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Principles of a Geometry Program for Architecture — Experiences, Examples, and Evaluations

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Abstract. Geometry is a basic science for architecture. During the past centuries there were various evaluations of the role of geometry for architecture. Especially in the last decades the importance of descriptive geometry was pushed back more or less in most countries. Some countries try to keep the traditional descriptive geometry discipline, others replace it by training CAD-systems. Both ways have some understandable reasons, but both let missing important points of an actual geometry education. On the one hand it is not enough to keep only the traditional discipline, on the other hand geometrical knowledge is still a necessary condition for architecture, and this geometrical basis is not reached only by training CAD-systems. As responsibles for the education of the students of architecture we have to react to the challenges of our times and we have to be aware of the new, but also the remaining requests of teaching geometry for architecture.
In this paper the principles of such a geometry program will be presented and explained by examples from the past teaching experiences at University of Kaiserslautern in Germany. The concept is exposed in a textbook and exercise material for the students.

Key words: descriptive geometry, geometry, architecture, architectural design, teaching concept, new media

MSC 2000: 51N05

1. Introduction

Principles of the geometry program for architecture will be pointed out in this paper illustrated by examples and evaluated by reactions of the students. There are four fundamental principles which characterize our geometry program for students of architecture:

1. The geometrical representation methods have to be included in an understanding of visual perception and communication about spatial objects (Fig. 1a).

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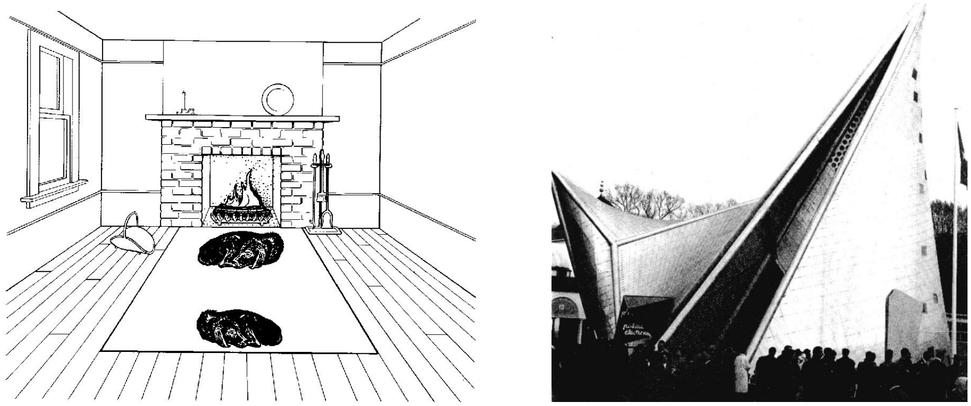

(a) (b)

Figure 1: (a) *"Ponzo"*-deception, (b) hyperbolic paraboloids, Philips-Pavillon, Brussels 1958 (Le Corbusier)

2. Geometrical forms and transformations provide the basis for architectural design (Fig. 1b).
3. The improvement of spatial visualization abilities is an overall goal of the geometrical education.
4. Geometrical knowledge forms the basis of working with CAD-programs and producing simulations and animations by new media (Fig. 2).

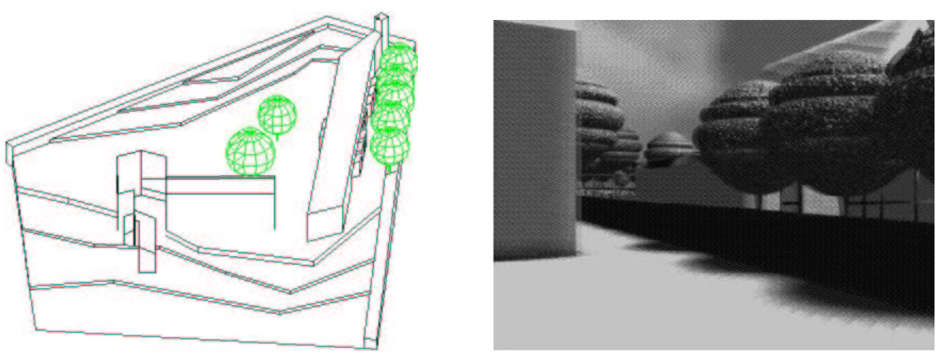

Figure 2: Geometric model in a CAD-program and stereo animation of a design project

The analyses of the relation between different geometrical space concepts and the architectural design processes are fascinating fields and become more and more important in recent architectural designs. The historical developments from Euclidean to projective, non-Euclidean or topological geometries find their correspondences in architectural developments. By showing the connections between the geometrical space concept and the architecture we become aware of the geometrical conditions of architecture.

The geometry program for architecture has also some didactical requests. Concrete and touchable spatial models or already built architecture help to achieve the spatial translation process between 3-D object and 2-D representation. By integrating geometrical tasks in architectural projects in collaboration with the teaching staff of other architectural disciplines we reach a high motivation for geometrical knowledge and the students become aware of the relevance of geometry for architecture.

The experiences with this geometry program during several years will be shown by some works of the students. Questionnaires at the end of the semester help to get a differentiated evaluation of the geometry program.

2. Geometry as part of Visual Science

In the architectural education there is a need to reflect visual perception and visual communication. In the architectural design process sketches, drawings and models are used to develop or specify the ideas and imaginations. At the end of the design process the result has to be presented the decision-maker or the performer of the planing with the suitable medium in each case. Drawings get the role of design and presentation media. The designed architecture is to be comprehended out of the representations. Visual perception influences the interpretation of the presented architecture by the drawings or other media. According to G.R. Bertoline [2] geometry is part of a new necessary science *"Visual Science"* besides spatial cognition and imaging. The teaching practice shows the demand for such a discipline, where geometry is combined with visualization methods and techniques in an artistic and technical way, psychology, philosophy, and other related fields. Geometry has to be taught as a part of such a visual science for architecture. Visual science is not introduced as a subject at our university, but we try to include the references and connections with the related fields in our geometry program for the students of architecture.

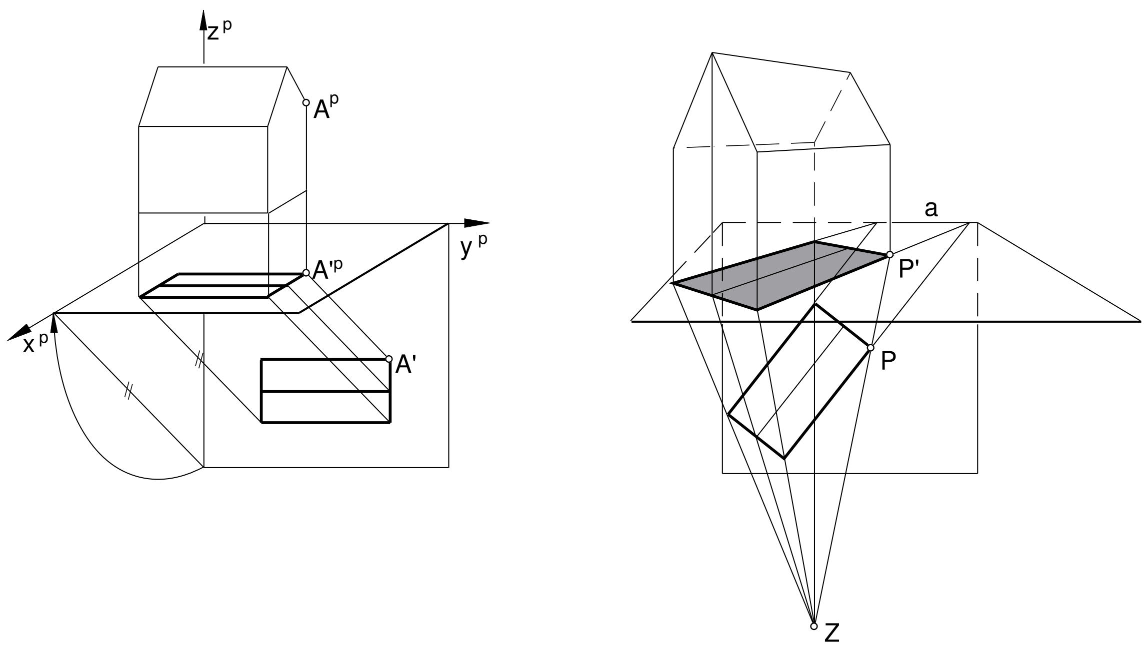

Figure 3: Developing the axonometric ground figure by an affinity and the perspective one by a collineation

In this context geometry has to provide the projection methods to achieve suitable 2-D representations of the 3-D architectural design, methods where it is also possible to reconstruct the spatial object out of the 2-D representation. In the basic geometrical education for architects we have to introduce the projection methods of parallel and central projection and to teach the characteristics of the multiview method as well as the characteristics of affinity and collineation used for example to get the axonometric or perspective ground figure of a house respectively (Fig. 3). The students get aware of the transformations of figures by parallel or central projection and understand the invariant characteristics of each transformation.

Such basic geometric background knowledge of the visual representation theory is supplemented by more complex geometrical detail knowledge in the advanced studies [6].

A favourite application of geometrical knowledge for visualization of architectural projects is for example photo-reconstruction and -montage. Fig. 4 shows two photomontages of a student's architectural design project from outside and inside where also reflections on the window are considered.

Figure 4: Photomontages of a student's architectural design project — outside and inside

3. Geometry as a background for architectural design

Geometry provides an understanding of forms, creation of forms, and transformations. Architectural design finds a starting point in this understanding. In the last semesters we worked with our students at the very beginning to create spatial configurations out of plane images. The connection between plane structural design and spatial design concepts can be studied by developing spatial configurations and architectural ideas out of an image. At the same time this design project helps to develop the spatial visualization ability. El Lissitzky developed similar ideas with his *"Proun"* paintings and spaces (pro unowis: for renewal of art). *" Proun"* is for him an interchange station between painting and architecture (Fig. 5). These new ideas of art and architecture were also described by K. Malevic (1927) in *"Suprematismus — Die gegenstandslose Welt"* [8]. El Lissitzky combined suprematism with constructivism.

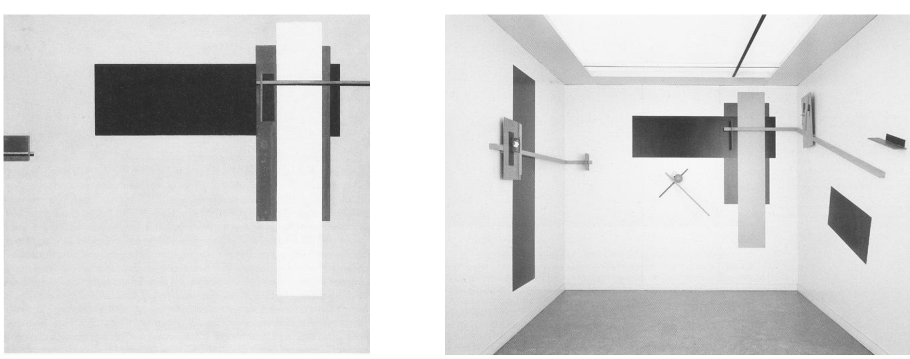

Figure 5: El Lissitzky: *"Proun"* — painting and space

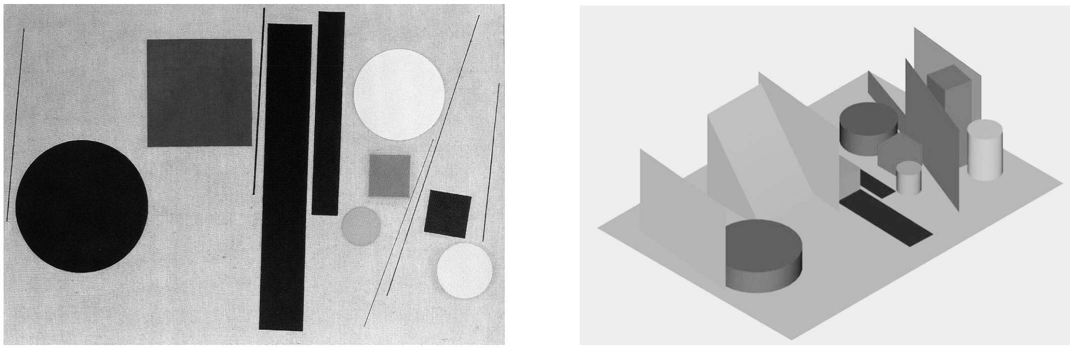

Figure 6: G. Fruhtrunk: *"Pour Jean Arp et Marguerite Hagenbach"*
and a derived spatial configuration

In the last semester we took the painting of G. Fruhtrunk [3] to develop spatial ideas for a living room (Fig. 6). This was the very first project for the students to create spatial imagination and at the same time to learn to work with the basic projection methods which descriptive geometry provides.

First the students created spatial ideas by sketching (Fig. 7). Then they had to work out their ideas in multiview and axonometric drawings.

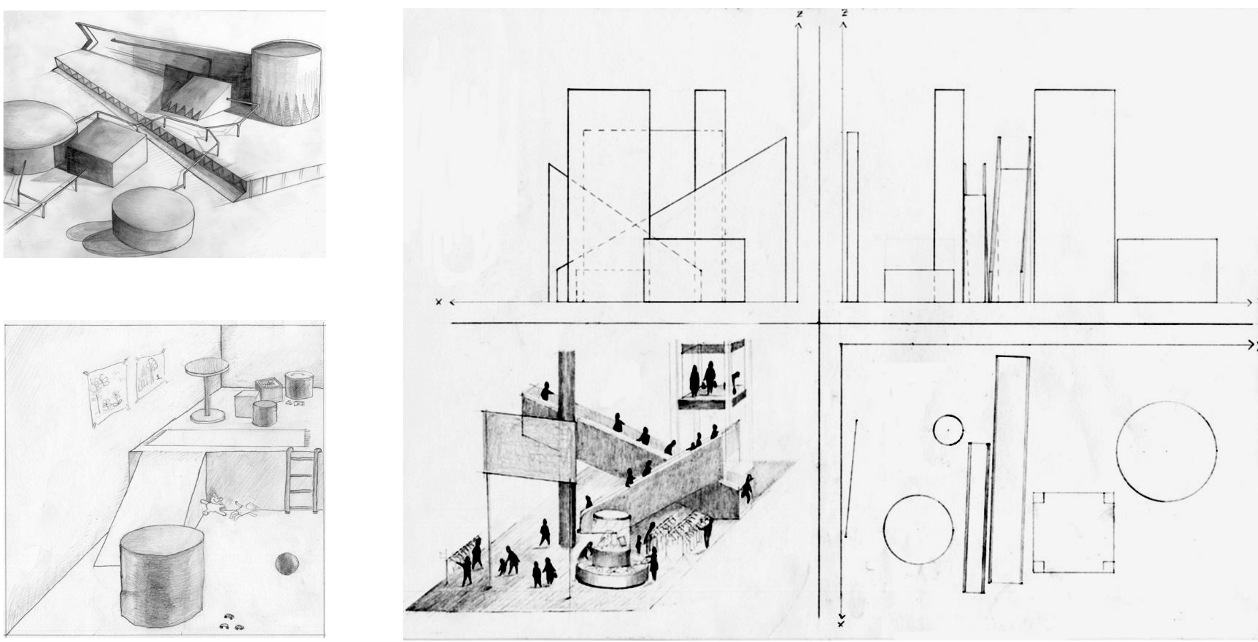

Figure 7: Sketches of spatial ideas from the image

This procedure helps to create spatial imagination out of a 2-D image. The various solutions of the students show that there are many possible interpretations of the 2-D image. The students get aware of the problem to fix a 3-D object by 2-D drawings. The methods of multiview drawings and axonometric drawings can then be applied to represent the ideas of 3-D configurations (Fig. 8).

These are basic examples to integrate geometry in architectural design. The creating conditions and characteristics of forms and shapes are wide application fields for geometry. Form experiments by models and 3-D modelling software are important for developing new architectural forms. Also other geometrical space concepts like topological space are able to

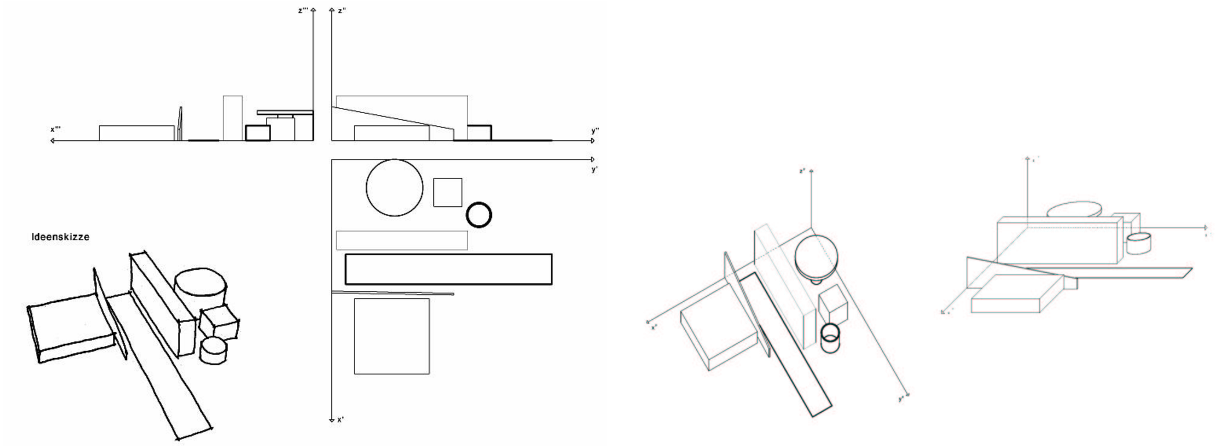

Figure 8: Sketch, multiview and axonometric drawings of a spatial configuration

give ideas for new architectures as for example Ben van Berkel and Caroline Bos [1] with their *"Moebius"* house.

4. Elements of the geometry program

The Geometry course consists of three parts: lectures, labs, and homework projects. In the lectures the theoretical background is presented, always explained by examples. The

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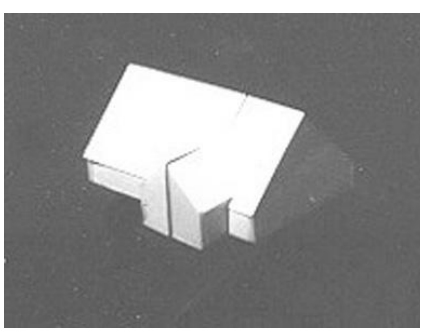
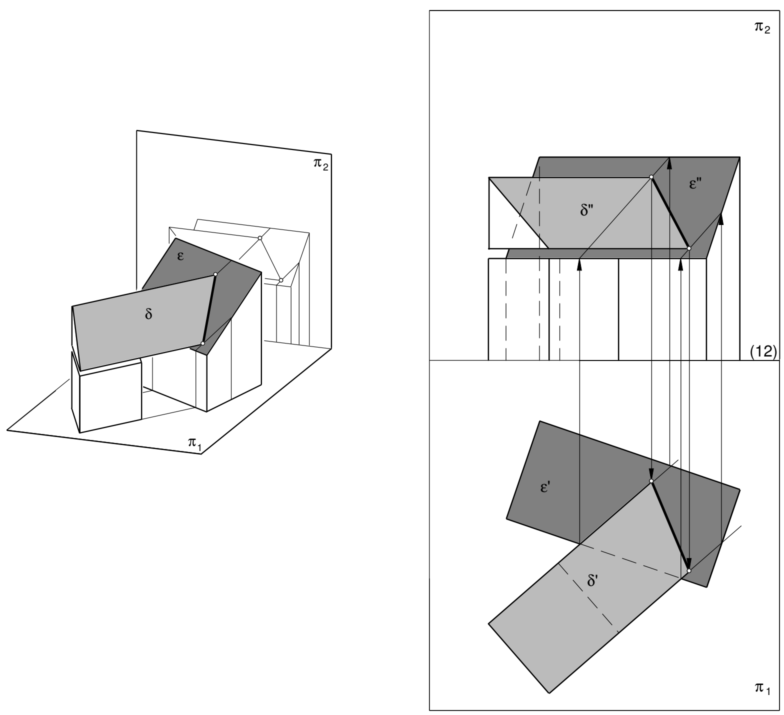


(b)



(c)

Figure 9: (a) Architecture photo, (b) model of an intersection problem, (c) basic geometrical task of the intersection between roof planes

textbook *"Geometrische Grundlagen der Architekturdarstellung"* (Geometric Foundations of Architectural Representations) [4] accompanies the students in the presented topics. The geometrical theory is illustrated by touchable and virtual models, axonometries, photos of built architecture, and finally exemplified by drawing examples. In Figs. 9–11 the various elements of the geometry program are presented along the topic intersection of planes.

After understanding the geometrical problem of the intersection of two planes as an architectural spatial problem students get the motivation and the spatial imagination to look for a solution. The problem can then be reduced to the basic geometrical task. The students have always be encouraged to create spatial imaginations when they solve the geometrical tasks. Therefore the architectural object remains also visible in the basic geometrical tasks (Fig. 9c).

In the labs the students get increasing complex problems to which they have to transfer the basic geometrical solution they have learned in the lectures. These examples for transfer work are most important for the learning process. After the presentation of the problem the students try to solve it for their own. We try to support them individually in this process, but we have not enough teaching persons to reach a didactically optimal situation. Fig. 10 shows such a task in the labs where a prism intersects a pyramid.

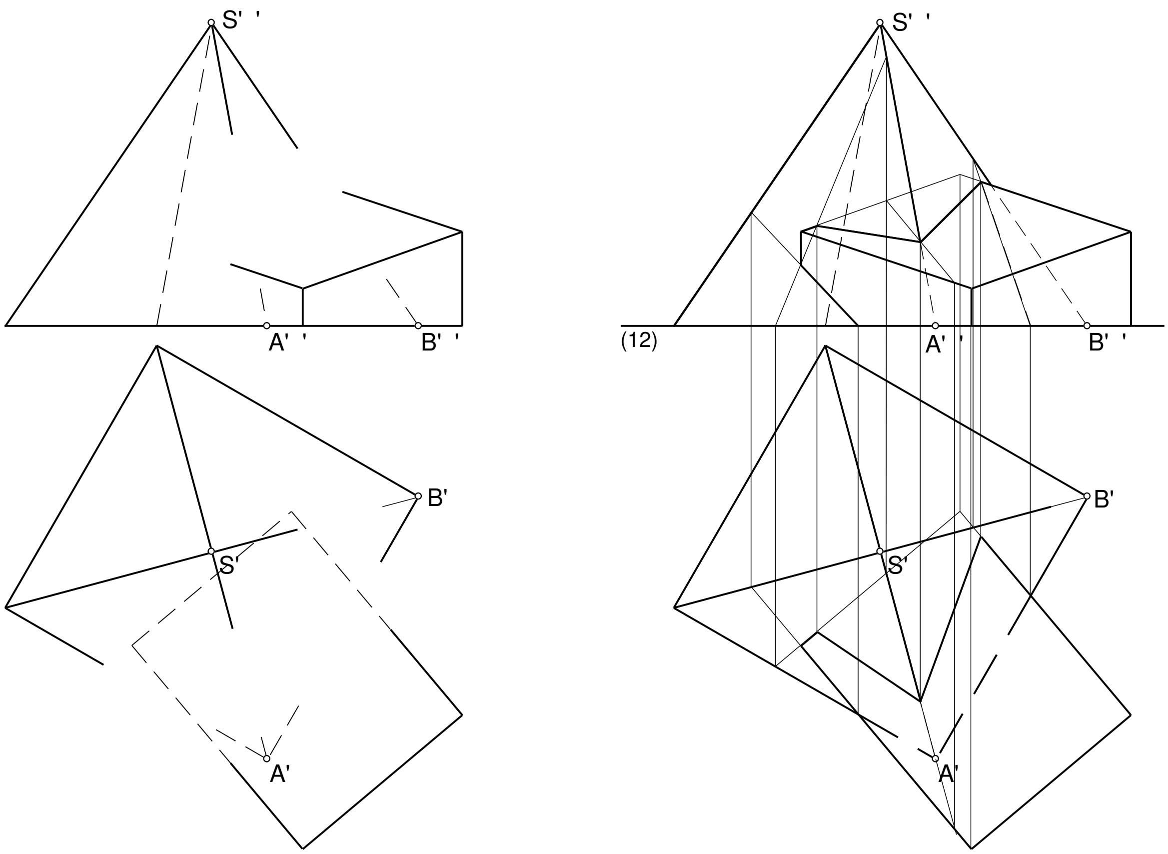
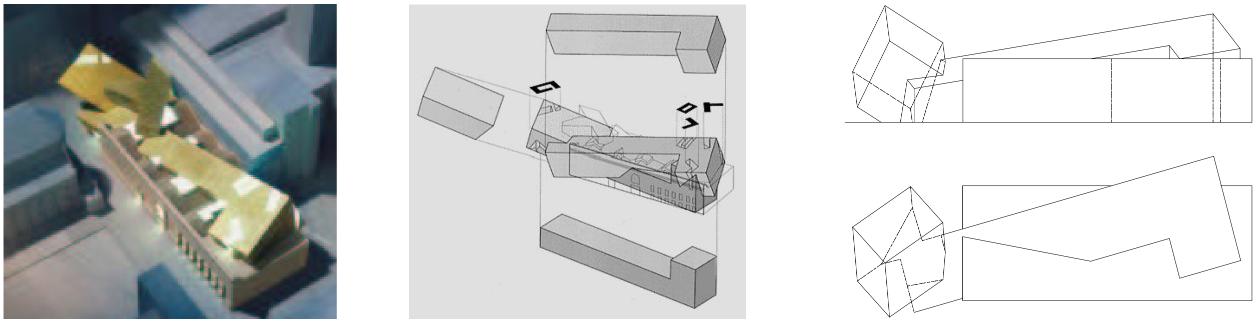


Figure 10: More complex task in the labs for transfer work

During the semester the students get 4–5 homework projects where they have to apply the learned methods to an architectural problem. The last project always stands in context with their design project in collaboration with the teaching staff of other architectural sections in our department. In winter semester 2000/2001 we applied the geometric problems to various museum buildings. The problem of the intersection of planes (Fig. 11c) was applied to the Jewish Museum in San Francisco by Daniel Libeskind [7]. Figs. 11a and b show the model and the visualization of the design conception which is derived from the Hebrew word *l'chaim* with the two letters of *chai*, which means "to live".



(a) Model (b) Design conception (c) Intersection problem solved by students as a homework project

Figure 11: Jewish Museum in San Francisco by Daniel Libeskind

5. Evaluation of the geometry program

At the end of the first semester the students were asked in a questionnaire about their experiences and assessment of the geometry teaching program. 55 students of architecture filled out the questionnaire. Some of the results are listed in Table 1.

	very helpful	*partly helpful*	*hardly helpful*	*not at all*
drawing examples in the lectures	32	18	5	0
spatial visualization is supported by models	31	15	5	2
by axonometries	17	32	5	1
by examples from architecture	10	28	16	1
by sketches	12	30	9	1
drawing examples in the labs	31	24	0	0

Table 1: Frequencies of answers in the questionnaire

The answers show that it is especially helpful and necessary for the students in the learning process to draw examples for their own. The students expressed the position that it is very important for them to work on the exercise material independently and then get supported in finding and working out the solutions. Our method in the labs to distinguish *two phases* in the learning process, first looking for an approach to the problem by the students for themselves and then presenting them a way to the solution and the solving procedure including their perceived problems, was confirmed by the students in the questionnaire. Besides the geometrical drawings we try to support and develop the spatial visualization ability by touchable and virtual models, axonometries, photos of architecture, and sketches. Especially the touchable models were assessed as very helpful for the spatial visualization processes.

For assessing the spatial visualization abilities, the students were tested by the MRT (Mental Rotation Test) of Vandenberg and Kuse [9] in pre-tests at the very beginning of their studies and in post-tests at the end of the first semester in descriptive geometry lectures and labs. With this pre- and post-testing we can have a look on the development of the spatial visualization abilities levels after one semester taking part in the geometry program. The results in pre- and post-tests, divided in men and women, are presented in Table 2.

	MRT pre-test	*(Std.Dev.)*	*MRT post-test*	*(Std.Dev.)*
women $(n = 28)$	61.3393	(16.9077)	76.4286	(14.3925)
men $(n = 46)$	80.5435	(13.8936)	90.7065	(8.5904)

Table 2: Mean pre- and post-test results in percent correct

The results are shown separately for men and women because a previous research and also this one determined significant gender differences in spatial visualization abilities. The students started in winter semester 2001/2002 on a higher level in the pre-tests as in a previous study where we compared test results of our university with Cracow University of Technology, Poland, and Michigan Technological University, USA [5]. There it was obvious that gains in the tests depend on the pre-test levels of the students. It is not such a high gain reachable when the student start already at a high level in the pre-tests. Therefore the absolute gains in the test scores can not be a measurement for the development of spatial visualization abilities. The gains depend strongly on the pre-test levels. Fig. 12 shows the comparison of pre-/post-test results in two semesters 1999/2000 (48 women, 62 men) and 2001/2002 (28 women, 46 men) of our first semester students.

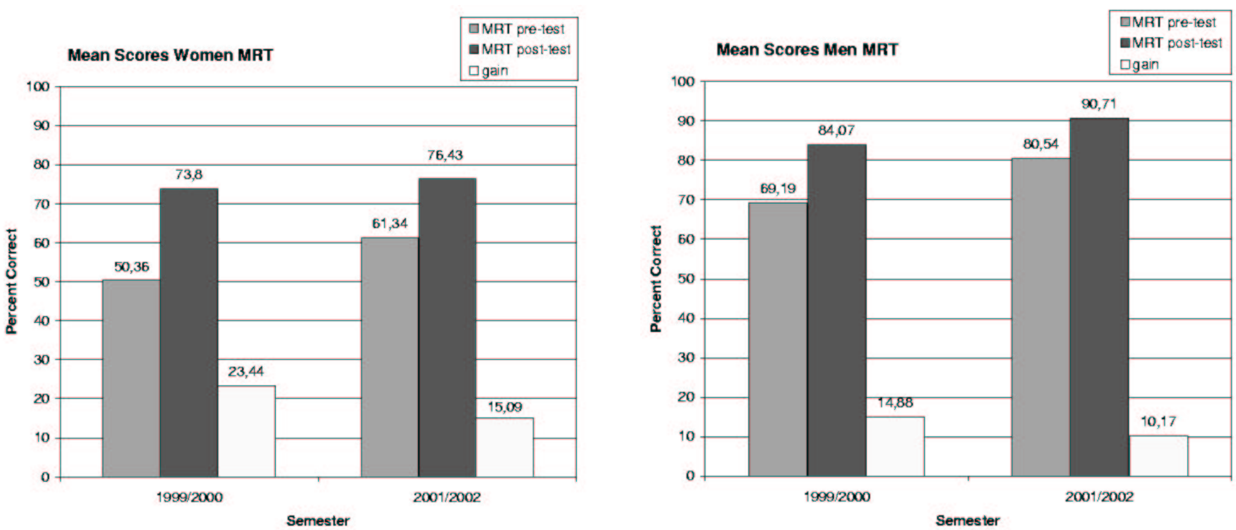


Figure 12: Mean MRT pre-/post-test and gain scores in percent correct in 1999/2000 and 2001/2002

In all cases the gain scores in the MRT were statistically significant as analyzed by a dependent t-test ($p < 0.0001$). Women reached higher gains in the MRT but there are still significant gender differences in the post-tests ($p < 0.0001$). In semester 2001/2002 women did not even reach the pre-test level of men in the post-tests. However, women improved their spatial abilities not so much as it is desirable. Our group sizes in lectures with $n = 140$ and in labs with $n = 70$ are too high for making an optimal individual support in the solving processes and especially this individual working method was accessed as very important by the students in the questionnaire. Supporting the students by models in their spatial visualization processes as the students assessed as very helpful (see Table 1) also requires smaller group sizes. To realize an optimal teaching situation we need more teaching persons in our section.

6. Conclusions

There are new challenges for geometry in the field of architecture. As part of a visual science geometry provides the projection methods to achieve 2-D representations of the 3-D architectural design. Geometry gives in this context also the foundations for simulations and animations by new media. Knowledge about geometrical forms and transformations as well as different geometrical space concepts form background for architectural design. Spatial visualization abilities can be developed by working on spatial geometrical problems with the aid of models and drawing examples. The spatial imagination gets supported by referring to concrete architecture. Different parts and elements in the geometry program set up on each other, help to optimize the learning process of the students.

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