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Rectification Note to the Paper "Gergonne and Nagel Points for Simplices in the n-Dimensional Space"

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Abstract. This is a short rectyfying note to the paper entitled "Gergonne and Nagel Points for Simplices in the n-Dimensional Space" and published in the Journal for Geometry and Graphics **4**, no. 2 (2000). The problem discussed in the paper concerns tetrahedra, which have Gergonne and Nagel points.

Key Words: 3-dimensional geometry, n-dimensional geometry, polar transformation, Gergonne point, Nagel point

MSC 2000: 51M04

1. Rectifying Note

In [2] the authors discussed the problem concerning tetrahedra, which have Gergonne and Nagel points.

The authors referred in this paper to the relation between a tetrahedron and a sphere inscribed into this tetrahedron in the 3-dimensional Euclidean space. In particular, the authors concluded that the only Gergonne tetrahedra are regular triangular pyramids. They also gave some conditions necessary and sufficient for a simplex to satisfy the Gergonne and Nagel property and generalized the obtained results for simplices in n-dimensional geometry.

The first conclusion has been questioned in the discussion on the international forum by the Internet. Prof. Aleksey A. ZASLAVSKY together with his pupils dealt with the same problem 2001. The results published on the Internet [3] show that not each Gergonne tetrahedron is a regular pyramid, as the authors state in [2]. One pupil of Prof. ZASLAVSKY, namely

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D. Kosov (Moscow), proved that a tetrahedron has a Gergonne (Nagel) point if and only if the products of cosines (sines) of half opposite dihedral angles are equal (so a tetrahedron which has both points is a regular pyramid). In 2003 two other pupils of Prof. Zaslavsky, namely M. Isaev (Barnaul) and V. Filimonov (Ekaterinberg), found another necessary and sufficient condition. This condition refers to the relation between the Torricelli point and a tangency point of the insphere. Next, M. Hajja and P. Walker described how to construct inspherical tetrahedra (cf. [1]).

Taking the above into consideration, the authors of the discussed paper [2] suggest that by the removal of the section starting with the words “The question arises ...” (p. 122) to the end of the section will rectify this discrepancy of the proof and will comply with [3].

References

[1] M. Hajja, P. Walker: *The Inspherical Gergonne Center of a Tetrahedron*. J. Geometry Graphics **8**, no. 1, 23–32 (2004).

[2] E. Koźniewski, R.A. Górska: *Gergonne and Nagel Points for Simplices in the n-Dimensional Space*. J. Geometry Graphics **4**, no. 2, 119–128 (2000).

[3] http://www.mccme.ru/olympiads/lktg/2003/treutetr.en/index.htm.

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