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The Composition of Decorative Art in Lao Traditional Monastic Architecture — a Computational Geometry Approach

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Abstract. This paper deals with the curvilinear parts of Lao motifs, decorative arts that are usually used at decorations of Lao traditional monastic architecture. Referring to the Roundness Method, the paper examines six samples of curvilinear parts of Lao motifs in comparison with those of Thai, which is said to be very similar to Lao motifs. The analysis yields that

(1) the inner location of Karn Lai components in the Lao motif is more round or blended than that of Thai. However, the roundness feature at the other locations of Karn Lai in Lao motifs are hardly differing from those of Thai.

(2) In all locations of Bark Lai motifs the curvature of curvilinear lines of Lao is hardly differing from those of Thai.

(3) In the upper location of Nhod Lai motifs the curvature of curvilinear lines of Lao is more round than that of Thai. In the other locations of Nhod Lai motifs the curvature of curvilinear lines of Lao is hardly differing from those of Thai.

According to these results, most of the curvilinear lines in Lao motifs are considerably more round that those of Thai.

Key Words: Curvature, roundness, Lao motifs

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1. Introduction

The art of Laos owes its reputation to achievement in the field of decorative arts. Typically, religious buildings are making use of these arts in their ornamentation (see Fig. 1). Lao decorative arts, rather often referred as *Lai Lao* or Lao motifs, are said to be similar to their neighborhoods, especially to the Thai, such that make confusing and misidentifying. These motifs are composed with curvilinear lines that run counter one another in such a way to form foliage shape, like fern, cotton leave, lotus and so on.

Often, proportional compositions are used to describe the ways Lao motifs are composed (as shown in Fig. 1). However, the curvilinear parts, which are dominating the motifs, have never been expressed in geometrical ways.

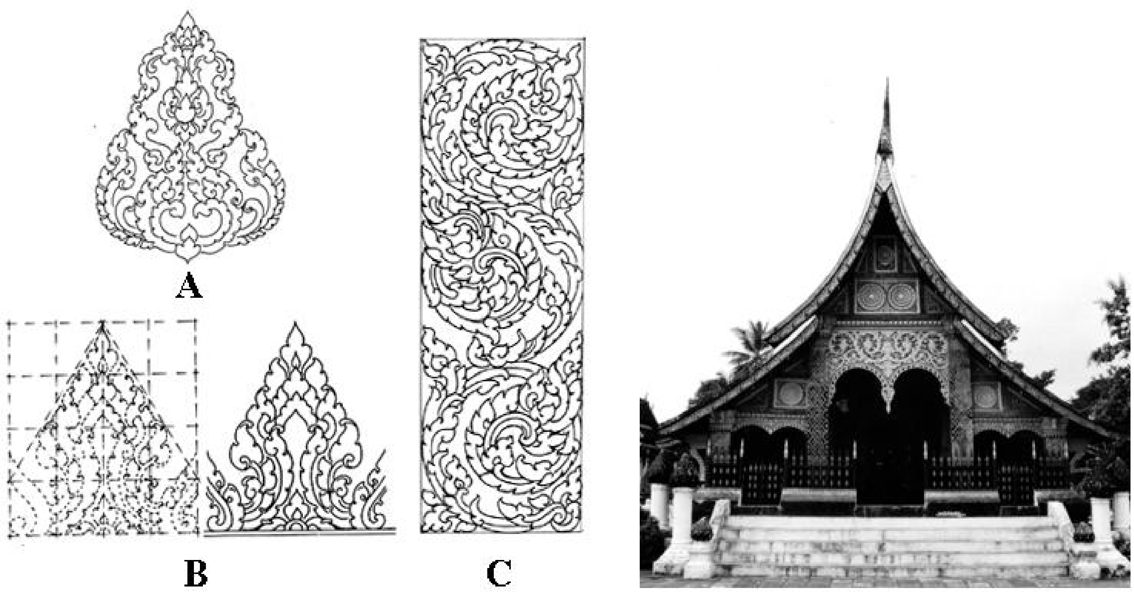


Figure 1: Lao motifs (A: single motif, B: parted motif, C: continuous motif) in Phutthasima at VatXiengthong (built in 1560 AD., Louang Phrabang, Laos)

In order to measure the curvature of curves that differ in scale, the paper adopted the *Roundness Method*. Utilizing this method, we examine the variation of curvature of the curvilinear lines in Lao motifs in comparison with those of Thai counterparts, in order to clarify the identical features in curvature of Lao and Thai motifs.

The curvature feature of a curve is usually expressed in numerical way by using the radius of curvature. In order to measure the curvature of a given curve, circles of curvature are placed along the curve. The radius of these circles represents the curvature of the measured curve. For a flattened curve next to a straight line or for a curve bended like a circular arc, the curvature features remain still unchanged when these curves are enlarged or reduced. However, it is not appropriate to compare curves in different scales by their curvature radius only.

To solve this problem, the Roundness Method has been proposed by S. Nakamura [3] – [6]. The concepts and the calculation process of this method are summarized in the next section.

2. Method

2.1. Roundness value

Roundness is a method of measuring the curvature of any curvilinear line with a numerical value. This method is based on the rotation of conic sections, particularly of a circle and a hyperbola. The conic section, for example a circle, is rotated from it upright position through the angle θ about either its vertical axis that passes through the center point, or about the horizontal axis, which touches the circle at the intersection with its the vertical axis (see Fig. 2). Afterwards the rotated circle is projected into a projection plane parallel to the initial position. Thus we obtain different curves, so called *EEL* or *EFL*, respectively. Doing the same way for a hyperbola, we obtain curves of types *HEL* or *HFL*.

The curvature features of these curves cover the curvature features of various curves and can be used to represent all curves [3] – [6]. The declination of the conic sections, i.e. $90 - \theta$, is referred as the *"roundness value"*, marked with ϕ_{EEL}, ϕ_{EFL}, ϕ_{HEL} and ϕ_{HFL} for a vertically or horizontally rotated circle or a vertically or horizontally rotated hyperbola, respectively.

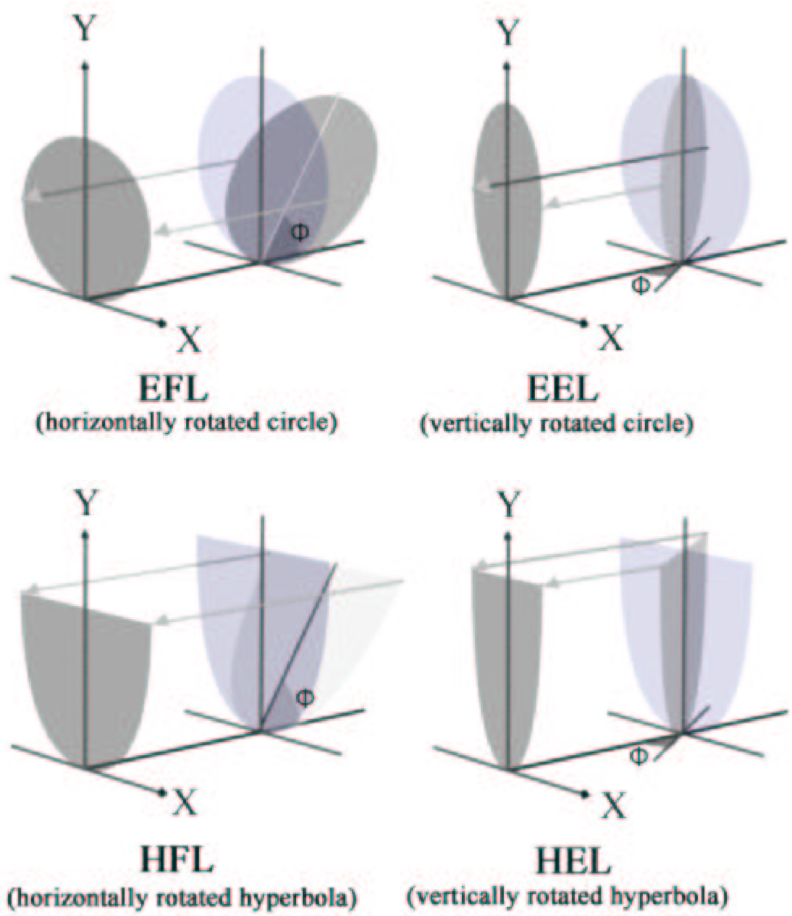


Figure 2: Roundness of a curve obtained from rotating conic sections

Any given curve with continuously changing radius of curvature can be matched and represented by a rotated conic section and may fall into one of the cases in Fig. 2. The *roundness value* can be calculated from an equation of type $\sin\phi = f(X/R, Y/R)$ where X, Y, R are the horizontal and the vertical coordinate of a specific point and the radius of curvature at the vertex point, the so-called *"standard radius"* of the curve, respectively.

For a curve with given equation, the first and the second derivatives can easily be calculated. At each point of the curve the radius r and the center $C(x_c, y_c)$ of the circle of curvature can be obtained by

$$r = F(x) = \frac{(1 + f'(x)^2)^{3/2}}{f''(x)} \tag{1}$$

$$x_c = G(x) = x - \frac{f'(x) + f'(x)^3}{f''(x)}, \quad y_c = H(x) = y + \frac{1 + f'(x)^2}{f''(x)}. \tag{2}$$

The maximum or minimum radius of curvature, which is supposed to be taken at the point with $f'(x) = 0$, is called the standard radius of curvature. At any specific point of the curve with given distance from the center point the *roundness value* can be calculated as follows:

$$\text{vertical rotation of a circle:} \qquad \sin^2(\phi_{EEL}) = \frac{2(Y/R) - (X/R)^2}{(Y/R)^2}, \tag{3}$$

$$\text{horizontal rotation of a circle:} \qquad \sin^2(\phi_{EFL}) = \frac{(Y/R)^2}{2(Y/R) - (X/R)^2}, \tag{4}$$

$$\text{vertical rotation of a hyperbola:} \qquad \sin^2(\phi_{HEL}) = \frac{(X/R)^2 - 2(Y/R)}{(Y/R)^2}, \tag{5}$$

$$\text{horizontal rotation of a hyperbola:} \qquad \sin^2(\phi_{HFL}) = \frac{(Y/R)^2}{(X/R)^2 - 2(Y/R)}. \tag{6}$$

The method of changing the curve is decided by the following conditions from formulas (3) – (6). The classification of ϕ_{EEL}, ϕ_{EFL}, ϕ_{HEL}, and ϕ_{HFL} is shown in Fig. 3.

$$\begin{aligned}
&\text{for } \phi_{EEL}: \quad (X/R)^2 < 2(Y/R), \quad |(X/R)^2 - 2(Y/R)| < (Y/R)^2,\\
&\text{for } \phi_{EFL}: \quad (X/R)^2 < 2(Y/R), \quad |(X/R)^2 - 2(Y/R)| > (Y/R)^2,\\
&\text{for } \phi_{HEL}: \quad (X/R)^2 > 2(Y/R), \quad |(X/R)^2 - 2(Y/R)| < (Y/R)^2,\\
&\text{for } \phi_{HFL}: \quad (X/R)^2 > 2(Y/R), \quad |(X/R)^2 - 2(Y/R)| > (Y/R)^2.
\end{aligned}$$

2.2. Roundness measurement of curvilinear lines

In order to measure the roundness of any curvilinear line, we first imput the scanned image into an image processor software to measure the horizontal and vertical coordinates of points along the curve. Then, five points $A(x_a, y_a)$, …, $E(x_e, y_e)$ of this curve are selected such that the horizontal distances between consecutive points $x_a - x_b$, …, $x_d - x_e$ are equal to a real constant s. Then by numerical manipulation we approximate this curve by a polynomial function using Gram's method

$$y = f_0 + f_1 p_1(t) + f_2 p_2(t) + f_3 p_3(t) + f_4 p_4(t) \tag{7}$$

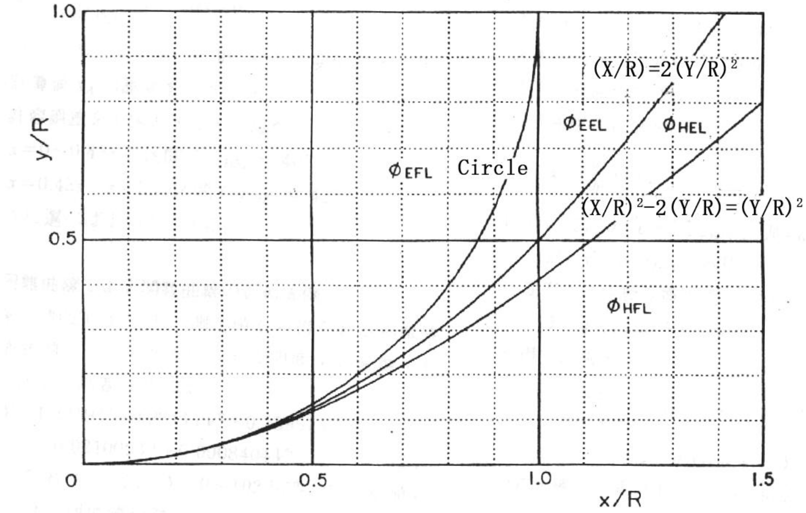


Figure 3: Classification of EEL, EFL, HEL, and HFL

where

$$f_0 = \frac{y_a + y_b + y_c + y_d + y_e}{5} \qquad p_1(t) = \frac{t}{2}$$

$$f_1 = \frac{-y_a - 0.5y_b + 0.5y_d + y_e}{2.5} \qquad p_2(t) = \frac{t^2 - 2}{2}$$

$$f_2 = \frac{y_a - 0.5y_b - y_c - 0.5y_d + y_e}{3.5} \qquad p_3(t) = \frac{5t^3 - 17t}{6}$$

$$f_3 = \frac{-y_a + 2y_b - 2y_d + y_e}{10} \qquad p_4(t) = \frac{35t^4 - 155t^2 + 72}{12}$$

$$f_4 = \frac{y_a - 4y_b + 6y_c - 4y_d + y_e}{70} \qquad t = \frac{x - x_0}{s} - 2$$

Here we use the following notation: x, y for the horizontal and vertical coordinates, x_0 for the horizontal coordinate of origin of our coordinate system, and s for the x-interval of selected points used for numerical manipulation to find the polynomial function.

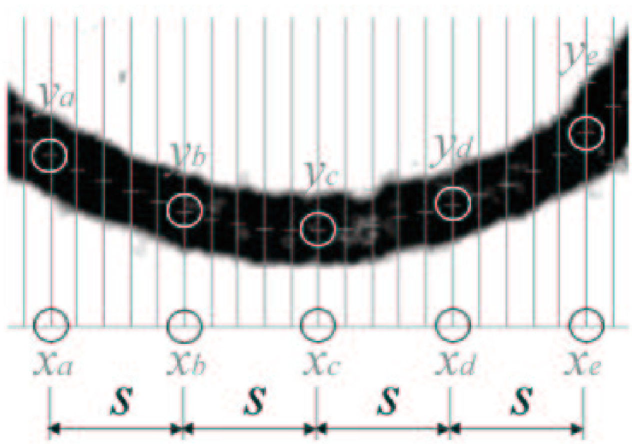


Figure 4: Measurement of a curvilinear line

Next, the first and second derivatives at specific points of the curve can be calculated by

$$y_i' = \frac{y_i - y_{i-h}}{2h}, \quad y_i'' = \frac{y_{i+h} - 2y_i + y_{i-h}}{h^2} \quad \text{for} \quad h = x_{i+h} - x_i = x_i - x_{i-h}\,. \tag{8}$$

Then, the radius of curvature can be calculated with the equation above, replacing $f'(x)$ by y' and $f''(x)$ by y''. Point $P(x_p, y_p)$ is a vertex point of the curve, at which the radius of curvature has a maximum or minimum R. Meanwhile, the center of the standard circle of curvature is $Q(x_q, y_q)$ obeying

$$x_q = x_p - \frac{(1+{y_p'}^2)y_p'}{y_p''}, \quad y_q = y_p + \frac{(1+{y_p'}^2)y_p'}{y_p''}, \quad R = r_p = \frac{(1+{y_p'}^2)^{3/2}}{y_p''}\,. \tag{9}$$

Taking the touching point of the standard circle of curvature and the measured curve, i.e., the vertex point of the curve, as the origin of a new coordinate system, and specifying the tangent line at the vertex as new X-axis and the perpendicular Y-axis, we transform the coordinate system by

$$\begin{aligned} X_i &= (x_i - x_p)\cos\theta + (y_i - y_p)\sin\theta, \\ Y_i &= -(x_i - x_p)\sin\theta + (y_i - y_p)\cos\theta, \end{aligned} \qquad \text{where} \quad \tan(\theta) = \frac{x_p - x_q}{y_q - y_p}\,. \tag{10}$$

Here θ denotes the angle between the former and the new coordinate system, (X_i, Y_i) are the new coordinates of point i, (x_i, y_i) are the horizontal and vertical coordinate of point i referring to the former coordinate system.

Then the ratios X/R and Y/R can be calculated in order to find the *roundness value* along the measured curvilinear line.

2.3. Motifs of Lao and Thai

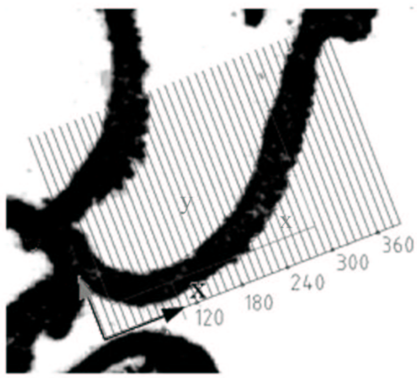
Karn Lai *Bark Lai* *Nhod Lai*

Figure 5: Type of motif component

The samples of motifs are photos and illustrations collected from textbooks on decorative arts [1, 2, 7, 8, 9, 10]. The analysis is conducted comparatively between Lao and Thai motifs in order to clarify the characteristics of the motifs, in particular the curvature of some components of motifs, such as *Karn Lai* — the stem, *Bark Lai*, and *Nhod Lai* (see Fig. 5).

First, the sample motifs are scanned by Canon FB680S with 1200 dpi gray scale. The scanned images are inserted into CAD application for coordinate measurements. The display solution is 1024×678 pixels. Secondary, the measured data are utilized for numerical manipulations as described in Section 2.1. Fig. 6 shows an example of coordinate measurement and the result of calculation.

As mentioned in Section 1, there are several sorts of motifs, such as single, repeated or continuous motifs. In this paper we deal with the continuous motif, which is mainly composed from curvilinear lines and usually utilized for decorating large surfaces such as door panels or pediment. Figs. 7 to 9 show samples selected for roundness measurement. We place a focus on some curvilinear parts of motifs that are apparently similar in Lao and Thai counterparts. In this subsection we examine whether there are systematic relations of curves in different components of motifs. Six pairs of Lao and Thai sample motifs are analysed comparatively. To compare the curvature of curves in different scale, we standardize all curves by using Y/R (ratio of vertical coordinate to the radius of curvature) and X/R (ratio of horizontal coordinate to the radius of curvature).



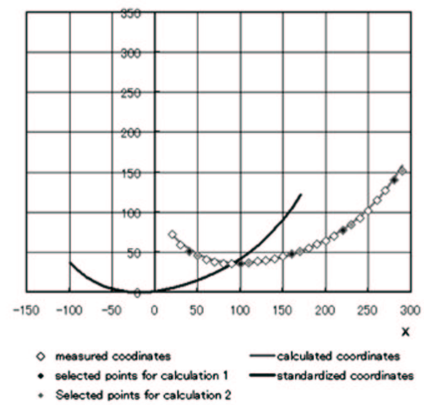


Figure 6: Type of motif component and curve by location

3. Results and Discussion

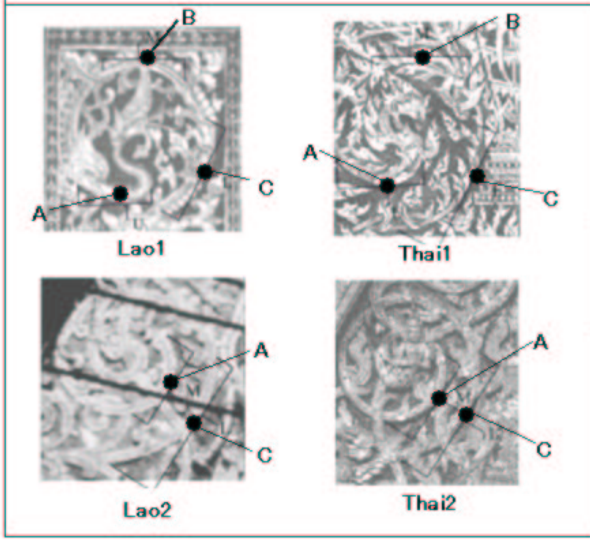
3.1. The roundness value in Karn Lai motifs

First, we examine the roundness at different locations of the Karn Lai component, namely the inner location (A_l, A_r), the middle location (B) and (C_l, C_r). Fig. 7 shows the *roundness value* of each location of Karn Lai of both Lao and Thai. The left part shows the examined samples, and the right side shows the *roundness value*. As indicated on the right side of the figure, the *roundness value* varies depending on the ratio X/R. We focus on the range $0.5 < X/R < 1.0$. At the inner location of the Karn Lai component, the variation of the *roundness value* of Thai is from 3 to 15 (ϕ_{HFL}) or from 1 to 16 (ϕ_{HFL}), and Lao is 44 (ϕ_{EEL}) or from 12 to 63 (ϕ_{EFL}) ($0.5 < X/R < 1$). At the middle location (C), the *roundness value* varies from 38 to 72 (ϕ_{EFL}) for the Thai, and Lao from 17 to 56 (ϕ_{HFL}) ($0.5 < X/R < 1$). As for the outer location, the *roundness value* varies from 2 to 8 (ϕ_{HFL}) or from 12 to 25 (ϕ_{EEL}) for the Thai, and from 16 to 28 (ϕ_{EFL}) or 4 (ϕ_{HFL}). According to variation of the *roundness value*, the inner location of the Karn Lai component in Lao motif is more round or blended than that of Thai. On the other hand, the roundness feature at the other locations of Karn Lai in Lao motifs is hardly differing from those of the Thai counterpart.

3.2. The roundness value in Bark Lai motifs

Fig. 8 shows the *roundness value* in the Bark Lai component for each location and each country. For the upper location of Bark Lai, the variation of the *roundness value* is from 42

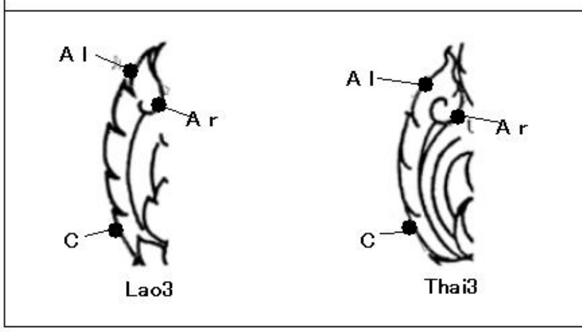
Karn Lai



Location	Sample	X/R 0.2	0.4	0.6	0.7	0.8	0.9
Inner part A	Lao1	HFL 2	HFL 27	EEL 44	EEL 59	EEL 64	EEL 61
	Thai1	HFL 6	HFL 3	HFL 3	HFL 7	HFL 11	HFL 15
	Lao2	EFL 12	EFL 28	EFL 41	EFL 48	EFL 55	EFL 65
	Thai2	HFL 6	HFL 4	HFL 1	HFL 1	HFL 4	HFL 6
Middle part B	Lao1	HFL 2	HFL 5	HFL 17	HFL 25	HFL 37	HFL 56
	Thai1	HFL 12	EFL 24	EFL 38	EFL 46	EFL 57	EFL 72
Outer part C	Lao1	EFL 16	EFL 28	EFL 40	EFL 47	EFL 54	EFL 65
	Thai1	HFL 8	HFL 21	HFL 44	-	-	-
	Lao2	HFL 4	HEL 42	EFL 44	EFL 48	EFL 55	EFL 65
	Thai2	EFL 12	EFL 25	EFL 42	EFL 54	EFL 70	EEL 70

Figure 7: The *roundness value* in Karn Lai motifs for each location and each country

BarkLai



Location	Sample	X/R 0.2	0.4	0.6	0.7	0.8	0.9
Upper part A	Lao3 l	EFL 12	EFL 25	EFL 49	-	-	-
	Thai3 l	EFL 15	EFL 24	EFL 42	EFL 55	EFL 80	-
	Lao3 r	HFL 17	EFL 69	EFL 53	EFL 57	EFL 63	EFL 73
	Thai3 r	HFL 1	EFL 24	EFL 58	EFL 66	EFL 48	EFL 39
Lower part C	Lao3 l	HFL 1	HFL 5	HEL 5	EFL 49	EFL 53	-
	Thai3 l	HFL 2	HFL 3	HFL 7	HFL 12	HFL 21	HFL 50

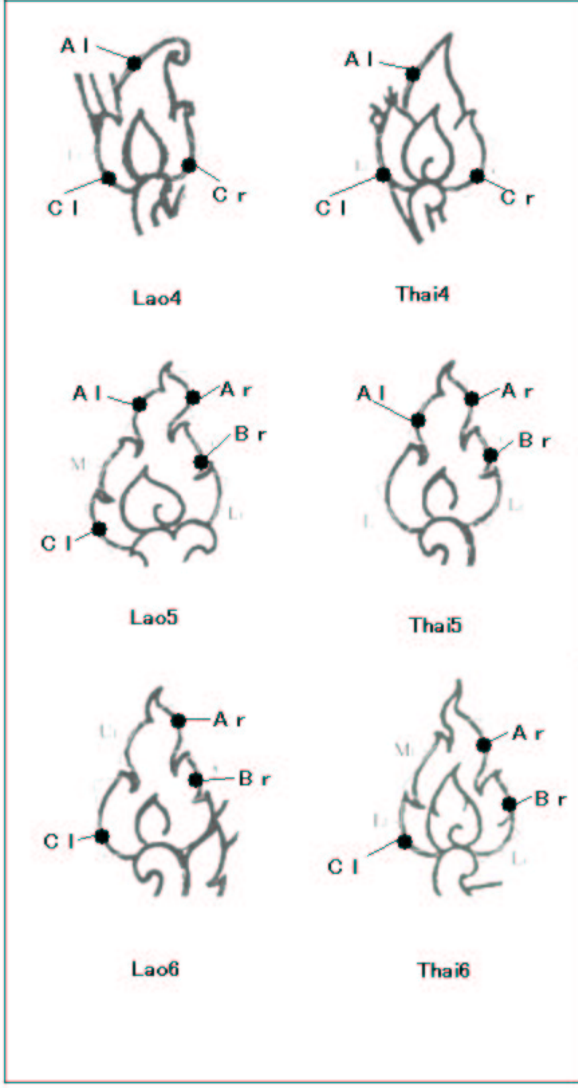
Figure 8: The *roundness value* in Bark Lai component for each location and each country

to 80 (ϕ_{EFL}) or from 39 to 85 (ϕ_{EFL}) for the Thai, and 49 (ϕ_{EFL}) or from 53 to 73 (ϕ_{EFL}) for the Lao ($0.5 < X/R < 1$). At the lower location, the variation of *roundness value* of Thai is from 7 to 50 (ϕ_{EFL}) and that of Lao is 5 (ϕ_{HEL}), from 49 to 53 (ϕ_{EFL}) ($0.5 < X/R < 1$). Therefore, in all locations of Bark Lai motifs, the curvature of curvilinear lines of Lao is hardly differing from those of Thai.

3.3. The roundness value in Nhod Lai motifs

Fig. 9 shows the *roundness value* in the Nhod Lai component for each location and each country. In the upper location of Nhod Lai, the *roundness value* varies from 37 to 79 (ϕ_{EFL}) in examples of Thai and that of Lao is from 34 to 74 (ϕ_{EEL}) ($0.5 < X/R < 1$). In the middle location, the variation of *roundness value* of Thai ranges from 37 to 60 (ϕ_{EFL}) or 74 (ϕ_{EEL}) and that of Lao from 38 to 62 (ϕ_{EFL}) or 77 (ϕ_{EEL}) ($0.5 < X/R < 1$). In the lower location of Nhod Lai, the *roundness value* of Thai is from 23 to 59 (ϕ_{EFL}) or 21 (ϕ_{HEL}) and that of Lao is from 50 to 80 (ϕ_{EFL}) or from 60 to 49 (ϕ_{EEL}) ($0.5 < X/R < 1$).

NhodLai



Location	Sample	X/R 0.2	0.4	0.6	0.7	0.8	0.9
Upper part A	Lao4 l	EFL 11	EFL 25	EFL 46	-	-	-
	Thai4 l	EFL 12	EFL 24	EFL 44	-	-	-
	Lao5 l	HFL 1	HFL 7	HFL 28	HFL 64	EEL 11	EEL 44
	Thai5 l	EFL 13	EFL 24	EFL 42	EFL 55	EFL 80	-
	Lao5 r	HFL 5	HFL 1	HFL 4	HFL 7	HFL 11	HFL 14
	Thai5 r	HFL 5	HFL 19	HFL 53	HEL 52	HEL 23	EEL 19
	Lao6 r	HFL 3	HFL 21	EEL 34	EEL 67	EEL 79	EEL 74
	Thai6 r	EFL 12	EFL 24	EFL 37	EFL 45	EFL 57	EFL 79
Middle part B	Lao5 Mr	HFL 1	HFL 13	EEL 6	EEL 63	EEL 76	EEL 71
	Thai5 Mr	HFL 1	HFL 8	HFL 20	HFL 30	HEL 50	HEL 48
	Lao6 Mr	HFL 5	EFL 24	EFL 38	EFL 49	EFL 62	EEL 77
	Thai6 Mr	EFL 11	EFL 24	EFL 37	EFL 47	EFL 60	EEL 74
Lower part C	Lao4 Ll	EFL 12	EFL 26	EFL 44	EFL 54	EFL 67	EEL 77
	Thai4 Ll	EFL 12	EFL 25	EFL 50	EFL 74	-	-
	Lao4 Lr	EFL 12	EFL 52	EEL 83	HFL 69	HFL 81	HEL 62
	Thai4 Lr	HFL 4	HFL 16	HFL 46	HFL 53	EEL 7	EEL 35
	Lao5 Ll	EFL 12	EFL 24	EFL 37	EFL 47	EFL 61	EEL 69
	Thai5 Ll	HFL 2	HFL 32	EFL 41	EFL 46	EFL 54	EFL 64
	Lao6 Ll	EFL 12	EFL 30	EFL 55	EFL 80	EEL 60	EEL 49
	Thai6 Ll	EFL 30	HFL 17	HFL 23	HFL 32	HFL 59	HEL 21

Figure 9: The *roundness value* in Nhod Lai motifs for each location and each country

Therefore, in the upper location of the Nhod Lai component of motifs, the curvature of curvilinear lines of Lao is more round of blended than that of Thai. In the other locations of Nhod Lai the curvature of curvilinear lines of Lao is hardly differing from those of Thai. According to these results, most of the curvilinear lines in Lao motifs are considerably more round that those of Thai.

4. Conclusion

In this paper, we analysed the characteristics of the Lao and Thai motifs by the Roundness Method. The findings of our analysis are:

1. The inner location of the Karn Lai component in Lao motifs is more round or blended than that of Thai. On the other hand, the roundness feature at other locations of Karn Lai in Lao motifs are hardly differing from those of the Thai counterpart.

2. In all locations of Bark Lai motifs, the curvature of curvilinear lines of Lao are hardly differing from those of Thai.
3. In the upper location of Nhod Lai motifs, the curvature of curvilinear lines of Lao is more round than that of Thai. In other locations of Nhod Lai motifs the curvature of curvilinear lines of Lao is almost the same as that of Thai.

In this paper, the curvature feature of motifs used in decoration of door panel and pediment of monastic architecture are quantified by the Roundness Method. According to these results, most of the curvilinear lines in Lao motifs are considerably more round than those of Thai. In order to clarify the design characteristic of Lao traditional architecture, we would next examine the roundness curvature features of some curves of the roof and of ornaments in architecture.

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