Geometry and Graphics in Spatial Invention: Among Mind, Hand, and Digital Means

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Abstract. Aiming to discuss about the 'spatial invention' in Architecture, Environment, and Design, we have to consider, at least, the related domains of Art, Science, Technique, and then, we have to accept the risk of slipping in a way apparently not strictly connected with Mathematics. As an architect, in fact, I am firmly persuaded of the old concept regarding the 'complexity' of the life space. Which is not, as Christian NORBERG-SCHULZ said, a simply abstract space, but an "existential" space for the human beings, in other terms, a physical space enriched with material and spiritual traces and works of the human presence [15]. Maybe, an 'old fashion' question but, with the university educators, it is deeply felt the difficulties of the students in front of a white paper, or of a black screen, at the beginning of a new project, and must of all, a sort of cultural fracture between tradition and the innovative approaches to the knowledges in the field of visual representation. At last, I think, two good reasons, both from a disciplinar and from a didactical point of view, to insist on these topics

Key Words: Manual sketch, digital sketch, project, Geometry and Graphics, representation, spatial configuration.

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1. Introduction

On the purpose, I would like to hereby submit two questions. The first: how to fix, on a physical or on a digital support, the image of a 'not yet' existing space? In other terms, how Geometry, generally considered as a 'deductive' tool, can support us in an 'inductive' way, that is in 'inventing' a spatial shape. The second: may the new means 'kill' the so called 'creativity'? In other terms, if digital Graphics could induce architects, designers, planners, to jump across the first steps of the work, compelling them to define too much quickly the shape.

About the first question, if we accept all Geometry — i.e., euclidean, projective, topological, and so on — both as a 'cognitive' and 'inventive' model of the world, and if we accept all Graphics — i.e., physical, iconic and non-iconic systems of signs — both as properties of the world, and of its description, then we have to detect, if possible, where these tools can be joined each other, at least in the mind of an architect, during the 'invention' of a new existential space, and how they can be used at the beginning of a new project. As we can note, strong relationships connect each other Geometry and Graphics, with Architecture, Design and Planning, because of their common three-dimensional structure (Fig. 1). So, even if, from a strictly deductive point of view, it seems to be impossible to fix a homological or homomorphic image of a not yet existing space, on the other hand, if not in Geometry, wherever else has an architect to recognize his intellectual and practical means? Maybe, nowhere.
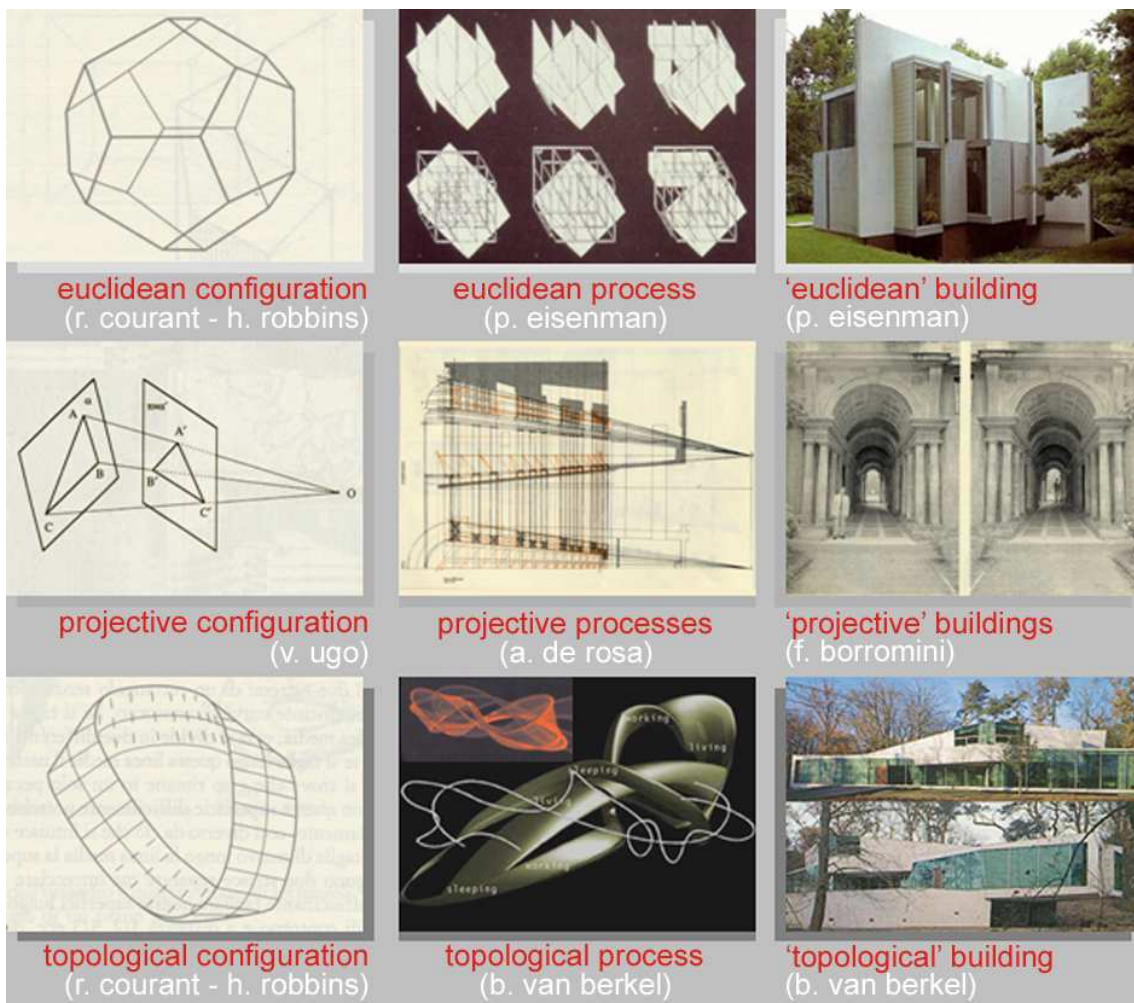

Figure 1: Geometry, Graphics, Architectural Space

What news? In fact, the architecture is a spatial event, and the Space is 'the' argument of Geometry. And the same Geometry (as well every science) shouldn't really a science without a strong 'inductive' potential. Thus, we could easily extend similar considerations to Graphics. Then, just the 'combination' of Geometry and Graphics, helps us not only in 'deriving', but also in 'thinking' new models of space, where the several and several aspects of the project just 'coagulate' around the visual representation. And by now, on my opinion, the largely familiar projective images, because of they are able to represent various kinds of geometrical
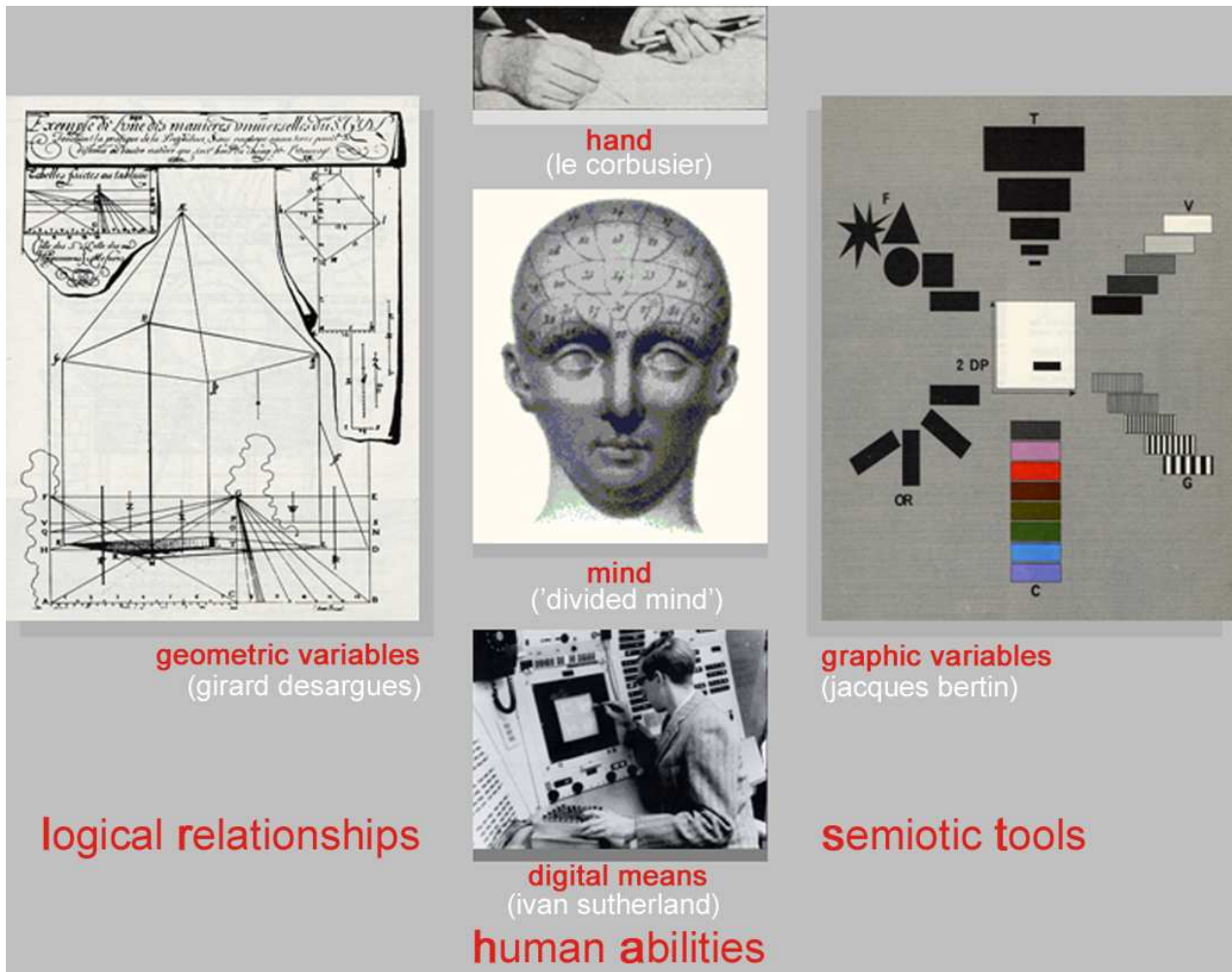
Figure 2: The drawing set: 'menù' and 'ingredients'

configurations (i.e., euclidean and non-euclidean), as well in manual as in digital Graphics, continue to hold a sort of 'primacy' for the our field. In every case, to put in contact theory and praxis, we need the support of a representation, and notwithstanding the space can be described by analytical equations or by visual images, the architect generally prefers the second, that is the most immediate way. Remarking this point, Rudolph ARNHEIM defined the "visual thinking" as well one of the most important human aptitudes [1]. For this activity, we have, at our disposal, something similar to a 'menù', consisting of the various expressive opportunities, and some 'ingredients', consisting of the basic elements, logic (geometric) and semiotic (graphic) of drawing [6] (Fig. 2).

2. The birth of space, between science and art

Well, where these 'menù' and 'ingredients' begin to help us in forming the first images of a certain space? Maybe, in the sketch. In fact, as we can note, we generally use the sketch at the beginning of our work, to capture and encapsulate, since the first intuitive steps, several and heterogeneous variables, by means of complex and metamorphic syntheses of Geometry and Graphics, as in surveying well in projecting. In the first case, in measuring or studying an existing space, when we intend to translate reality into theoretical and conceptual models,
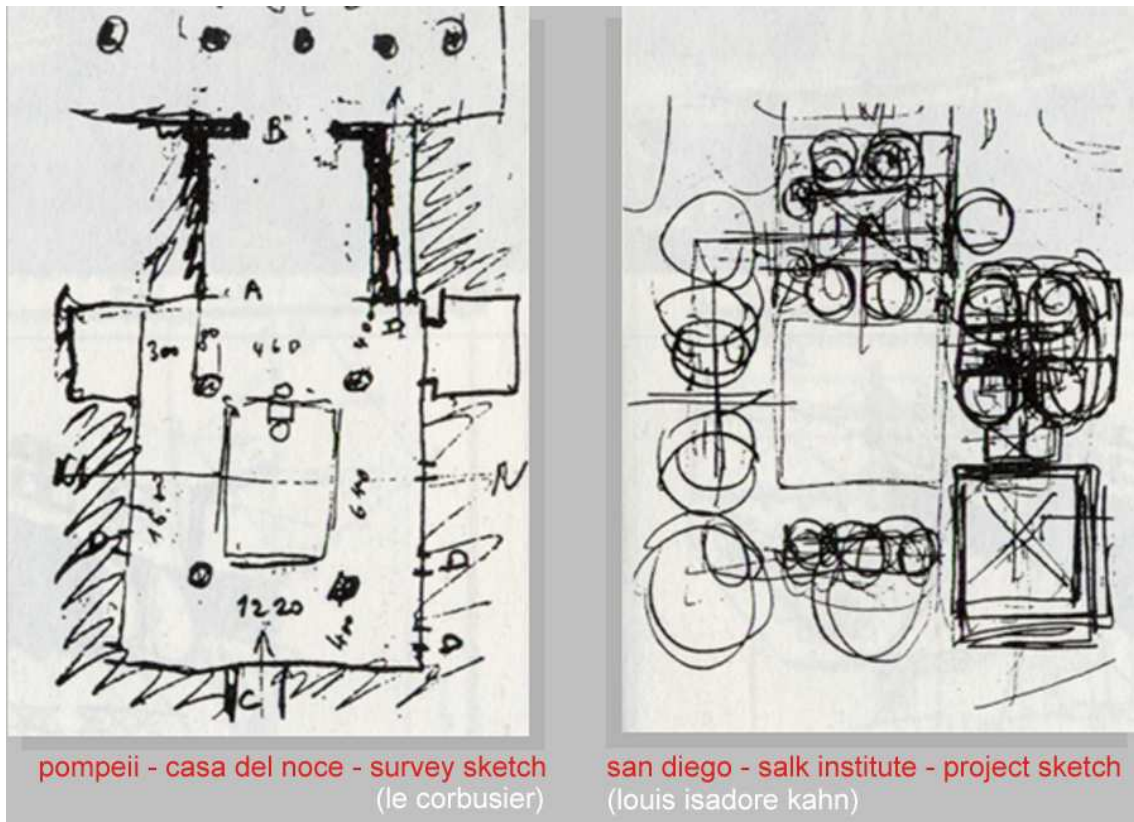
pompeii - casa del noce - survey sketch
(le corbusier)

san diego - salk institute - project sketch
(louis isadore kahn)

Figure 3: Spatial 'cognition', spatial 'invention': manual method

which make us able to describe and recognize an absent true space, either a far away or a no more existing one. In the second, in projecting a new space, when we proceed, on the contrary, trying to elaborate and to translate and idea into reality, according to projectual goals, that is, when the absent space is a not yet existing space. As we can see, two symmetrical ways, but each one strongly based on visual devices, and intended to capture the 'genetic code' of a space, the 'embrionic' structure of a configuration (Fig. 3).

As architecture is based on Science, Technique and Art, so the sketch is historically used in these fields, as well confirms us the long series of the Leonardo DA VINCI sketches and drawings, by the means of them he represented things and sentiments, space and time, and in short, we should affirm, he tried to study and to invent an entire world. And in spite of the various applications, we can note that the sketch has got a typical iconography, a sort of 'proper form', coming to the point that we frequently can't quickly distinguish a studying sketch from a projectual one: for example, comparing the couples of Leonardo — artistic, technical and scientific — sketches, of study and of project, here presented, who could distinguish them only basing on the visual iconography? (Fig. 4) But in a certain sense, it is right so. After all they are, in both the cases, sort of 'explorative portraits' of objects and properties not completely discovered. Similar examples we could find in the architectural iconography. And just with regards of architecture, it seems particularly emblematic, for our topic, an interesting and rich series of sketches of the *Breda Pavilion* project by Luciano BALDESSARI. As we can see, they don't show only combinations of geometrical and no geometrical 'primitive blocks', but they are more 'plastic' and 'uncodifiable', maybe as well the first mental elaborations connected to the spatial imagination. In other words, they represent a crucial and complex phase at the beginning of the work. In fact, as Giancarlo
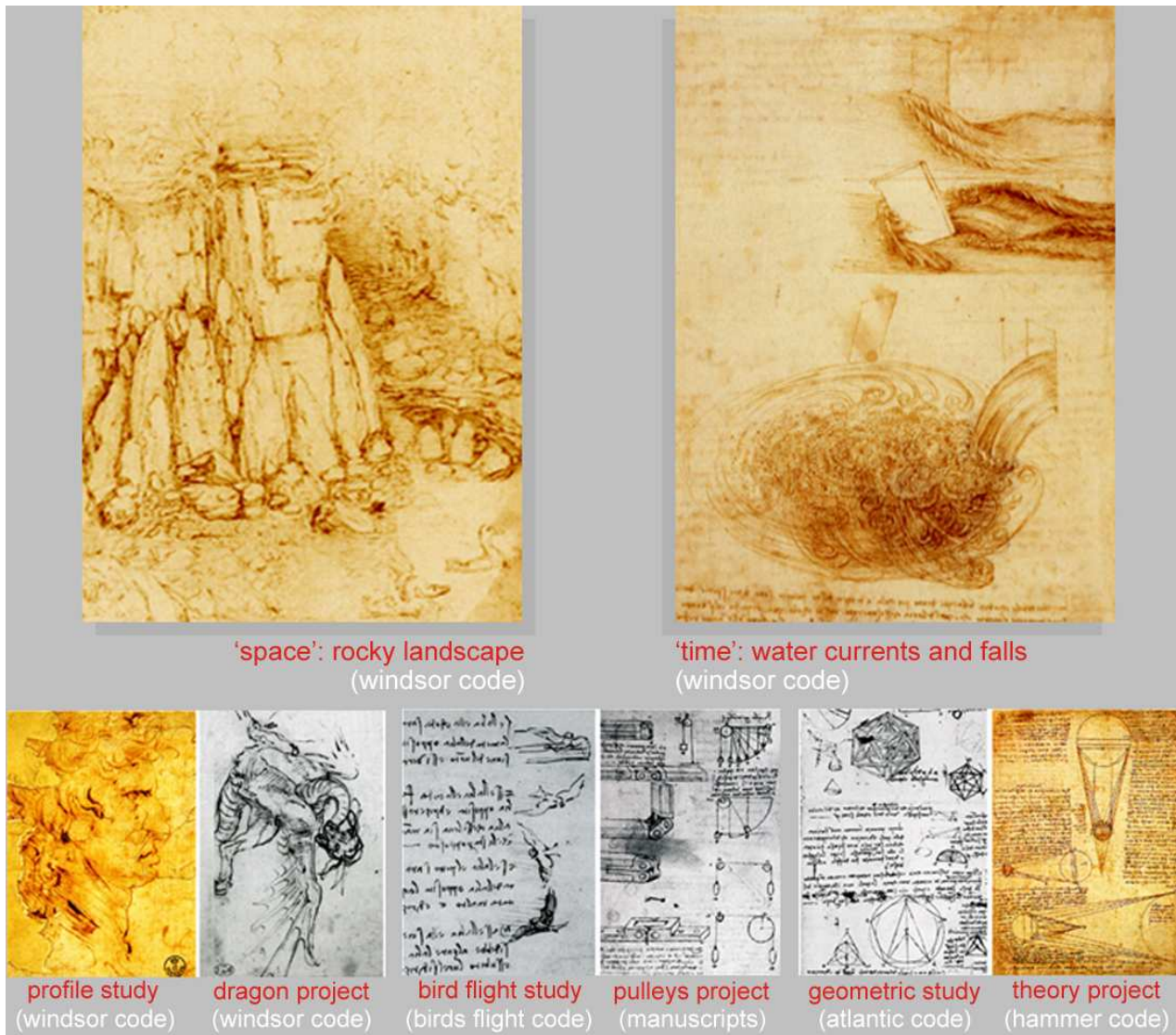
Figure 4: Leonardo DA VINCI: studies *vs* projects. Space and Time,
among Art, Science, Technique

CARNEVALE noted in an interesting and inspired article [2], the sketch of project shows us a sort of "hazard" in searching new solutions, nearly as well as it was "invoking" them in a prayer, by means of "drawing and redrawing" the crucial points of the problem. This kind of "litany" produces a distorted image, that is to say, a sort of "parody" of the shape, right to better emphasize and visualize several selected main aspects. In this game, the operator and the problem become strictly connected in a reciprocal "fascination", and so the drawings also narrates a sort of "autobiography". On the opinion of the author, this phase of the work comes out "after theory" and "before" project, that is, between thought and action, so representing a genuine and irreplaceable "human" experience (Fig. 5).

Unfortunately, just this subjective character, and its implicit form, seem to prevent us to completely 'decode' a sketch, and nearly as well as in biology happens, this 'embrionic' structure is not necessarily homomorphic or omological similar to the adult organism, so the sketch contains, but doesn't necessarily resemble, the final shape. Then, we have found the most universal tool, but we have not its 'access key' language, and its treasure of contents seems
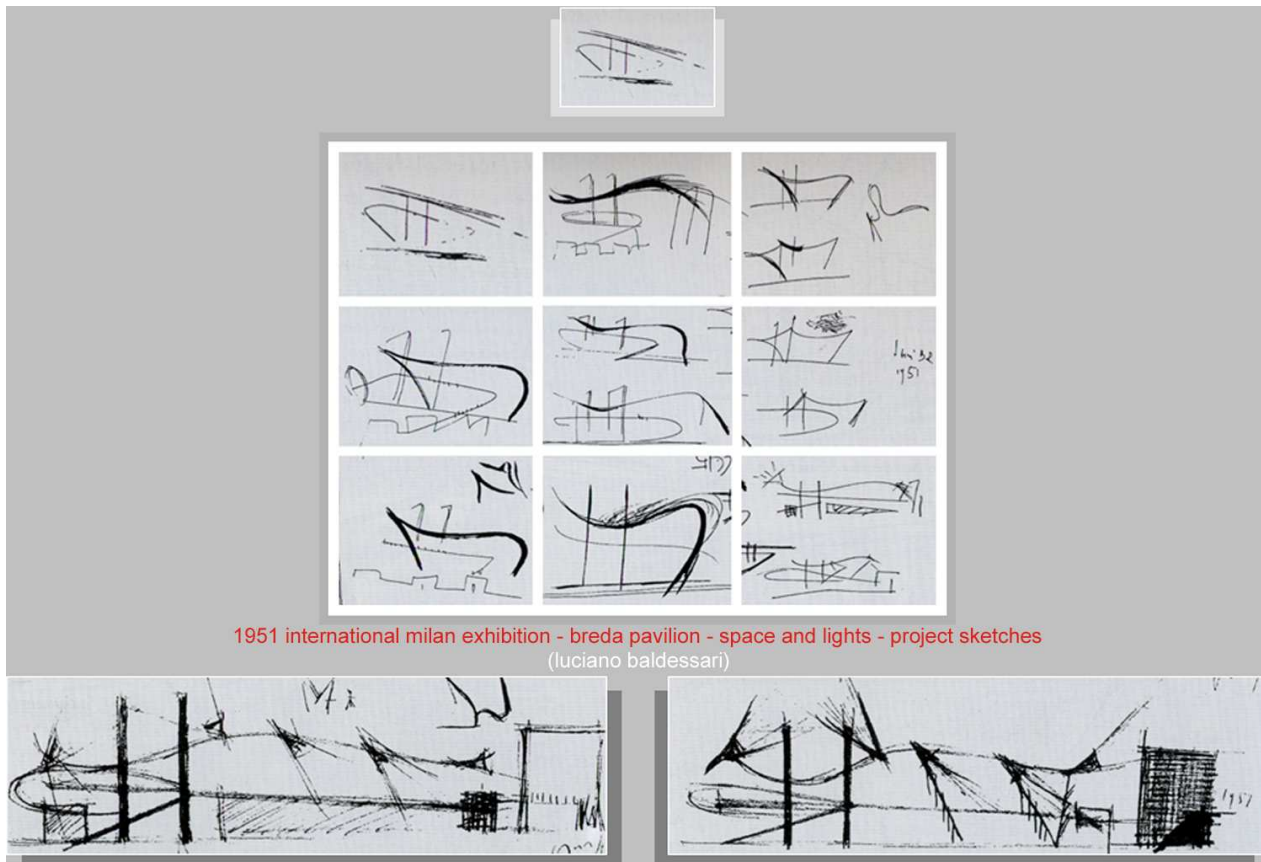
1951 international milan exhibition - breda pavilion - space and lights - project sketches
(luciano baldessari)

Figure 5: The 'birth' of a space

to largely remain a cryptic scheme, a sort of 'private property' of the inventor. Nevertheless, against this apparent contradiction, we have not to surrender, but we have to consider this cryptic property, not as well a 'defect' or an 'obstacle', but only as a 'peculiarity' [7].

The sketch, in fact, is a high 'sensitive' product, with a higher informative richness. It contains all the energy that the mind and the hand spread, in searching and capturing the intimate idea of a spatial conformation [16]. Consequently, every alteration or adjustment, in its apparent 'confusion', may produce several and several different results. Then, the sketch needs an attentive and respectful regard. The proof is the pride with several famous architects use it on the architectural magazines. Obviously, as well in literature we have preliminarily and 'non-conventional' notes and schemes, in which Grammar and thought are highly implicit, and only developable by the author, so in the preliminary sketches, Geometry and spatial ideas, are. Obviously, no written and no drawn notes are possible without literary, and in our case, architectural culture, because of every language acquires different semantic senses according to the disciplinary field in which it is used. In every case, a classification of drawings, basing on their semantic characters, seems to be possible, as Vittorio UGO have demonstrated. On the author's opinion, in fact, architectural drawing is comparable to the "auxiliary verbs", helping us in elaborating and visualizing what we "want/or want not" realize (i.e., our concept sketches/or oneiric visions), what we "can/or can not" build (i.e., our technical drawings/or our utopias), and what to which we "must/or must not" give a concrete form (i.e., our building plans/or theoretical symbols) [16]. Then, because of several branches take part in defining Architecture, Environment and Design, these fields and their project processes, escape form every strictly mathematician or artistic, or technical, or social
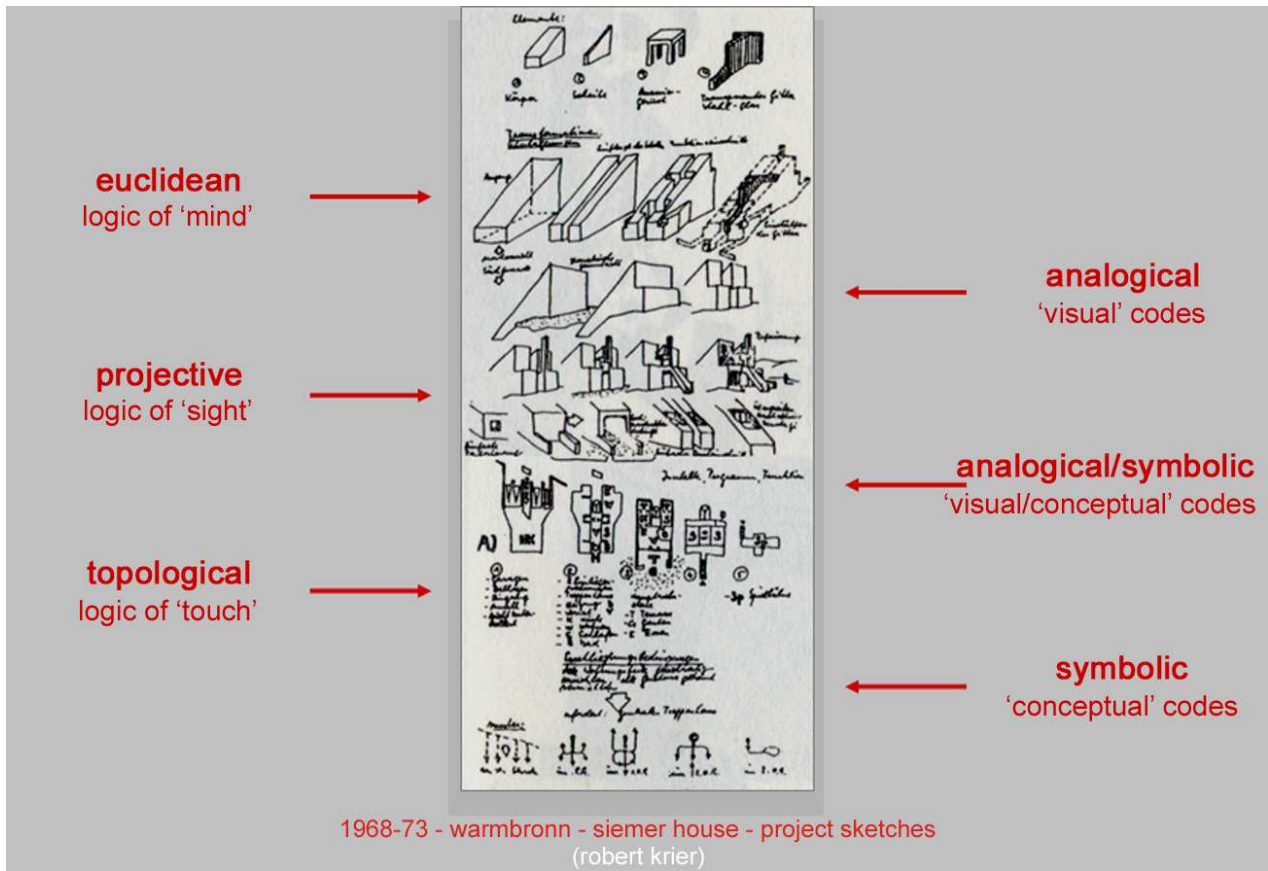
euclidean logic of 'mind'

projective logic of 'sight'

topological logic of 'touch'

analogical 'visual' codes

analogical/symbolic 'visual/conceptual' codes

symbolic 'conceptual' codes

1968-73 - warmbronn - siemer house - project sketches (robert krier)

Figure 6: To 'dismantle' a sketch

classification, and they are near to be just considered as a proper 'branch' of the human knowledge. Similarly, in the 'branch' of drawing, related to the named fields, the sketch seems to from a specific category.

But, what a sketch consists of? In other terms, what we can find, if we 'dismantle' a sketch? Pieces of Geometry, and pieces of Graphics, mixed in a sort of 'synopsis', such as we can see in the exemplar case of the *Siemer House sketches* by Robert KRIER (Fig. 6). Then, a sketch contains, in an implicit key, formal, functional, technical data, based on different kinds of geometrical relationships and graphic codes. And even if, in it we cannot lightly identify metrical and non-metrical properties, analogical and symbolic signs, we can, however, easily pass from one to another group of properties; we can develop them under few visual guidelines; we can discover them in the final project; and finally, we can completely explicit and test them in the architectural building. In a gradual process, toward the 'conquering' of the space. Something of similar happens in the children drawings, as well George LUQUET noted [13]: first, only a sort of data collection, but a general "inability to reach a synthesis"; after, an evident "intellectual and non-iconic realism"; finally, a "visual realism" begin to appear, in our case, just at the beginning of a morphological organization of the geometric structure of the space. However, as we can show in the enclosed scheme (Fig. 7, left), all the processes converge in a polysemeiotic image, a sort of 'flywheel' among the several languages that provide to graphically translate the several aspects of the 'complex' space [3]. On the purpose, i wish hereby highlight as the original meaning of the word 'image' derives from the Indo-European term *yem*, that is "to hold", to "connect". And more precisely, in our
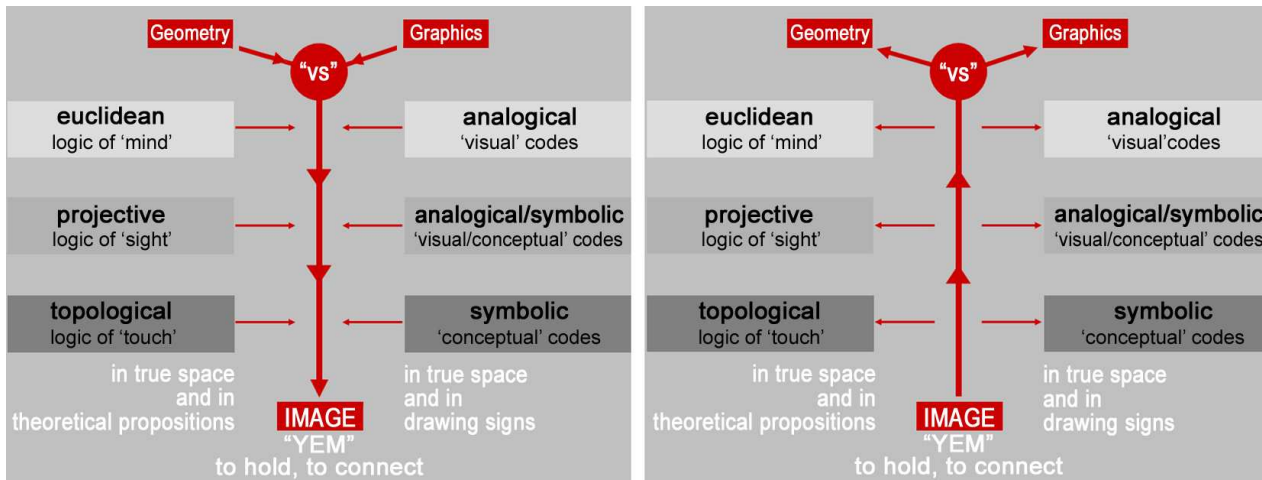
Figure 7: The 'ways' of the image

case, the image really connects the structure of the space to the structure of the drawing: as should have said René DESCARTES, *"res cogitans"* and *"res extensa"*, or, if we prefer the Martin HEIDEGGER formula, the *"think"*, and the *"thing"* (Fig. 8).

The answer to the first interrogative seems to be more evident, at this point. But, before passing on the second, maybe it should be useful to briefly highlight further aspects of the relationships between Science and Art. On the subject, Thomas Samuel KUHN notes that the sketches carry out different tasks in the two fields [10]. In fact, while in Science they are generally considered "by-products", in Art they are generally considered "documents". Furthermore, about the specific "paradigms", while in Science a theory "dies" when new theories take its place, in Art the style "survives" in spite of the advent of further new styles. Moreover, passing from 'work' to 'worker', he highlights an evident inversion between
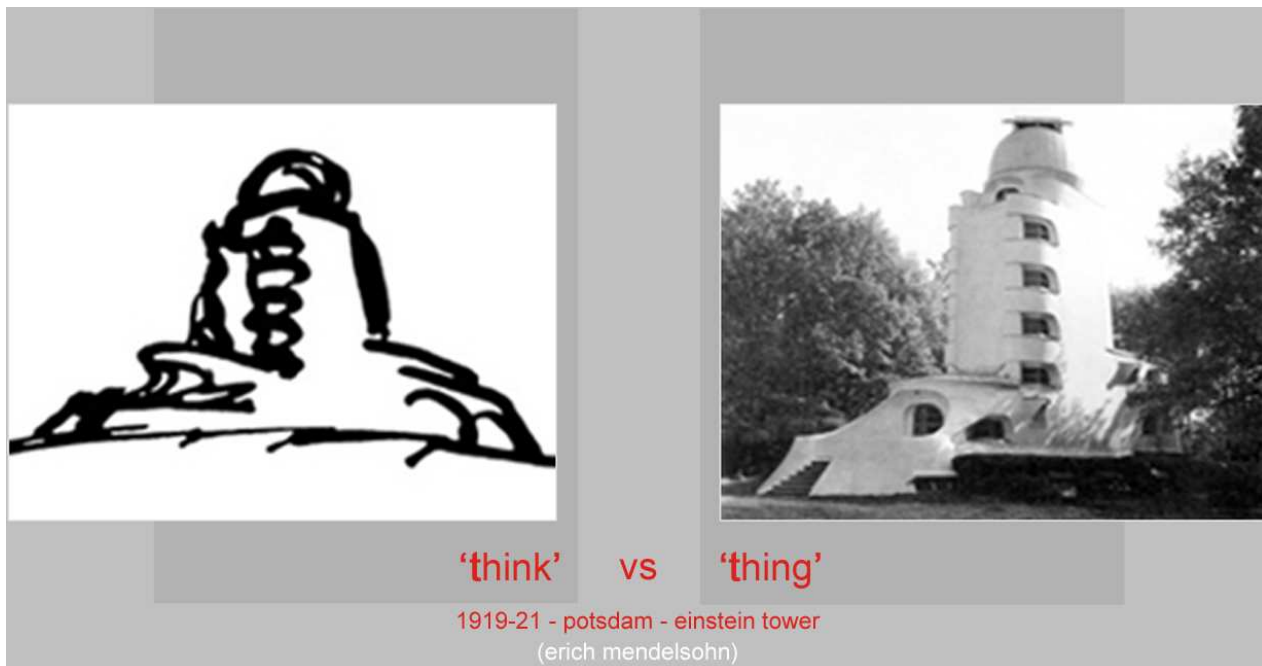

Figure 8: In the image, in the world

"means" and "goals": while in Science, in fact, technical goals can be obtained and presented by means of aesthetically refined statements, in Art, on the contrary, aesthetical goals can be obtained and presented by means of technical devices. So, if in Art, the sketch increases the comprehension of the work, in Science it should help us, at the most, in the comprehension of the worker. On my opinion, saving the evident differences, there are some important similarities. In every case, in fact, the sketch represents a sort of common tool to test new ideas; secondarily, in both the cases, as the most private document, it reveals and the work, and the worker, as Sigmund FREUD remarked, because of the strong connections between inwardness and reality, although frequently unconscious; third, even if a paradigm dies, it will be at least usefully preserved in History of Science or in History of Art. Or, in the absence, in the mind of an architect, or in the body of an architecture, where every kind of aspect, material, spiritual, symbolic, will converge in a synthesis.

3. Among mind, hand and digital means

'image-model': borgomaggiore church project, manual sketch (giovanni michelucci)

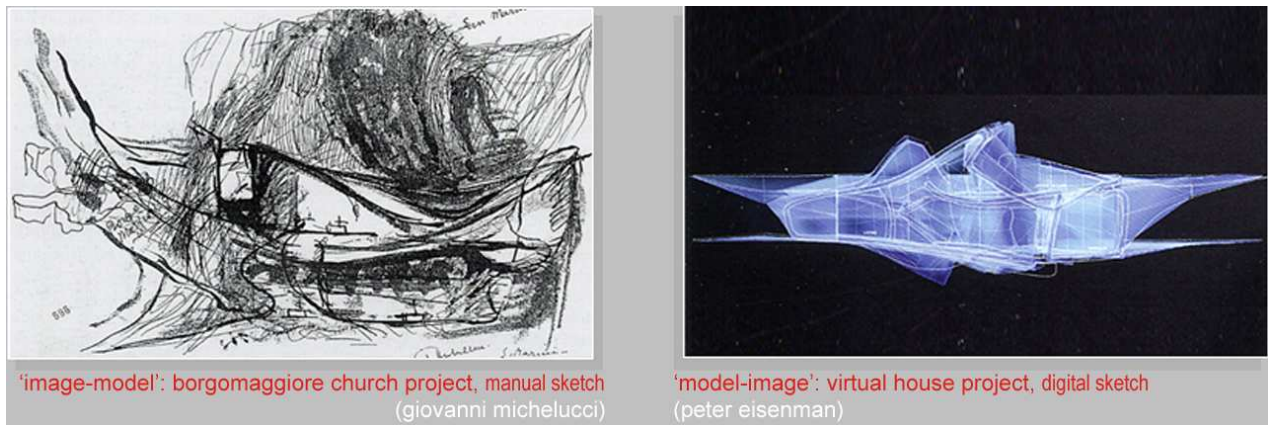'model-image': virtual house project, digital sketch (peter eisenman)

Figure 9: Image-model *vs* model-image

Passing now on the our second interrogative, let we give a look to other two architectural sketches herewith enclosed: a 'manual' sketch of the *Borgomaggiore Church project*, by Giovanni MICHELUCCI, and a 'digital sketch' (I would like to consider it so) of the *Virtual House project* by Peter EISENMAN (Fig. 9). Each one represents a 'section' of an architectural space: what is the difference? The first of them is only *one* of a limited series of fixed images, that form a 'discrete' model of the space, which we have to complete in our mind: as the matter, I named it *image-model*, that is an image that 'alludes' to a model. The second, is only *an* image, of a potential unlimited and dynamic series, extracted from a 'continuous' 3D digital model, exploreble in all its corners and characters: as the matter, I named it *model-image*, that is a model that 'alludes' to a fixed image. Comparing the figures, and overtaking the prejudice about the nature of the means, it is clear how much this difference involves not only the visual results, but more deeply, two different approaches to the project [14]. We can hypothesize, on the purpose, that, while the Renaissance established a visual and 'pictorial' supremacy in the project, our era seems to establish a more general supremacy of all the senses, aiming to recovering the pleasure to manipulate the space, at least, in a 'sculptural' mode. Obviously, in a virtual way. And a great goal, on this point, was the re-connection between 'visualizing' and 'modelling' abilities, historically strictly connected to each other, but for a long time divided in the last centuries. We are well conscious of this difference,

Figure 10: Spatial 'cognition', spatial 'invention': digital method

whether in survey or in project, especially today, when we are more and more supported by three-dimensional models, since the beginning of the work, so nearly 'inverting' at all the traditional praxis (Fig. 10).

However, the same computer is an historical result of our culture, and maybe, there are at least three good reasons connected to the wide diffusion of the 'digital way': first of all, the great visual analogy of the computerized images with the traditional iconography (i.e., projecting, topological, movie, and so on); secondarily, the power of its algorithms to ascribe physical properties to the geometrical model (i.e., spatial, temporal, static, acoustic, and so on); besides, the possibility of a higher interaction between the architect and the space, by the means of virtual reality (in multi-sensorial terms, connecting image and action) [5]. But, notwithstanding under a conceptual point of view it seems to be some curious, we can still distinctly recognize a 'sketching phase', also in this unusual wedding between the 'first idea' of a spatial configuration and its 'algorithmic' reproduction. Maybe, nothing strange, if we can less or more consider the same 'mental images' as the result of a largely unknown 'natural algorithm', although different from an artificial software.

Furthermore, this new kind of graphic tool, seems to have demolished the historical barrier
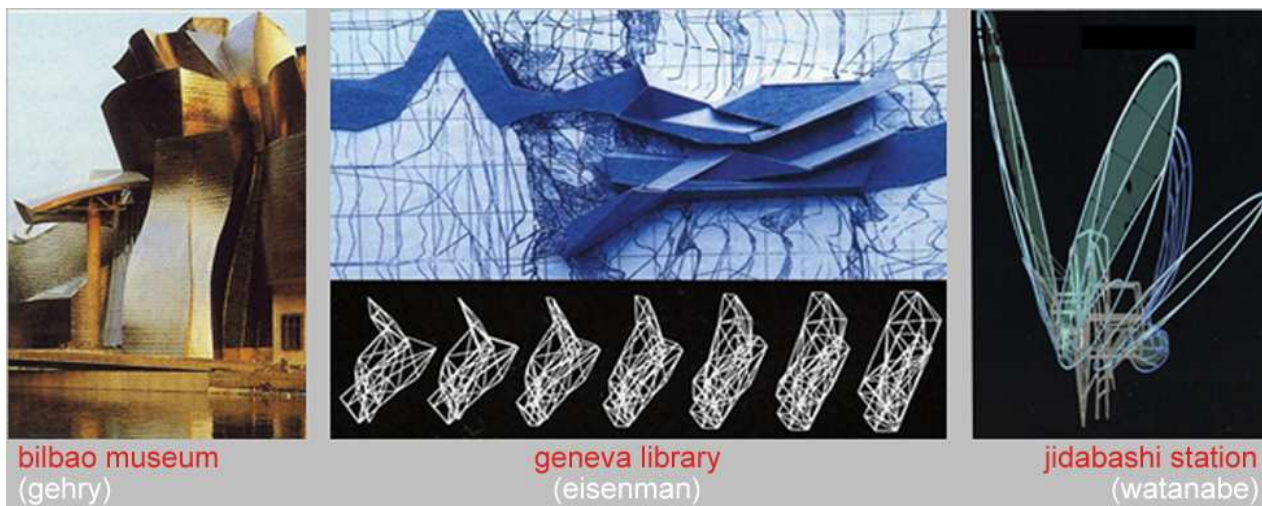

Figure 11: Numeric forms *vs* visual forms

between iconic and non-iconic codes, at long last connected in a sort of reciprocal expressive continuity [4], such as another series of three famous architectural examples shows us (Fig. 11). Basing on these instances, it is evident how it is possible to deduce the analytical form from a graphic configuration, or from a solid maquette (i.e., in the structural calculation of the Frank Owen GEHRY's *Bilbao Museum project*); how it is, on the contrary, possible to deduce graphical configurations and metaphoric cues from analytical diagrams (i.e., in the cerebral waves diagrams of the Peter EISENMAN's *Geneva Library project*); how it is, furthermore, possible to obtain a graphical configuration from the auto-generative action of the most opportune inductive algorithms (i.e., in the digital auto-generation of the Makoto Sei WATANABE's *Jidabashi Station project*) [17]. Then, maybe we are in presence of a new type of sketch, that permits us to elaborate every kind of data in a sort of dynamic and multilingual synthesis, during all the phases of the project, without abandoning the original file. So, we could provisionally name this new kind of visual product, *3D-data-sketch*, or more briefly, as we pointed before, *digital sketch*, just to distinguish it from the traditional *manual sketch*, but again and even more able to serve artistic, technical, scientific, and architectural representative scopes (Fig. 12).
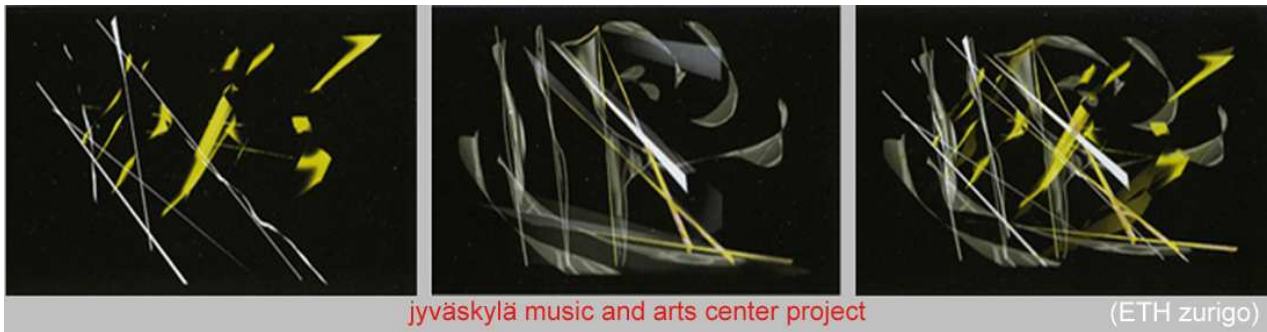

Figure 12: Digital sketch progression

Consequently, it seems quite reasonable to reckon that the new digital means shouldn't 'kill', but maybe 'increase' the creativity [8]. At the moment, only modifying some steps of the traditional work. But, if the means and the human brain are strictly connected each other, as well as history teaches us, it should be possible that, when we will become more and more able in using and inventing new instruments, we'll really be modified, also in our 'way of thinking' and 'conceiving' the space.

4. Conclusions

Basing on the above lines of reasoning, by the moment we can only consider that the more refined digital models increase the possibility to control the global impact of the architectural space on individuals and society; that the 'image' is acquiring a bigger and bigger power, which is, according to Michel FOUCAULT, one of the most revolutionary events of the contemporary culture, evident in every branch of the human knowledge; an that, in this sense, our mentioned conceptual scheme seems to be still valid, but only 'inverted' (Fig. 7, right). So, attempting a conclusion, maybe one of the great goal in the next future, more and more will consist in continuing to connect knowledge and languages, or, if we prefer, as Jurij LOTMAN should confirm us, in homogeneously enlarging our "semiosphere". Obviously, if one could agree with LOTMAN when he highlights the great semantic potentialities deriving from the
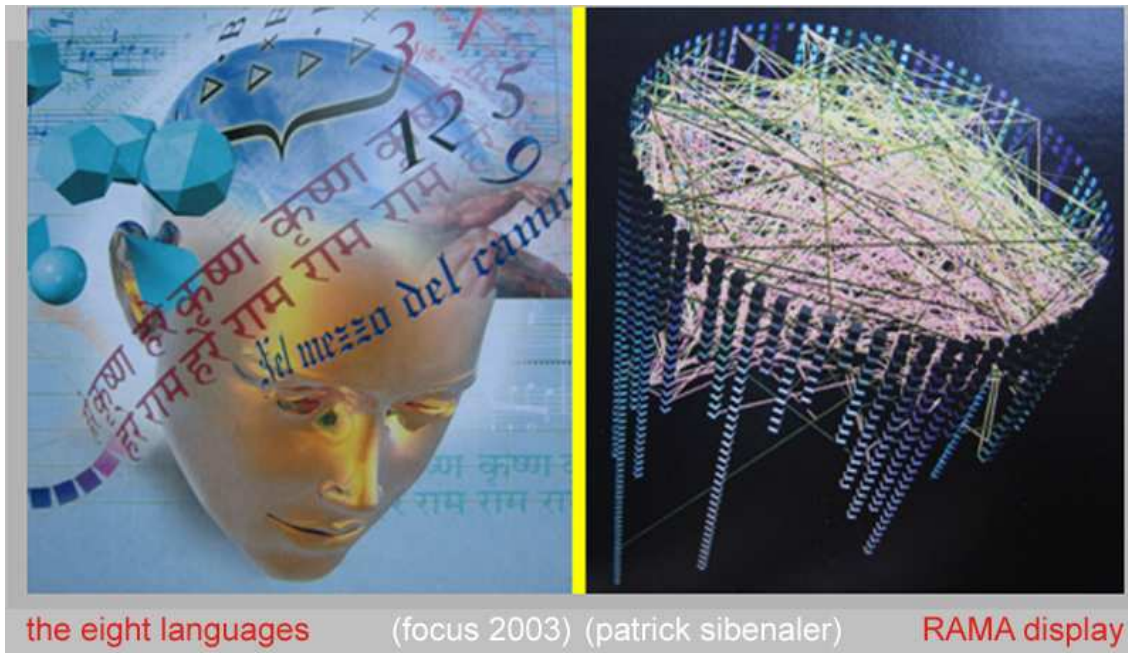
Figure 13: Linked languages

juxtaposition of 'heterogeneous' expressive elements [12], we have to not forget the risk that the communication should become "nebulous" without an appropriate control, as Umberto Eco should remark [2] (Fig. 13). In every case, with reference to our peculiar field, we should think, at the same time, of something less 'hybrid' than a *hyper-text*, and more similar to an *hyper-image*: in other terms, not of a simple addition of different 'linked' languages, but really of a graphic synthesis of them in a 'polysemeiotic' geometrical model, or, in a sort of 'visual esperanto'. And, most of all, dynamic.

On this point, we can note that digital drawing seems to be based on a singular paradox, a sort of 'paradox of the instability'. The great modifiability of the digital models, in fact, constitutes an undeniable weakness of the system, implying some considerable problems in order to preserve databases and, as probably Walter Benjamin should have said, in order to go back to their original 'paternity'. Nevertheless, just this circumstance, offers the most important resource, making models dynamically developable, and then continuously adaptable to every stimulus of the external context. In other terms, interactive, or, if we prefer, virtual (Fig. 14). In fact, whereas the topological articulations generally regard our 'tactile' use of the space, and whereas the fixed projective images regard some moments of our visual or abstract 'perception' of the space, the digital sketch finally seems to connect and dynamically involve all the geometrical branches, offering four -and maybe more- virtual dimensions to the architectural drawing, introducing the parameter 'time' by means of 'four-dimensional' visualizations, providing more and more refined simulations of our five — or maybe six — senses, and aiming to generate more and more complete models of the true space, that was in our topic since the beginning.

But it is also important to remark, that even the traditional drawing, although in a different way, consists of a very flexible tool, and must of all, it defines a special kind of virtual device. According to Pierre Lévy, in fact, we have to remember that the term "virtual" is not at all synonymous with "digital", but it responds to the Latin word *"virtus"*, which we can translate as "strenght", "power", potentiality, that is to say, the capability
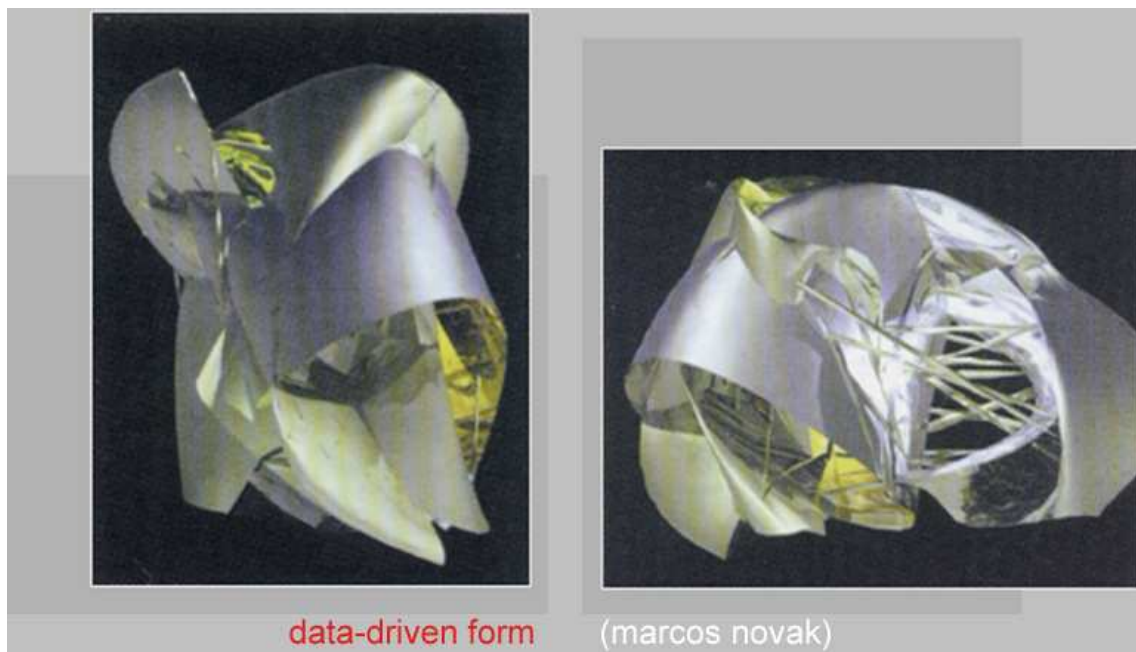
Figure 14: The 'paradox of the instability'

to pre-figure, to foresee, in short, to project, a certain thing or event [11]. Because of this fact, philosophy largely retains "virtuality" as one of the fundamental engines of the human evolution. Consequently, we can affirm that virtual reality in Architecture, Design, and Planning, might concern the convergence of all our competences in preparing and realizing our physical and spiritual daily space. So, we could rightly consider visual representation,
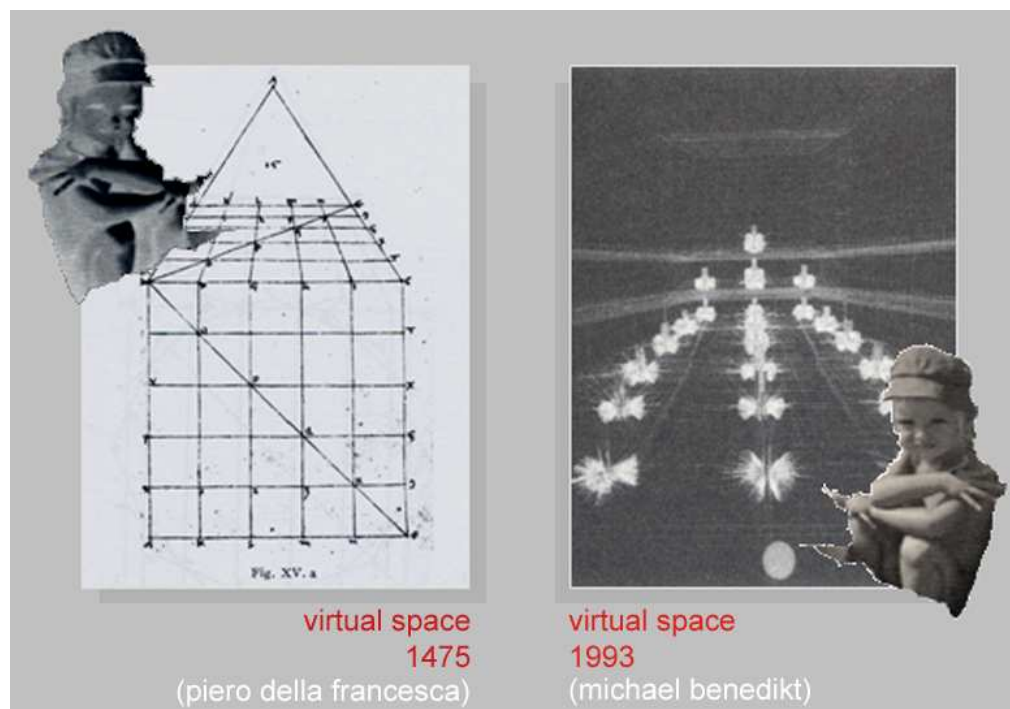
Figure 15: Virtual: what news?

and then, Geometry and Graphics, as *the* virtus, in providing us the must generative power, so in inventing, thinking and building our "existential" space, since the first elaboration of the spatial ideas (Fig. 15).

Last, it is equally evident that, even if virtuality doesn't strictly depends on manual or digital mode, on the other hand, as Alexandre Koyré noted, we have also to consider that means and methods are not completely indifferent in the human activities, generally suggesting us new operative modalities, new possibilities and -maybe- a new concept of space, such as the logical and technical instruments up to now invented by Art, Science, Technique [9]. In other words, our tools, our sight (Fig. 16).
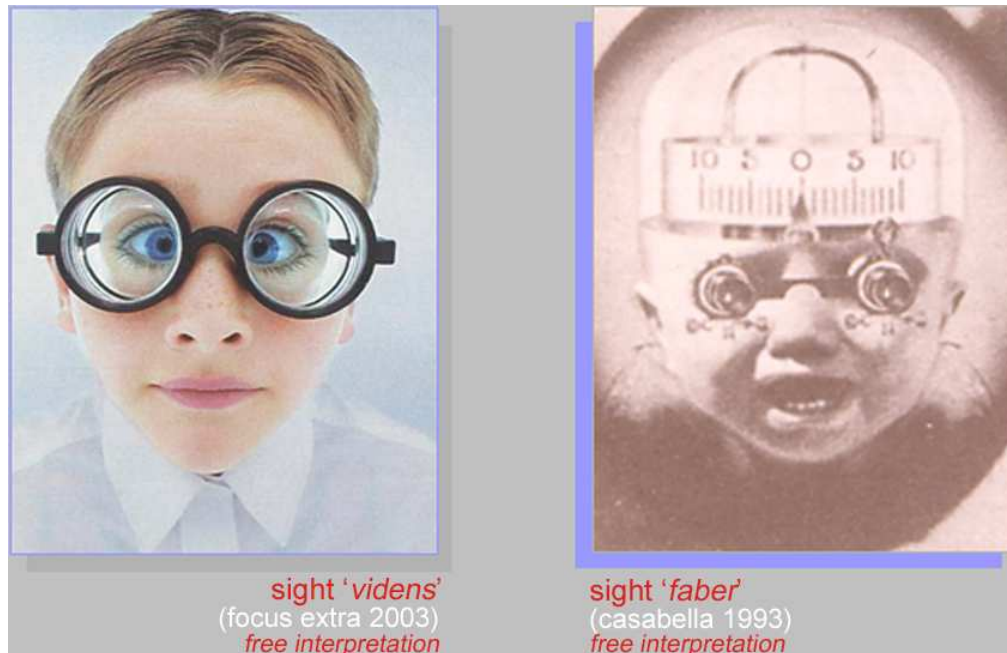

sight '*videns*'
(focus extra 2003)
free interpretation

sight '*faber*'
(casabella 1993)
free interpretation

Figure 16: Our tools, our sight

Acknowledgements

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