivial ordering exists if the decision space is given a norm. Analogous to the approach of Arrow and Hahn (1971, [2]) for the standard case, a single utility function representing Ψ_l is just the map which associates with each pair (x, w) the distance (with reverse sign) from x to $\Psi_l(w)$. For each w, this (negative) distance is single-valued and continuous by virtue of the fact that $\Psi_l(w)$ is convex. More generally, for each $w \in \mathcal{W}$, L^* utility establishes a complete pre-ordering on $\mathcal{X}$ whose upper and lower continuous sets defined by an $x \in \mathcal{X}$ are closed. Then, using Debreu (1983, [9], Chapter 6),

III. *A single-valued continuous real order preserving function exists that represents an L^* pre-ordering.*

4. Opportunity Cost of High–Order Satisficing

At each stage in the lexicographic sequence, choices are constrained to be feasible, and satisfice all higher order utility indexes. Consequently, the more criteria are satisficed, the more constrained the choice. This implies that high order satisficing bears an opportunity cost in terms of the first nonsatisficed criterion. This yields interesting insights in various applications.

To formalize this idea, let us introduce a family of *constraint functions* $\beta_i : \mathcal{X} \times \mathcal{W} \to \mathbb{R}$, $i = 1, \ldots, m$, and a family of *limitation functions* $\gamma_i : \mathcal{W} \to \mathbb{R}$, and assume that $\Gamma(w)$ defined by

$$\Psi_0(w) \equiv \Gamma(w) := \Big(x \mid \beta_i(x,w) \leq \gamma_i(w),\ i = 1, \ldots, m\Big). \tag{10}$$

For each value $w \in \mathcal{W}$, the L^* choice correspondence gives the solution set of the constrained optimization problem

$$\text{maximize } \varphi_{l(w)}(x,w) \tag{11}$$

subject to

$$\beta_i(x,w) \leq \gamma_i(w),\ i = 1, \ldots, m \tag{12}$$

and subject to

$$\varphi_i(x,w) \geq \sigma_i(w),\ i = m+1, \ldots, m+l(w)-1 \tag{13}$$

and to the non-negativity restriction $x \geq 0$. This problem is equivalent to the Kuhn-Tucker problem:

$$\begin{aligned} \pi_{l(w)}(w) = \max_{x \geq 0} \min_{y \geq 0} \Big(\varphi_{l(w)}(x,w) &+ \sum_{1}^{m} \Big[\gamma_i(w) - \beta_i(x,w)\Big] y_i \\ &+ \sum_{m+1}^{m+l^*-1} \Big[\varphi_i(x,\bar{w}) - \sigma_i(\bar{w})\Big] y_i \Big). \end{aligned} \tag{14}$$

Let $\big(x(w), y(w)\big)$ be a solution of this primal-dual problem where $y(w) = (y_1, \ldots, y_{m+n})$. The dual variables $y_i(w)$, $i = 1, \ldots, m$ have the usual interpretation: they are the marginal payoffs in terms of the objective function $\varphi_{l(w)}(x,w)$ of a marginal increase in the i^{th} constraining factor $i = 1, \ldots, m$, and the marginal increase in payoff in terms of a marginal decrease in the i^{th} satisficing level, $i = m+1, \ldots, m+l(w)-1$. Decreasing the satisficing level of the determining criterion expands the region of choices that are both feasible and $l(w)-1$ satisficing.

IV. *Corresponding to every L^* decision $x^*(w)$ is a dual imputation vector $y^*(w)$ defined by* (14) *which gives the marginal opportunity cost of each resource and each satisficing coefficient in terms of the determining criterion.*

5. Competitive Equilibrium with L^* Agents

Could an economy of lexicographic agents possess a competitive equilibrium? Certainly, excess demand functions will be more complicated than is usually represented in texts, as illustrated in Day and Robinson (1972), but the continuity of choice with respect to parameter perturbations suggests that the answer is affirmative anyway. To verify this, let us outline the relevant considerations for the Arrow-Debreu pure exchange economy.

For consumers, the choice variable x^k is a consumption vector of commodities in a commodity space $\mathcal{X}$, the datum is a price vector $w \equiv p$, which appears in the consumer k's budget set, say $\Gamma^k(w) \equiv B_k(p) := \Big\{x^k \in \mathcal{X} \mid p(x^k - \xi^k) \leq 0\Big\}$ where ξ^k is the vector of (positive) initial commodity endowments. It is assumed that agents' preferences are independent of prices.

Now suppose each consumer, k, has an L^* family, $\big\{\mu_i^k(x) := \min\{\mu_i^k(x),\, \sigma_i^k\},\, i = 1, \dots\big\}$ where the σ_i are satisficing constants and where the utility functions μ_f and satisficing constants do not depend on prices. Assume that each $\mu_i^k(\cdot)$ is strictly quasi-concave and assume the nonsatiation requirement: that for each $x^k \in \mathcal{X}^k$ there exists an $l \geq 1$ (depending on k) and a consumption vector $y^k \in \mathcal{X}^k$ such that

$$\varphi_l^k(y^k) > \varphi_l^k(x^k). \tag{15}$$

Remark 2.2 implies that $\varphi_i^k(x^k) = \sigma_i^k = \varphi_i^k(y^k)$, $i = 1, \dots, l-1$. If the L^* family is strong, each criterion can be satisficed but the consumer's list of needs and wants is never exhausted. If we assume a weak L^* preference ordering, it means that the last l^{th} criterion is nonsatiable and cannot be satisficed.

The excess demand correspondence is $z(p) = \sum_k \Psi_l^k(p)$. Assume p belongs to the unit simplex. Then a competitive pure exchange equilibrium is a vector $\tilde{p}$ and a set of vectors $\tilde{x}$ such that for each k

$$\tilde{x}^{k^*} = \Psi_l^k(\tilde{p}),\ z(\tilde{p}) \leq 0,\ \tilde{p}z(\tilde{p}) = 0.$$

Each consumer's demand correspondence defined by $\Psi_l^k(\tilde{p})$ is an upper semi-continuous correspondence by virtue of **II**. Given this, the conditions of existence are met. The argument can be extended to the case of a production economy where the firms have the single criterion of maximizing profits. Without going into details, we have:

V. *Consider an economic system whose firms and households are described by Assumptions* I, II, *and* IV *of Arrow-Debreu. Assume the households are strong L^* agents who satisfy the nonsatiation requirement* (15) *and the assumptions of* **II**(ii). *This economic system possesses a competitive equilibrium.*

Remark 5.1. A competitive equilibrium depends on all the parameters of preference and technology, in particular on both the order and satisficing coefficients of the several utility functions. Any change in priorities or satisficing levels will change the equilibrium solutions.

6. Characterization of an Abstract Adaptive L^* System

Now consider the implications of L^* preferences when agents adapt out of equilibrium in response to experience and to changes in the environment. For this purpose, I will formulate an abstract adaptive society along the lines used in Day (1996, [6]) made up of interacting agents with L^* preferences and a co-evolving environment. The agents take action based on plans which are based on information obtained by observing the state of the environment. The emerging environmental state depends on its previous value and on the various agents' actions.

6.1. An Agent/Environment System

For simplicity, let us begin with a single agent interacting with an external environment (that perhaps includes other agents). The environmental state is z_t. The agent observes this state and derives information, w_t, about it. On the basis of this information, plans are formed. Actions are not the same things as plans. In reality, they often involve a more or less conscious attempt to control behavior according to plan, but they are also contingent on the current state of the environment and on information that unfolds after formulation of the plan. The dependence of actions on information and states via the controlling function can cause deviations from plans.

In formal terms, let

$\mathcal{A}$	be the action space,
$\mathcal{X}$	be the plans space,
$\mathcal{W}$	be the information space,
$\mathcal{Z}$	be the state space.

Define the operators

$\delta : \mathcal{A} \times \mathcal{Z} \to \mathcal{W}$,	an information function;
$\Gamma : \mathcal{W} \to \mathcal{P}(\mathcal{X})$,	a feasibility correspondence;
$\psi^* : \mathcal{W} \to \mathcal{X}$,	a strong L^* planning operator;
$\zeta : \mathcal{A} \times \mathcal{X} \times \mathcal{W} \times \mathcal{Z} \to \mathcal{A}$,	a control operator;
$\omega : \mathcal{A} \times \mathcal{X} \to \mathcal{Z}$,	a state transition operator.

Begin with a state $z_t \in \mathcal{Z}$. This leads to an information $\delta(a_t, z_t) = w_{t+1} \in \mathcal{W}$. The planning correspondence has the structure of equation (1). Given a plan, given the available information, and given the state, an action emerges $a_{t+1} = a_t + \zeta(x_{t+1}, w_{t+1}, z_t)$, which we consider to be a modification or departure of the previous action a_t. This is just a discrete counterpart of the concept of action as a continuous trajectory in an action space. We can think of the action as a continuous flow in the interval $[t, t+1)$. Then a new state unfolds $z_{t+1} = \omega(a_t, z_t)$. In this formulation the information, planning, and control functions are known by the agent. The agent's knowledge of a state z and the state transition operator ω is incorporated in w and the structure of the feasibility operator Γ.

By composition we have an action *operator* $\alpha : \mathcal{A} \times \mathcal{Z} \to \mathcal{A}$,

$$a_{t+1} = \alpha(a_t, z_t) := a_t + \zeta\Big(\psi^*(\delta(a, z), z),\ \delta(a, z), z\Big). \tag{16}$$

The co-evolution of the agent/environment system is given by the discrete dynamic system

$$\begin{aligned} a_{t+1} &= \alpha(a_t, z_t) \\ z_{t+1} &= \omega(a_t, z_t). \end{aligned} \tag{17}$$

Define a *region of potential action* defined by a *potentiality correspondence*,

$$F : (a, z) \longrightarrow F(a, z) \subset A,$$

which describes the set of potential actions given current behavior and the environmental state. Actions are *potential* if

$$F(a_t, z_t) \neq \emptyset. \tag{18}$$

If

$$a_{t+1} = \alpha(a_t, z_t) \in F(a_t, z_t), \tag{19}$$

then the action operator α is *practical* and the agent is *viable* at time t.

The feasibility operator, $\Gamma(w_t)$, can be thought of as providing an estimate of the set $F(a_t, z_t)$. Even if the agent has an optimal feasible *plan*, a potential *action* may not exist. What the individual considers to be feasible may be distinct from what is actually the set of viable actions. It is essential that the control function make up for this possibility by being practical.

In addition to the viability conditions the environmental feedback effect of action must return the system to a state in U. In this case, we will say that the *system is environmentally friendly.*

VI. *If there exists a nonempty set $U \subset \mathcal{U}$ such that*

(*i*) *for all $u \in U$, $F(u) \neq \emptyset$ and $\alpha(w) \in F(a, z)$ (control is practical),*
(*ii*) *and $\big(\alpha(u), w(z)\big) \in U$ (the system is environmentally friendly),*
then for all $u_0 \in U$, $u_{t+1} = \Phi(u_t) := \big(\alpha(u_t), \omega(u_t)\big) \in U$, $t = 1, 2, 3, \ldots,$
and the agent/environment system is viable on U.

6.2. An L^* Agent/Environment System

Next, consider the implication of L^* objectives on behavior in the agent/environment system just described. It was noted above that the L^* choice problem could be characterized as the solution of a programming problem. This implies that an L^* plan satisfies a dual system of equated constraints associated with that solution. If the information on which plans are based changes from period to period, this system of equated constraints may change. The characterization of Ψ_l in terms of this system of equated constraints will change correspondingly. There are a finite number of possible equated constraint systems. Let us index these sets by a subscript $p \in [1, \ldots, \bar{p}]$. Then associated with each w there is at least one system of equated constraints, $p = p(w)$, and a single-valued map $\pi_p : (\mathcal{W} \to \mathcal{X})$,

$$x(t) = \pi_p(w_t) \in \Psi_l(w_t).$$

Define

$$W_p = \Big\{w \in \mathcal{W} \mid x(t) = \pi_p(w) \in \Psi_l(w)\Big\}.$$

Let us refer to π_p as the p^{th} *planning rule* and the set W_p as the p^{th} *planning information zone* and define W_0 as the set such that for all $w_t \in W_0$, $x_t = \pi_0(w_t) = 0$ where 0 is the null vector.

As $w = \delta(a, z)$, each action/state pair will lead to an information in one or the other of these information zones. This induces a partition $\big\{U_1, \ldots, U_p\big\}$ on $U \subset \mathcal{A} \times \mathcal{Z}$ with $\bigcup_{p=1}^{\bar{p}} U_p = U$ such that for each $u \in U$ there exists a p such that

$$u \in U_p \Longrightarrow \delta(u) \in W_p.$$

Now define the p *action rules*

$$\alpha_p(u) := a + \zeta\Big(\pi_p(\delta(u)),\ \delta(u), u\Big),\ u \in U_p,$$

where $u = (a, z)$, and let $\theta_p(u) := \big(\alpha_p(u), w(u)\big)$. Then we arrive at:

VII. The trajectory of the L^* agent/environment system is described by the multiple phase dynamical system

$$u_{t+1} = \theta(u_t) := \theta_p(u_p) \in U_p.$$

As the system evolves, from time to time u_t may cross the boundary from one phase zone U_t to another implying that the system of equated constraints will have changed. The phase structure switches endogenously. The implication is that different constraints (associated with different resources, for example) may be binding or – and this is the point in the present context – a *different set of needs and wants may be satisficed. The particular preferences governing choice at any time and the values imputed to the various constraining factors will therefore change.* The map $\psi(w)$ may not be continuous even if the assumptions of **III**(i) are satisfied. This means that trajectories can exhibit discrete jumps from time to time even when change may have been very gradual in between times.[1]

6.3. Evolving Preference Orderings

In reality, people change their minds about priorities with the result that they are re-ordered from time to time. To account for this phenomenon within the agent/environment system framework, assume that not only the objective and satisficing functions depend on states, but their order does also. Let $N(w) := \{i_1(w),\, i_2(w), \dots\} \subset N := \{1, 2, \dots\} \subset \mathbb{N}^+$. We can then refer to the j^{th} element in $N(w), i_j(w)$ by the index j.

Consider the L^* family defined in (8) except replace N by $N(w)$. For any information datum w_t, the L^* planning correspondence is well defined and can be characterized by Kuhn-Tucker optimization problem (10–14) with the proviso that now the criterion $l(w_t)$ is the $l(w_t)^{\text{th}}$ element in $N(w_t)$.

VIII. *Given this interpretation, the number* <u>*and*</u> *order of criteria satisficed and the decision rules governing behavior evolve.*

7. An Abstract Adaptive L^* Society

If there are a number of L^* agents interacting with each other and their common environment, and if each is characterized as in §6, then we have an abstract adaptive L^* society. Identify each agent by an index $k \in K := \{1, \dots, \bar{k}\}$ and the information, plans, and action spaces associated with each by superscripts, k. Similarly, identify each agent's information, plan, and action space likewise and define the *social spaces*:

$$\begin{array}{lcl} \mathbf{A} & := & \mathcal{A}^1 \times \cdots \times \mathcal{A}^{\bar{k}} \\ \mathbf{X} & := & \mathcal{X}^1 \times \cdots \times \mathcal{X}^{\bar{k}} \\ \mathbf{W} & := & \mathcal{W}^1 \times \cdots \times \mathcal{W}^{\bar{k}} \\ \mathbf{U} & := & \mathbf{A} \times \mathcal{Z} \end{array}$$

[1]For a general characterization of the structure of multiple phase trajectories, see (Day (1994, [7], Chapters 6 and 9). The discussion there is in terms of single-valued maps but all of the concepts hold for the more general dynamical systems described here.

with a social action $\mathbf{a} := (a^1, \ldots, a^{\bar{k}})$, social plan $\mathbf{x} := (x^1, \ldots, x^{\bar{k}})$, and social information $\mathbf{w} := (w^1, \ldots, w^{\bar{k}})$. Define the corresponding social action, planning, information, and state transition functions on the *product spaces* appropriately:

$$\begin{array}{lcl} \delta(\mathbf{a}, z) & := & \big(\delta^1(\mathbf{a}, z), \ldots, \delta^{\bar{k}}(\mathbf{a}, z)\big) \\ \psi(\mathbf{w}) & := & \big(\psi^1(w^1), \ldots, \psi^{\bar{k}}(w^{\bar{k}})\big) \\ \alpha(\mathbf{a}, z) & := & \big(\alpha^1(a^1, z), \ldots, \alpha^{\bar{k}}(a^{\bar{k}}, z)\big) \\ \omega : (\mathbf{a}, z) & \longrightarrow & \omega(\mathbf{a}, z) \end{array}$$

Notice that each agent may form information about the other agents as well as the environment.

The existence of potential behavior for each agent as described in §6 must hold here but in addition the several agents must also satisfy inter-agent compatibility. Such a requirement can be described by a *compatibility region* $\mathcal{C} \subset \mathbf{A} \times \mathcal{Z}$.

IX. *Define* $\mathbf{u} := (u^1, \ldots, u^{\bar{k}}) \in \mathbf{U} := \times_{k=1}^{\bar{k}} U^k$ *where* $u^k = (a^k, z)$, $k = 1, \ldots, \bar{k}$.

(*i*) *Define the multiagent potentiality correspondence mapping* $\mathbf{F} : \mathbf{A} \times Z \longrightarrow \mathbf{A}$,

$$\mathbf{F}(\mathbf{a}, z) := \times_{k=1}^{\bar{k}} F^k(a^k, z), \quad \text{where } F^k(a^k, z) \neq \emptyset$$

for all k.

(*ii*) *If there exists a set* $U \subset \mathbf{A} \times \mathcal{Z}$ *such that for all* $\mathbf{u} \in U$,

$$\alpha(\mathbf{u}) \in \mathbf{F}(\mathbf{u}),$$

then all agents are individually, potentially viable,

(*iii*) *and*

$$\Phi(\mathbf{u}_t) := \Big(\alpha(\mathbf{u}), \omega(\mathbf{u})\Big) \in \mathcal{C},$$

then for all $\mathbf{u}_0 \in U$,

$$\mathbf{u}_{t+1} = \Big(\alpha(\mathbf{u}_t), \omega(\mathbf{u}_t)\Big) \in U, \ t = 1, \ldots,$$

the system is viable.

Obviously, all of the properties described in §6 carry over to the present case. Thus, the trajectory of behavior will be characterized by multiple phase dynamics and evolving priorities where the various equated constraints at each time and specific order of priorities may vary among the agents. Consequently, the qualitative as well as the quantitative history of each agent may be unique. Also implied is an endogenous evolution in the distribution of satisficed wants and needs; a possible convergence – or divergence – of social values, a development of social accord or discord. These social developments could have an important feedback effect on action and on the possibility that the compatibility property will hold.

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