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Shape Grammar and Form Properties of Architectural Figures

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Abstract. Since the beginning of history, various forms of architecture have been designed in the world. The characteristics of those forms were considered in the figure of elevation. In this study, architectural figures as the simple expression of architectural forms are analyzed to understand the shape grammar, i.e., the geometrical composition of figures, and the form properties of architecture. Firstly, the shape grammar of architectural figures is verified by presenting 74 figures of representative architecture, and secondarily, a method to evaluate the form properties is shown.

1. Origamic Architecture as architectural figure

One of the authors of this paper, M. Chatani, born in 1934, is the creator of *Origamic Architecture*, which expresses architecture in a postcard sized pop-up white card. He travelled to a lot of countries since he was young, and took a careful look at worldwide architecture. Then he learned to express the images in pop-up white cards.

While Japanese traditional *Origami* is the art of folding paper in various figures with colored paper, Origamic Architecture uses cutters to get architectural figures as their image. Tracing of architectural figures and getting their image requires a sense of beauty, and M. Chatani was possessed of this sense with his love for architecture.

Origamic Architecture is very unique work of art and has 5 different types of work, which are 0° angle type, 90° angle type, two types of 180° angle and 360° angle type. 0° angle

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types use two sheets of paper, two halves of sheets are pasted together, and figures are cut. Then their sections are folded inward to come out with 3-dimensional effects. 90° angle types, shown in Fig. 1, are often used to express architectural figures. One sheet of paper is folded in two by bending or cutting. The figure will come out by opening it at 90° and the sheet keeps its original form by opening to 180° as shown in Fig. 2. 180° angle types are produced by 3-dimensional figures with attached pieces of paper on folding pasteboard. And another types of 180° will come out with two pieces of 90° angle types, which are pasted together in the center of the pasteboard. 360° types are 3-dimensional figures attached to pasteboard like 180° types.

Figure 1: Example of Origamic Architecture (church at Mykonos)

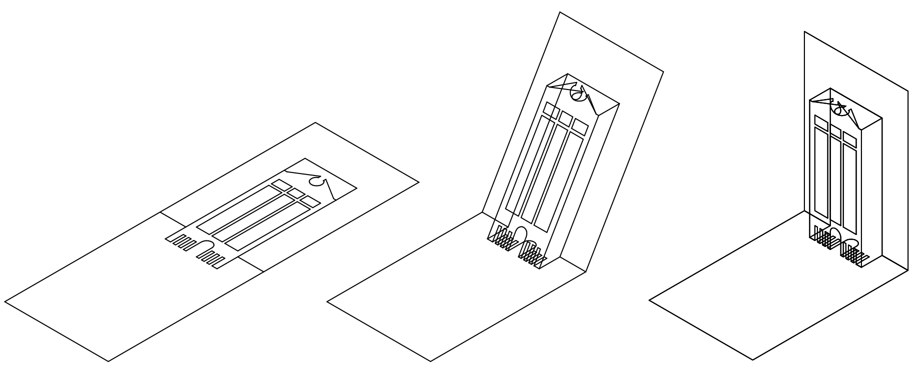

Figure 2: Folding process of Origamic Architecture (AT&T Building)

The first exhibition of Origamic Architecture was held 1982 in Tokyo, then the first book *Origami Kentiku [Origamic Architecture]* was published in the next year. Then exhibitions were held in many countries and books were translated into several languages, such as English, Chinese, German and Dutch. And now the books are published in varieties of 45 kinds with 6 languages.

While Origamic Architecture was originally the expression of architecture, this idea was applied to other figures, such as flowers, animals and letters. They are listed by the thousands, and some 300 works are figures of architecture or constructions. These works are simple expression of architecture, so they keep the characteristics of each architectural shape.

Well-proportioned beautiful Greek architecture, skyscraper towers of Gothic architecture, and modern architecture with a variety of shapes are expressed in the elevation of Origamic Architecture. Therefore, they are the models of architectural shapes that describe the characteristics of architectural figure.

In this paper, we take attention to Origamic Architecture as models of architectural shapes, and analyze the characteristics of composition with their elevation by grasping their distinctive feature. The analyzed objects of this study are the elevations of 74 pieces of Origamic Architecture shown in Table 1 out of some 300 works. They include a wide range of construction periods and types of architecture like churches, banks, bridges, residences, offices, and museums. And they include also a variety of countries, such as France, Italy, U.S.A., U.K., and Japan. The selected works can represent major architecture and constructions in history.

2. Shape grammar of architectural figures

As mentioned above, the projection of Origamic Architecture as architectural figure is a simplified model of architectural elevation. What are the rules of geometrical composition of these simple figures?

In this chapter, as a preliminary treatment, we propose a simple algorithm, a software program for personal computer, to distinguish architectural elements in the figure, mainly walls and openings (i.e., windows and doors).

2.1. Algorithm to distinguish the architectural elements

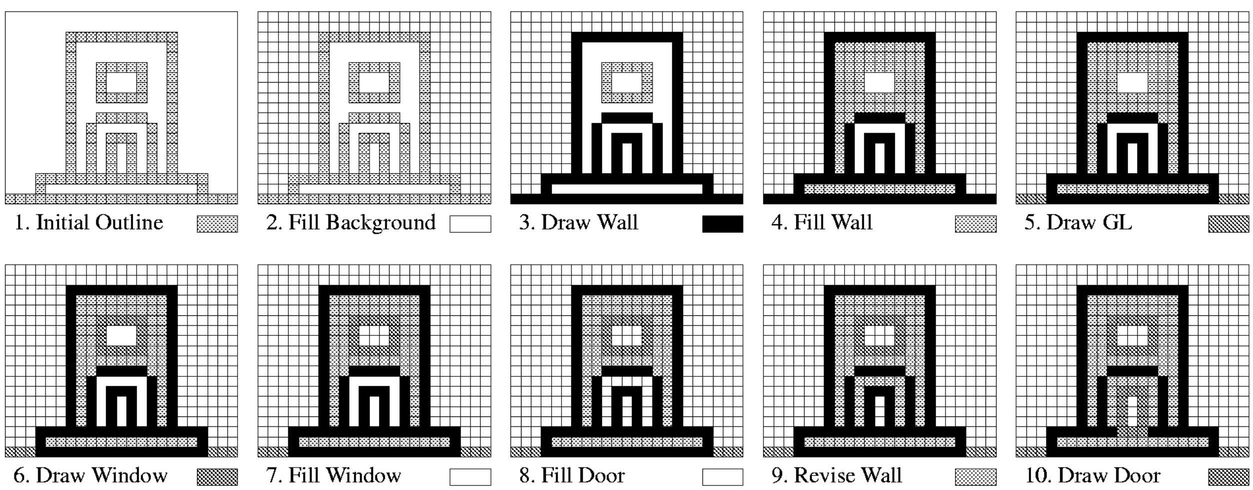

Figure 3: Process of identifying elements

On the graphic screen of a personal computer, an architectural figure is expressed as a collection of pixels. We can find some rules of composition in this set of pixels. For example, pixels adjacent to the background are mostly the wall, and an isolated pattern inside the wall is presumed to be the window. The process shown in Fig. 3 and the following is an algorithm to identify background, walls, openings, and GL (i.e., ground line).

1. Initial figure, only an *outline* is drawn.
2. Fill the outer field of pixels as *background*.

name	*type*	*country*	*const. period*	*source, see final book list*
01. Akasaka Prince	hotel	Japan	20th century	syo04, syo08
02. Alberobello	residence	Italy	16th century -	kou01
03. Alhambra	palace	Spain	13th-15th cent.	ond03
04. Amsterdam	residence	Netherlands	—	kou01
05. ATT	office	U.S.A.	20th century	syo01, syo04, syo08
06. Bay Bridge	bridge	Japan	20th century	kou01
07. Big Ben	tower	U.K.	19th century	ond06
08. Borobudur	monument	Indonesia	8th century	kou02
09. Chenonceaux	castle	France	16th century	ond03
10. Chrysler	office	U.S.A.	20th century	syo06
11. Crown Hall	institute	U.S.A.	20th century	frn01
12.Daisekiji	temple	Japan	20th century	syo06, syo08
13. Diet Bldg	public bldg.	Japan	20th century	pop01
14. Eiffel	tower	France	19th century	syo06
15. Empire State	office	U.S.A.	20th century	syo06
16. Falling Water	residence	U.S.A.	20th century	frn02
17. Gallaratese	residence	Italy	20th century	syo08
18. Giza Pyramid	monument	Egypt	26th cent. B.C.	kou02
19. Glass Pyramid	museum	France	20th century	syo01
20. Golden Gate	bridge	U.S.A.	20th century	syo04
21. Grundtvig	church	Denmark	20th century	syo01
22. Guggenheim	museum	U.S.A.	20th century	frn01
23. Himeji Castle	castle	Japan	17th century	ond03, syo07
24. H.K.Shanghai Bank	bank	China	20th century	syo08
25. House 6	residence	U.S.A.	20th century	frn02
26. Hyatt Regency	hotel	U.S.A.	20th century	syo08
27. Johnson Pavillion	office	U.S.A.	20th century	unpublished
28. Johnson Wax	office	U.S.A.	20th century	frn01
29. Kappel Bridge	bridge	Switzerland	14th century	kou01
30. Kings Cross	station	U.K.	19th century	syo06
31. Kintai Bridge	bridge	Japan	17th century	syo05
32. Ledoux	residence	France	18th century	syo02
33. London Bridge	bridge	U.K.	20th century	ond06
34. Loyola	institute	Italy	20th century	syo06, syo08
35. Marina City	residence	U.S.A.	20th century	syo08
36. Maya	monument	Mexico	6th century	kou02
37. Megane Bridge	bridge	Japan	17th century	ond06, kou01
38. Milano Duomo	church	Italy	14th-19th cent.	ond03
39. Minneapolis F.R.B.	bank	U.S.A.	20th century	syo08
40. Mobile Home	residence	U.S.A.	20th century	unpublished
41. Morris Shop	shop	U.S.A.	20th century	unpublished
42. Mykonos	church	Greece	—	syo01
43. NEC	office	Japan	20th century	syo08
44. Notre Dame	church	France	12th century	unpublished
45. Pacific Design	office	U.S.A.	20th century	syo06, syo08
46. Paris East	station	France	19th century	syo06
47. Parthenon	monument	Italy	5 cent. B.C.	kou02, syo01, syo02
48. Pazzi Chapel	church	Italy	15th century	unpublished
49. Pompidou	museum	France	20th century	syo06, syo08
50. Pont du Gard	bridge	France	1st century	kou02, syo02
51. Ponte Rialto	bridge	Italy	16th century	kou01, syo05
52. Ponte Vecchio	bridge	Italy	14th century	kou01, syo05
53. Potala Palace	palace	China	17th century	kou01
54. Republic Bank	bank	U.S.A.	20th century	syo04, syo08
55. Retti Candle	shop	Austria	20th century	syo04, syo08
56. Sagrada Familia	church	Spain	19th century	syo05
57. Salk Institute	institute	U.S.A.	20th century	unpublished
58. San Giorgio Maggiore	church	Italy	16th-17th cent.	unpublished
59. San Marco Duomo	church	Italy	11th century	unpublished
60. Statue of Liverty	monument	U.S.A.	19th century	syo06
61. Taj Mahal	monument	India	17th century	kou02, syo01, syo02
62. Taranto Duomo	church	Italy	20th century	syo06, syo08
63. Tepee	residence	U.S.A.	-19th century	frn02
64. Triumphal Arch	monument	France	19th century	syo06
65. Tucker House	residence	U.S.A.	20th century	syo08
66. Venturi House	residence	U.S.A.	20th century	syo08
67. Vesta	temple	Italy	3rd century	kou02
68. Villa Poiana	residence	Italy	16th century	unpublished
69. Villa Rotonda	residence	Italy	16th century	unpublished
70. Villa Valmarana	residence	Italy	16th century	unpublished
71. Whitehouse	residence	U.S.A.	19th century	syo01
72. Winter Garden	atrium	U.S.A.	20th century	syo06, syo08
73. Wright Studio	office	U.S.A.	20th century	unpublished
74. Yakushiji Pagoda	pagoda	Japan	8th century	syo06

Table 1: List of architectural figures

3. Take the pixels adjacent to the *background* and the continuous pixels to them, in 8 directions, as the *outline of walls.*
4. Fill the blank fields adjacent to the *outline of walls* as *walls.*
5. If the upper pixel of the lowest position pixels are *background*, take them as *GL.*
6. Take the continuity of black pixels adjacent to *walls* as the *outline of openings (windows).*
7. Fill the blank fields adjacent to the *outline of openings (windows)* as *openings (windows).*
8. Fill the remained fields as *openings (doors).*
9. Modify the outer fields of stacking or recurring *openings (doors)* to *walls.*
10. Take the pixels adjacent to *openings (doors)* as the *outline of openings (doors).*

2.2. Finding the architectural elements

By adapting this algorithm to 74 figures shown in Table 1, 34 of 74 (45.9%) were completely recognized. In detail, there were 2254 blocks, i.e., divided fields of pixels by the outline, in 74 figures, 30.5 blocks in a figure on average. 1899 blocks out of 2254 (84.3%) were distinguished as proper architectural elements. Thus, the most part of the blocks inside the figures were identified as architectural elements by the geometrical process.

Fig. 4 shows the examples of mis-distinguished elements. 325 *walls*, out of the 355 misidentified elements, were incorrectly distinguished as *openings*, and 30 *openings* were incorrectly recognized as *walls.* At the Maya Pyramid, the rectangular fields of the base were misidentified as *openings (doors).* And the highest *opening (door)* was mis-modificated as *wall*, because it was located on the upper part of a misidentified *opening (door).* At the San Giorgio Maggiore Church, the ornamental wall patterns inside the pediment were incorrectly distinguished as *openings (windows).* In addition, at figures with complicated composition like *House IV* and *Akasaka Prince Hotel*, *walls* and *openings* were confused in identifying.

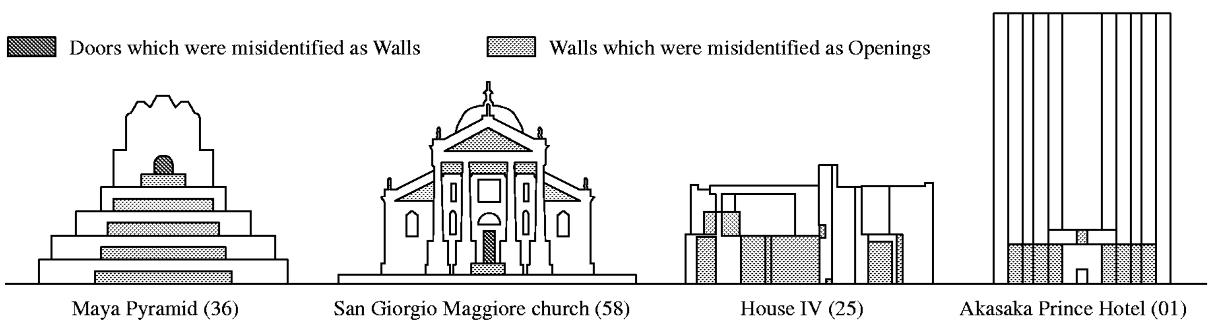

Figure 4: Misidentified elements

3. Form properties of architectural figures

3.1. Calculated data

We calculated the following 9 data, which indicate the characteristics of architectural figures (see Fig. 5). The 9 calculated data were as follows:

1. Height of architecture (HGT): Maximum height of architecture (ratio to the standard height of pasteboard (= 100 mm))

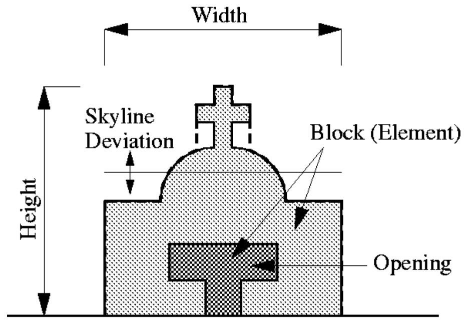

Figure 5: About calculated data

2. Width of architecture (WID): Maximum width of architecture (ratio to the standard width of pasteboard (= 150 mm))

3. Skyline deviation of architecture (SDA): Deflection of rooftop skyline within architecture's width

4. Number of blocks (NBK): Number of divided elements, such as windows, doors, wall elements, base, etc.

5. Entropy of blocks (EBK): Complicated degree of architecture by calculating the entropy of blocks

6. Occupance of the largest block (OBK): Occupied ratio of the largest block's area to the area of architecture

7. Number of openings (NOP): Number of windows and doors

8. Quantity of openings (QOP): Ratio of total area of openings to the area of architecture

9. Occupancy of the largest opening (OOP): Occupied ratio of the largest opening's area to the area of architecture

	1. HGT	*2. WID*	*3. SDA*	*4. NBK*	*5. EBK*	*6. OBK*	*7. NOP*	*8. QOP*	*9. OOP*
08. Chrysler	1.38	0.26	0.41	105	0.68	35%	98	30%	6%
12. Himeji Castle	0.94	0.87	0.24	108	1.57	34%	61	7%	0%
14. H.K. Shanghai Bank	1.32	0.43	0.04	119	1.69	23%	72	6%	2%
09. Notre Dame	1.19	0.35	0.19	92	1.04	13%	51	17%	1%
11. Ponte Rialto	0.59	1.00	0.16	12	0.96	25%	8	43%	26%
13. Triumphal Arch	0.59	0.46	0.02	2	0.46	78%	1	20%	20%

Table 2: Example of calculated data

Table 2 shows the data from 6 examples, figures of them are shown in Fig. 6: Chrysler Building[1], Himeji Castle[2], Hong Kong Shanghai Bank[3], Notre Dame Cathedral[4], Ponte di Rialto[5] and Triumphal Arch[6]. The upper figures in the table have qualitatively complicated forms, and the lower figures have simple forms.

The correlation coefficients between the 9 data are shown in Table 3. The greatest correlation is 0.71 between *skyline deviation* and *height*. We also calculated the following 6 data. But, as the 6 data below had a strong correlation between one of the above 9 data in 0.76–0.91, those data were considered to be able to replace one of the above 9 data.

[1] Modern high-rise building, 20th century
[2] 17th century Japanese historical castle
[3] Contemporary high-tech building
[4] 12th century French cathedral
[5] 16th century Italian bridge
[6] 19th century French monument

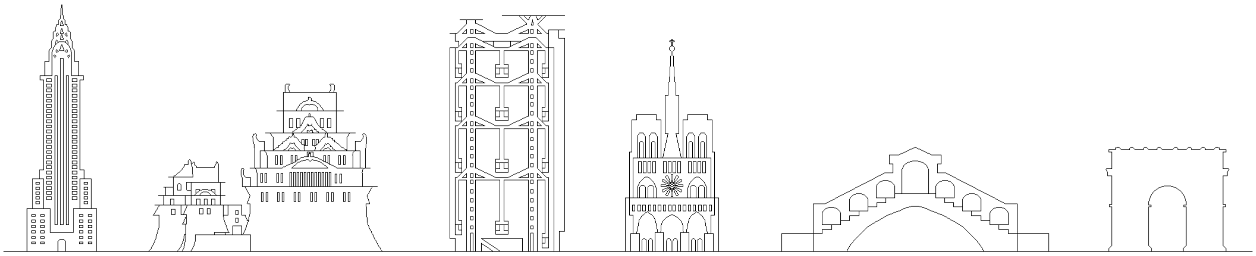

Figure 6: Example figures of calculated data

	1	2	3	4	5	6	7	8	9
1. Height of architecture	1.00	-0.63	0.71	0.21	0.05	0.10	0.27	-0.20	-0.16
2. Width of architecture		1.00	-0.36	-0.02	0.36	-0.26	-0.11	0.18	0.10
3. Skyline deviation of architecture			1.00	0.07	-0.25	0.13	0.07	-0.29	-0.18
4. Number of blocks				1.00	0.53	-0.44	0.61	-0.12	-0.28
5. Entropy of blocks					1.00	-0.64	0.24	0.00	-0.21
6. Occupancy of the largest block						1.00	-0.15	0.02	0.21
7. Number of openings							1.00	0.25	-0.07
8. Quantity of openings								1.00	0.67
9.Occupancy of the largest opening									1.00

Table 3: Correlation analysis

10. Occupancy of architecture (SIZ): Ratio of occupied area for architecture to the size of background paper, i.e., size of architecture. Correlate to EBK in 0.76

11. Slenderness of architecture's maximum height (SMX): Ratio of maximum height to width, i.e., slimness of architecture. Correlate to HGT in 0.83 and to WID in -0.84

12. Slenderness of architecture's average height (SAV): Ratio of average height to width, i.e., slimness of architecture. Correlate to WID in -0.82

13. Skyline deviation through background width (SDB): Deflection of skyline including ground line. Correlate to HGT in 0.87

14. Entropy of openings (EOP): Indication for complicated degree of openings' composition. Correlate to QOP in 0.91

15. Alienation from rectangular (REC): Ratio in area of architecture to the area of maximum height $*$ width, indicate the alienation of architecture's shape from rectangular shape. Correlate to SDA in -0.79

3.2. Principal component analysis

By using the above 9 data, 3 principal components shown in Table 4 were extracted by the factor analysis.

The first principal component shows a great number of blocks, a great number of openings, a complicate composition of figure and a lack of the large element. We named this component *rhythm property*. This property is expressed by the arrangement or distribution of plural openings or the continuous motif of the walls.

The second shows great height, less width, and great skyline deviation. We named this *silhouette property*. This property is expressed by the outer outline of the figures. For instance, Gothic architecture with high towers would have a strong silhouette property.

	rhythm	*silhouette*	*symbol*
4. Number of blocks	*0.85*	0.20	-0.07
5. Entropy of blocks	*0.80*	-0.29	-0.14
6. Occupancy of largest block	*-0.70*	0.31	0.20
7. Number of openings	*0.67*	0.37	0.38
1. Height of architecture	0.16	*0.88*	-0.10
2. Width of architecture	0.17	*-0.78*	0.03
3. Skyline deviation	-0.07	*0.77*	-0.22
8. Quantity of openings	0.06	-0.18	*0.91*
9. Occupancy of largest opening	-0.27	-0.13	*0.82*
Eigen value	2.55	2.49	1.57
Pct of var	28.4%	27.7%	17.5%
Cum Pct	28.4%	56.0%	73.5%

Table 4: Principal component analysis

The third shows a great quantity of openings, a great number of openings, and a great occupancy of the largest opening, i.e., at least one of the openings' areas is distinctly large. We named this *symbol property*, as this is mainly evaluated by the area and the geometrical characteristics of openings.

Characteristics of architectural figures are explained by these 3 properties.

4. Typology of architectural figures

To find the types of architectural figures, a cluster analysis by 3 properties, i.e., by the score of 3 principal components, was carried out. As shown in Figs. 7 and 8, 74 figures were classified under 5 types and 1 peculiar case.

Type-A: Silhouette Type. In this type, the silhouette property is considerably high. High-rise buildings that have a conspicuous silhouette in downtown, and mausoleums or churches that have individual external forms belong to this type.

Type-B: Symbol Type. In this type, the symbol property is considerably high. Bridges with unique openings under their long span, and architecture with an originally designed entrance or with arranged openings belong to this type.

Type-C: Simple Type. In this type, the rhythm property, i.e., the complexity of figurative composition, is excessively weak. Modern architecture with simple formed walls and rare openings is classified in this type.

Type-D: Medium Type. In this type, 3 properties are in the intermediate range, yet the silhouette property is slightly in the minus range. Architecture with various forms is included in this type.

Type-E: Hybrid Type. In this type, the rhythm property and the silhouette property are both considerably high. Architecture with unique silhouette and complicated composition, like Japanese castles, Gothic churches, contemporary high-technology buildings, were classified in this type.

Type-F: a peculiar case. Such as the Chrysler Building, a masterpiece of Art Deco architecture built by 1930 in New York. All 3 properties are expressively high in this figure.

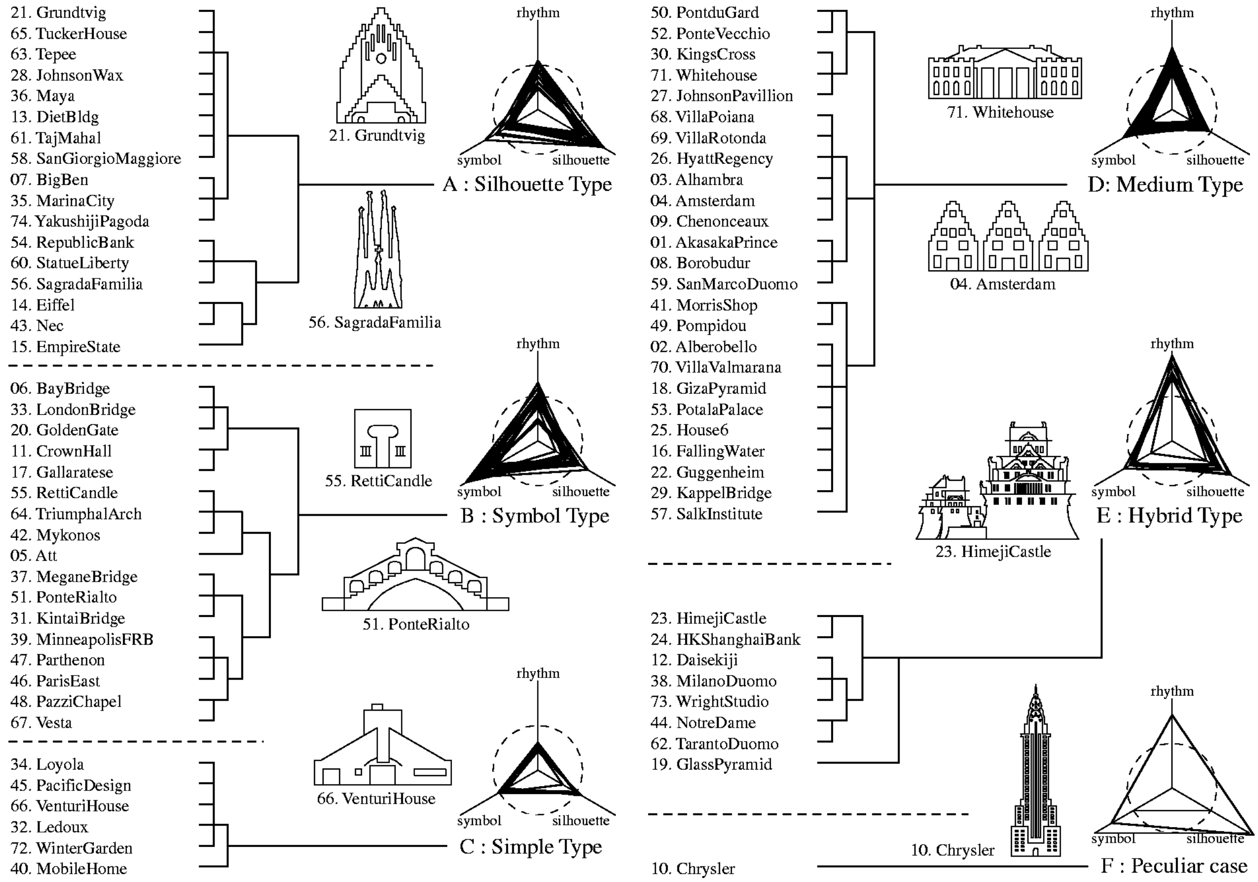

Figure 7: Cluster analysis

5. Conclusion

The form analysis of architectural figures was carried out by using the simple architectural elevations of Origamic Architecture. The conclusions of this analysis are as follows:

1. Origamic Architecture as a simple expression of architectural figures is suitable for the database of architectural forms.
2. The algorithm to find the elements of architecture made it possible to comprehend the shape grammar of architectural forms. However, the proposed algorithm is not completed yet. We would like to develop this technique for the next argument.
3. Any architectural form has 3 properties such as rhythm, silhouette and symbol. The rhythm property indicates the range of complexity, the silhouette property is that of the outer form, and the symbol property concerns to the design of the openings.

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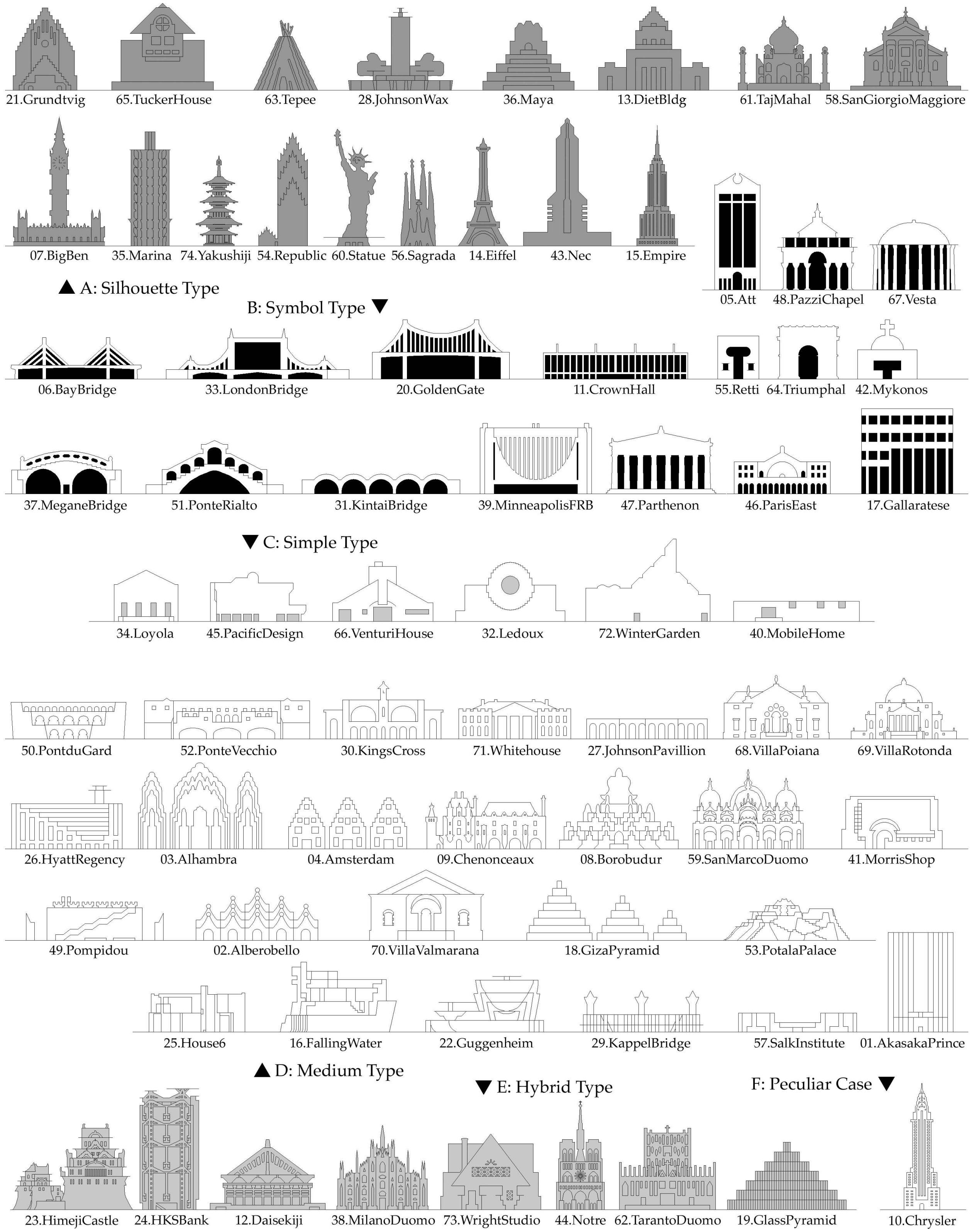

Figure 8: Types of architectural figures

References

List of source books written by M. Chatani in Japanese

ID	*Name of source book*	ISBN
ond01	Magic House	4-277-75301-9
ond02	Heartful Card	4-277-75306-X
ond03	Paper Magic no Sekai	4-277-75307-8
ond04	Pop-up Card	4-277-75308-6
ond05	Precut no Greeting Card	4-277-75309-4
ond06	Igigen Greeting Card	4-277-75317-5
ond07	Magic House 2	4-277-75321-3
ond08	Tukuru Tanoshisa Okuru Yorokobi no Greeting Card	4-277-75322-1
kou01	Super Origami	4-06-100382-8
kou02	Super Origami Iseki-hen	4-06-100393-3
syo01	Origami Kentiku	4-395-27011-5
syo02	Origami Kentiku Katagami-syu	4-395-27012-3
syo03	Origami Kentiku Syunkasyutou	4-395-27013-1
syo04	Origami Kentiku Toranomaki	4-395-27014-X
syo05	Origami Kentiku Katagami-syu 2	4-395-27016-6
syo06	Origami Kentiku Sekai no Kentiku-meguri	4-395-27017-4
syo07	Origami Kentiku Kachou no Maki	4-395-27018-2
syo08	Origami Kentiku Gendai no Meikentiku o Tukuru	4-395-27040-9
syo09	Origami Kentiku Shiki Oriori no Card o Tukuru	4-395-27042-5
syo10	Origami Kentiku Nara no Tabi	4-395-27043-3
syo11	Origami Kentiku Kyo no Tabi	4-395-27044-1
syo12	Origami Kentiku Greeting Card	4-395-27045-X
syo13	Origami Kentiku Sekai no Meikentiku o Tukuru	4-395-27046-8
pop01	Tobidasu Origami	4-591-01610-C8772

List of books written by M. Chatani in English

ID	*Name of source book*	ISBN
frn01	Great American Buildings	4-7700-1538-O
frn02	Origamic Architecture American House	4-7700-1337-X
frn03	Pop-up Origamic Architecture	0-87040-656-6

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