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Evaluation of Students' Spatial Abilities in Austria and Germany

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Abstract. The Mental Cutting Test was administered as a pre- and post-course test by students in Austrian and German universities. In these countries, almost half of all students start learning Descriptive Geometry when they are around thirteen years old. The subject groups were five Descriptive Geometry courses at universities in Austria and Germany. One reference group in Austria was also examined as a control. Among the five courses, students in two courses already had experience in fundamental Descriptive Geometry learning at the beginning of the course. The results were as follows:
(1) There were statistically significant differences between males and females in almost all groups except one.
(2) Apparent significant differences were observed between experienced and non-experienced groups.
(3) The problem solving process suggested the possibility that through the Descriptive Geometry education not only the intuitive spatial recognition ability, but also some logical thinking ability might be progressed, because experienced groups solved some of the difficult problems which certainly require the process of logical judgment.

Key Words: Mental Cutting Test, spatial ability, Descriptive Geometry education
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1. Introduction

In courses on graphics at the undergraduate level of study, 3-D spatial abilities have received much attention lately. In fact, for the past decade, several tests have been given to evaluate the spatial abilities of students. The MCT (a sub-set of CEEB Special Aptitude Test in Spatial Relations, 1939) was used by Suzuki et al. [1] (1990) for measuring spatial abilities in relation to graphics curricula. Since then, the improvement of students' spatial ability has been discussed quite often among graphics educators, and various experimental studies have started to evaluate the spatial abilities of students from the viewpoint of how they are related to the graphics education.

The Mental Cutting Test (hereafter, MCT) consists of 25 problems for which a full score is 25 and test time is 20 minutes. In each problem, subjects are given a perspective drawing of a test solid which is to be cut with an assumed cutting plane. The subjects are asked to choose one correct cross section among 5 alternatives (Fig. 1).

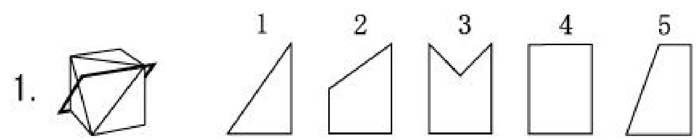


Figure 1: An example of the Mental Cutting Test

MCT is sometimes used as a one time test in order to give screening for the spatial ability of the students before starting graphics curricula. However, usually MCT is performed twice by the same student, at the beginning and the end of the course as pre- and post-course test. In this case MCT is used to evaluate the course effect in relation to the enhancement of spatial abilities.

Up to the present, not only the statistical results of the average scores but also various aspects of the problem solving process of the MCT have been analyzed. As a result, some strategies are becoming clearer which show how subjects solve the MCT problems.

Here we will show some results of MCT in Austria and Germany. In these countries, almost half of all students start learning Descriptive Geometry when they are about 13 years old.

2. Method

As shown in Table 1, the paper-pencil MCT was administered by students in five Descriptive Geometry courses (dg) at the Vienna University of Technology (TUW), the University of Applied Arts Vienna (UAK) and the Dresden University of Technology (TUD). Among the five "dg" courses, students in two courses had already learned fundamental Descriptive Geometry at the beginning of each course (TUW_dg_exp, UAK_dg_exp) because they are majoring in teaching Descriptive Geometry. The MCT has also been administered to the Design course students at UAK (UAK_ref) as a reference group.

We performed the pre-test at the beginning of the course and the post-test at the end of the same course. In case of the experienced group at UAK (UAK_dg_exp), we could only perform the pre-test.

3. Results of pre-tests

3.1. Mean scores

Table 1 shows the mean MCT scores and standard deviations of the pre-test in five "dg" courses and the reference group. The mean values are about 18.7, 18.3 for experienced groups, 15.3, 14.5, 15.4, for non-experienced groups and 13.7 for the reference group, respectively. Here, the full score is 25.

Concerning the inter-group differences, we could observe apparently significant differences between the experienced group (TUW_dg_exp) and non-experienced groups. Significant levels, i.e., probability values, are all less than 1%.

From these results, we may consider that the Descriptive Geometry education has enhanced the spatial abilities evaluated by spatial tests such as MCT, although the results are calculated using cross-sectional data in this case.

Table 1: Group mean scores and statistical differences

Course		*ALL*	*Female*	*Male*	*Sex difference*	*Group differences*[3])
TUW_dg_exp[1])	×	18.67	15.67	20.92	**	
	SD	4.13	3.24	3.23		
	N	21	9	12		
UAK_dg_exp[1])	×	18.29	17.11	19.17		
	SD	5.34	4.51	5.92		
	N	21	9	12		
TUW_dg	×	15.28	13.76	16.63	**	**
	SD	5.27	4.76	5.34		
	N	241	113	128		
UAK_dg	×	14.48	13.28	17.33	($P < 0.1$)	**
	SD	5.75	5.54	5.45		
	N	29	18	9		
TUD_dg	×	15.39	13.98	17.77	**	**
	SD	5.15	4.88	4.63		
	N	194	103	77		
UAK_ref[2])	×	13.67	13.50	14.00		**
	SD	5.40	5.82	3.27		
	N	24	18	4		

1) Course for teaching profession of "Elementary Geometry" and "Descriptive Geometry". The subjects have already learned fundamental DG.

2) Reference group (Design course)

3) Differences between the mean score of TUW_dg_exp and other groups.

** $P < 0.01$

3.2. Sex differences

As Table 1 shows, the mean scores for females seem to be apparently lower than those of males. The sex difference for each group score was about 5.25, 2.06, 2.86, 4.06, 3.79, and 0.50, respectively.

Concerning results of t-tests, there were statistically significant differences between male and female in almost all "dg" groups except for UAK_dg_exp.

4. Results of pre-test and post-test

4.1. Mean scores of pre-test and post-test

Fig. 2 shows the differences in scores between pre-tests and post-tests. As the pre- and post-tests were administered to the same students, the results could be treated as a longitudinal data. Namely, we can investigate the course effect directly as an experience of each student.

Except for the reference group, i.e., UAK_ref, post-test scores have increased and moderate significant differences were found between both tests. Only in the case of UAK_dg a distinct statistical difference was found between pre-test and post-test. There was no increase for the control group.

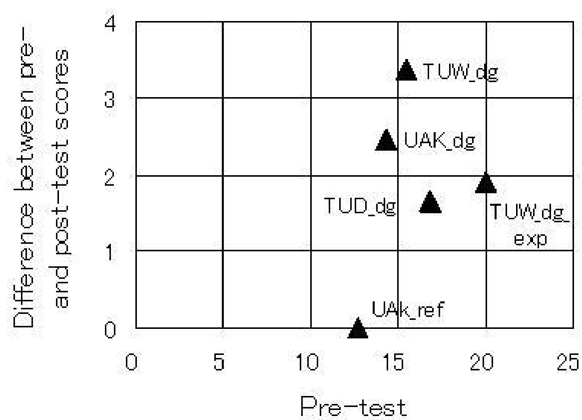


Figure 2: Pre-test score and difference

4.2. Effect of Descriptive Geometry education

In order to investigate the effect of the Descriptive Geometry education, the post-test mean scores of "dg" courses were compared with the pre-test mean score of the experienced group (TUW_dg_exp). As far as the knowledge of Descriptive Geometry is concerned, the knowledge level of the students at the end of the "dg" course should be almost the same as that of the experienced students at the period of pre-test.

Table 2 shows that there were no significant differences between the pre-test result of the experienced group and post-test results of "dg" courses, except for UAK. But even in case of UAK, the statistical difference was not so distinct ($P < 5\%$). In case of the control group, i.e., UAK_ref, there was a considerable significant difference ($P < 0.1\%$).

The results may indicate the fact that by the Descriptive Geometry education for beginners the students normally enhance their spatial ability as evaluated by the MCT, and they

Table 2: Comparing the post-test mean score with the pre-test mean score of the experienced group (TUW_dg_exp)

Course	*Pre-test*	*Post-test*	*P (t-test)*
TUW_dg_exp	20.0	(21.9)	
TUW_dg	(15.4)	18.8	–
UAK_dg	(14.4)	16.8	∗
TUD_dg	(16.8)	18.4	–
UAK_ref	(12.7)	12.7	∗∗

∗∗: $P < 0.1\%$ ∗: $P < 5\%$ –: no significant difference

obtain a way of solving problems or strategies similar to the advanced learners of Descriptive Geometry.

4.3. Correct response rate

Fig. 3 shows the correct response rates of 25 problems. The upper graph shows the results of pre-tests and the lower graph shows that of post-tests. The problems are sorted in the order of correct response rates of the experienced group which is shown on the white surface, supposing that the experienced group has a rather high spatial ability. Striped and black bars show the correct response rates of the "dg" courses at TUW and TUD, respectively. White dots and lines show that of the "dg" course at UAK.

The results of the post-tests are also sorted in the order of the pre-test results of the TUW experienced group.

These graphs show that although the tendency of each correct response in the pre-test differs from that of the experienced group, i.e., TUW_dg_exp, the results of the post-test are similar to that of the experienced group. This means that after learning Descriptive Geometry, the students not only increased their scores but might obtain a way of solving problems or strategies similar to those of the advanced learners of Descriptive Geometry.

In case of UAK, the problems with extremely low correct response rates in the pre-test have increased their scores in the post-test, e.g., problems no. 4, 24, 12, 17, 21, and 25.

In case of the "dg" courses at TUW and TUD, the correct response rates of slightly difficult problems have increased up to those of the TUW experienced group, however, those of relatively easy problems did not increase so much.

These results may suggest the possibility that through Descriptive Geometry education, not only the intuitive spatial recognition ability, but also some logical thinking ability may be progressed. We can surmise that for solving some of the difficult problems certainly a process of logical judgment is needed.

In case of the reference group, which was not shown in this graph, some correct response rates decreased and no regular tendency could be found.

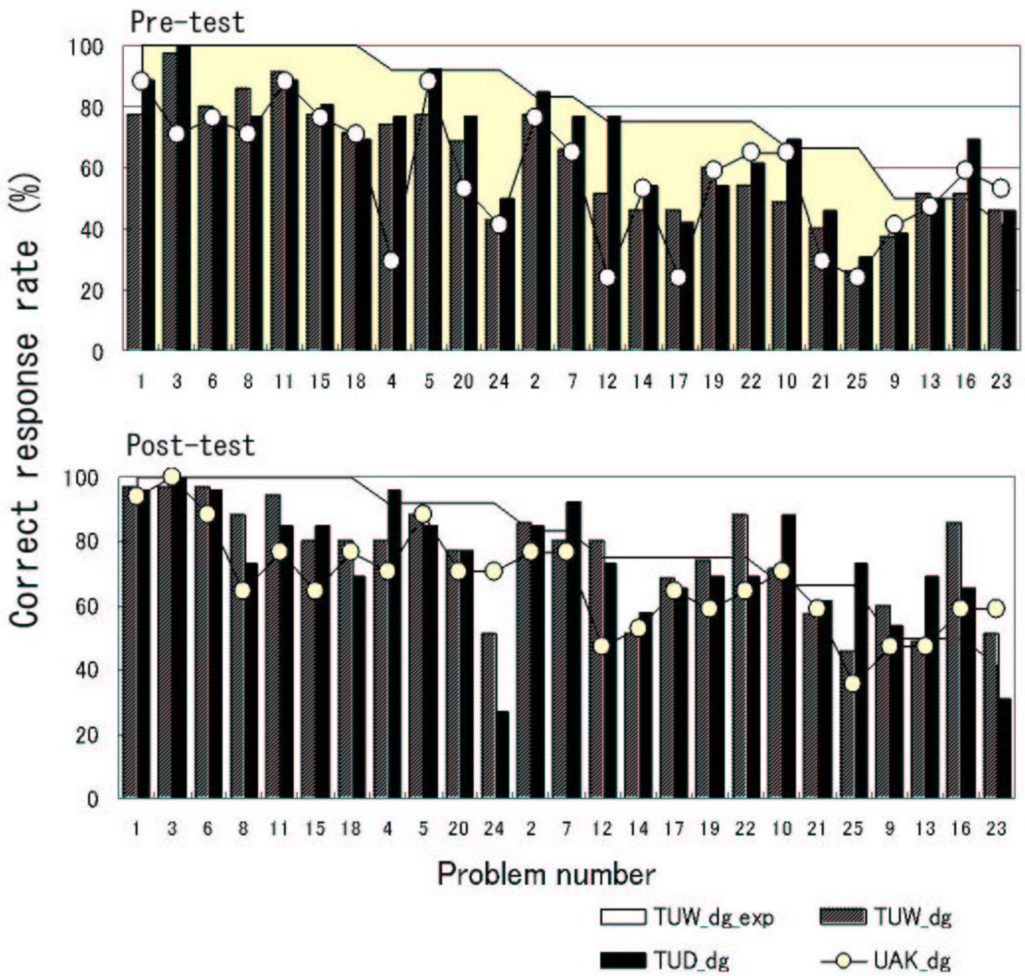


Figure 3: Comparison of correct response rates — pre-test and post-test

4.4. Selection rates of 5 alternatives

Next, the selection rates of the 5 alternatives in the pre-tests were investigated to know how subjects recognize each problem.

Fig. 4 shows the selection rates of the 5 alternatives in two sample problems. The black bar indicates the selection rate of the correct alternative. In these problems, the selection rates of the correct alternative in the non-experienced groups were rather low compared to those of the experienced group.

Thus we might explain that the scores of these problems are easily increased through the Descriptive Geometry education. The object of problem no. 25 is defined by two cylinders intersecting with their axes at 90 degrees to form a combination of two semi-circular cylinders. We would have difficulty in recognizing this shape because of its unfamiliarity. Thus in order to solve problem no. 25, we need not only the intuitive spatial ability but some geometrical consideration as well.

Fig. 5 shows the selection rates of 5 alternatives in another two sample problems. In this case, there are not so many differences in the selection rate of the correct alternative in both experienced and non-experienced groups. In problem no. 11, the selection rate of the correct

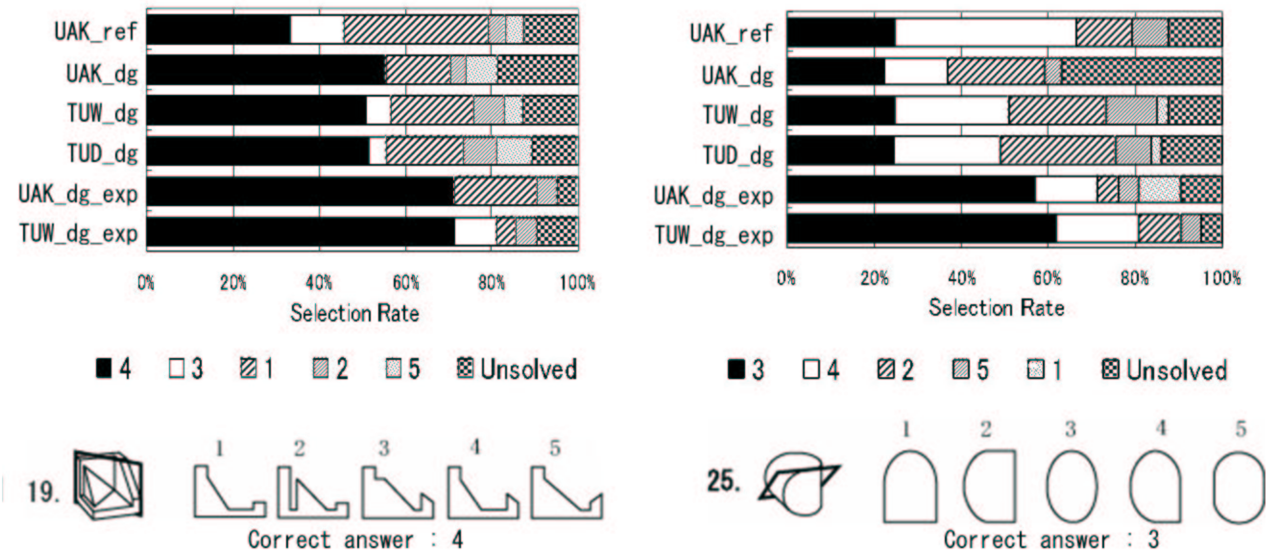


Figure 4: Selection rates of 5 alternatives — experienced vs. non-experienced. Example of problems with different selection rates

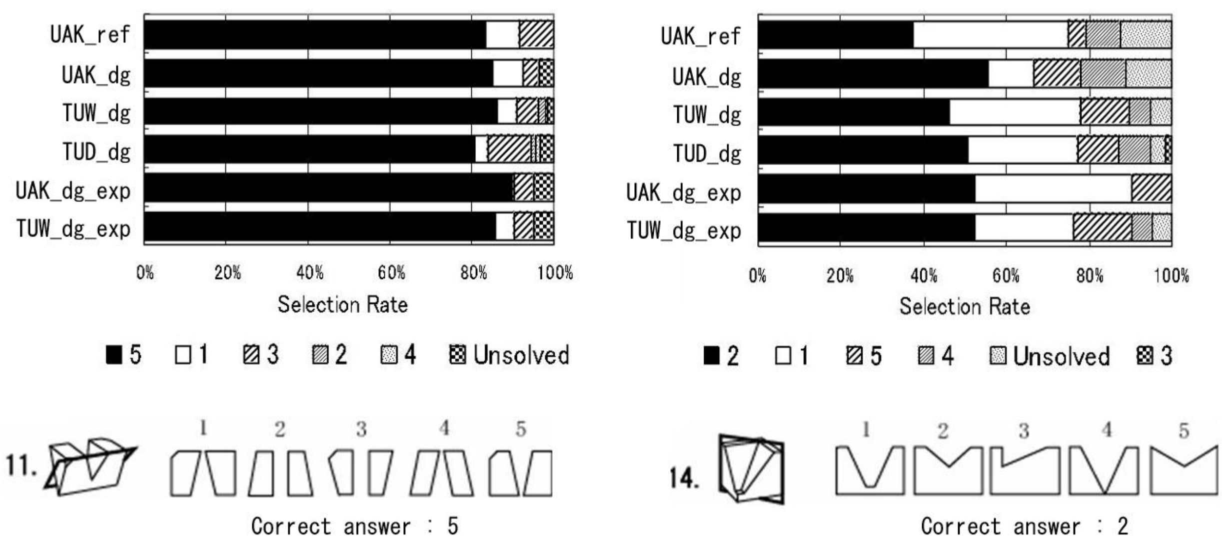


Figure 5: Selection rates of 5 alternatives — experienced vs. non-experienced. Example of problems with similar selection rates

alternative (no. 5) was more than 80% for all. Meanwhile, in problem no. 14 the selection rates of the correct alternative (no. 2) were about 50%.

Thus we might explain that a problem like no. 11 is fairly easy to recognize. Although the solid in this problem has a complex shape, the correct alternative (no. 5) includes a shape like the letter "V". This characteristic shape is similar to the shape of the front surface of the problem object. So even if the subject cannot recognize the 3-D form or doesn't have geometrical knowledge, it would be able to solve this problem, if only he/her can notice this symbolic shape.

On the other hand, for a problem like no. 14, in which the precise recognition of the relative location of the cutting plane against the object is required, a high order spatial recognition skill may be needed. So even with geometrical knowledge, these problems might be difficult to solve.

4.5. Changes in individual scores

Let us analyze the change in individual scores: In the "dg" course at UAK seventeen students have taken both the pre- and post-test. The mean scores of these students in each test were 14.4 and 16.8, respectively. There was a clear statistical difference between the two tests ($P < 1\%$). The result indicates that the spatial abilities such as spatial visualization abilities are enhanced to some extent by the Descriptive Geometry education of the non-experienced group in UAK.

Fig. 6 shows the changes in the individual scores for the "dg" course at UAK. The subjects are sorted in the order of their pre-test scores. We can notice that the scores increased in the post-test as a whole. We also notice that in the post-test there appear two clusters, one with 10–15 points and the other with 20–15 points.

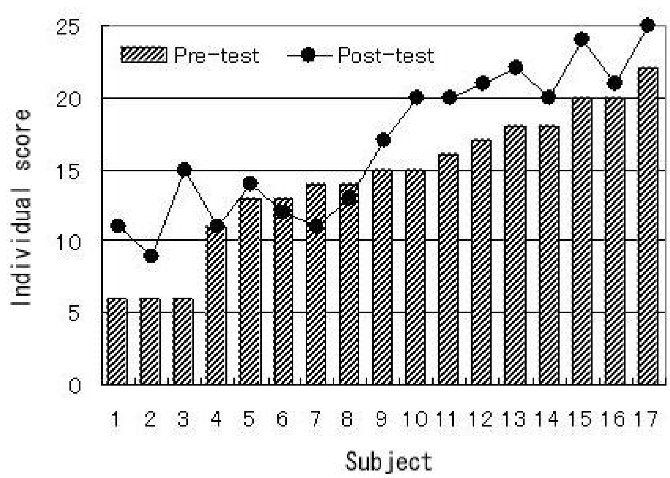


Figure 6: Changes in individual scores

It was clear that higher scoring students in the pre-test further increased their scores in the post-test. On the contrary, middle score students (10–14 points) could not increase their scores, although extremely low score students in the pre-test increased their scores in the post-test.

Consequently, it seemed as if students were branching off into two different level groups in the post-test. So, this is the reason that there appeared two clusters.

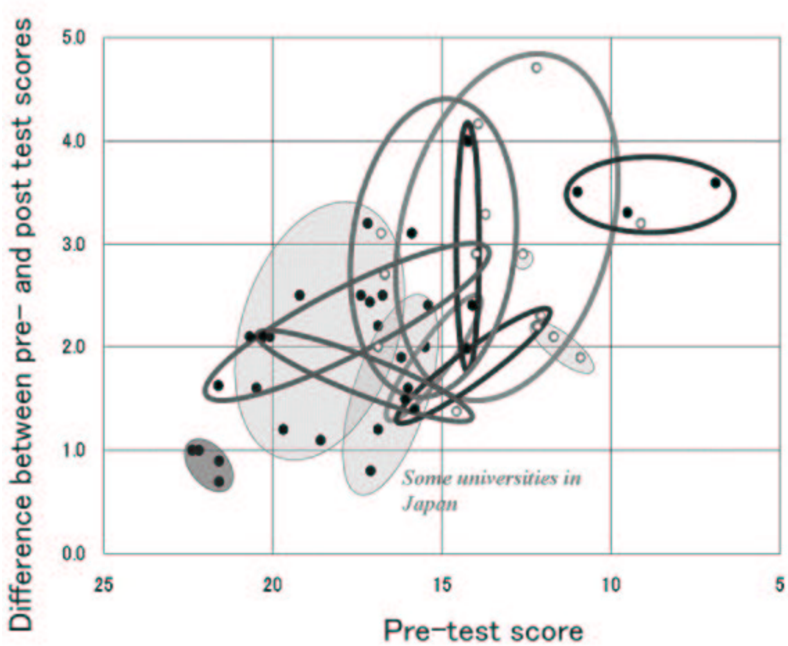
5. Comparison of intercollegiate MCT results

Suzuki [2, 3] has compared the intercourse differences of MCT results of 53 groups with the distinction of sex at 16 universities in 5 countries in which 4 courses were included, i.e., Descriptive Geometry, Engineering Graphics, Computer Graphics and 3-D CAD. Although we added the results of 25 new groups including the results in this paper, the intercourse tendency did not change so much the previous data. Thus it might be concluded that these relations between each course are a rather consolidated trend.

The analysis of intercourse differences [2, 3] suggested that the spatial ability as evaluated by the MCT was enhanced most through the Descriptive Geometry courses.

Fig. 7 shows the pre- and post-course MCT results [4]–[18] of Descriptive Geometry courses (47 groups with distinction of sex at 17 universities in 6 countries). It was interesting that the results with regard to the same university form a cluster as indicated with ellipses.

It might be noted that most of intercollegiate differences may arise due to the differences of course contents and curricula among the subject groups. Thus it is required to examine not only the solving process of MCT but also syllabus, course contents and textbooks in detail.



Pre-test	Gain	Pre-test	Gain
6.9	3.6	16.2	1.9
9.1	3.2	16.7	2.7
9.5	3.3	16.8	2.5
10.9	1.9	16.8	3.1
11.0	3.5	16.9	2.2
11.7	2.1	16.9	1.2
12.1	2.3	16.9	2.0
12.2	4.7	17.1	0.8
12.2	2.2	17.1	2.4
12.6	2.9	17.2	3.2
13.7	3.3	17.4	2.5
13.9	4.2	18.6	1.1
14.0	2.9	19.2	2.5
14.1	2.4	19.7	1.2
14.3	4.0	20.1	2.1
14.3	2.0	20.3	2.1
14.4	2.0	20.5	1.6
14.6	1.4	20.7	2.1
15.4	2.4	21.6	0.7
15.5	2.0	21.6	0.9
15.8	1.4	21.6	1.6
15.9	3.1	22.2	1.0
16.0	1.6	22.4	1.0
16.1	1.5		

Figure 7: Comparison of intercollegiate MCT results in Descriptive Geometry courses. Each ellipse encloses the results in the same university. Shaded ellipses show the results of courses in Japan. Black dot: Male, outline dot: Female

6. Summary and Conclusion

The Mental Cutting Test was performed as a pre- and post-course test by the students in five Descriptive Geometry courses at universities in Austria and Germany. One reference group in Austria was also examined as a control. Among the five courses, students in two courses already had experience of fundamental Descriptive Geometry learning at the beginning of the course. The results were as follows;

(1) There were statistically significant differences between male and female in almost all groups except one.

(2) Apparent significant differences were observed between the experienced group and non-experienced groups.

(3) The problem solving process suggested the possibility that through the Descriptive Geometry education not only the intuitive spatial recognition ability, but also some

logical thinking ability might be progressed, because the experienced groups have solved some of the difficult problems which certainly require the process of logical judgment.

(4) Intercollegiate differences of MCT scores may arise due to the differences of course contents and curricula among the subject groups.

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