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# Upside Down and Inside Out – How to Systematically Generate Transformed Space in Images

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**Abstract.** Various artists and architects show a special interest in issues related to projection. Borromini, El Lissitzky, M.C. Escher, Peter Eisenman, and Patrick Hughes, to name a few, have used and modified the regularities of linear perspective to match their intentions. Furthermore we will focus on paintings, that are actually in conflict with projection as taught in Descriptive Geometry. Consequently the topic is dealing with perception and optical illusion.

The dominant picture-creating method of today is photography. To keep up with the quality of photographs, rendering software is conventionally used to create correct images – correct with regard to only using the rules of central and parallel projection. We will extend the range of achievable representations by implementing other projective concepts into a common software package. By this way we want to get closer to some phenomena in art and architecture as mentioned above. In order to keep the balance between "acceptable" and "chaotic" we will discuss and classify the results in relation to comprehensibility. We ask: "Can the image or model be seen as a – more or less – strange representation of a familiar object or is it a strange object in itself?"

The different deformations applied are systematically based on linear and quadratic spatial transformations, for example relief perspective and inversion. Although even more chaotic mappings could be taken into account, this restriction to classical geometrical methods guarantees legibility as desired in this context. At any time the observer should be able to understand and reconstruct the represented space. Starting from a relief perspective we will invert the visibility on the viewing rays of a central projection. The result reminds us of some paintings of the Middle Ages. Models of the spatial transformations are created by means of a rapid prototyping system. Such a system is existing at the department of geometry in Dresden.

*Key Words:* Geometry in arts, constructive geometry, computer aided design, relief perspective, inversion, inverted perspective, inverspective, CAM

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## 1. Introduction

The history of the art of painting can be understood as a step by step discovery of linear perspective, finally leading to digitally generated representations with a high degree of perfection. In this reading of the past we disregard any concept that consciously does not obey the one-eyed look from a fixed point of view, the motive of the camera obscura.

The captivating illusionary pictures produced synthetically by computers are in one line with the quality of photography. Nevertheless linear perspective is a cultural setting, how ever natural it may look to us. This setting is subject to many experiments of art, testing the borders and possibilities of linear perspective since its invention.

We mention several examples of such experiments before we try to transfer an alternative concept of pictures into digital images. But first of all we have a look at the history of the art of painting with our theme in mind.

Figure 1: G.F. Ziebland, wall painting of Pompeii, detail, about 1828 (from [8])

## 2. The art of painting

The paintings of different epochs show the artists' understanding of visual perception and what was known about optical and psychological phenomena. While the cave paintings of prehistoric time are narrative, comparable to a written text *(glyph)*, the stage designs of the Greeks intend to produce an optical effect *(image):* the scenery evokes a virtual reality right on the stage. The necessary techniques of illusionary art can be derived from exact observation of reality. The purpose is not an idealizing documentation of objects but their mapping by optical means.

The famous frescoes of Pompeii start from a frontal representation of architectonic parts in real size. The painted structural elements fit to the plane of the wall. The spatial impression results while the third dimension of each object is drawn with a certain angle to the rectangular front (Fig. 1). The different angles are chosen with respect to the spectator. Objects under the eye-level expand upside and those above expand at the lower side. A similar construction is known from isometric perspective: the third dimension is added in the drawing plane. The frescoes of Pompeii do not have a vanishing point as used later in linear perspective but a good mimicry of the natural impression.

Figure 2: Pope Gregory the Great by the Master of the Registrum Gregorii, Trier, about 983 (from [6])

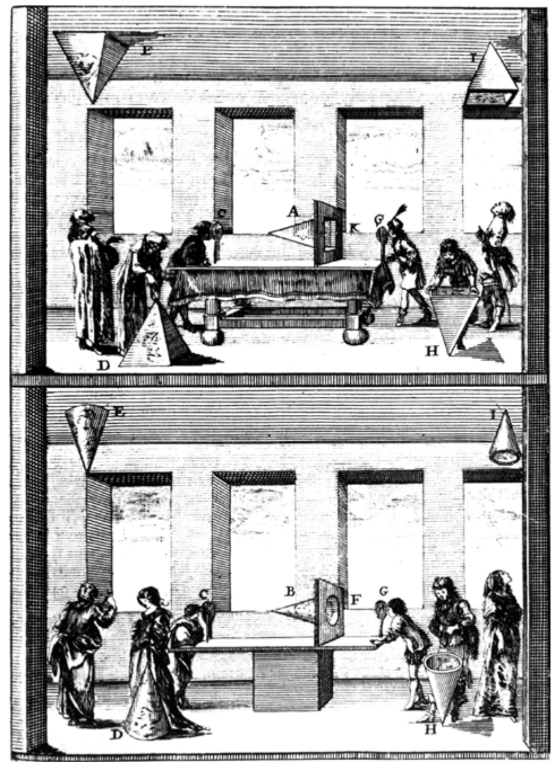

Figure 3: DuBreuil, *La Perspective pratique*, 1649 (from [3])

Illusionary painting seems to have no importance in the Middle Ages. Subordinated by religious values, architectural elements in pictures are only decorative expansions of selected persons. Like a third skin architecture is used to illustrate state and dignity of a high official (Fig. 2). *Perspective as symbolic form* (Panofsky) refers to a divine order rather than the vanity of this world. On the canvas the objects orient towards the dignitaries. The observer of the painting has no comparable importance; the objects do not align to his viewing direction.

The artists of the renaissance pick up the art of the ancient world again and are highly interested in problems of projection. Their aim is a picture that, while neatly placed in the bundle of the viewing rays, evokes the same impression as the pictured objects themselves. The mechanism of viewing from a fixed point of view, the looking with one eye, is in the focus of interest. Thus the individual is a researching and recognizing mind, surveying the world with the help of optical rules. This aspect of drawing is best seen in the graphic work of A. Dürer [2]. In the course of time the regularities of linear perspective were discovered. Consequently the regularities totally dominated the composition of pictures. With perspective in mind any picture could be estimated *"right"* or *"wrong"* until the end of the 19th century.

But even with a geometrically correct usage of linear perspective, this system produces astonishing problems and apparent contradictions.

These conflicts and phenomena can be demonstrated in set-ups for experiments as done by the 17th century's optical cabinets of curiosities (Fig. 3). Since then a great number of artists deal with anamorphic tricks and optical illusions. Fine examples can be found in the work of Hans Holbein, Francesco Borromini, Andrea Pozzo, Père DuBreuil, Jean-François Niceron [3], M.C. Escher, Patrick Hughes, and Stefan Mauck.

## 3. Wrong perspective

In the 20th century, painters like Picasso break up the construction of linear perspective. The enlightened, seeing human being is no longer the conceiving pole in a world that is to be investigated [10]. The new topics of this time — motion of bodies in space, dynamics, ontological doubts about the objective, synchronicity of events — find their counterpart in an unlimited picture-language. Moreover, photographic machines fulfill the task of realistic reproduction. As a consequence of this process, Kasimir Malevich leads the art of painting to point zero with his famous painting *"Black Square"* (1915).

In some strange way, Picasso's representations of moving space seem to continue medieval traditions. Evaluating these pictures with the use of the academical categories "right" or "wrong" perspective, they must be rated "wrong". This means they follow an unusual logical pattern [11]. A logical pattern unfamiliar to us, keeping in mind that we are permanently confronted with photos or renderings.

If we technically want to get closer to this logical pattern, let us assume that the "wrong" perspective is the opposite of a usual linear perspective. We can then construct an *inverted* perspective such that the viewing rays are oriented towards the centre, and the visibility is defined accordingly (Fig. 4). Everything that is usually covered because it is far away from the centre, is now visible and covers those elements close to the centre. This way we must deal with completely new aspects in the design of the picture, the suitable position of centre and screen. Nevertheless the construction of an inverted perspective works just like the construction of a linear perspective.

The inverted perspective looks quite similar to a relief perspective viewed from the reverse or a theatre scenery seen from backstage (Fig. 5). Here lies the key to a simple implementation

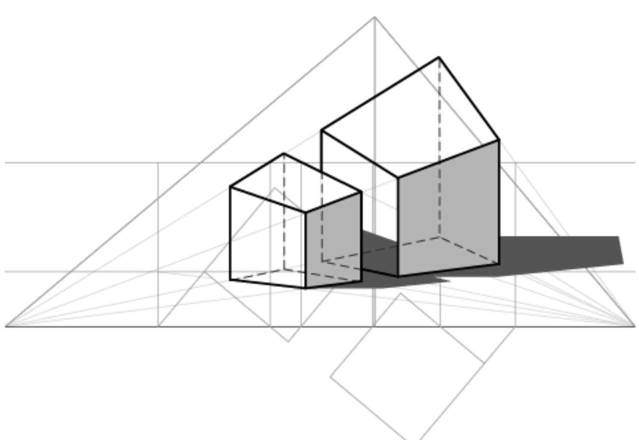

Figure 4: Inverted perspective of two cubes, 2005

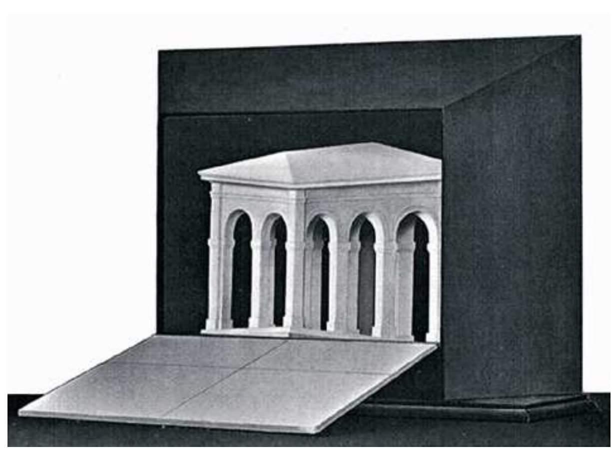

Figure 5: Burmester, relief model from behind (from [1])

of the inverted perspective into standard rendering software (e.g. Cinema4D), without the need to manipulate the rendering engine.

## 4. Inverspective

A relief perspective is a central collineation of three dimensional space, where the plane $\pi$ of traces and the plane $\varphi$ of vanishing points are arranged in order to serve the purpose. That is, $\pi$ is in between the centre of projection and $\varphi$ [9]. Relief perspectives in Cinema4D can be created with the "Formula Deformation Object". A linear algorithm produces the spatial transformation [7]. We now want to interchange $\pi$ and $\varphi$ and by this invert the sequence of points on the mapped lines. So the visibility compared to the original object is inverted. If the distance between $\pi$ and $\varphi$ vanishes, the relief changes to an inverted linear perspective. But instead of this approach we keep the relief, because by this we can make advantage of the shading qualities of Cinema4D. We simply map the inverted relief by a normal projection [9] and the result is the desired picture, which from now on will be called an *"Inverspective"*.

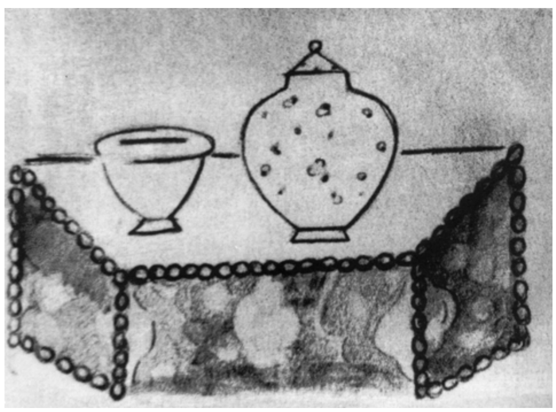

Figure 6: Persian miniature, 15th century, detail (from [11])

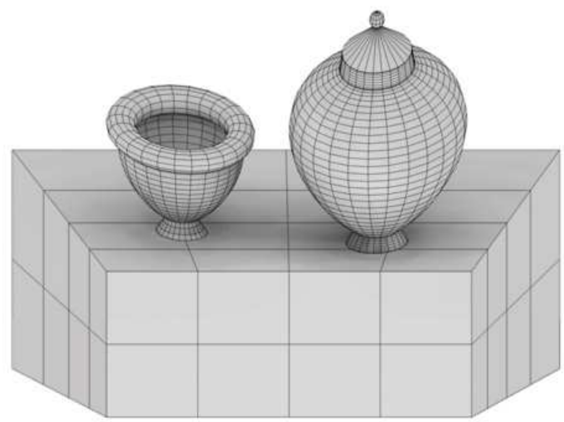

Figure 7: Rendering 2005

Two motives from different ages serve as testing objects in the following inverspectives. The first is a detail of an old painting with a still life (Fig. 6). On a rectangular coffer showing four sides are placed two vases. The vases are shown in a frontal projection with the small modification, that some lines of latitude are bent to reflect the essence of the rotational surface. With a little imagination the spatial situation can be reconstructed. Of the supposed situation we produced the rendering in Fig. 7.

The second motive (Fig. 8) is not a "wrong" perspective but an isometric drawing. Understanding an isometric projection as the borderline between linear perpective and inverspective we can now make the free decision to where the parallel lines shall align – to the vanishing points or the traces. Figure 9 shows the corresponding inverspective.

## 5. Final remarks

How to read an inverspective? How to get the right impression of the objects represented by the image? In the case of the well-known linear perspective the spectator should take the position of the point of view on which the construction is based. He should look with one eye closed and he should not move. The eyeball turns and scans the picture. In the case

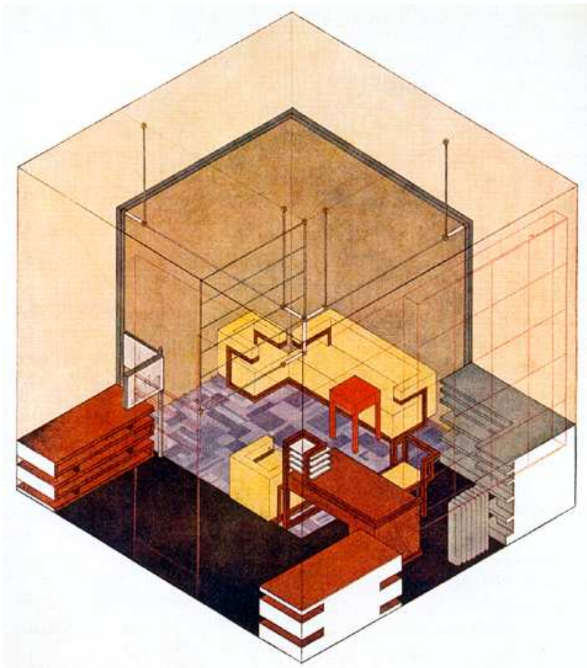

Figure 8: Herbert Bayer, *Atelier of Walter Gropius*, 1923, Bauhaus Weimar (from [4])

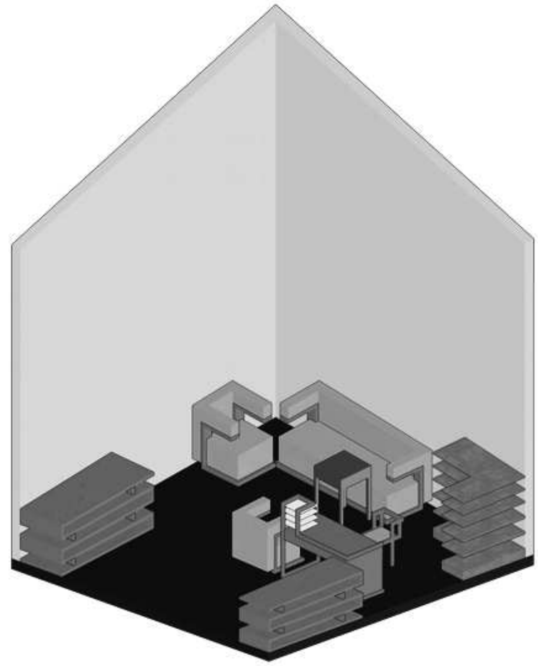

Figure 9: Rendering 2005

of the inverspective the spectator in theory has to keep his look orientated to the centre of projection while moving constantly. He may scan only one pixel from each position. The single impressions would then sum up and, in his mind, generate the original space and objects. This is impractical. Nevertheless, we are in general able to understand such an inverspective both in its meaning and topology. That indicates the picture is readable.

Let us finally have a closer look into our eyes. The viewing process in our eyeball is not very different from the theoretical method of reading an inverspective as described above. The photoreceptors in the retina are located around the intersection of the viewing rays in the lens. In our mind we generate the picture from a series of single points. In every eyeball the viewing rays are orientated radial to the centre of projection: a natural setup for an inverspective.

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