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Constructional Graphics Application in Engineering Computer Graphics

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Abstract. In studying the drawing there is even today a tendency to concentrate upon what is marked on a piece of paper, and to forget that much of drawing that was marked in the past on the actual stone or wood. As the education and books on drawing were increasingly developed and used by academia, the drawing techniques became theoretically supported by developed geometrical principles extracted from basic empirical constructions. Geometrical concepts that provide developments of such empirical constructional methods, applications of which were utilised in the pre-Descriptive Geometry era, are denoted as constructional geometry. As the construction of 3D parametric solid models becomes recognised as the skill modern engineers need to posses, the ability to spatially construct and manipulate virtual geometrical elements will unavoidably become an elementary part in the engineering educational system. With the application of a vector space in computer graphics, the introduction of graphical techniques that conduct 3D problem solving by spatial construction instead of the planar projection becomes essential for engineering spatial graphics. Discussion of this need for introducing graphical concepts that deal with spatial relationships in a computer graphics vector space and subsequent application of such a constructional method in modern engineering computer graphics are presented in this paper.

Key Words: Theoretical graphics, computer aided geometric design, engineering computer graphics

MSC 2000: 51N05

1. Introduction

Professor Steve M. Slaby in his 1996 discussion of the statement that more of us should be involved in doing theoretical work in geometry and graphics, stressed out that: *"It is crucial that research is continued and enlarged in the field of theoretical graphics and that our students, whether they are undergraduate or graduate students be exposed to established and*

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developing theories. If this is not done then shifts or changes in our paradigm will not be solidly based. The changes will tend to be faddish in nature" (see [5]).

In his 1998 discussion of the fact that the computers are introduced into the curriculum at all university levels, Slaby further exploits that: *"It is very important to deal with education in a more inclusive, integrated, and 'global' manner, in all subject matter, to enable students to acquire a deeper understanding and knowledge of geometry, graphics, science, and technology"* ([6]).

Going further Slaby argues that, possibly because of the lack of funding for research in geometry and graphics, the academics are ignoring their responsibility of pursuing theoretical research in the area. A consequence of, as Slaby declares, a too much of devotion of educators to learn and subsequently teach the use of CAD tools developed by corporations whose basic interest is not in education but in maximising their profits [5], is the affect: *"The explosion in information technology (will) fundamentally affect universities — and (probably) not for the better"* (brackets added) (Noam [4], 1999). In the same context and making once more an emphasis on the importance of further research in theoretical graphics — the issue addressed in this paper, a final citation from [6], where, by concluding his query on defining the connecting links between the geometry and graphics with the technology of the past and the present, Slaby questions: *"In what detailed forms these links will take place (in the future)?"*. Slaby concludes that whatever forms the geometry and graphics will take, they both will continue to underpin science, technology and engineering design.

The constructional graphics — discussed in the next section is such a form of graphics, which needs to be developed as an application of constructional geometry in computer graphics setup. Such computer supported constructional graphics when used in the engineering graphics instruction provides good visualisation ability improvements — the abilities crucial to future technological developments. Importantly, it also supports an advanced learning and understanding of the computer graphics modelling for engineering design.

2. Constructional Graphics

The drawing principles dealt with in the engineering communication are concerned with both the graphics representation of objects and with deriving information which is inherent in the objects represented but is not immediately accessible. A cylinder represented by a circle for its plan view and a rectangle for its front view, is one simple example of the need for deriving information from a drawing. If, for example, one needs to make such a cylinder from a piece of sheet metal or paper, one has to make still another drawing — a development of the surface. A shape is then needed which is implied in the representational drawing but is not explicitly given. Both of these uses, the graphics representation of objects and deriving the information from a drawing, are of invaluable importance in the engineering and science communication.

So long as one deals with plane geometrical shapes, there is no difficulty in thinking about them, representing them or making them. Both the drawing and the object in that case are geometrically same except that the object has thickness whilst the drawing in theory has none. The difficulties arise when one needs to consider spatial graphical problems or interpret 2D representations of 3D objects.

Historically looking, to develop methods for solving spatial problems meant throwing over graphical representation techniques and using constructional ones. Constructions are, of course, solid things and not drawings, but from these constructional solutions, all today's graphical techniques emerged. To brother ideas for further engineering graphics analysis, the terms of graphical and constructional geometry needs to be explained. Geometrically speaking it can be said that the drawing of a circle and making a disc or wheel are the same in character. To draw the circle one uses a pencil in pair of compasses. To make a wheel one substitutes a cutting edge for the pencil. To make distinction between the two a drawing is called graphical geometry and its physical counterpart constructional geometry (Booker [1]).

The principles of Descriptive Geometry form the foundation for traditional engineering graphics methods — the methods founded exclusively on graphical geometry. To solve a spatial graphical problem the Descriptive Geometry defines relationships between adjacent orthographic planar representations and the related graphical methods utilise these definitions to describe spatial relationships of concerned 3D objects.

To familiarise the reader with the constructional graphics the types of spatial construction, which have been used before the introduction of Descriptive Geometry in 18th century, will be considered. The engineers at that time (artists as they were called) practiced a mixture of empirical constructions and graphical techniques. Gerard Desargues, (1593-1662), whose Projective Geometry theory is described in numerous books on geometry, tackled many of these empirical problems from which he produced his universal graphical techniques. These mixed methods, difficult to understand and not having graphical applications in engineering after the Descriptive Geometry is introduced, are today known under the names of *The Art of Sun Dialling, Shipbuilding, Carpentry and Stone-Cutting.*

One example from this before the era-of-exclusively-graphical-geometry-methods presented in (Booker [1]) on the use of mixt constructional and graphical methods for solving three-dimensional problem used by stonecutters is illustrated in the diagram of Fig. 1. The figure shows a typical stone shape used in masonry with bevels and squares placed on the various angles.

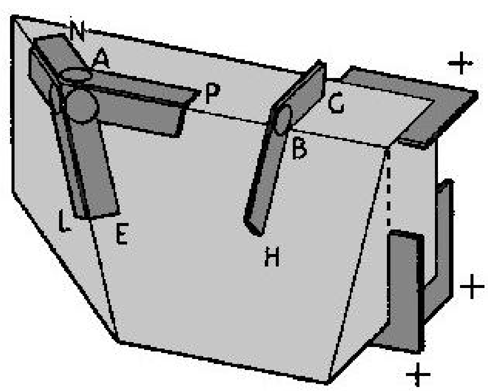

Figure 1: Typical stone shape used in masonry

The stone is basically a rectangular prism, except that it has the front face inclined to the vertical and has the left vertical face cut across obliquely. Angle CBH is a primary one as is angle NAP, and both may be marked directly upon the stone. Angle LAN, however, is a compound angle formed by the intersection of two inclined planes, as is angle EAP. The stone can be cut without pre-determining these angles. But supposing that another stone is to be cut to about on to the left face, one either has to wait until the first stone is actually cut so that angle LAN can be measured, or one can work out this angle so that both stones can be cut simultaneously and then fitted together.

In principle, the method to solve this problem described by Desargues, depends solely upon isolating a number of triangles, some of them given at the start and the others derived from the drawing. Instead of drawing these triangles separately, Desargues showed them by revolving each into the ground plane, that is, into the plane of drawing.

Fig. 2 shows the solution described by Desargues for which the procedure is explained step by step but no rational explanation is given. For many of those mixed empirical and graphical methods described in the old books, the constructions of which are predominantly extremely complicated, no rational explanation has been recorded.

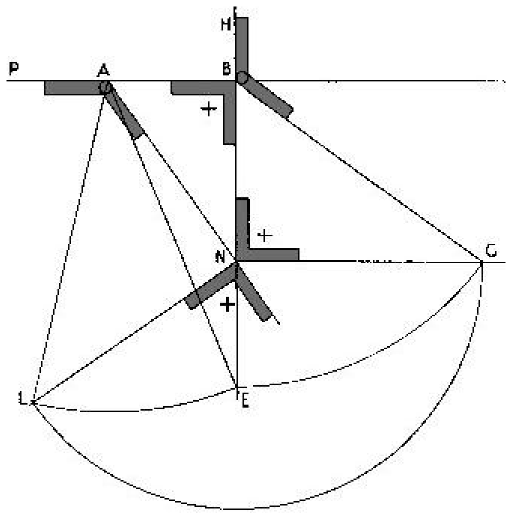

Figure 2: Graphical construction of compound angles

It seems likely that Desargues arrived at the solution to this spatial problem in much the same way as most of the sun dialling problems at his time were tackled. Fig. 3 shows the three-dimensional reasoning behind the solution to this problem.

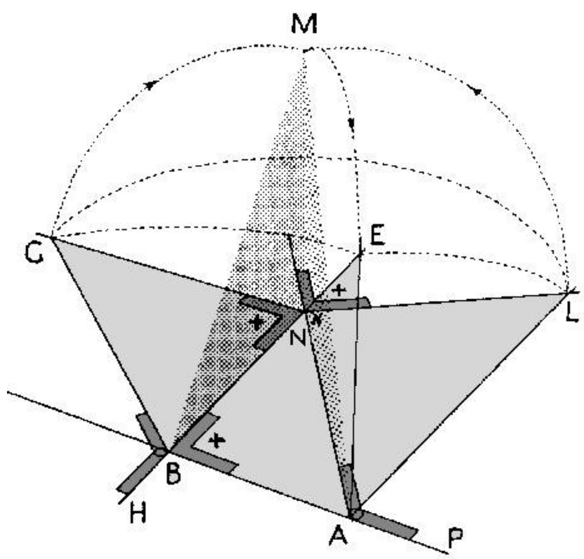

Figure 3: Revolving triangles flat for graphical construction

The triangular plane *MBN* in Fig. 3 represents the slope of the front face of the stone and revolved about *BN*, this becomes the triangle *BNC*. The plane *MAN* likewise is the true shape of the obliquely cut vertical face which, revolved about AN, becomes the triangle ANL. Since NC and NL are, in reality, MN they must be the same length, hence the arc about centre N joining C to L. In a similar way the triangle ABM can be laid down to become triangle ABE; BE and BC are then really BM and so are of the same length and so on.

The shortcoming of these mixed methods is that they led to problem solving by rote. It seems fairly obvious that DESARGUES had himself worked out a number of generalised drawing doctrines and by using these, he could produce solutions to most of constructional problems of his time. However, only the final solutions got recorded not the generalisations which lay behind them.

In studying the graphics and drawing there is still a tendency to concentrate upon what is marked on the paper, and to forget detail drawing that was in the past marked on the actual wood or stone as described in the previous example. As academia took over the development of graphics and drawing the marking on the paper became theoretically supported by developed geometrical principles extracted from the actual empirical constructions. With the introduction of Descriptive Geometry the constructional methods were not utilised in engineering and education, and consequently were not fully developed or geometrically supported.

By the introduction of vector space in computer graphics the design of "virtual stones" is coming back to the working benches of engineers and academics. Techniques that introduce spatial constructions and not exclusively planar projections are therefore becoming relevant for engineering spatial graphics design. As the construction of 3D parametric solid models is the skill a modern engineer needs the spatial construction and manipulation of virtual geometrical elements becomes an important part of engineering practice and therefore engineering education.

Because the projective mapping introduced in the computer graphics automatically solves the planar graphics representations (orthogonal and perspective), the principles of a modern engineering computer graphics do not need to repeat planar projections. The geometrical notions forming geometrical base for such new mixed constructional and graphical methods needs to be developed in a way that a direct application of projection onto the spatial geometrical figures can be produced. Starting from the linear algebra definition of a point space, the elemental direction and point position should be defined first. The definitions of a body, surface and a curve as basic geometrical elements should be then given as a number of positionaly predefined points. The general and orthographic projections are then defined using point relationships between those basic geometrical elements. All other geometrical elements (plane, sphere, line, circle, etc.) may be defined then by using basic point relationships and predefined geometrical elements.

Such a geometry uses a plane to define the projecting direction, but this plane, called projecting plane, is not an image (projection) plane as it is defined in Descriptive Geometry. The projection is still described by a light source casting shadows of an object, but this time the shadow is not projected on an imaginary plane but directly onto the surface of another object.

Such geometry corresponds to the constructional techniques used in pre-Descriptive Geometry era and should be denoted as constructional geometry. This modern constructional geometry instead of dealing with the real physical objects in a workshop deals with the virtual geometric elements defined in the computer graphics. Such defined modern constructional geometry would form a foundation for the constructional graphics to be developed for applied engineering computer graphics. The constructional graphics — a group of mixed constructional and graphical methods, defines spatial solution to geometrical problems by using spatial projection of 3D geometrical objects without manipulating orthogonal representations of these objects.

One such mixed constructional graphics method has been developed by author for which the rational explanation and a generalised drawing doctrine is presented in [3]. The method, called *Projecting Plane method*, is an application of mixed graphical and constructional geometry. To illustrate this spatial constructional approach the applications of *Projecting Plane method* on solving traditional and practical spatial graphics problems are described below.

3. Applications

First application is on solving a traditional graphics problem of finding the intersection point between a line and a sphere. The task is to find the two intersection points between the given line AB and sphere S, sphere being defined by the centre point C and radius r, as shown in Fig. 4. The intersection points between an intersection curve, in this case the circle f_1 as shown in Fig. 4, and the actual line AB defines the intersection points between the line AB and the sphere S.

By defining a projecting plane P_p coplanar with the line AB and cutting the sphere and then determining the intersection circle between that projecting plane P_p and the sphere S, the projection curve (the circle f_1) of the line AB onto the sphere S is defined. The solution is then given by determining two intersection points of the line AB and projection curve (circle f_1), the points E and D in Fig. 4.

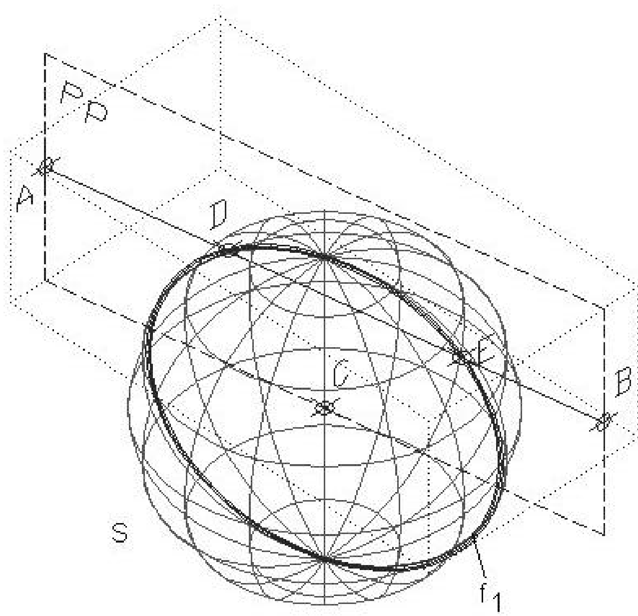

Figure 4: Using three existing points for defining projecting plane

The points A, B and the centre point of the sphere, point C, as shown in Fig. 4, are defining the three required points of the projecting plane P_p. For this particular position of projecting plane the radius of the intersection circle f_1 equals to the radius of the given sphere S, and the centre of the intersection circle f_1 coincides with the centre of the given sphere S.

However, another intersection circle, the circle f_2 for example in Fig. 5, defined by another projecting plane P_{p1} can be used. In general this or any other projecting plane to which the given line is coplanar would determine the two intersection points between the given line and the given sphere.

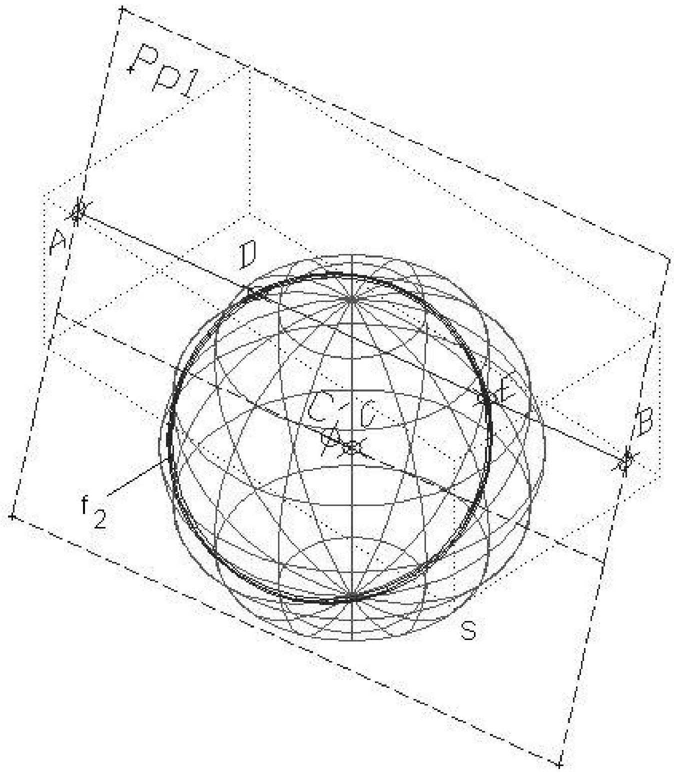

Figure 5: Using two existing and a free point for defining projecting plane

The described procedure for finding the intersection between a line and a sphere is also applicable for a more general problem of finding the intersection of a free positioned curve and the sphere. This can be illustrated by the task of finding the two intersection points between a circle and a sphere. The sphere is defined by the position of the centre point C and the radius r. The circle is defined by the position of the centre point C, radius R, and the position of a coplanar plane-*Ppcircle*, shown in Fig. 6, defined by three points.

Applying *Projecting Plane method* first the intersection point between an intersection curve, in this case circle *ISECTIONcircle* shown in Fig. 8, of a projecting plane and the sphere needs to be defined. The intersection point between the given *CIRCLE* and this new circle *ISECTIONcircle* is also defining the intersection between the given *CIRCLE* and the given sphere, points *ISECTION1* and *ISECTION2* in Fig. 8.

To define the radius of the curve of intersection *ISECTIONcircle* two points are needed. First point is the centre point and is defined as the projection of the centre point of the sphere onto the projecting plane *PPcircle*. The second required point is the point of intersection between an arbitrary line *LppCIRCLE* and the given sphere.

To define this second point, an arbitrary line *LppCIRCLE* is constructed coplanar with the projecting plane *PPcircle* and cutting the given sphere, see Fig. 7. This line intersects the sphere in a point that defines the radius of the intersecting circle *ISECTIONcircle* between the projecting plane *PPcircle* and the given sphere.

To determine the intersecting point between the line *LppCIRCLE* and the given sphere new projecting plane *PPline* needs to be defined (see Fig. 7). The projecting plane *PPline*

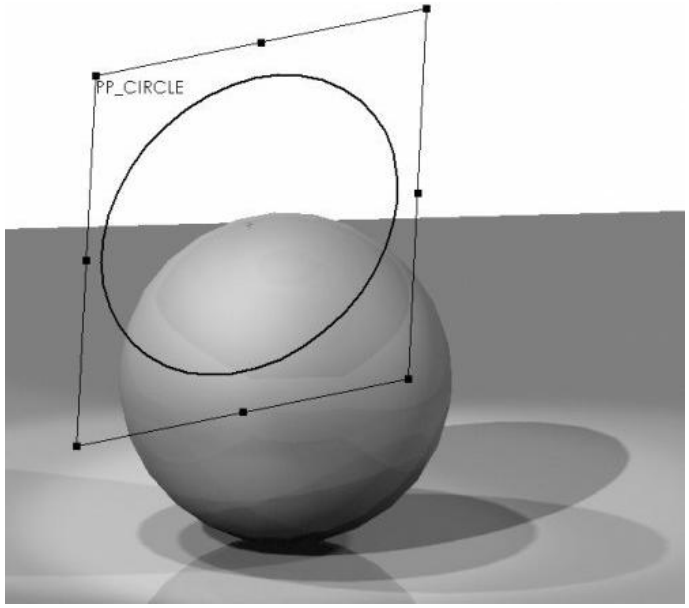

Figure 6: Circle–sphere intersection problem setup

is defined by the two endpoints of the line *LppCIRCLE* and the centre point of the sphere. The intersecting curve, in this case circle *CIRCLEline* in Fig. 7, between the projecting plane *PPline* and the given sphere, intersects the line *LppCIRCLE* in the point *ISECTIONline*.

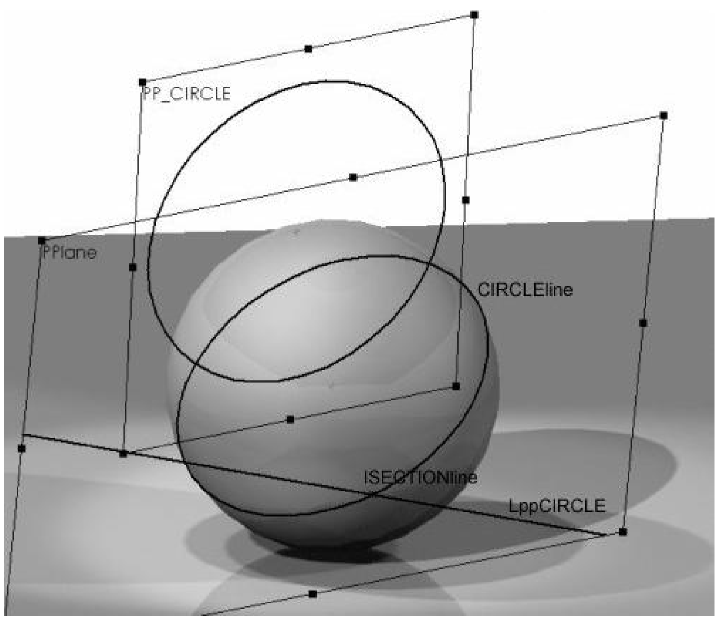

Figure 7: Line–sphere intersection

Now, the intersecting circle *ISECTIONcircle* (see Fig. 8) between the projecting plane *PPcircle* and the sphere can be constructed coplanar with the plane *PPcircle*, having the centre point coincident to the orthographic projection of the sphere centre onto the plane *PPcircle*, and the radius defined by the point *ISECTIONline*. The intersecting circle *ISECTIONcircle* and the given circle are intersecting each other in two points *ISECTION1* and *ISECTION2* were the given circle intersects the given sphere.

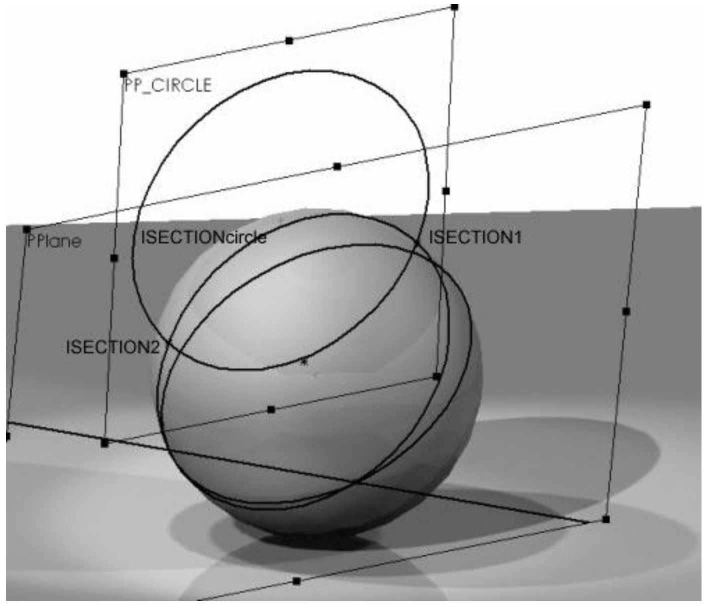

Figure 8: Final diagram for the circle-sphere intersection problem

Second application is a practical example, which by only applying the existing CAD tools cannot be solved in a 3D vector space of computer graphics. The solution to the problem is possible either by applying Descriptive Geometry principles using traditional planar representations or by the *Projecting Plane method*. Note also that, because of the need to construct planar representations, more time is needed for solving the problem by traditional Descriptive Geometry.

The solid models of a magnification exchanger and four lenses of a microscope assembly are given as shown in Fig. 9. The magnification holder is a dome shape with the holes created at certain angles for holding the lenses.

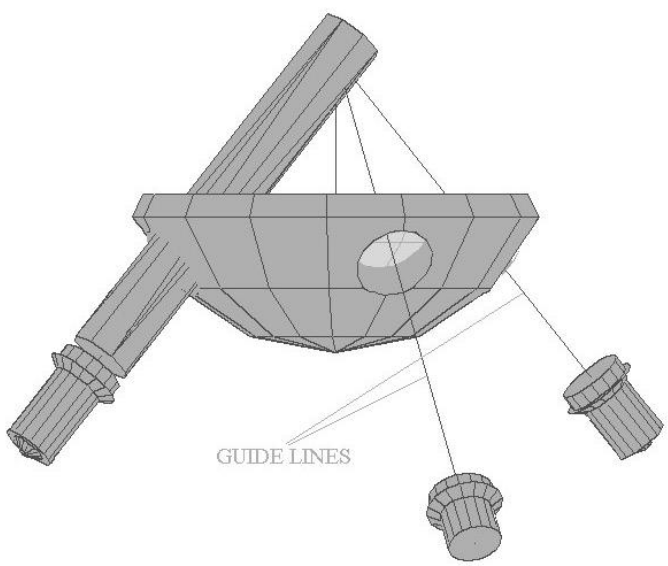

Figure 9: Initial position of holes and lenses

The holes were created by first constructing the lines perpendicular to the dome surface at the lengths that give the right incident angles to the lens position, labelled as the guide lines in Fig. 9. Four free length cylinders (only one is shown in Fig. 9) used for cutting the holes in the dome were then created along these constructed guidelines. The lenses were created at the base of those four cylinders. The task is to assemble the magnification exchanger by positioning each of the four lenses such that the centre of the base circle of each lens coincides with the centre point of the hole in the magnification holder at the level of the external dome surface.

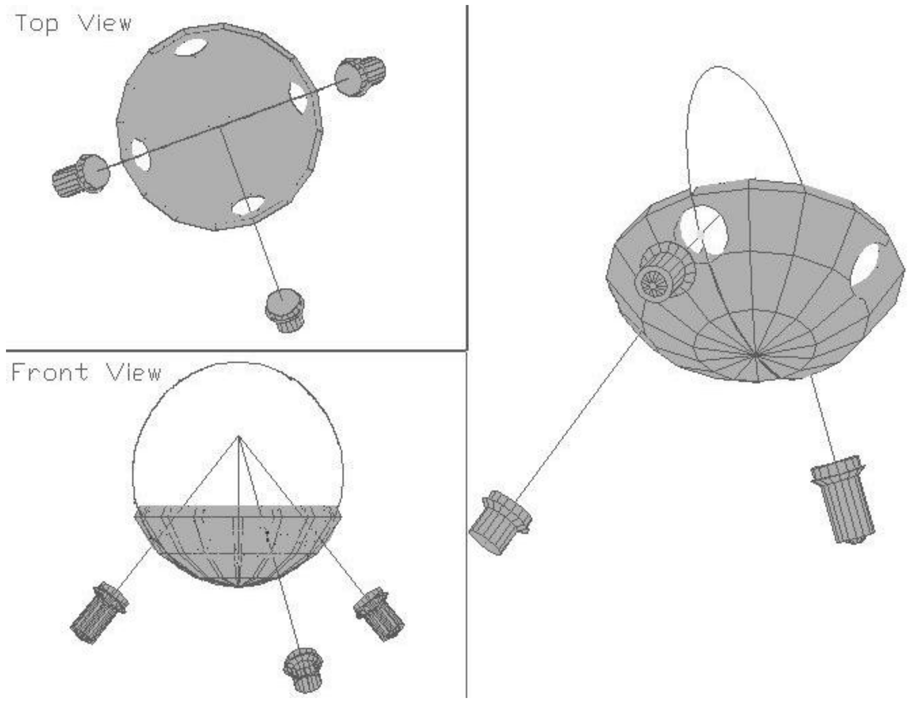

Figure 10: Solution for magnification exchanger assembly

The solution to this problem is to create a projecting plane defined by two end points of one of the guidelines and centre of the dome, shown in Fig. 10. Constructing a circle coplanar with the projecting plane and the radius and centre point identical as the radius and centre point of the dome defines the intersection curve between the projecting plane and the surface of the dome. By finding the intersection point between constructed circle and given guideline the reference point for one lens position is defined. Repeating the same technique, the reference points for other lenses can be constructed.

The task for third application is to find the two intersection points between a circle and a cylinder. The circle is defined by position of the centre point C, radius R, and the position of the coplanar plane- *PPcircle* shown in Fig. 11, defined by three points. The cylinder is defined by the given position of the centre point, coplanar plane to the base circle and its radius and height.

To solve this problem by using *Projecting Plane method* the ellipse of intersection between the plane *PPcircle* shown in Fig. 15, coplanar with the given circle and the given cylinder is required. The intersection points between the given circle and found ellipse defines the two required intersection points.

To construct the ellipse one needs to define three parameters: centre, major and minor radius of ellipse. The minor radius of an ellipse is always same as the diameter of the cylinder

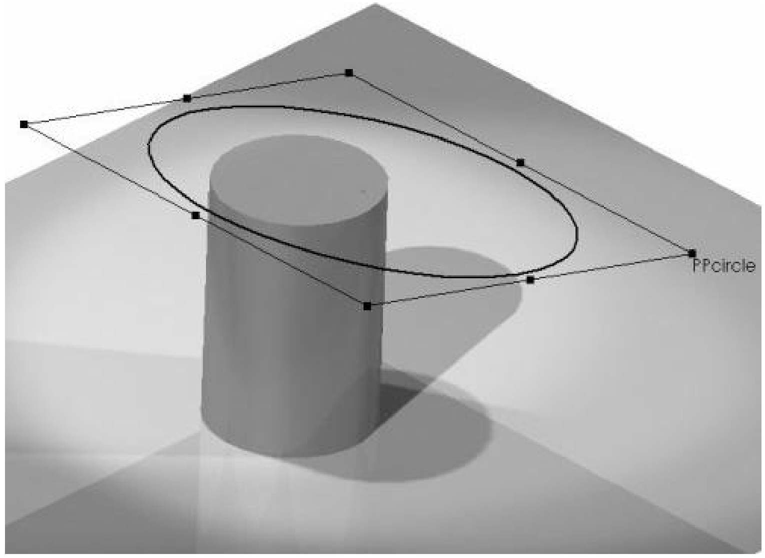

Figure 11: Circle–cylinder intersection problem setup

to which it belongs, which reduces the whole problem on finding only two parameters: centre point and major radius of ellipse.

To find the centre of ellipse, the intersection point between the centreline of the cylinder and the plane *PPcircle* needs to be defined. By defining a line-*LINEcircle*, shown in Fig. 12, coplanar with the plane-*PPcircle* and constraint to be coincident to the orthographic projection of the centreline of cylinder onto the plane, the centre of ellipse can be defined as the intersection of this line and the centreline of cylinder.

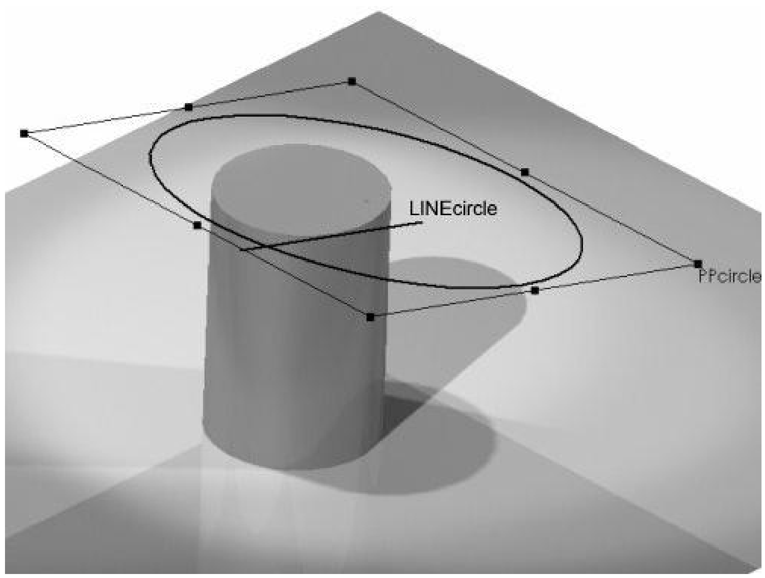

Figure 12: Line that defines centre and major radius of ellipse

The major radius of ellipse is defined as the distance between the centre of ellipse and the intersection point of the line-*LINEcircle* and the given cylinder, point *ISECTIONlincyl* shown in Fig. 14.

To define this *ISECTIONlincyl* point a new line which is the orthographic projection of the line-*LINEcircle* onto the base plane is constructed. Using this new line-*LINEprojection* shown in Fig. 13, and the line-*LINEcircle* a projecting plane-*PPline*, perpendicular to the base circle of the cylinder and containing both lines is defined.

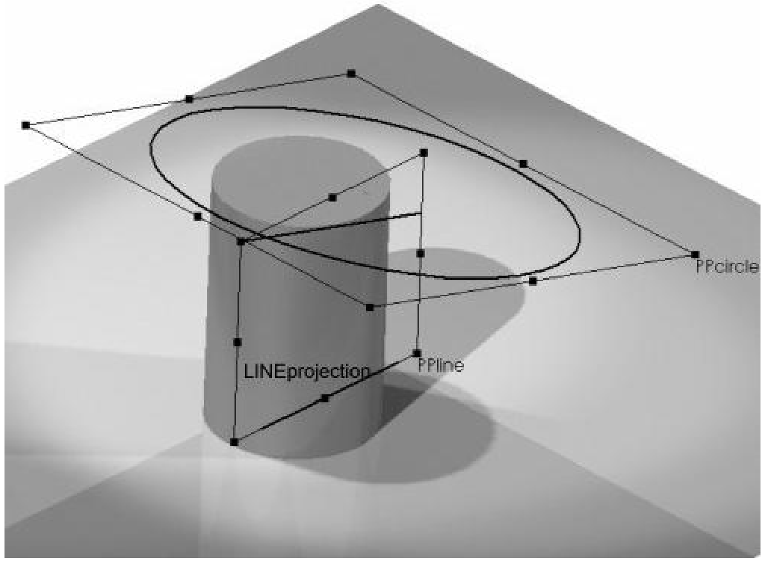

Figure 13: Projection of the line onto the base of cylinder

Then the intersection curve between this projecting plane-*PPline* and the cylinder is defined, line *VERTICALprojection* in Fig. 14. The intersection point between the first constructed line-*LINEcircle* and the cylinder is found as the intersection between the line-*LINEcircle* and the intersecting curve-line *VERTICALprojection*. The point *ISECTIONlinecyl* in Fig. 14 is defined as coincident to the intersection point of line-*LINEcircle* and line-*VERTICALprojection*.

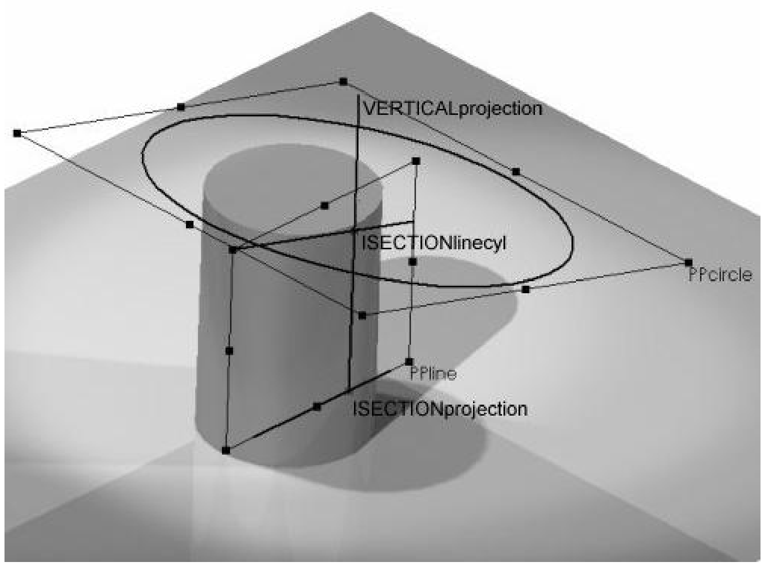

Figure 14: Projection of the point of intersection back onto the line

Now, the ellipse can be constructed coplanar to the plane-*PPcircle* and using point-*ISECTIONlinecyl* for the major radius. Fig. 15 shows the final construction for the circle-cylinder intersection problem.

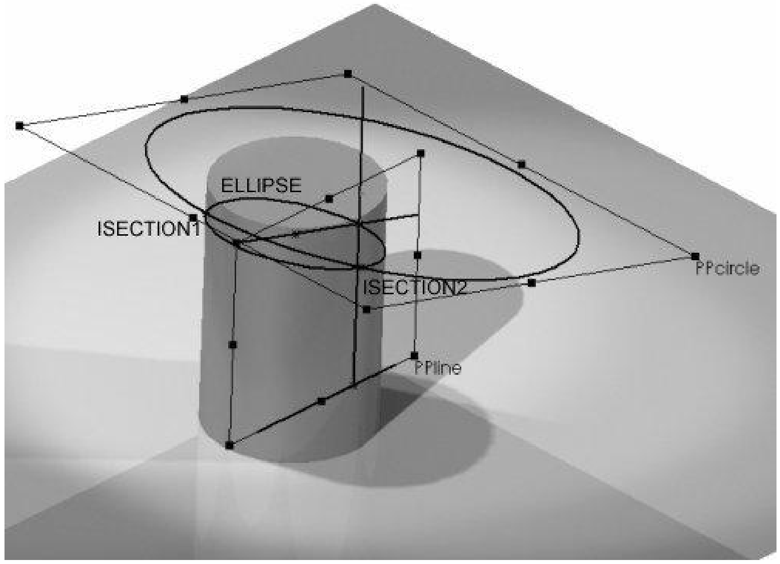

Figure 15: Final diagram for the circle–cylinder intersection problem

4. Concluding Comments

It is difficult and possibly unwise to speculate too much about future trends in graphics and drawing, but recording ideas in some form or other will presumably always be with us. There is, indeed, considerable scope for improving general spatial cognition as a mean of graphical thinking in the aid to the imagination. Design has been said to be the process of taking a vague, imagined solution to an engineering problem and giving it more certainty through making drawings, calculations and tests. As the drawings constructed in computer graphics can move and can behave ever more closely like the objects themselves depicted, the drawing, as an ultimate design tool becomes more and more reality.

Mathematicians had to work out a new discipline, computational geometry, which is an extension of the analytical geometry of Descartes, Monge, and many others, but more suitable for computer manipulation. A shape defined in the projective space of a computer in digitised coordinate form can be convert via transformations to give parallel projections — orthogonal or axonometric views, or central projections — perspective views. These are all carried out in terms of the real spatial geometry, referred to as the primary geometry, and not in terms of vanishing points, or the general secondary geometry.

The application of spatial manipulation of primary geometry, called here constructional graphics, in engineering computer graphics is emphasised by the need to exploit spatial construction in engineering graphics problem solving and computer graphics modelling.

The described *Projecting Plane method* represents such a constructional technique. As the *Projecting Plane method* was generalised and problems developed it was gradually introduced in classrooms at RMIT. Fully developed method has been integrated in the CAD part of the graphics course starting from 1997 academic year. The effectiveness of graphics instruction, with and without the introduction to *Projecting Plane method*, on students' visualisation abilities was monitored throughout a six years long study showing encouraging results.

By using the method presented the problem of teaching computer-based engineering graphics to a class of students with different background and learning styles is significantly improved. The students using this method in a computer graphics based engineering graphics course are more focused on representation of geometric solutions than on the mechanics of geometric construction. The available evidence indicates that the use of the presented spatial graphical method significantly improves student's visual abilities (Gradinscak, Lewis [2]).

As engineers in the future will need to model and mark the details on their "virtual stones" for building their "virtual mansions", the important theoretical change in engineering computer graphics paradigm for today, as discussed in the paper, is the introduction of applied modern constructional geometry. The use of constructional graphics in engineering computer graphics education promotes a deeper understanding of geometry and graphics and eventually will produce an advanced knowledge of applied computer graphics in engineering design.

As the computer technology advances and become more accessible, from the engineering professional and educational stand point and in relation to the applied engineering computer graphics, further considerations, investigations, developments and analysis of theory, methods and effects of geometry and graphics will inevitably become essential.

The principles of constructional graphics applied in a vector space of a modern computer graphics makes the basis for a systematic problem solving in engineering computer graphics. Implementing those principles in the engineering graphics instruction endorses an effective engineering education program.

The Descriptive Geometry provided a convenient method for graphical representation based on projection planes. Because of having in the past paper as the only medium for engineering graphics representations the planar projection is accepted and used as the only pertinent solution to spatial problem solving. The Desargues method, being the 3D solution to spatial problems, was not pursued by geometers after discovery of Descriptive Geometry, and no significant invention, supported research or further use of non-planar graphical techniques has been made since.

The latest developments in computer graphics introduce new media based concepts of a virtual point space that can be applied to engineering graphics problem solving. By implementing projective geometry in computer graphics a need emerges for constructional techniques to be re-introduced and further developed. This solicits for further geometrical developments in support of new spatial constructional techniques.

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