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A Photographic Method for Panoramic Sequence with a Regular Camera
Part 3: Application to Sky Photographs

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Abstract. In parts 1 and 2 [1, 2] the authors presented a method to produce a panorama by connecting photographs. In part 3 this method is applied to sky photographs. For this purpose a computer program is presented to calculate the relative position of the two photoprints. An example for this will be introduced, too.

Next, a method for estimating the center of a sky photograph and the focal length of the lens of the camera will be presented. This method is based on the celestial coordinates of fixed stars being displayed on the sky photograph. The center and the focal length are important data but often there are gaps in the data when this method is used. The new method offers a more accurate placement of the two photoprints. This method can be applied to measuring the data needed to derive the positions of celestial bodies.

Key Words: sky photograph, celestial coordinate, center of photoprint, focal length

MSC 2000: 51N05

1. Introduction

In order to combine two photographs into one panorama photograph, in previous papers [1, 2] numerical methods were presented, the compass method, the stereoscopic method and others. However, in all these cases the scenery was connected and it was possible to move the two photographs manually until they match along a common borderline. However, in sky photographs, the different stars are isolated; so there are no markings available which allow one to move the two photographs until they match. In order to solve this problem, in this paper a new numerical method is presented to derive the two bisecting points of two photoprint centers. Moreover, regarding the joining of a regular panorama photograph, the position of the center of each photoprint and the value of the focal length of the lens used

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when taking the photograph are needed. This time the author reports that in the case of a sky photograph, even if the focal length is not known, from the true position of the stars and their images, a calculation as well as a graphical method can produce the required results.

2. Connecting sky photographs

2.1. The joining line and matching of positions

2.1.1. Method of calculation

The principles involved in calculating the joining line are shown in Fig. 1. Table 1 shows the method of calculation used in the program. Originally, in order to derive the joining line, as shown in Fig. 2, the distances $a_1, b_1, c_1, a_2, b_2, c_2$ on the back of the photoprint are given. Next, the distances u_1, u_2, v_1, v_2 are derived. From these values, the joining line 1-2 on the left side and 3-4 on the right side were derived. After the photoprints were cut, they were turned face up. Adjusting the two photoprints by looking at the patterns leads to the right placement where they match. In Table 1, lines 10–200 represent the original program.

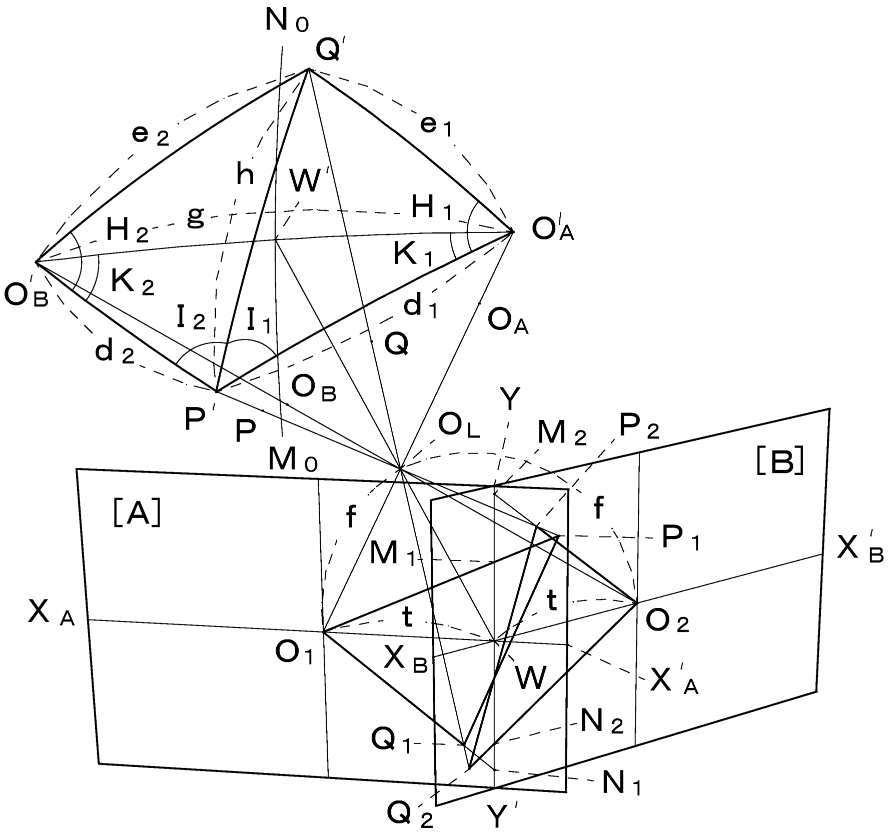
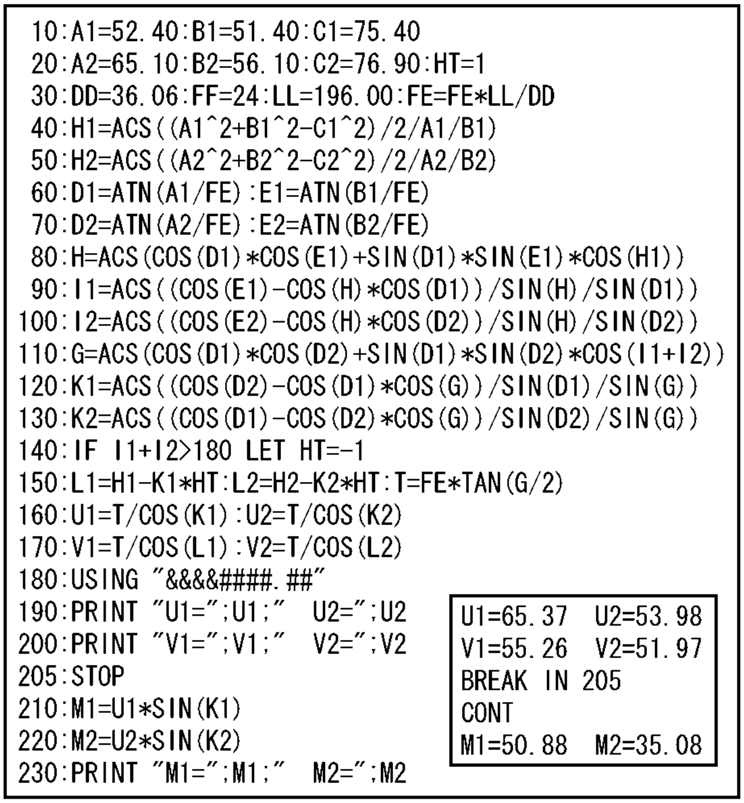


Figure 1: Image of outside field and photographs

However, as individual fixed stars in the sky photograph are isolated, the markings to be used when moving the joining line cannot be found. For this reason, this time, as shown in Fig. 3, formulae (1) are used and m_1 and m_2 are found. The addition of this method allows one to find the positions W_1 and W_2. Formulae (1) are shown in Table 1 on lines 210–230.

$$m_1 = u_1 \sin K_1\,, \quad m_2 = u_2 \sin K_2 \tag{1}$$



```
 10:A1=52.40:B1=51.40:C1=75.40
 20:A2=65.10:B2=56.10:C2=76.90:HT=1
 30:DD=36.06:FF=24:LL=196.00:FE=FE*LL/DD
 40:H1=ACS((A1^2+B1^2-C1^2)/2/A1/B1)
 50:H2=ACS((A2^2+B2^2-C2^2)/2/A2/B2)
 60:D1=ATN(A1/FE):E1=ATN(B1/FE)
 70:D2=ATN(A2/FE):E2=ATN(B2/FE)
 80:H=ACS(COS(D1)*COS(E1)+SIN(D1)*SIN(E1)*COS(H1))
 90:I1=ACS((COS(E1)-COS(H)*COS(D1))/SIN(H)/SIN(D1))
100:I2=ACS((COS(E2)-COS(H)*COS(D2))/SIN(H)/SIN(D2))
110:G=ACS(COS(D1)*COS(D2)+SIN(D1)*SIN(D2)*COS(I1+I2))
120:K1=ACS((COS(D2)-COS(D1)*COS(G))/SIN(D1)/SIN(G))
130:K2=ACS((COS(D1)-COS(D2)*COS(G))/SIN(D2)/SIN(G))
140:IF I1+I2>180 LET HT=-1
150:L1=H1-K1*HT:L2=H2-K2*HT:T=FE*TAN(G/2)
160:U1=T/COS(K1):U2=T/COS(K2)
170:V1=T/COS(L1):V2=T/COS(L2)
180:USING "&&&&####.##"
190:PRINT "U1=";U1;"  U2=";U2
200:PRINT "V1=";V1;"  V2=";V2
205:STOP
210:M1=U1*SIN(K1)
220:M2=U2*SIN(K2)
230:PRINT "M1=";M1;"  M2=";M2
```

```
U1=65.37  U2=53.98
V1=55.26  V2=51.97
BREAK IN 205
CONT
M1=50.88  M2=35.08
```

Table 1: Pocket computer program in Basic

As the program runs, the first result is that u_1, u_2, v_1, and v_2 are shown. Below that, as BREAK IN 205 is shown, CONT RUN is input, m_1 and m_2 are shown. The places for the derived values W_1, W_2 are moved up and down after they were written on the backs of the photoprints. Then the photoprints were cut and they were made to meet. When the photoprint is placed face up, the panorama photograph has been completed. Still, in the sky photograph, the points P_1, Q_1, P_2, and Q_2 are positions given to fixed stars, and thus it is easier to work with than a photograph of ordinary scenery. In this way, the method of working with a sky photograph until a joining line is found is shown in Fig. 4, (a) and (b). The resulting panorama photograph is shown in Fig. 5. In Table 1, the input data and the answer values are shown. The computer used was a Sharp product, Pocket Computer PC-G820.

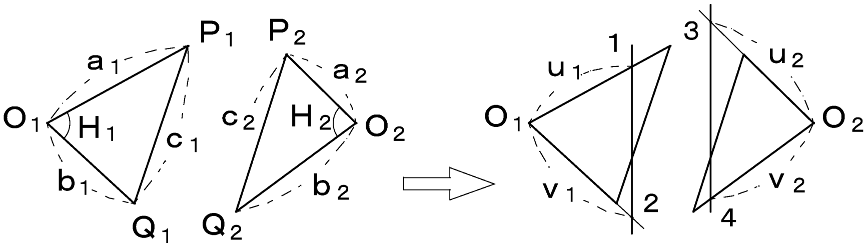


Figure 2: Given triangles and joining lines

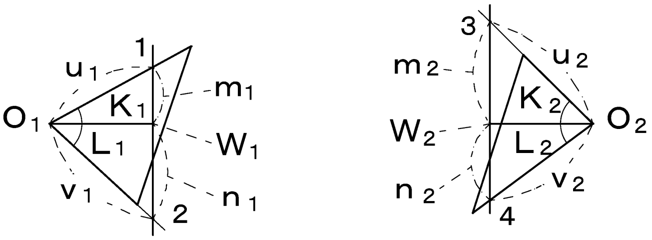


Figure 3: Intersecting points on joining line

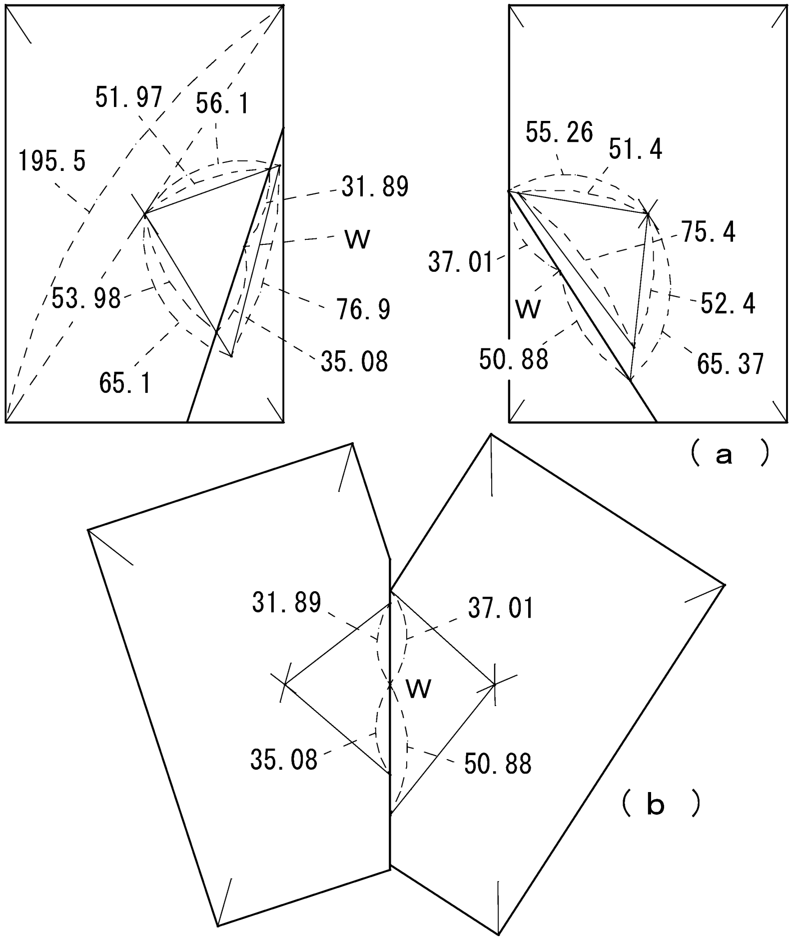


Figure 4: Drawing process and connection

2.1.2. Method of drawing

Another method of drawing is shown in Fig. 6: W_1 and W_2 are the pedal points of O_1 and O_2 on the lines 1-2 and 3-4, respectively. W_1 and W_2 derived in this way, as in 2.1.1, are also found by moving the pictures until they match.

3. The center and the focal length

In the case of a panorama built by connecting two photographs, the position of the center of each photoprint and the focal length of the lens of the camera used are important data.

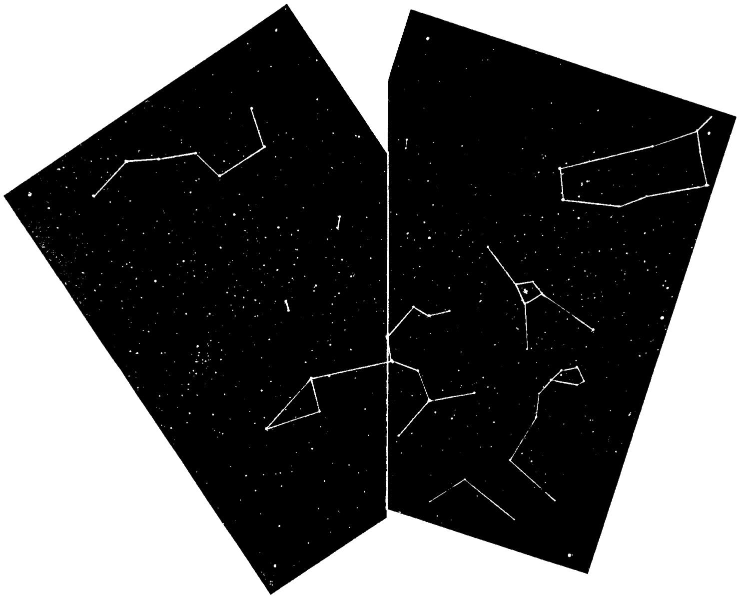


Figure 5: Sky photograph in spring

Figure 6: Joining points from drawings

After the film is developed and the photoprint is enlarged into a full frame, the center of the photoprint can be obtained by intersecting the two diagonal lines. However, when the film was developed in a regular photo shop, the edges are trimmed and one cannot estimate the center precisely. Moreover, if the focal length of the used lens is recorded, there is no problem, but if it is lost, the focal length is uncertain. For sky photographs in particular, one wants to connect photographs that have been taken in the past. In this case, as the needed parts have been trimmed and the percent of expansion is unknown, the center of the photoprint is also unknown. Accordingly, it becomes impossible to connect the photographs.

However, in this research, the author takes the case of a sky photograph. Then the center and the focal length can be calculated as the positions of the fixed stars are available using a star-atlas [3] and a star-catalogue [4]. Using this method, even very old photographs can be joined with unknown percentage of expansion or position of the center.

3.1. Principle of measurement

3.1.1. Basic principle

Fig. 7 shows the relationship between the photoprint and the lens of a sky photograph. C is the position of the lens. X and Y are the straight lines that go through the center of the

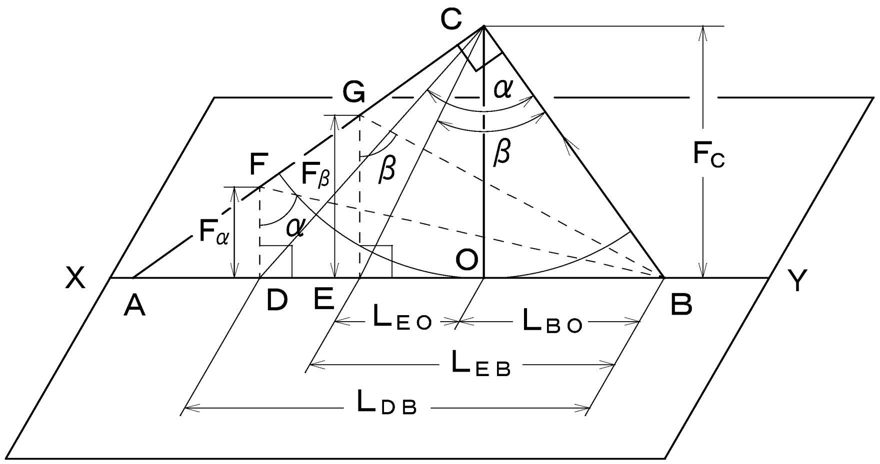


Figure 7: Center of photoprint and focal length

photoprint O. On this straight line XY the points A, B, D, and E are taken. Let

$$\angle ACB = \angle R = 90^\circ, \quad \angle BCD = \alpha, \quad \angle BCE = \beta.$$

Next, draw a line through D perpendicular to AB and intersect AC at point F. In the same way, draw a line through E and get G. Here, as $\angle FDB = \angle FCB = \angle GEB = \angle GCB = \angle R$, from the inscribed angle theorem we obtain $\angle BFD = \angle BCD = \alpha$ and $\angle BGE = \angle BCE = \beta$. This gives $CO = F_c$, $FD = F_\alpha$ and $GE = F_\beta$, and

$$F_\alpha = \frac{L_{DB}}{\tan\alpha}, \quad F_\beta = \frac{L_{EB}}{\tan\beta}. \tag{2}$$

3.1.2. How to avoid the influence of distortion

In the case of distortion, as for the position of O and the appearance of errors in the values, the method below can be used to minimize errors:

In Fig. 8, from A, B, C, D, and E, with OC as a symmetrical axis, the points A', B', C', D', and E' are derived. B and D are used together. In other words, $B' = D$, $D' = B$. The position of E' is added. Let the position of A' be $\angle B'C'A' = \angle R$. In cases like this, if there is no distortion, C and C' meet, but if there is distortion, as shown in Fig. 9, the intersecting point of the straight lines FG and $F'G'$ is taken as C'' and becomes the center of the photoprint O'. The focal length of the lens F_c is $C''O''$ and about the same value is derived.

3.2. Measurement

3.2.1. When the middle horizontal line is understood

At a regular camera store the film is developed in such a way that the distance the photoprint is shifted up or down is small, but right or left great. Accordingly, here a method for deriving the center of the screen after the photograph has been moved right and left and for deriving the focal length is discussed.

(1) Rectangles such as those shown in Fig. 7 are halved (into the upper and lower part) by a straight line.

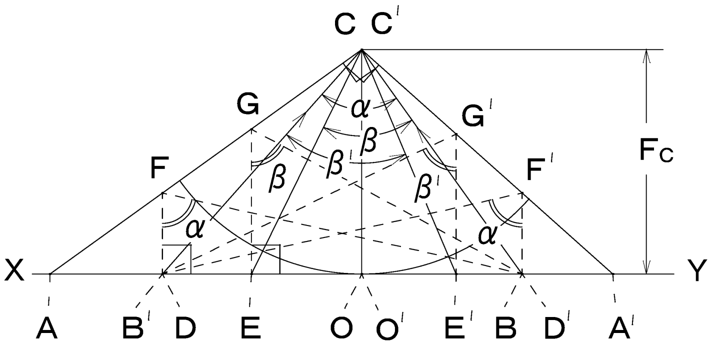


Figure 8: Method of avoiding distortion

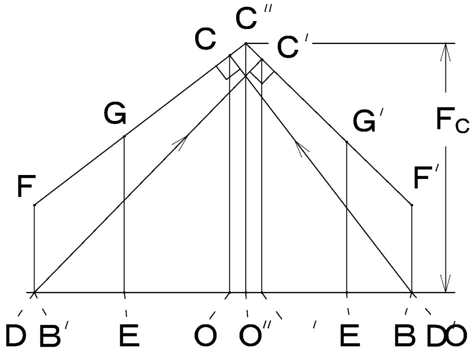


Figure 9: Influence of distortion

(2) On the straight line, on the left two points D, E and on the right one point B is specified. A star-atlas and a star-catalog are used to derive the right ascension (RA) and declination (dec.). If on the straight line no star is available from the atlas, a hypothetical star can be used.

(3) For these three aligned points on the celestial sphere the angle distances a_{DB}, a_{EB} (deg.) and the lengths L_{DB}, L_{EB} (mm.) can be computed according to (3)–(8).

$$
\begin{aligned}
b_1 &= 90 - (\text{dec. of } D), \quad c_1 = 90 - (\text{dec. of } B) && (3)\\
A_1 &= (\text{RA of } D) - (\text{RA of } B) && (4)\\
a_{DB} &= \cos^{-1}(\cos b_1 \cdot \cos c_1 + \sin b_1 \cdot \sin c_1 \cdot \cos A_1) && (5)\\
b_2 &= 90 - (\text{dec. of } E), \quad c_2 = 90 - (\text{dec. of } B) && (6)\\
A_2 &= (\text{RA of } E) - (\text{RA of } B) && (7)\\
a_{EB} &= \cos^{-1}(\cos b_2 \cdot \cos c_2 + \sin b_2 \cdot \sin c_2 \cdot \cos A_2) && (8)
\end{aligned}
$$

(4) F_α, F_β are derived in equation

$$
F_\alpha = \frac{L_{DB}}{\tan\alpha} = \frac{L_{DB}}{\tan a_{DB}}, \quad F_\beta = \frac{L_{EB}}{\tan\beta} = \frac{L_{EB}}{\tan a_{EB}}. \tag{9}
$$

(5) Through point D a line F_α perpendicular to AB is drawn and F is derived. Similarly, the line F_β through E gives G. The straight line from F to G is extended.

(6) Point C is the pedal point of B with respect to the line FG. C is the position of the lens. Moreover, the line through C being perpendicular to AB gives O, and this is the

center of the photoprint. The focal length of the lens F_c is CO. However, if the film is enlarged K times, the focal length is $F = F_c/K$ (mm). If the distances L_{BO} and L_{EO} are measured, the center of the photoprint can be fixed.

3.2.2. An actual example of a sky photograph

This method was applied to a sky photograph:

	RA	*dec.*
point D	149.542°	+ 7.3199°
point E	140.523°	+ 15.9760°
point B	110.511°	+ 36.7606°

The results are: The angle distance $DB = 46.091°$, the distance $DB = 158.0\,\text{mm}$, $F_\alpha = 152.09\,\text{mm}$, the angle distance $EB = 33.731°$, the distance $EB = 115.7\,\text{mm}$, $F_\beta = 173.28\,\text{mm}$. When the above results were drawn, the following values were derived.

$$L_{BO} = 92.6\,\text{mm}, \quad L_{EO} = 23.1\,\text{mm}, \quad F_c = 184.9\,\text{mm}.$$

3.2.3. When the position of the center is unknown

In previously published materials containing sky photographs, only what was needed was trimmed. In many cases, the position of the center was not clear. A general method for dealing with cases like this is discussed below. As in Fig. 10, the points 1, 2, 3 and 4 are connected by a straight line. We set $1 = D$, $2 = E$, $4 = B$ and $4 = D'$, $3 = E'$, $1 = B'$.

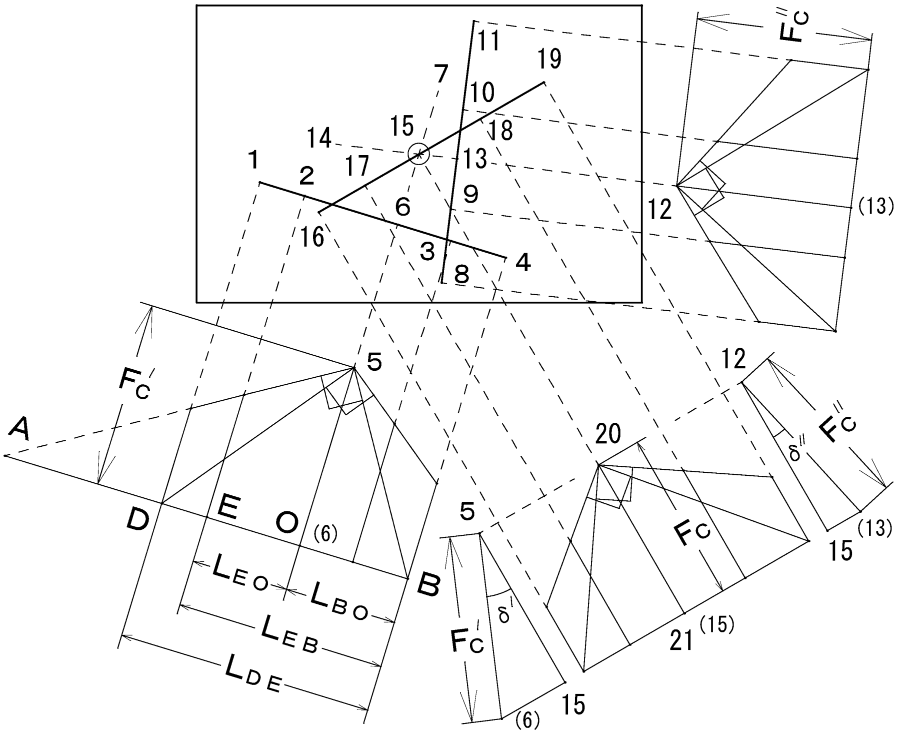


Figure 10: General method: deriving center and focal length

Using these results, the method of 3.2.1 produced points 5, 6 and (6). The straight line (6)-5-6-7 is drawn. Next, in the same way, 8, 9, 10 and 11 are set on a line which is roughly perpendicular to 1, 2, 3, and 4. Points 12, 13 and (13) are derived and the straight line (13)-12-13-14 is drawn. The intersecting point of lines 6-7 and 13-14 is point 15, the center of the photoprint. Next, the focal length of the lens, F_c, is derived by considering the straight line 16, 17, 18, 19 that passes through the center of the photoprint, and points 20 and 21 are also derived. $F_c =$ 20-21. Moreover, $F_c' =$ 5-6, $F_c'' =$ 12-13, accordingly,

$$F_c = F_c' \cdot \cos\delta', \quad F_c = F_c' \cdot \cos\delta''. \tag{10}$$

Moreover, from the center of the photoprint, the straight lines 1, 2, 3, and 4 and 8, 9, 10 and 11, the minimum distances 15-6 and 15-13 are given by

$$\text{15-6} = F_c' \cdot \sin\delta', \quad \text{15-13} = F_c'' \cdot \sin\delta''. \tag{11}$$

A part of Fig. 10 is taken and shown in three-dimensional form in Fig. 11.

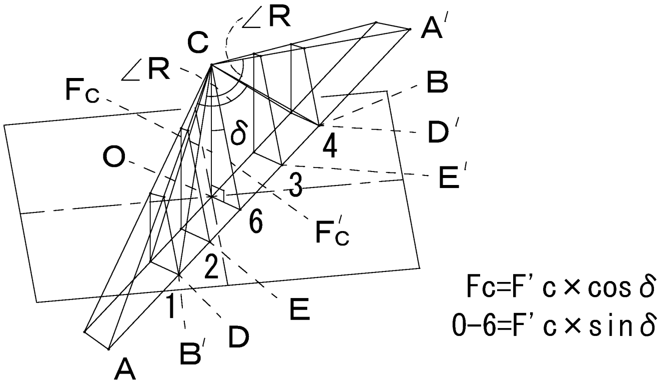


Figure 11: Relationship between on-center, off-center

4. Consideration of results

In the process of connecting the sky photographs, the center of the photoprint and the focal length of the lens could be estimated by calculation. Thus the above results were derived. It is the expectation of the author that in the future this method can be used in the field of star photographs. However, this time, from the sky photograph it should be possible to calculate the distortion curve of the lens, although it has not yet been calculated. In this aspect, this work is incomplete. It is the intention of the author to finish this aspect by the next International Conference of Geometry and Graphics.

5. Conclusion

In order to develop a method for connecting two photographs into one panorama photograph by making their photoprint centers meet, a computer program was presented, the problem was solved, and an example was given. A method of estimating the placement of the center of a photograph of celestial bodies and the focal length of the lens was developed and the theory and an example were shown. It was suggested that from a sky photograph, the distortion curve of the lens can be calculated.

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