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Spatial Imagination Among Students Commencing the Course of Descriptive Geometry at Technical Studies

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Abstract. In the years 1996–2001, a new research on geometrical predisposition was conducted among the students of Environmental Engineering at the Faculty of Environmental Engineering and Land Surveying at the Agricultural University of Cracow. The research started in the years, when the entrance examination in mathematics has been cancelled. Students, who lack geometric knowledge, have also great problems to complete courses both in descriptive geometry and engineering drawing. The research continued in the following years, when exam in mathematics was brought back to life. The goal of the research was to provide evaluation of students' spatial abilities and to indicate conclusions for future use.

Key Words: geometric predisposition, descriptive geometry, graphics education

MSC 2000: 51N05

1. Research description

In the years 1996–2001 a research was carried out on geometric predispositions among students enrolling on the first year of studies at the Department of Environmental Engineering of the Faculty of Environmental Engineering and Land Surveying at the Agricultural University of Cracow. Such predisposition may be viewed as psychophysical traits favouring or predetermining success of technical studies. They are indispensable especially in the pursuit of the subject of descriptive geometry and mechanical drawing conducted in the above-mentioned department during the winter and summer semesters of the first-year study involving 72 units of teaching time.

The research was launched at a time when persons commencing the studies were found to present lack of geometric predisposition. The observation may have come from the author's subjective perception relating, for instance, to the students' lower level of knowledge in the basic topics of geometry covered by the teaching curricula for secondary-school mathematics.

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The lower knowledge indicators may have stemmed from the fact of suspension of mathematics entrance examinations to the above studies in those years. Admissions to the studies in that period were made on the basis of grades awarded on secondary-education certificates and results obtained from qualificatory interviews. Mathematical topics were subsequently incorporated into the subjects of those interviews.

Probably the reasons for the lower level of knowledge in geometry among persons admitted to the studies lay in the absence of selectivity previously exercised during mathematical entrance examinations, as well as in the lack of specification, by defining an adequate scope of the entrance examination, of the necessary mathematical requirements including, admittedly, geometry. The abandonment of the aforementioned examination resulted in a rise of interest in this line of studies and a larger number of aspiring candidates. However, it may be surmised that it was a consequence of the facilitation of admission to the studies, and that the candidates did not necessarily have adequate knowledge of geometry. The mathematics entrance examination was brought back in 1998, but in the year 2002 it was discontinued again.

As a comparison, during the last year of research a similar experiment was carried out among the students of the Geodesy and Cartography Department at the same faculty where, due to a very large number of candidates, a mathematical entrance examination has always been organized.

2. The research methodology

The studies were carried out in the years 1996–2001 among students embarking on the first year of their academic education at the Department of Environmental Engineering of the Faculty of Environmental Engineering and Land Surveying of the Agricultural University in Cracow. The survey was conducted during the opening classes in the subject of descriptive geometry and mechanical drawing, with over 90 percent of students taking part. The figure represented at least 100 persons in the successive years of the research when the number of candidates taking up studies ranged between 111 and 170 individuals. The control group was formed by 126 students (from among 130) commencing studies in the year 2001 at the Department of Geodesy and Cartography of the same faculty. Each of the subjects answered 10 simple questions within the field of geometry, which allowed, within boundaries of possible means, evaluation especially of spatial (three-dimensional) imagination, although knowledge pertaining to the basic problems of geometry was equally essential. The subjects were allowed to execute auxiliary drawings.

2.1. Exemplary set of problems

Below there are some exemplary questions included in the *Test on spatial abilities*.

1. How many various length segments one may draw between various vertices of a cube (the same length segments are considered to be one segment)?
2. How many faces (maximum) one may see from an optionally chosen viewpoint?
3. How many squares one may create using various vertices of cube?
4. How many cubes one can insert into a cube of a side-length equal to triple-length of the searched cubes?
5. If four points neither lie on a single straight line nor make a plane — what geometrical shape do they create?

6. An intersection of a cylinder with the base of diameter D and height h may be a rectangle. When does this case occur and what are the dimensions of this rectangle?
7. An intersection of a cube with a plane may be a square. When does this case occur?
8. A sphere with radius R and a plane have two common points. What is the shape of a line of intersection and what is the special dimension of this line of intersection. Please give the name of this special case's shape.
9. Taking into account the property, that a sphere has the largest ratio between its volume and surface if compared to the other 3D solids, please choose the solid with the largest volume among: a sphere with diameter $D = 8\,\text{cm}$, a cube with the side length 8 cm or a cylinder of revolution with the base's diameter $d = 8\,\text{cm}$ and height $h = 8\,\text{cm}$.
   What solid can be circumscribed or inscribed into another solid?
10. If a cone of revolution with a base of radius R and height h is intersected with a plane perpendicular to its axis of revolution at 1/4 height h, what shape will we appear and what dimensions in relation to the base circle will be obtained?

3. Results of the research

As a result of the studies carried out, each student under examination received a value within the range 0–10 corresponding to the number of correct answers. The results indicating particular years as well as those representing the control group are presented in Table 1.

	1996	*1997*	*1998*	*1999*	*2000*	*2001*	*control group*
number of subjects	154	155	109	137	145	144	128
arithmetic mean	5,27	4,67	5,50	5,35	5,32	4,92	5,63
standard deviation	1,88	1,38	1,84	1,54	1,84	1,81	1,72
median	5	5	6	5	5	5	6
bias	0,185	0,132	-0,102	0,515	-0,234	0,050	0,119
standard bias error	0,195	0,195	0,231	0,207	0,201	0,202	0,214

Table 1: The number of correct answers among the students of particular years

By analyzing arithmetic means of scores gained by the students commencing their studies in particular years, certain variations between them were observed. The relevance of those mean variations between particular groups (years) was analyzed by means of the t Student's test, as the mean values approximated the median and, except for two groups (1999 and 2000), the results distribution may be considered normal (the absolute value of the bias index being lower than the standard bias error). The results of the testing are grouped in Table 2.

Table 2 shows that, in comparison with the control group, the students of Environmental Engineering exhibited variations on a relevance level not higher than $\alpha = 0.05$ only in two study commencement years, i.e., 1997 and 2001. The 1997 group differed significantly ($\alpha < 0.05$) from almost all the other examined groups (except for the year 2001). Another student year group differing substantially, albeit only from half of the other groups, were those who began their studies 2001. Besides the earlier-mentioned year groups of 1997 and 2001, the other groups differed significantly only from those specified.

The distribution of frequency of attained particular results (scores, points) in the successive years of research is depicted in Fig. 1. The thick continuous line represents the results of

	1996	*1997*	*1998*	*1999*	*2000*	*2001*	*control group*
1996		0,002	0,387	0,675	0,813	0,109	0,095
1997	0,002		0,000	0,000	0,001	0,178	0,000
1998	0,387	0,000		0,594	0,520	0,020	0,501
1999	0,675	0,000	0,594		0,870	0,033	0,173
2000	0,813	0,001	0,520	0,870		0,068	0,155
2001	0,109	0,178	0,020	0,033	0,068		0,001
control group	0,095	0,000	0,501	0,173	0,155	0,001	

Table 2: Relevance levels for variations between particular groups (according to the t Student s test)

the control group students, the thin continuous lines refer to students from the year groups undergoing an exam in mathematics, while the dashed line reflects the results pertaining to the years when admissions to the studies were made on the basis of school certificate competition.

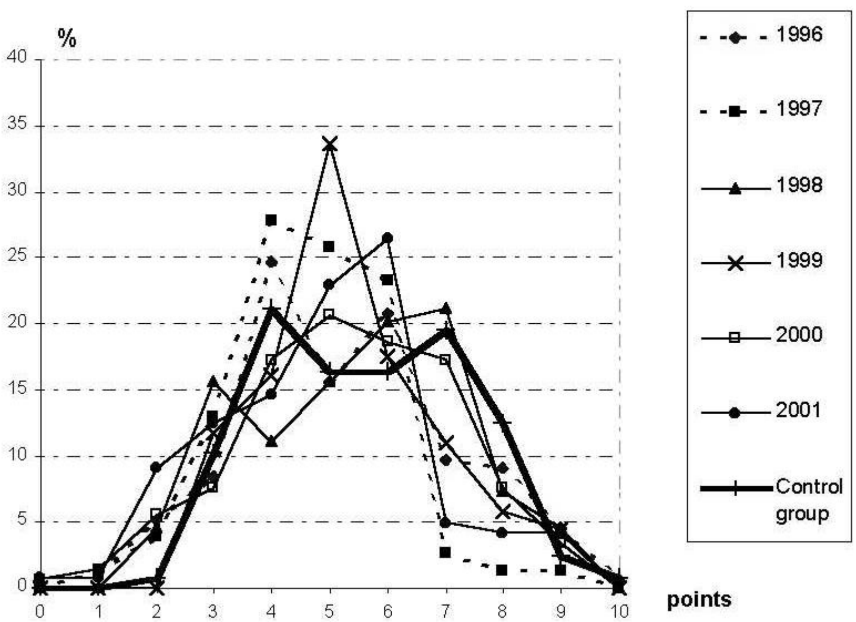

Figure 1: A percentage frequency distribution for attainment of particular scores (points) in the successive years of research and in the control group

The graph in Fig. 1 shows that the control group which drew a large number of candidates, i.e., involved more stringent requirements on admission to the studies, exhibited the lowest percental proportion of persons scoring less than 3 points, while attaining a comparatively high proportion among those who scored more than 7 points. The best approximation to normal distribution was achieved in 1999 — every third person gained 5 points. The largest score variations occurred with the control group members and the participants from the 1998 and 2000 year groups. The least favourable scores belonged to the 1997 and 2001 year groups in which cases the scores exceeding 7 points were attained by the fewest number of persons.

Those tendencies may be rendered more accurately by the graph in Fig. 2 displaying the percental proportion of persons scoring not less than the specified numbers of points.

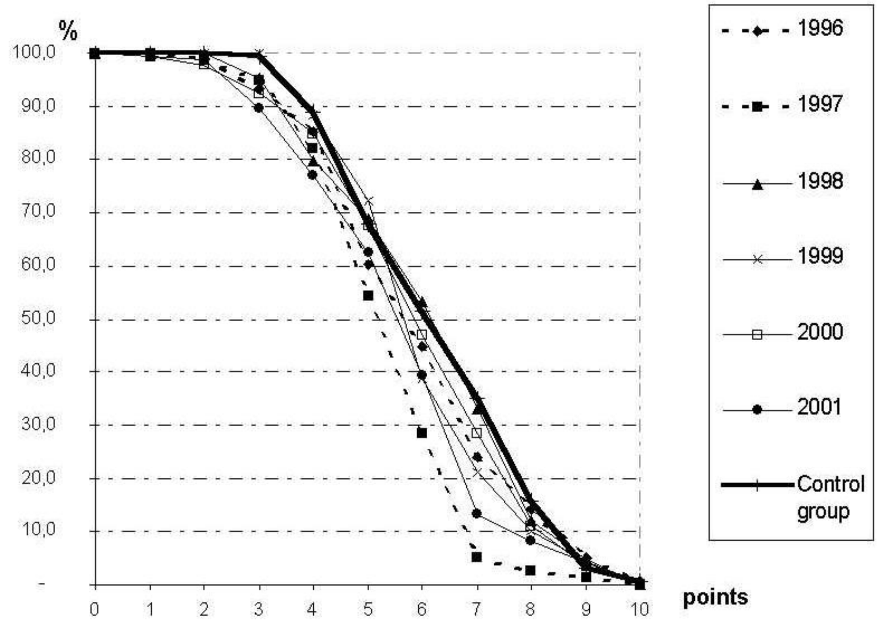

Figure 2: Percental proportion of persons scoring not less than the specified numbers of points

The graph depicted in Fig. 2 shows the difference between the groups evident from the shapes of respective lines. Variation in the line upward extent represents differences in the scores gained. In the case of the control group it corroborates the nearly 100 percent proportion of persons attaining not less than three points. Worth noting is also the very little proportion (approx. 5 and 13 percent, respectively) of persons gaining at least 7 points, who came from the "weakest" year groups, i.e., 1997 and 2001.

In order to account for the lower scores in those years, it was necessary to analyze both the form of enrolment to the studies and the number of the candidates. In one case it was a school certificate contest, while in the other one a mathematics examination. The number of candidates in 1997 was relatively high (more than 3 persons for each vacancy). However, this may be explained by reference to the "easy availability" of enrolment to the studies, yet the school-leaving certificate grades were low. In the year 2001, when an entrance exam in mathematics was carried out, the number of candidates was smaller (a little more than 2 for one spot). It may be presumed, however, that the prevailing number included those who were well prepared for the mathematics examination.

4. Conclusions

To summarize, therefore, one may concede the thesis that the predisposition for spatial imagination among students are not directly related to the form of enrolment to the studies, or to the number of candidates. Such a conclusion may be confirmed by the following observations:

1. In some years the results were poorer (involving a significant statistical difference), both when the mathematics examination was held and when it was absent.
2. The number of candidates to the studies need not have determined the scores gained, since with both a higher and a lower number of candidates applying for each vacancy there were instances of lower year results.

3. The control group, composed of students from a department with more demanding admission criteria deriving from more than twice the larger number of candidates (six for one entry) as well as from the fact of staging the mathematical exam, differed substantially in respect of the obtained scores only from the (weaker) 1997 and 2001 year groups. The control group did not differ markedly from the students from other groups, although its results were among the highest.

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