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Studies of Geometry Integrated in Architectural Projects

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Abstract. Geometry plays the role of a basic science in engineering, especially in architecture. In the past years the importance of geometry was pushed back because of a wrong estimation of new technologies like CAD. Geometry researchers and teachers failed to convince the practical engineers of the importance of geometrical reasoning, even in the use of computerised methods. This paper will show the concept of an integrated teaching of geometry in architectural projects which is able to point out the importance of geometry in practising architecture as well as in the use of new technologies. Descriptive Geometry turns out not to be an antiquated science but a current one. In our integrated geometry concept we start with the architectural project that leads us to geometrical problems to be solved. The experiences with project-oriented studies for students of architecture in their major courses at our university in the past years are presented in this paper. In the examples like developing geometrical forms, projection methods for representations of architecture, photoreconstruction and photomontage, we reflect the way from the geometrical task, arising from the design project, to the geometrical solution.

Key Words: architecture, Descriptive Geometry, teaching methods, computer-based construction

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1. Role of Geometry in Architecture

There are two main application fields for geometry in architecture: developing architectural forms and representing architecture.
First, geometry offers a system of order to describe the design out of fundamental geometrical forms. Therefore geometry is a helpful science to develop architectural structures and forms. Second, geometry, especially Descriptive Geometry, plays the fundamental role in producing two-dimensional representations of three-dimensional objects by providing various projection

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methods. Geometry gives the foundation for communicating about architecture by means of drawings. For choosing an appropriate projection method like orthogonal parallel projection, oblique parallel projection, or central projection for creating representations of the architectural project, the purposes of the representations in the communication process about designing and constructing architecture have to be reflected. The formula of H.D. Lasswell [2] summarises the different components of a communication process which is also applicable for architecture in a mostly visual communication in the question: *"Who says what in which channel to whom with what effect?"* Answering this question leads, for example, to different application fields of axonometries and perspectives in architecture [5].

2. Integrated Teaching Method

An important element in studies for architecture is working on design projects. These projects give the students the opportunity to deal with various subjects in relation to each architectural project. The studies of the subjects are motivated by a practical viewpoint referred to the chosen project. There is a chance for geometry to show the relevance of geometrical reasoning and proceeding for architecture. At the 6$^{\text{th}}$ ICECGDG Conference in Tokyo in 1994, C. Leopold presented a paper with the title *"Geometrical Problems of Architectural Objects"* [4] in which an integrated teaching of geometry in architectural projects in major course studies was suggested. At that time the idea and some examples of interested students were given, but it was not realised in the curriculum of architecture at our faculty.

In 1995 we managed to introduce this concept and to fix it as an optional subject in the major course studies in the curriculum of architecture with four hours a week. The students have to select four optional subjects, after the basic courses in which they have learned the principles of geometry, especially Descriptive Geometry. The fundamentals of Descriptive Geometry are illustrated by applied examples as shown in [3]. Descriptive Geometry has to be comprehended as a basic science to understand geometrical space and form as well as the projection methods. We start with an integrated project in Descriptive Geometry already in the first semester, where the students have to draw an individual axonometry of their own designed projects in the subject *building construction*. In context with the design projects in major course studies we begin with the architectural project and then work out the included geometrical problems. Often the opposite way is used by studying geometry as a theoretical science and then looking for applied examples, but then there is no direct motivation for the studied subject. By turning this procedure in the opposite direction we reach a high motivation and gain the experience that geometry is closely connected with the practical work of an architect.

After analysing the geometrical problem students work on a possible solution. In their project presentation the students demonstrate their ways to the solution, not only the results. This concept of an integrated teaching of geometry in architectural projects points out the importance of geometry in practising architecture. Thereby the international trend of reducing lecture hours of geometry in the curricula could be stopped or even turned to the reverse by the integrated teaching concept.

3. Embedding Computer-based Working

Within the architectural projects students use the possibilities of various computerised methods for solving geometrical problems and presenting the solutions. The selection of the applied computer program is motivated by the respective architectural project and its requests. New technologies like CAD, rendering, image editing, or animation are today unrenounceable tools. But this estimation should not lead to a disapproval of geometrical knowledge. There is a need of geometrical reasoning and knowledge more than ever before. The content weight changes. A clear understanding of geometrical space is especially necessary by working with CAD: world and user-based coordinate systems, navigation in space. As in earlier times we did not teach how to use a pair of compasses at our universities, we should nowadays avoid simply the teaching of CAD-programs. We give our students a good foundation for computer-based working by imparting an understanding of the geometrical space and forms as well as by solving geometrical problems. Manual and computerised drawings complement one another. By integrating computerised working in geometrical problems of architectural projects we emphasise the necessity of geometrical reasoning. With the aid of computer-based working we get the possibility to go on to more complex and sophisticated geometrical questions.

4. Examples of Architectural Projects

4.1. From Model to Drawing

Architectural designs are often developed on the model. Therefore it becomes necessary to transfer the geometry from an intuitively developed model to an exact drawing. Especially complicated forms and arrangements of solids demand basically geometrical knowledge about assigned orthographic views. For the required spatial reasoning it does not matter for the first step if we work on the transformation process manually, or with the aid of a computer. The example by a student shows the design of a *Nibelungen museum* in the German city Worms, consisting of two cuboids, doubly inclined and twisted against one another on a basic plane (Fig. 1).

For transferring the model-based design in a geometrical drawing, first of all we have to derive the geometrical parameters of the position of cuboid I and II from the model. The solution is developed by applying geometrical basic tasks in assigned orthographic views at first to each cuboid separately.

1. Definition of plane ε by two lines $\mathrm{f} \cap \mathrm{g} = \mathrm{X}$.
2. Transfer of an angle α between two lines (for example between the edge BG and the line $\mathrm{f} \subset \varepsilon$).
3. Definition of line g through two points X and C of the cuboid side BCGH.
4. Definition of the position of point X on a line by means of a distance a.

After these reflections it is possible to apply the distance a to the point X on the edge BG as well as angle α between cuboid edge BG and line $\mathrm{f} \subset \varepsilon$. After introducing a new basic line $(23) \perp \mathrm{f}$, the second line g of ground plane ε can be drawn with the aid of point C. A new view parallel to line g shows cuboid I in a top view. The cutting of cuboid I with ground plane ε shows the ground rectangle on level 0. After constructing cuboid II in a similar way, the two resulting ground rectangles I and II can be brought in position to each other with the aid of two measured points Y^{IV} and Z^{IV} (Fig. 1).

4.2. Work with Complex Geometry

Conceptional characteristics and constructive reflections leading to the founding of the architectural design can bring the architect to work with complex geometries. In the following

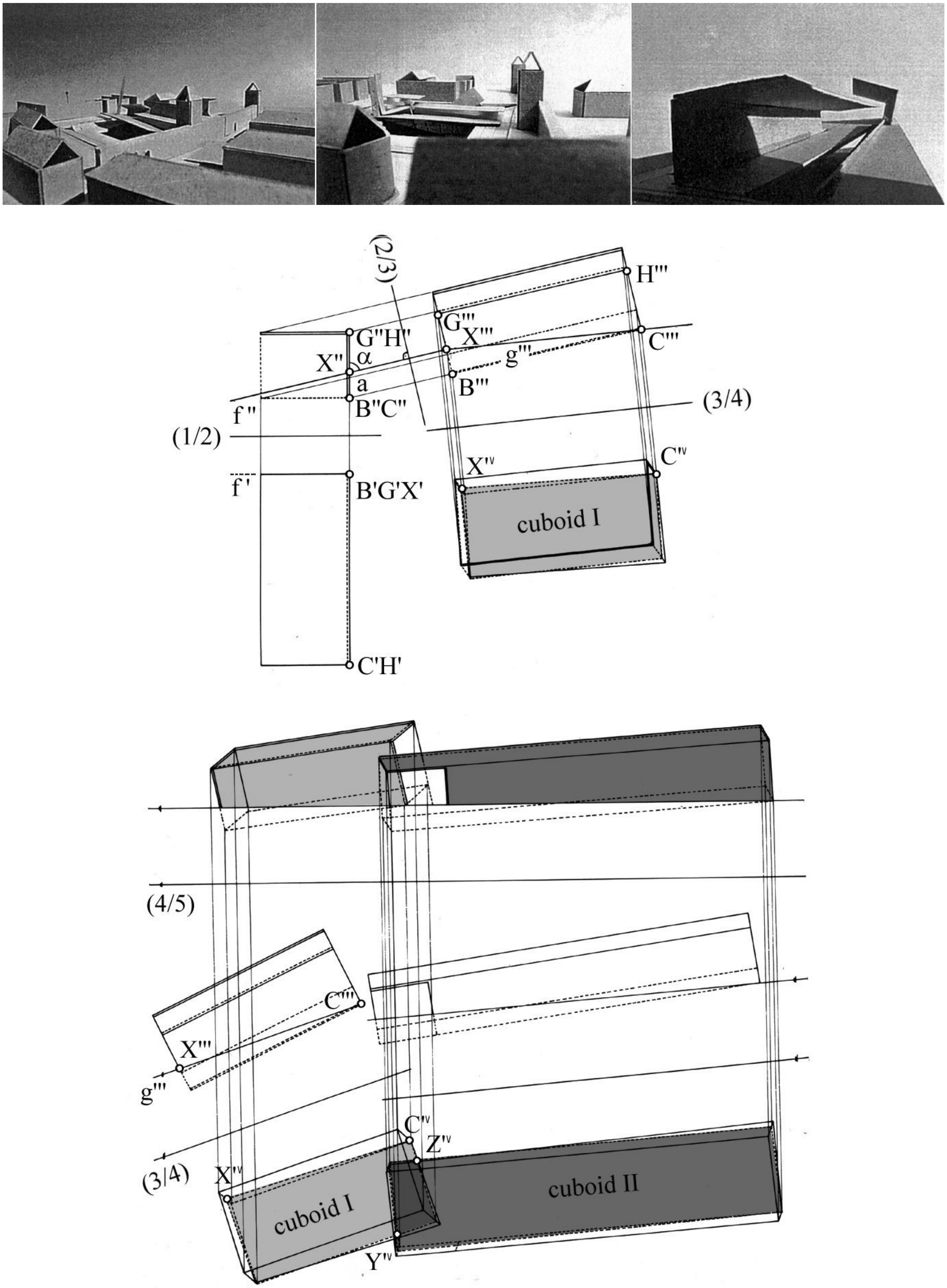


Figure 1: Model-based concept, developing cuboid I, and positioning the two cuboids

student example the geometrical question arose out of the architectural design to build up a climbing frame as a sphere. The elements of the frame should made as cast steel components. Therefore it is necessary to find a solution to build up the sphere out of as few different elements as possible to receive an economical solution. These basic reflections led the student to the studies of Fuller [1] about geometry of a sphere within his geodetic studies. In this example the geometrical achievement of the student is to understand the development of a complex form and to utilise it for architecture. Buckminster Fuller used, among others, the icosahedron to approximate a sphere. It is possible to construct the icosahedron out of one element length because it consists of equilateral triangles. By dividing the triangles of the icosahedron in smaller triangles and projecting them onto the sphere surface, non-regular polyhedrons are developed with many different element lengths according to the grade of refinement (Fig. 2).

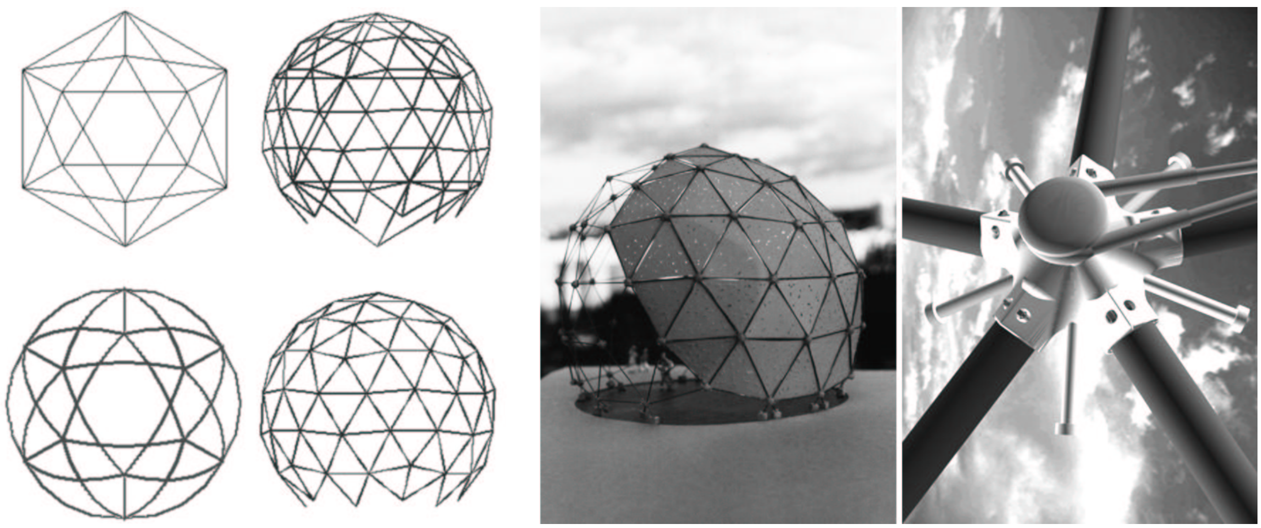


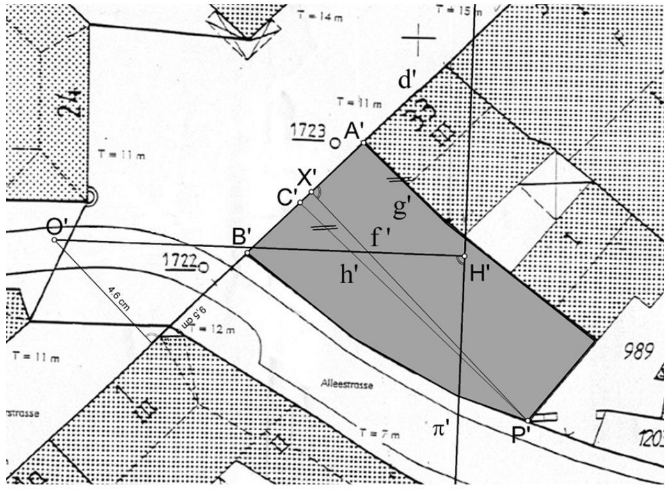
Figure 2: Refinement of the triangles of the icosahedron and the result of the sphere construction

Therefore there is an optimal refinement dependent on the extension of the sphere. It takes time to project the edges on the sphere surface by manual drawing. The use of a computer is helpful to produce the geometrical form exactly and quickly. The geometrical knowledge plays an important role also in the computer-aided solution. Navigating in space by user coordinate systems and snap points helps to construct the geometrical characteristics of the geodetic sphere. After the determination of the grade of refinement and with it the number of different element lengths, the developed wire frame can be used as the geometrical structure to generate the elements of the sphere construction (Fig. 2).

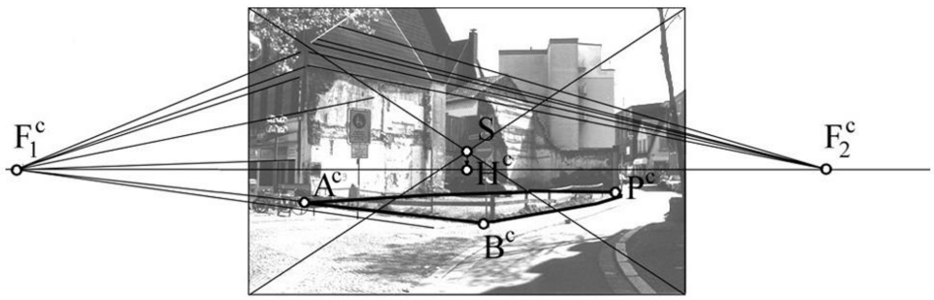
4.3. Photoreconstruction and Photomontage

The photomontage achieves a high performance by showing the designed architecture in combination with the existing surrounding and planning area. To produce a photomontage it is first necessary to reconstruct the photo of the planning area, for example, by means of a known horizontal quadrangle (Fig. 3a). This is an often applied task for Descriptive Geometry to get presentations of designed architecture.

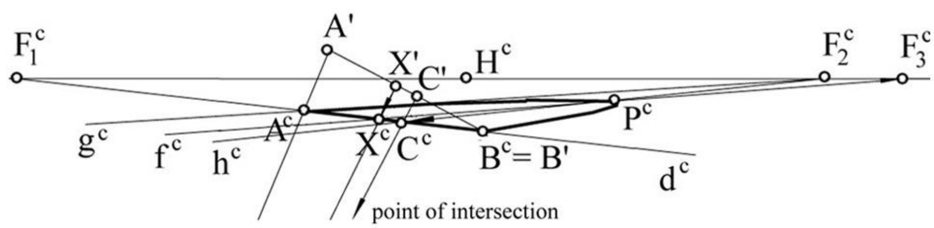
For solving the reconstruction problem a detailed knowledge about geometrical parameters and their effect on the change of the position is necessary. To achieve a correct solution of



(a)



(b)



(c)

Figure 3: (a) Ground plan with the horizontal quadrangle, (b) First vanishing points of the photoreconstruction, (c) Construction with the projective pencil of lines

the photoreconstruction goal-directed dealing with inaccuracies is required. Especially an exact determination of the point of view is important. There is an interface for computerized drawing methods when adjusting the perspective drawing of the designed architecture to the photo. In the present case of a building gap in the centre of Kaiserslautern the camera goal direction is slightly inclined to the top. The vertical edges of the houses are slightly vanishing to the top, and then the horizon moves a little down in relation to a perspective with a vertical projection plane (Fig. 3b). This movement is indicated in the following as a manageable inaccuracy within the assumption of a vertical projection plane. To receive a reconstruction of the point of view as exactly as possible we take striking points which are spatially lying far away in the photo combined with points in the foreground (point A and P). The vanishing points of lines d and g are well determined in the photo. However, these two directions are not exactly orthogonal to each other, which would lead to contradictions in the following processing. Therefore, we construct the vanishing point of line f orthogonal to line d with the help of a projective pencil of lines through point $X \in f$:

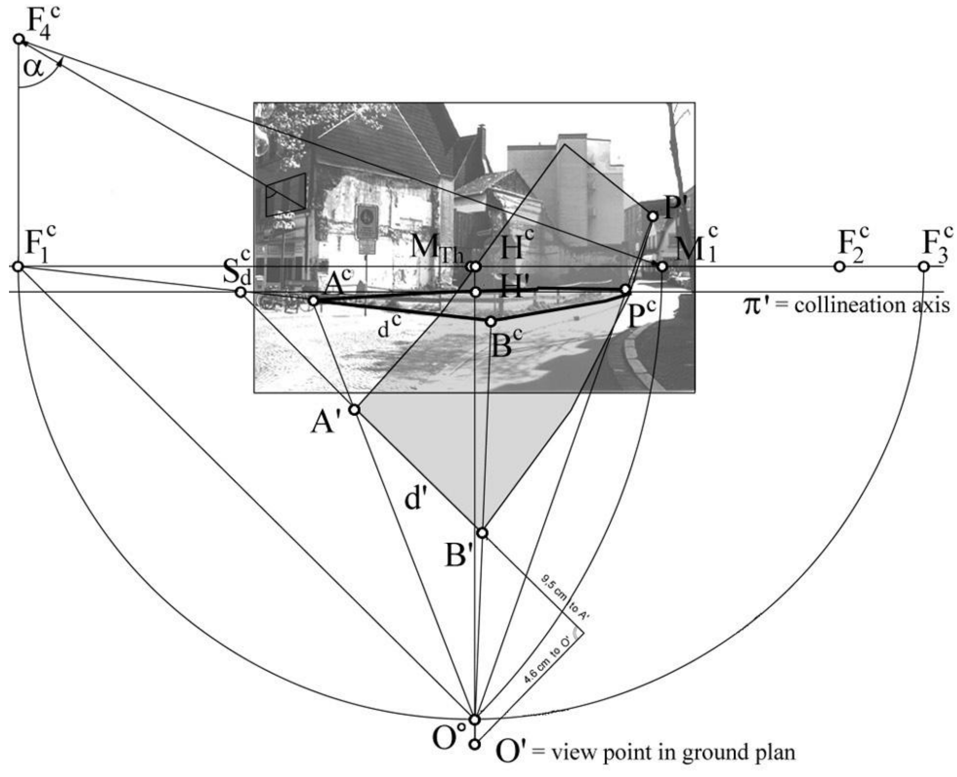
1. A parallel line to g' through point P results in line h' and cut with d' in point C'.
2. The perpendicular f' from point P' to line d' results in point X' (Fig. 3a).
3. Accordingly line h^c and point C^c are received in the perspective with the aid of the vanishing point F_2^c of line g and point P^c.
4. By transferring the proportional distances in the ground plan, point X^c on line d^c will be constructed by means of a projective pencil of lines through the common point S of lines $C'C^c \cap A'A^c$ and point X' [7].
5. Point X^c combined with P^c gives line f^c and with it the searched vanishing point F_3^c of line f (Fig. 3c).

After this helping construction the collineation centre O^0 is determined by the circle of Thales over the line between F_1^c and F_3^c. Another vague piece of information about an angle in a vertical rectangle helps to find out exactly enough the position of the point of view. The determination of the inside parameter of the perspective follows the outside determination by means of a known distance, for example A^cB^c. Then the scale and a collineation axis for positioning the ground plan is fixed to mark the view point O' in the ground plan (Fig. 5a).

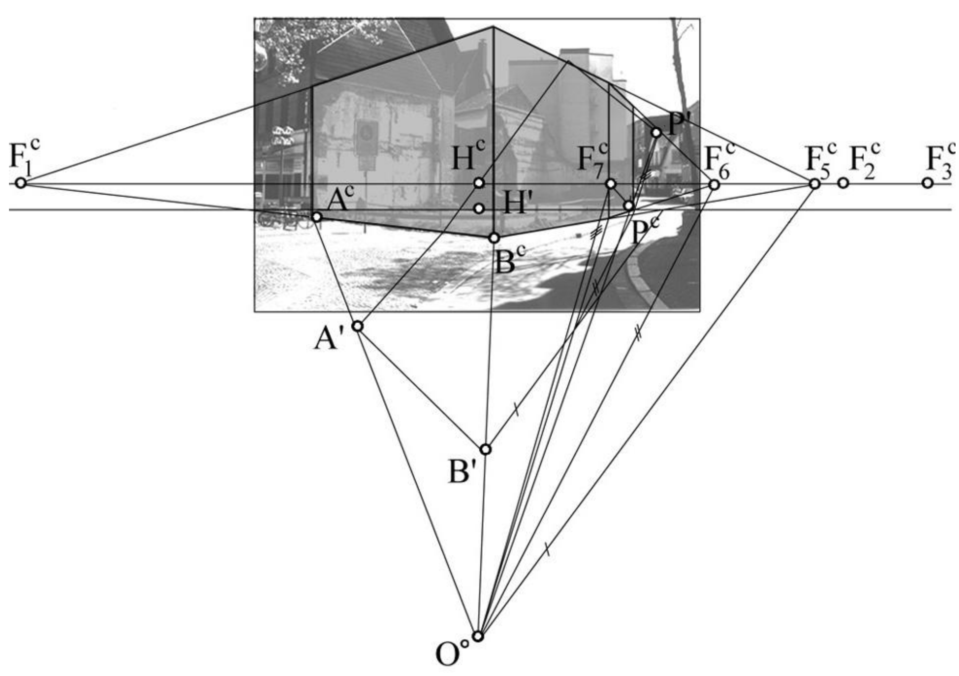
The perspective of the designed building can be sketched in the photo after these procedures (Fig. 5b). If the point of view is determined exactly, the three dimensional constructed design in a CAD-program can be fit into the photo by means of the found perspective parameters. Image editing programs help in the last step to receive a suitable photomontage (Fig. 4). Supplementary, the light reconstruction makes it possible to represent the newly-designed building with a real shadow corresponding with the existing shadow in the photo.

(a) (b) (c)

Figure 4: Montage with an image editing programm, (a) rendering, (b) rendering added to the photo, (c) ready photomontage



(a)



(b)

Figure 5: (a) Photoreconstruction with marked point of view, (b) Photomontage

4.4. Comparisons of Representation Types

Descriptive Geometry offers different projection methods for getting representations of architecture. There is a chance to compare the different types of representation in context with an architectural project.

The representations applied in the project can be checked regarding their contribution in transmitting information and their presentation quality. By applying different projection methods in the architectural project the students develop their geometrical skills and learn to use the various representation types in the communication process between the architect and the respective target group. They develop their communication ability through various representations and learn to select a specific representation in reference to the desired purpose. In the following example the student compared four types of representations. Besides the well-known different uses of axonometries and perspectives — axonometries refer to the object, perspectives refer to the viewer [8] — there is the opportunity to deal with the connection between seeing and drawing (Fig. 6).

When drawing a perspective we get distortions outside the circle of vision. Therefore we have to restrict our vision angle and we see only a small detail of the project in the perspective representation. In the examples the vision angle is in each case 60^0. Only by selecting another view direction, the observed detail in the perspective gets increased from the one-point perspective to the two-points perspective. In the third perspective, a *retina perspective* [9], we catch a larger undistorted detail of the project with a constant vision angle of 60^0 by simulating a projection on the inside of a sphere by means of two independent cylinders. With this method we receive an image approximating our seeing with an acceptable expenditure where lines are shown as lines instead of projections of circles. Certainly, this method does not provide an exact image and can be applied only in a limited way. But the example shows the combination between geometrical knowledge and aim-oriented economical use of geometrical construction methods.

To get a better simulation of seeing where it is possible to look around and not looking just in one direction like in a perspective, panoramical representations became more and more popular. The panorama is reached by a cylinder projection. With the aid of a computer program like Quick Time VR Authoring Studio™ a panoramic scene of the architectural project can be produced out of various images created as perspectives with turning view directions. The panoramic scene allows one to navigate by turning around and zooming. The following example shows such a panoramic scene of a design project created by students (Fig. 7).

The panorama Viewer makes it possible to go through the designed lobby of the hotel by turning around and zooming. By means of other linked panoramas the user can walk through the whole building.

These and other student projects have been prepared for a CD-ROM [6]. There the projects can be studied interactively and in animated sequences.

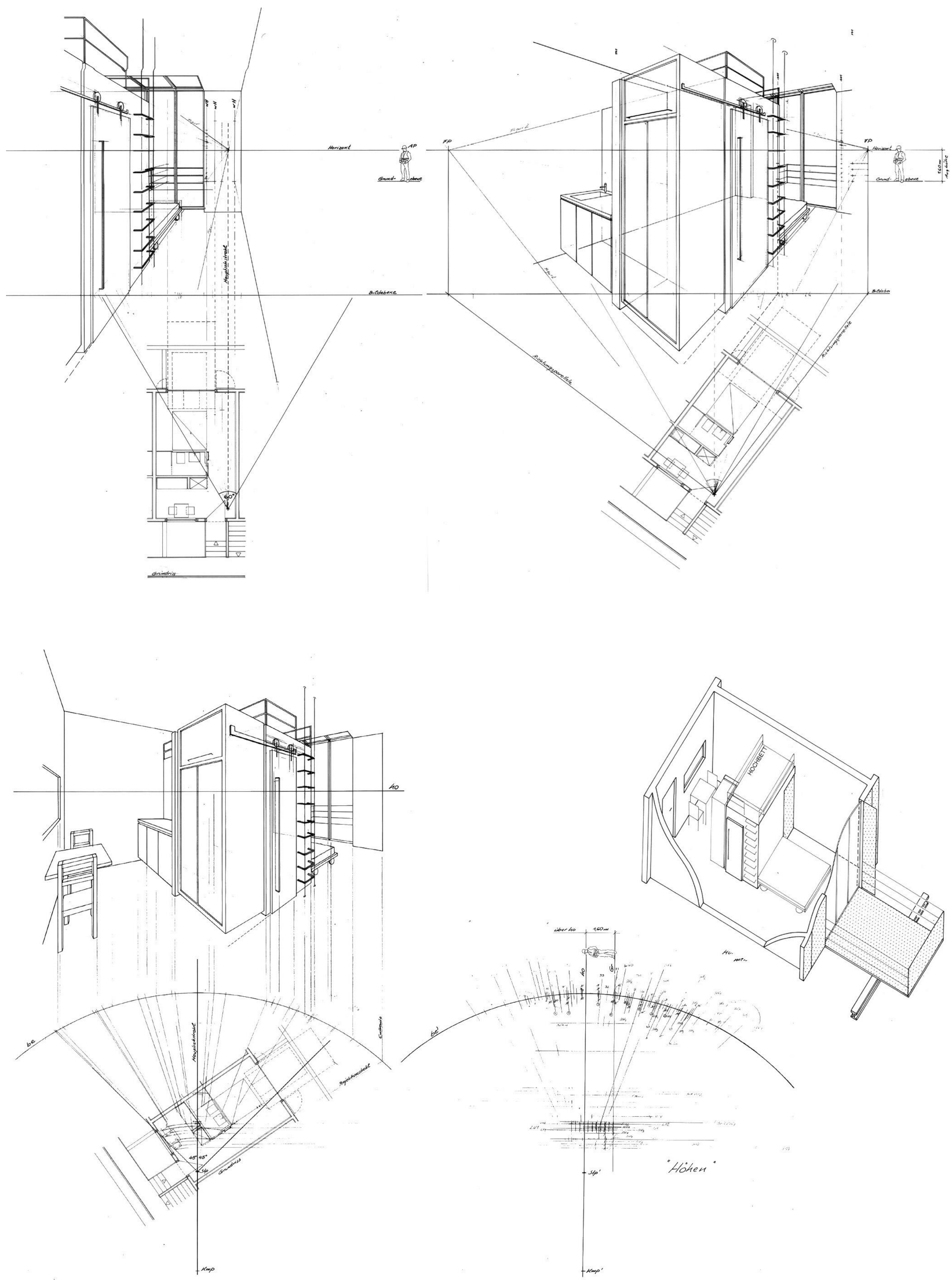
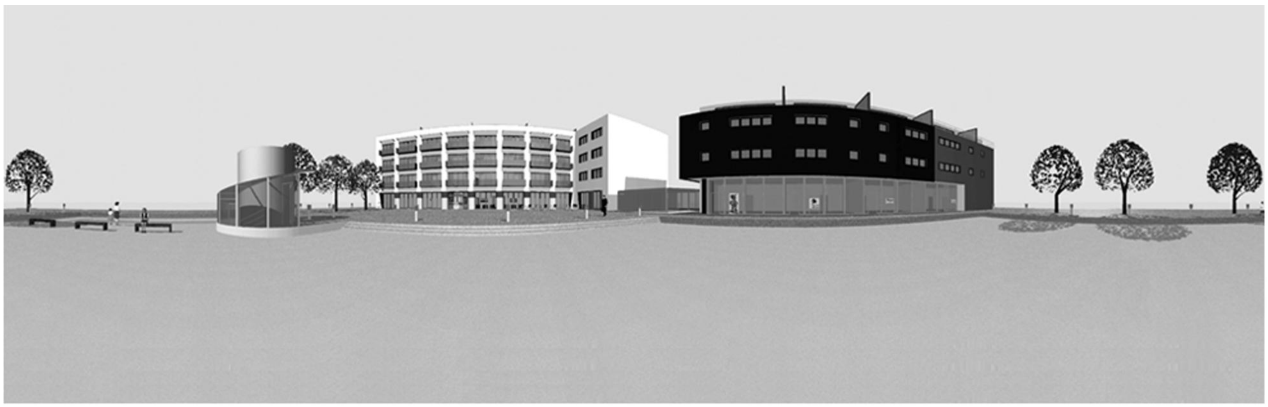


Figure 6: One-point perspective, two-points perspective, *retina perspective*, and axonometry

(a)



(b)

Figure 7: Panoramic scenes, (a) hotel lobby, (b) outside view of the building

5. Conclusions

Architectural projects offer numerous interesting problems which give the opportunity to acquire geometrical knowledge in various fields. A basic knowledge in Descriptive Geometry helps to grasp the geometrical problems. By beginning with the architectural project and not with the geometrical problem, we reach a high motivation to intensify and broaden geometrical knowledge and to be aware directly of the practical importance of geometry for architecture. The computer-based working which is embedded in the projects shows the various possibilities with the aid of new technologies and emphasises the necessity of geometrical reasoning by working with these new technologies. There is a way to achieve an understanding for more complex geometrical coherences. The large field of interesting geometrical application areas in architecture can be opened up by the presented integrated teaching method.

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