Journal for Geometry and Graphics
Volume 8 (2004), No. 1, 117–126.

A Mental Cutting Test Using Drawings of Intersections

Emiko Tsutsumi

School of Social Information Studies, Otsuma Women's University
2-7-1, Karakida/Tama, Tokyo 206-8540, Japan
email: tsutsumi@otsuma.ac.jp

Abstract. In order to examine the role of information processing in recognizing 3D-objects using 2D-projection drawings, a standard paper-pencil MCT was administered to female students, who were instructed to draw intersecting lines on the projection chart of a test solid before selecting alternatives. As a result, it is quite possible that the ability to draw intersections itself is related to the score of the MCT, as well as to spatial ability. However, it did not seem possible to stimulate the sense of recognizing 3D-objects through the process of drawing intersections directly on to the test solids. Thus we concluded that most of the subjects who made low scores in the MCT, couldn't envision the space itself that contained three-dimensional objects when they observed projection drawings.

Key Words: Mental Cutting Test, intersection, spatial ability

MSC 2000: 51N05

1. Introduction

In recent years, 3D spatial abilities have received much attention and the *Mental Cutting Test* (hereafter MCT), a sub-set of the CEEB Special Aptitude Test in Spatial Relations [1] (1939), has been used by Suzuki et al. [3] to measure spatial abilities in relation to graphics curricula.

The *'standard MCT'* consists of 25 problems. In each problem, subjects are given a perspective drawing of a test solid, which is to be cut with a hypothetical cutting plane. Subjects are then asked to choose one correct cross section from among 5 alternatives. There are two categories of problems in the MCT [3]. Those of the first category are called *'pattern recognition problems'*, in which the correct answer is determined by identifying only the pattern of the section. The other are called *'quantity problems'* or *'dimension specification problems'*, in which the correct answer is determined by identifying not only the correct pattern but also the quantity in the section, e.g., the length of the edges or the angles between the edges.

ISSN 1433-8157/$ 2.50

In the standard MCT, each of the test solids is given as a perspective drawing. A projection drawing is a chart consisting of a set of lines on a two-dimensional surface that represents an image of a three-dimensional object, which by its nature cannot exist on a 2D-surface. This paradoxical nature of a projection drawing may make it difficult to recognize.

The first barrier in recognizing the test solid from the perspective drawing is judging and clearly recognizing the constructional elements of the test solid such as vertex, edge and plane geometrically. Most of the test solids in the MCT problems have relatively unusual shapes, although they may have some common shapes in part such as a rectangular parallelepiped or triangular prism. Thus it might be difficult to recognize the test solids geometrically even if they have been recognized topologically.

Next, a complicating matter is the fact that it is extremely difficult to recognize the relative location of the cutting plane vis-a-vis the solid. Tsutsumi et al. reported in [4] that low scoring subjects did not seem to be able to correctly recognize the test solid and its cutting plane, and were unable to form correct images of objects, even when they used the stereographic system in the MCT.

Therefore, the question remains as to how subjects recognize the test solids and the sectional views. One apparently effective method of analyzing the problem solving process is to make the test subjects produce an auxiliary sketch. If the process of drawing intersecting lines directly onto the test solids is added to the MCT, it might be possible to stimulate the sense of recognizing a 3D-object. In this study, the standard paper-pencil MCT was conducted to female students who were instructed to draw intersecting lines on the surface of a test solid before selecting an answer.

2. Methods

2.1. MCT using intersection drawings

A standard paper-pencil MCT was conducted, in which the subjects were instructed to draw intersecting lines on the surface of a test solid with a red ball-point pen before selecting alternatives (hereafter *'drawing MCT'*, see Fig. 1).

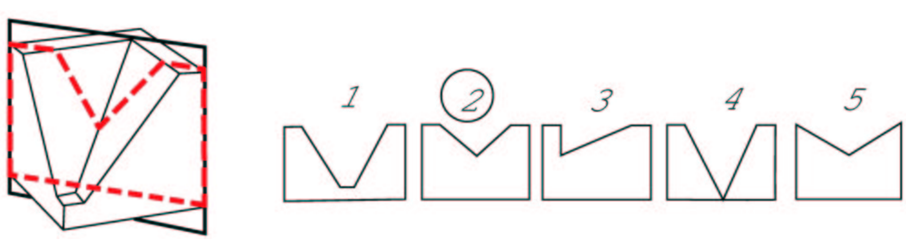

Figure 1: Example of the 'drawing MCT'

2.2. Subjects

Fifty-five subjects participated in the experiments in December 2000. All participants were first-year female students at the School of Social Information Studies, Otsuma Women's University. None of the subjects had taken any courses on graphics before.

2.3. Reference data

The standard MCT was conducted to the same subjects 8 months earlier in order to provide reference data.

3. Mean scores and unanswered problems

3.1. Mean scores

Mean scores for the drawing MCT and the standard MCT were 10.49 ($SD = 3.10$, $N = 55$) and 11.62 ($SD = 3.87$, $N = 55$), respectively. Though there was no significant difference between the mean scores of the two tests, the correct response rate in the drawing MCT tended to be lower than that in the standard MCT.

3.2. Unanswered problems

The number of unanswered problems was analyzed in order to examine whether the slightly lower mean score of the drawing MCT was caused by a shortage of time which might have been induced by making subjects draw the intersecting lines on the test solids. The unanswered problems increased in number in the late stages in both tests. Concerning the last 6 problems, there were two subjects who weren't able to answer them in the standard MCT, while there were seven subjects in the drawing MCT who had not reached that part of the test.

In the drawing MCT, the additional task of drawing intersections was required to solve the test problems. Thus, the slightly lower mean score of the drawing MCT may have partially been caused by a shortage of time. However, there might exist other factors for the lower mean score, because even in the early stage, there were some problems, like nos. 1, 2, 5, 7, 8, 9, and 13, for which correct response rates were fairly lower than in the standard MCT (Fig. 2).

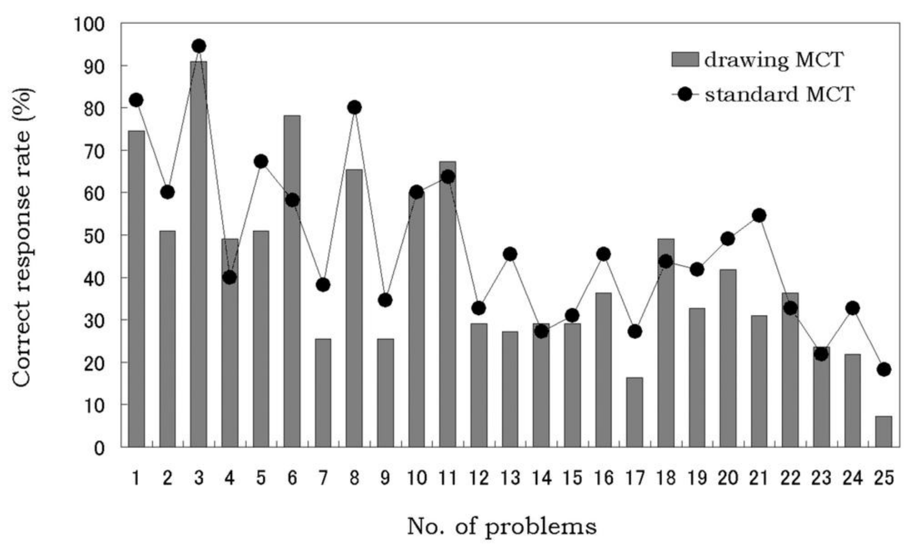

Figure 2: Correct response rates for each problem

4. The number of intersection drawings for each problem

There were some patterns seen in the relation between responses and intersection drawings in the test problems (Fig. 3), i.e.,

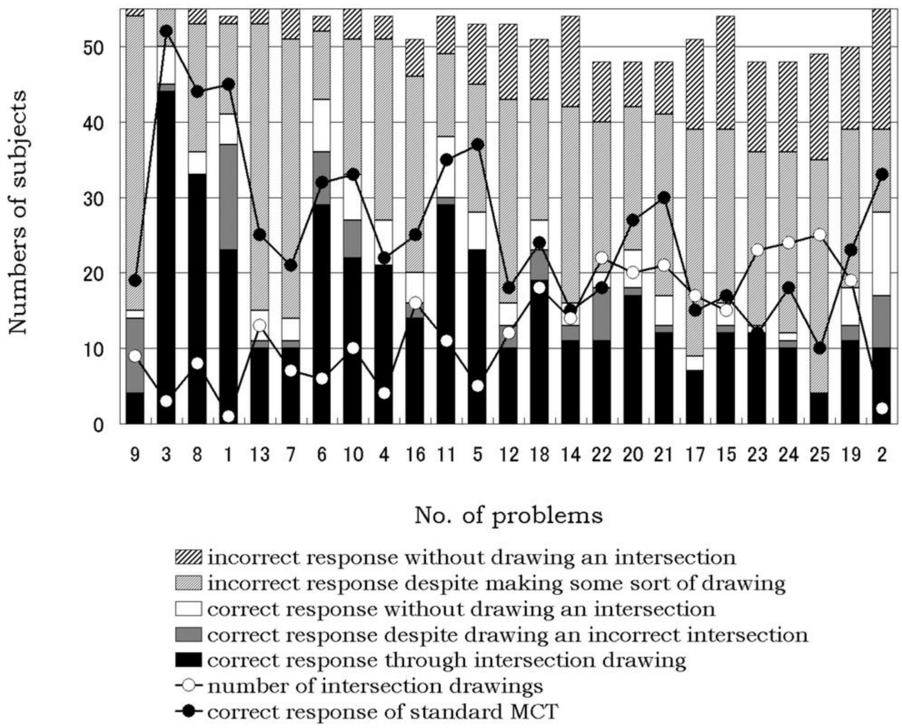

Figure 3: Numbers of correct responses and intersection drawings for each test problem

(1) making a correct response through a correct intersection drawing,

(2) making a correct response despite drawing an incorrect intersection,

(3) making a correct response without drawing an intersection,

(4) making an incorrect response despite making some sort of drawing, and

(5) making an incorrect response without drawing an intersection.

The correlation coefficient between the number of intersection drawings and correct responses for 25 problems was 0.423, indicating an intermediate correlation.

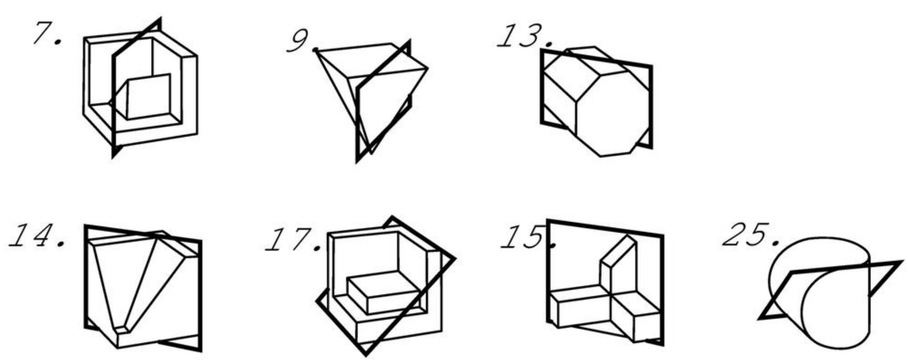

Figure 4: Specific problems with intersection drawings

Incidentally, detailed inspection of the figures revealed several specific problems. First, problems 9, 13, and 7 got very few correct responses, though more than 90% of the subjects drew intersections. These problems showed low rates of correct responses even in the tandard MCT. In problem 7, as shown in Fig. 4, there was an opening space behind a triangle prism that was to be cut by a cutting plane. Problem 13 was a dimension specification problem.

Problems 14, 17, 15, and 25, where many subjects couldn't draw the intersections or couldn't draw correct intersections, also had very few correct responses. Most of the subjects failed to recognize the relative location of the cutting plane with respect to the test solid (Fig. 4). Although nos. 14 and 15 were given with vertical cutting planes, the solids included unusual shapes near the cutting planes i.e., an inverted pyramid-shaped opening space (no. 14), a rectangular parallelepiped of unknown size (no. 17) and three perpendicular intersected solids (no. 15).

It was found that subjects failed to recognize the relative location of the cutting plane with respect to the test solid in some problems. Subjects also were influenced by the unusual shapes of the solids. Thus Chapter 6 will examine the inaccurate schemes in drawing the intersections.

5. The number of intersection drawings for each subject

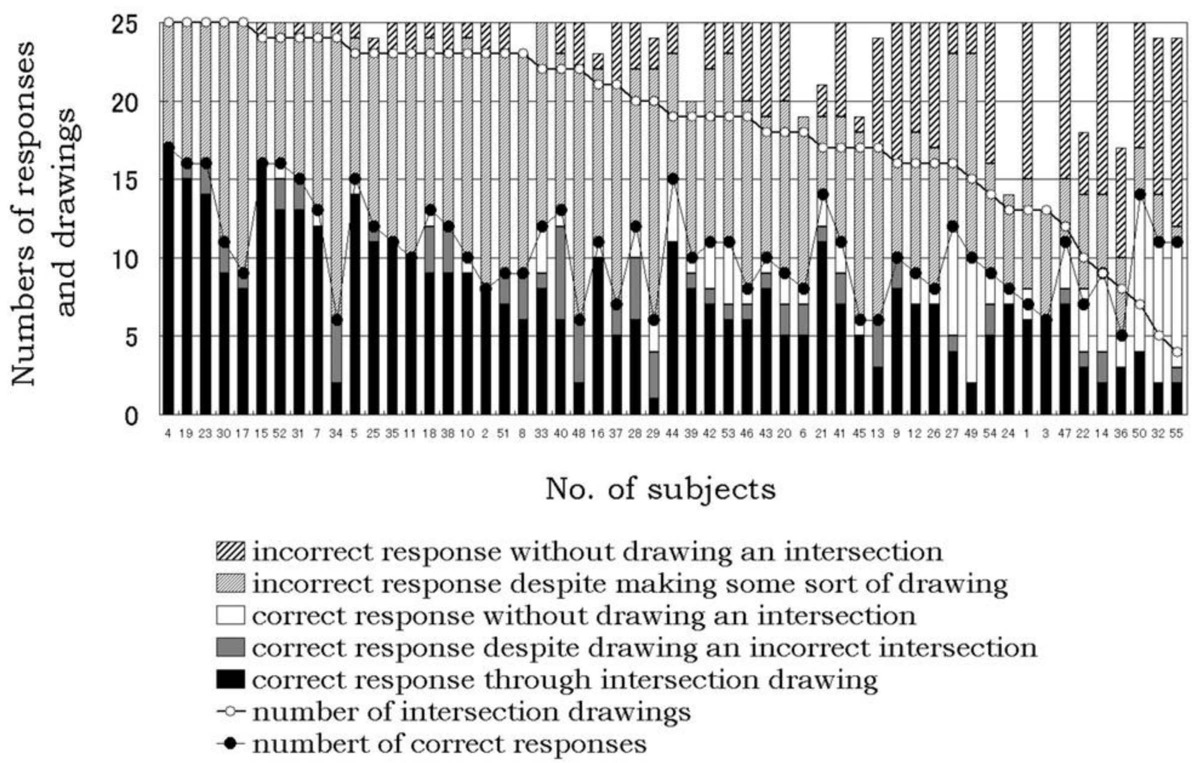

Figure 5: Numbers of correct responses and intersection drawings for each subject

Fig. 5 shows the numbers of responses and intersection drawings for each of the subjects. The correlation coefficient between the number of intersection drawings and correct responses for 55 subjects was 0.34, indicating an intermediate correlation. Several subjects can be seen to have distinctive characteristics. For example, subjects 34, 48, 29, and 13 showed extremely low rates of accurate responses though they could draw a fairly large number of intersections, exclusive of the accuracy of responses. In contrast, subjects 50, 32, and 55 showed relatively high accurate responses though they drew a small number of intersections (only 7, 5, and 4 problems, respectively).

If the results of these 7 special cases were omitted, the correlation coefficient would increase up to 0.61, indicating a relatively high correlation. Thus, it is quite possible that the ability to draw intersections may be closely related to the score of the MCT, as well as to spatial ability.

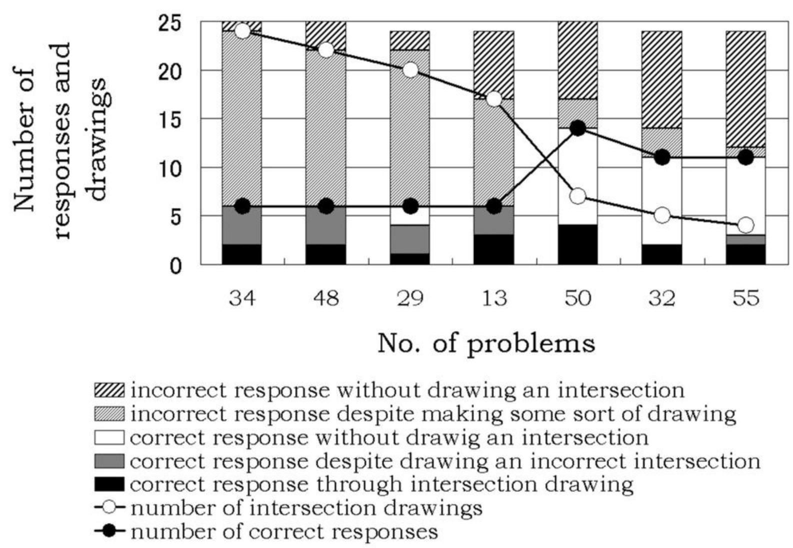

Figure 6: Numbers of correct responses and intersection drawings for the special-case subjects

Fig. 6 shows the relation between the numbers of correct responses and the intersection drawings for the 7 special-case subjects mentioned above. It was observed that the subjects 34, 48, 29, and 13, who had very few accurate responses even though they could draw a fairly large number of intersections, tended to draw intersecting lines either on the contours and edges of the test solid (Fig. 7a, b), outside the solid (Fig. 7c), or inside the solid (Fig. 7d). The results show that they made inaccurate intersections because they tried to draw intersections based on the prominent shapes on the projection chart. In other words, it appears that they didn't have a sense of depth or couldn't recognize the test solids as correct 3D-shapes.

In the case of subjects 50, 32, and 55, who only drew a few intersections, the rates of accurate responses ($\text{mean}_{\text{nos.50,32,55}} = 12.0$) for all 25 problems exceeded the mean value for all the subjects ($\text{mean}_{\text{all}} = 10.49$), and the amounts of correct responses which were obtained without drawing intersections (10, 9, and 8, respectively) were much higher than the mean value for all the subjects (mean $= 1.82$). This shows that they could make correct responses without drawing intersections using only their imagination to some extent, although the instructions specifically said to use intersection drawings. Even among those subjects, however, the problems that were answered correctly without drawing intersections were limited to those which others had answered correctly by drawing intersections. As to the problems to which most of the subjects had made inaccurate responses, no significant difference was found between those subjects and the other subjects.

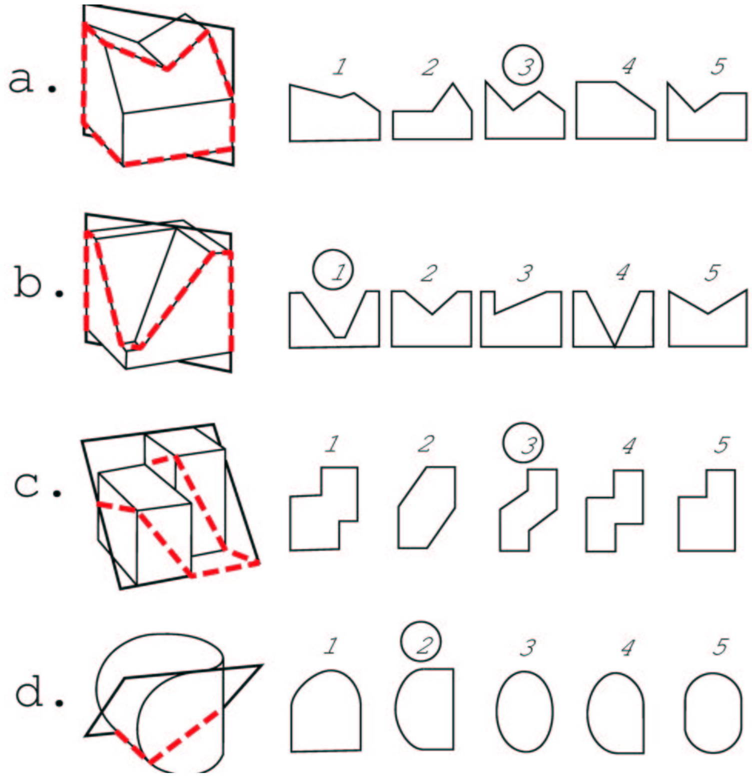

Figure 7: Samples of drawn intersections and selected alternatives of the special-case subjects 34, 48, 29, and 13

6. Inaccurate schemas for drawing the intersections

6.1. Analysis of schemes for drawing intersections

The schemes for drawing intersections were analyzed because it was found that some differences in the results of intersection drawings existed due to the characteristics of the test solids and the subjects. Fig. 8 shows samples of the drawings and the selected answers that represent interpretations of each scheme.

(1) Passing over the distinctive marks on the solid

- Drawing intersections on the contour or edges of the test solid (Fig. 8a).
- Drawing intersections through a pseudo-point which is represented on a projection plane as the intersecting point of the outline of the cutting plane and edge of the test solid (Fig. 8b).
- Drawing intersections through a pseudo-point which is represented on a projection plane as intersecting point of edges of the test solid (Fig. 8c).

(2) Drawing parallel lines

- Drawing intersections parallel to the edge of a plane surface of a test solid which is not parallel to the outline of the cutting plane (Fig. 8d).

(3) Failure to identify the quantity

- Failure to identify the quantity of the section though having been drawing correct intersections (Fig. 8e).

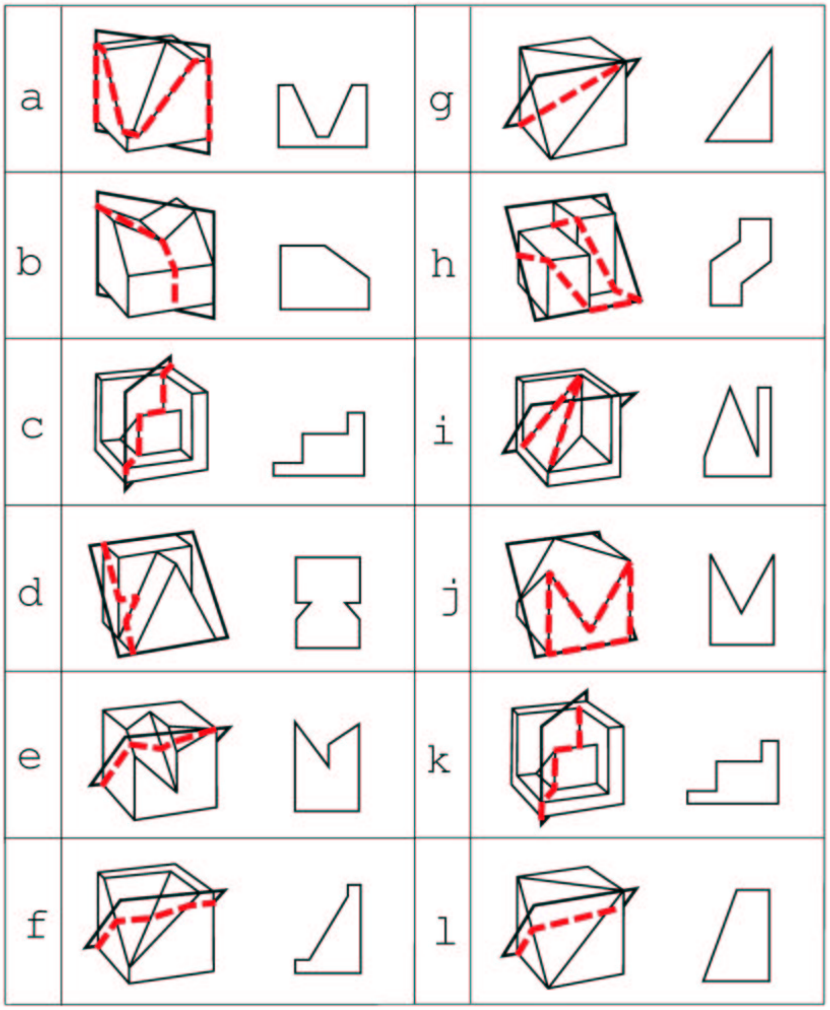

Figure 8: Samples of drawings and selected alternatives

- Neglecting the quantitative difference between the drawn intersections and alternatives (Fig. 8f).

(4) Deficiency of a sense of depth

- Drawing intersections that disregarded some edges of the solid (Fig. 8g).
- Drawing intersections outside or inside the solid, i.e., which don't make contact with the surface of the solid (Fig. 8h).
- Obviously disregarding the relative location of the cutting plane with respect to the test solid (Fig. 8i).
- Assuming one of plane surfaces of the solid to be intersections (Fig. 8j).

(5) Neglecting concavity in a solid

- Neglecting a space behind a salient in the foreground (Fig. 8k).

(6) Neglecting a difference in direction

- Selecting an alternative which runs in a different direction from that of the intersections (Fig. 8l).

6.2. Discussion

It has been definitively shown that the drawn intersections had various characteristics and there were some inaccurate schemes being used when subjects couldn't judge the intersections adequately. Most of the inaccurate schemes included steps in which subjects tried to draw intersections using distinctive marks on the test solid such as contours, edges, pseudo points, or parallel lines, and many such subjects were lacking a sense of depth, i.e., they could not visualize a three-dimensional object hidden in a solid, for example. From that point of view, it appears that most of the subjects who made low scores in the MCT couldn't imagine the space itself which contained three-dimensional objects when they observed projection drawings, and they seemed to have rather vague criteria for judging depth.

7. Summary and conclusion

In order to examine the role of information processing in recognizing 3D-objects using 2D projection drawings, a standard paper-pencil MCT was conducted to female students who were instructed to draw intersecting lines on the surface of a test solid before selecting alternatives. The main results were as follows.

(1) The mean score of the drawing MCT was slightly lower than that of standard MCT although there was no significant difference.

(2) The number of intersection drawings and correct responses showed an intermediate correlation ($R = 0.423$). Subjects failed to recognize the relative location of the cutting plane with respect to the test solid in some problems. Subjects also were influenced by the prominent shapes of the solids.

(3) When the results of certain subjects were omitted, the correlation coefficient between the number of intersection drawings and number of correct responses increased up to 0.61, indicating a relatively high correlation. This strongly indicates that the ability to draw intersections itself was related to the score of the MCT, as well as to spatial ability. On the other hand, it did not seem possible to stimulate the sense of recognizing 3D-objects through the process of drawing intersections directly on to the test solids. However, it might be possible that the subjects couldn't draw intersecting lines in 3D space by using projection drawings because they couldn't envision a correct image of 3D space.

(4) There were some schemes available for drawing intersections in case the subjects couldn't determine three-dimensional relations. Most of the inaccurate schemes included steps in which the subjects tried to draw intersections using distinctive marks on the solids, and many such subjects were lacking a sense of depth, i.e., the ability to visualize a three-dimensional object, which by its nature cannot exist on a 2D projection surface.

Thus we concluded that most of the subjects who made low scores in the MCT, couldn't envision the space itself that contained three-dimensional objects when they observed projection drawings. They seemed to have a somewhat underdeveloped sense of depth.

At first, it was assumed that if the process of drawing lines of intersection directly onto the projected test solids were added to the MCT, it might be possible to stimulate the sense of recognizing 3D-objects even when the subjects couldn't recognize the 3D space as a sensuous impression. However, in the case of low scoring subjects of MCT, they could judge and recognize the constructional elements of the test solid such as vertices, edge and plane distinctively to some extent, but they seemed to have difficulty in recognizing the test solid or drawing intersections on the perspective drawings, because they had had used rather vague criteria for judging the three-dimensional positioning.

Acknowledgement

The author would like to thank Prof. Kenjiro Suzuki of The University of Tokyo for his helpful discussions and constructive criticisms.

References

[1] *CEEB Special Aptitude Test in Spatial Relations*, developed by the College Entrance Examination Board, USA 1939.

[2] T. Saito, K. Shiina, K. Suzuki, T. Jingu: *Spatial Ability Evaluated by a Mental Cutting Test*. Proc. 7th ICECGDG, Cracow/Poland 1996, pp. 569–573.

[3] K. Suzuki, S. Wakita, S. Nagano: *Improvement of Spatial Ability through Descriptive Geometry Education* [Japanese]. Journal of Graphic Science of Japan **49**, 21–28 (1990).

[4] E. Tsutsumi, K. Shiina, A. Suzaki, K. Yamanouchi, T. Saito, K. Suzuki: *A Mental Cutting Test for Female Students Using a Stereographic System*. J. Geometry Graphics **3**, no. 1, 111–119 (1999).

Received August 1, 2002; final form February 23, 2004