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Knowledge, Repertoire and Ruled Surfaces in Oscar Niemeyer's Architecture

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Abstract. The aim of this article is to emphasize the importance of developable ruled surfaces in NIEMEYER's works. Our research intends to investigate some geometrical procedures in NIEMEYER's architecture and his architectural lexicon of form from the geometrical point of view, since geometry provides the means to comprehend architectural form. As a modern architect, his projects respect some values in terms of economical resources, simplicity and regular form. We analyzed case studies to understand how the architect obtains a unique form from simple geometrical sketches, based on second degree curves, the conics. We demonstrate that the 3D models of those buildings allow us to understand the geometry which underlies them.

In order to examine shape aspects of architecture through graphical analysis, we study drawings to comprehend the architect's compositional procedures. In addition, the surfaces analyzed are developable ruled surfaces, so we can unfold each surface into a two-dimensional plane. Thus, the Gaussian analysis on 3D models allows demonstrating the regularity of the surfaces. A detailed analysis indicates that NIEMEYER prefered to generate 3D shapes from 2D section shapes. The section allows the architect to understand both the interior and exterior of the building, the spans and the structural stability of construction. Hence, the examination of the drawings permits interpreting his main proposal: how to turn sculptural form into architecture. The final results of this research are summarized in this paper through diagrams that provide a visual comprehension of the geometrical operations developed by the architect.

Key Words: Developable surface, ruled surface, design process, graphical analysis
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1. Introduction

In some way, every architect makes use of geometric and mathematical concepts. As we know, geometry is a branch of math which studies shapes. Geometry discloses how form is

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produced, as well as the relations among elements that constitute it. Assessment of design projects depends on understanding of the concealed geometry. However, each architect has his own method of applying geometry. From an understanding of the applied geometry in the project, we can comprehend the specific way of spatial organization adopted by the architect. This is an essential concern on complex form analysis, where the organization parameters are widely different from those applied on simple form organization. The best examples of changeable curvature geometry in architecture are those which are generated from some premises, particularly the ones that provide a possibility of ruling and rationalizing. Thus, some precaution must be adopted at the moment of modeling curved surfaces, in a way to obtain a feasible proposal.

"Architects construct what they can draw and draw what they can build", affirmed William Mitchell. The best way to make the construction of higher complexity forms possible is to rationalize its geometry and, over all, to understand the constructive and material techniques adjusted for its construction. For the reason that double curvature surfaces are onerous to manufacture, the ruled surfaces are frequently prefered by architects. This is the case of Niemeyer's architecture. The work has been investigated in five stages:

1. Selection of five case studies;
2. Analysis of manual drawings to discover 2D curves used;
3. Creation of 3D digital models to comprehend 3D surfaces;
4. Data analysis of unfolding surfaces and Gaussian curvatures;
5. Compare and classify the results.

Our presentation is organized in six parts:

- The *conceptual ideas* that are behind the spatial form;
- *Sketches and drawings;*
- *Knowledge and repertoire;*
- *Ruled surfaces;*
- *Geometrical analysis;*
- *Digital models*

In recent years, the question how to build curvilinear surfaces has been a constant source of discussion. The objective of the current investigation is to provide means to assess curvilinear surfaces during the beginning of design process, as well as potential improvements to rationalize the new form in architectural buildings.

2. Conceptual ideas

In 1988, during the Oscar Niemeyer's *Acceptance Speech of the Pritzker Prize* the architect declared:

"(...) as Charles Baudelaire once said, 'The unexpected, the irregular, the surprise, the amazing are an essential part and characteristic of beauty' ".

This is a main idea about creation in Niemeyer's work. The sculptural form is part of justification of the project *"(...) when a form creates beauty it has its own beauty justification"* [3, p. 209].

The sketches allowed the architect to visually demonstrate the reasons for the intended solutions. Niemeyer explains that: *"The desire of discovering a new form became evident in all my work, form that the technique and the special conditions of each subject suggested. In* *order to reveal this process I presented drawings and 'sketches' which demonstrate how they were integrated in the internal conveniences of projects"* [3, p. 271].

The varied column formats allowed as well the great free spaces, as reducing the number of *pilotis*.

The geometry of concrete shells took place in conveniences and functional necessities, visual, aesthetic and techniques conjunction. In the auditorium of *Barra da Tijuca Center of Conventions* (1997) the slenderness of one of the extremities resulted from the intention to extend the view from inside of the auditorium towards the square and buildings of the set. *"Arcs, constructive vaults and other elements had been used by myself with the objective to abolish columns or to establish the desired contrast with straight lines surfaces"* [1, p. 112].

For the architect, the curved coverings allowed to get great free spaces with great sculptural plasticity, to launch great eaves of plain slab with no need of intermediate supports and, at the same time, to shelter items from the necessities program with great freedom inside the flexible space generated. Those great free spaces represented the freedom of people to put into motion and revealing themselves as free citizens.

Besides to impose dynamism to the façades, their inclinations allowed to expose or to protect the transition of the areas between covered and open spaces.

The origins of curvilinear form were explained as being a natural result from the topography analysis of the place, in some cases, as *"the beloved woman curves or even from mountains of Rio de Janeiro"*. In consequence, it is difficult to justify, fully and rationally, the architect's plastic intentions, fruits of a strong desire to express himself with free and organic form.

In the buildings adjacent to accented declivities, the architect planned great structures with overhangs on extremities. As much in *Museum of Caracas* as in *Museum of Niterói*, the central structure supports the great overhangs, with no columns at all.

In the *"Praça dos Três Poderes"* NIEMEYER affirmed that *"(...) I didn't intend it cold and technique, with the classic pureness, it lasts, already waited from the straight lines. I desired to see it, on the opposite, full of form, dream and poetry"* [1, p. 81]. The architect intentionally opposed himself to the rationalism and functionalism generated by pure and orthogonal form. For the architect, the geometry of curved form provided a dynamic and sensual composition, which defied the technique in a search for an innovative, poetical and dreamer solutions. Thus, in those palaces, the architect planned slim and re-curved columns, whose extremities merely seem to lightly touch the ground.

3. Sketches and drawings

Cognitive processes in design are directly dependent on our visual and perceptual mechanisms. Sketches, as external representations, allow accumulating ideas and cues for later inspection. The predominant technique revealed by the architect is the freehand drawing. During many years, NIEMEYER's drawings have being done with greater easiness and spontaneity, becoming more and more synthetic. Throughout this research, it has become clearer and clearer that the sketches, as embryonic drawings, allow developing graphically the space reasoning.

The sketches demonstrate different problem aspects, from aesthetical to technical problems, as well as the geometrical aspects.

The sketch is a fundamental element during the development of the project. The sketches allow tracking the rapid development of some ideas and verifying how the decisions had been made. Confirming the importance of the sketches, in his design's process, the architect Oscar NIEMEYER affirmed that *"(...) when in architecture it is possible to explain a project with a* *small sketch, this is a proof that it is well thought, as it always should be"* [1, p. 18]. In this manner the architect tries to persuade us that his solution is the best choice. Furthermore, the simplicity of the drawing, showing only contours of form traced by some gestures lines, tries to convince us that the proposal is both economical and beautiful as a sculpture.

Niemeyer stresses his preference for section and perspective drawings. While the section reveals the roof curves and big spans, the perspective allows persuading us the generous freedom from space, as well as the relationship between building and landscape. The first one shows different technical and functional aspects from the project, such as structure, geometry and heights of the ceiling, while the second one illustrates the aesthetics aspects of the project, such as visual field, proportions, rhythm and scale.

During the detailed examination of the drawings, one easily realizes the importance of the sections drawings, particularly the ones related to the spatial proposal in Niemeyer's work. The section sketches are useful to decide the correct curvature of the structure. Therefore, this kind of drawing allows geometrizing the roofs and to study the building's component details' dimensions.

4. Knowledge and repertoire

Problem-solving depends on the combination between experience, knowledge, abilities, skills and mainly on a lot of work. Although for Niemeyer, the central concept of the project comes up from a good balance between knowledge and intuition, it is possible to perceive that the architect is used to draw exhaustively and compulsively several variations of the same project. Normally Niemeyer creates lots of small sketches for the roof before he takes the final decision. This process is not magical. There is no intuition without professional experience and knowledge.

Design is a dynamic process of adaptation and transformation of the knowledge of prior experience in order to accommodate them to the different present situation [2]. Niemeyer has a large repertoire of forms and spatial solutions formed during his career. He reuses these experiences in a creative way. It is interesting to observe the sequential transformation of the several spatial and formal solutions as it is accommodated and adapted to other context.

5. Ruled surfaces

Ruled surfaces are those created from a generator and a line of direction. A ruled surface can be described by a straight line that goes through a trajectory in the space.

There are four possibilities to generate developable ruled surfaces:

1. a plain surface;
2. cylindrical surfaces;
3. conical surfaces;
4. tangent surface of a spatial curve.

All these surfaces share the property that there is always in at least one direction a straight line located on the surface generated.

Ruled surfaces are swept out by bars passing along director curves thus facilitating the definition of geometric and constructive elements. The hyperbolic paraboloid is an example of this. The surface is formed by a generator that passes along director lines. It is a non-planar surface where each point is the intersection of two straight lines. The parabola and hyperbola

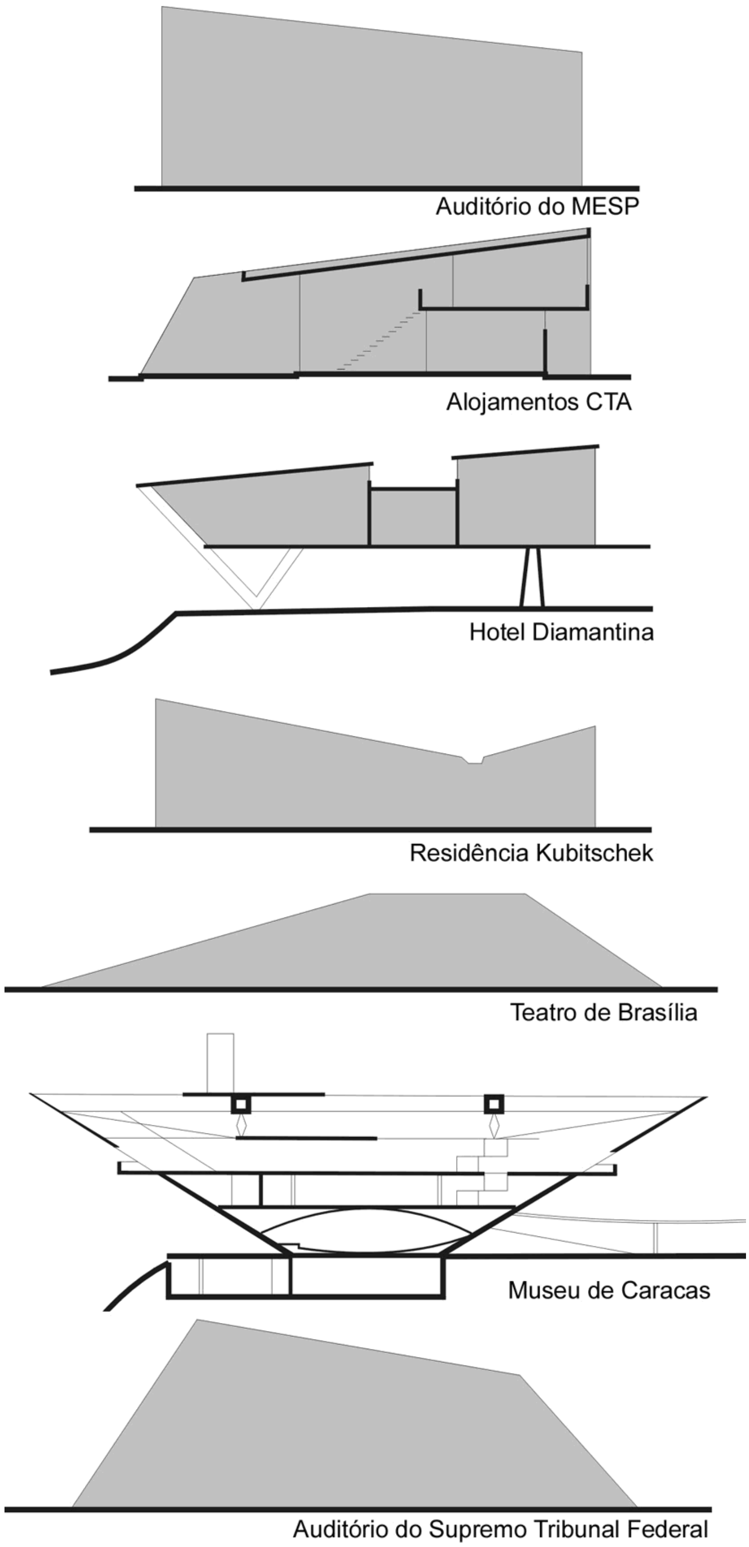

Figure 1: Trapezium metamorphosis

appear when the surface is intersected with a plane in arbitrary direction. Another example is the surface that is generated by a straight line that establishes a connection between two curved lines.

6. Geometrical analysis

In the beginning of Niemeyer professional career, from the second half of 1930 on, we can notice a strong influence from Lucio Costa as well as from Le Corbusier. The geometry of his first projects is clearly manifested by the statics of a regular form and straight lines. However, it resulted from the set of buildings carried out for Pampulha in Belo Horizonte (Brazil), that Niemeyer established the first lexicon of curved form which will follow him throughout his architectural work.

Initially the architect adopted a trapezoidal form in a way to break straight angles. The trapezoidal form is being transformed into more complex curvilinear form, successively each time. Soon the architect noticed that the catenary formed between abutments could be spatially explored, becoming an integral part of the architectural proposals.

Firstly the trapezium was intercalated by vault. After that the sets of vaults sequence soon were incorporated to his vocabulary of form. The following step was to mix a trapezoidal form with the vaults (Fig. 1).

At the same time where the architect developed a new form for closings and coverings, he also developed a new form for columns and porticos which provide sustentation. We can notice that if the initial columns were circular and perpendicular to the ground, the columns which came to follow had a trapezoidal format, inclined and in "V" format.

In the following phase Niemeyer starts to create parabolic coverings and porticos with great plasticity.

Since the project for the *São Francisco de Assis Church* (1940) in Pampulha, Belo Horizonte, the vaults were spatially explored in his work. Those ruled surfaces are generated from a straight line which covers the parabolic arcs giving origin to the vaults. Although it was initially materialized in the *Weekend Residence of Oswald de Andrade* (1938), in a form of abated arc, the vault was constructed, for the first time, in this Church. Later in his following works we notice that this sequence of vaults was incorporated to his architectural language. This research verified and classified three different applications of ruled vaults surfaces in Niemeyer's projects (Fig. 2):

Type 1 – The first one is characterized by a small sequence of vaults.

Type 2 – The second one is at buildings where only one vault achieves a great free space.

Type 3 – The third one is when one or two great vaults are supported by a central beam that constitutes an axle transversal line to the building.

Vaults type 1:

Igreja São Francisco de Assis (1940)
Estádio Nacional (1941)
Teatro Municipal de Belo Horizonte (1943)
Restaurante e Centro de Lazer da Lagoa Rodrigo de Freitas (1944)
Residência Burton Tremaine (1947)
Sede da ONU (1947)
Hotel Regente em São Conrado (1949)
Restaurante e Auditrio do CTA em São José dos Campos (1950)
Fabrica Duchen (1950)
Posto de Gasolina Clube 500 (1952)
Convento Dominicano em Saint-Baume (1968)

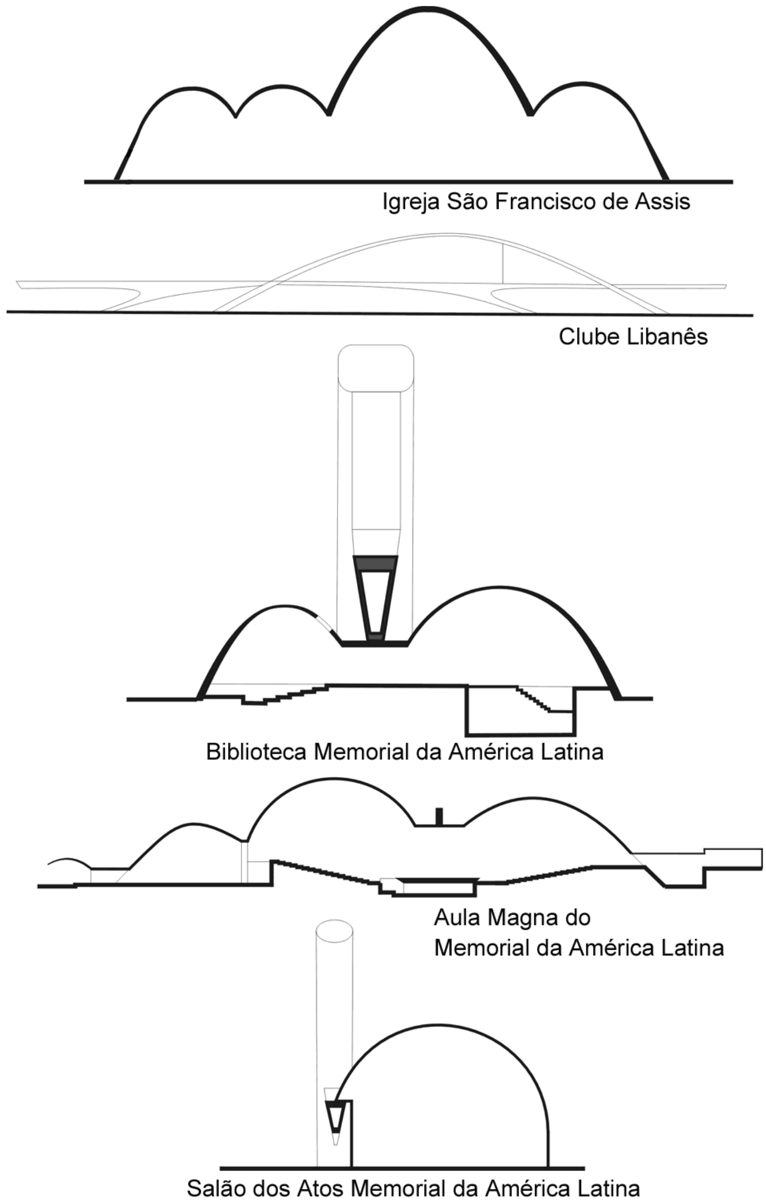

Figure 2: The three types of vaults

The diversity of uses sheltered by the same form demonstrates that, for Niemeyer, form does not necessarily follow the function. This type of covering was widely used in the decades from 40 to 60.

Vaults type 2:

Estádio Nacional (1941)
Clube Libanês em Belo Horizonte (1950)
Hospital Sul América no Rio de Janeiro (1952)
Colégio Estadual de Campo Grande (1953)
Colégio Estadual de Corumbá (1954)
Teatro Polivalente do Centro Cultural do Parque do Tietê (1986)
Estádio de Turim (1987)
Memorial Paranaense da Coluna Prestes em Santa Helena (1998)

We can suppose that with the technical mastery of small span curved roofs the architect starts to construct bigger roofs with greater spans from the decade of 1950.

Vaults type 3:

Universidade de Constantine (1969)
Portal da Casa do Cantador em Ceilândia (1986)
Salão dos Atos do Memorial da América Latina (1986)
Monumento ao Escravo em Gorée (1991)

The vaults which are self-support by beams and slabs renew the lexicon of innovative form, as well as they also show the importance of this type of covering throughout the entire architect's work.

The **concrete shells** constitute catenaries supported by columns:

Iate Clube Fluminense (1945)
Residência Edmundo Canavelas (1954)
Capela Nossa Senhora de Fátima (1958)
Auditório do Ministério do Exército (1967)

NIEMEYER's close familiarity with his calculating engineers, such as Emilio BAUMGART, Joaquim CARDOSO and José Carlos SUSSEKIND, brought him more knowledge about the way structure could perform architecture in a technical as well as in a plastic.

The **parabolic shells** supported by concrete porticos:

Auditório do Ministério da Educação e Saúde
Pública — MESP (1948)
Fábrica Duchen (1950)

The exposed structure (exo-structure), formed by the porticos, benefited the creation of rhythms in two directions — the shell in the longitudinal direction and the sequence of porticos in the transversal direction. Soon the architect noticed that the structural strengthen would be more efficient when they were used in cooperation.

It is clearly noticed that the geometry of certain curvilinear elements, used in whole NIEMEYER's work, is always a counterpoint to the rectilinear form. The balance between straight lines and curves is always present in NIEMEYER's projects.

High elaborated geometry is applied in the design of the columns. In this way, the formal simplicity of those buildings contrasts with the complexity of some columns. This is what we call *principle of contrast*.

This principle is applied as much to contrast buildings among themselves (straight and curved), as to contrast columns and roofs. Thus, when the architect utilizes pure form to design the building, the columns certainly have more complex geometry. On the other hand, when the building itself is composed by curved form then the columns have a simple shape. The same situation occurs in the case of coverings. Since the roof is curved, its perimeter, viewed in plan view, is relatively simple, while that, when the covering is plain, the perimeter is curvilinear or cut.

It is easy to note the strong contrast between interior and exterior. In Fig. 4 we can examine an exterior simplicity and interior complexity.

The same principle is also used to contrast materials employed. The glass fragility and transparency in the openings contrast with the stability and opacity of the concrete of the structure.

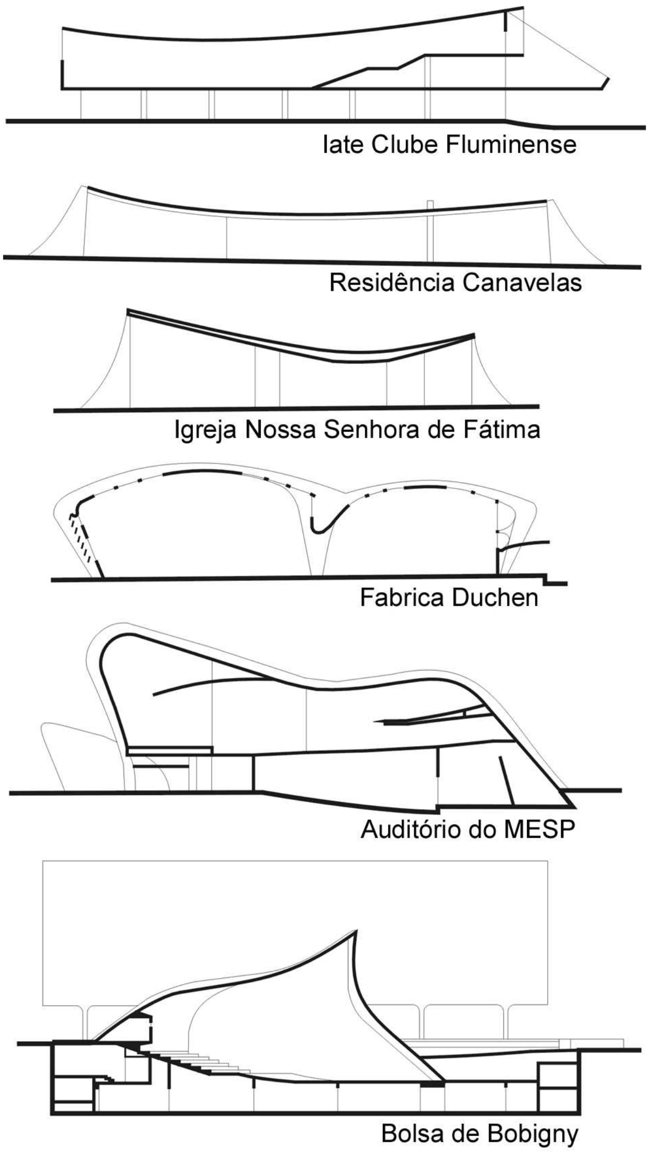

Figure 3: Flexible roofs

For the reason that the buildings can be conceived as sculptures, Niemeyer takes care with several details to preserve the formal integrity of the building, be in the drawing of the openings or the ramp access to the building. That is what we call the *principle of penetration*.

In this principle, the access ramps to the buildings are conceived in a way to allow a penetration in the building with no interference in its volumetric form. The way established by the architect was to detach this element from the main volume, so that the ramp becomes another element of the composition. Since the *Pavilion for the Fair of New York* project (1937), that the curvilinear ramp contrasts with the perimeter simplicity of the building. As much in the *Museum of Curitiba* as in *Museum of Niterói* (Fig. 4) Niemeyer creates curvilinear ramps detached from the building, such that they contribute to the composition without intervening in its integrity. Even in the *Palace of Dawn* the straight ramp in ascension is detached from the main volume form, creating an inclined plane without obstructing or "competing" with the form of the building.

In the first years of his professional activity, the use of trapezoidal form as a way to break

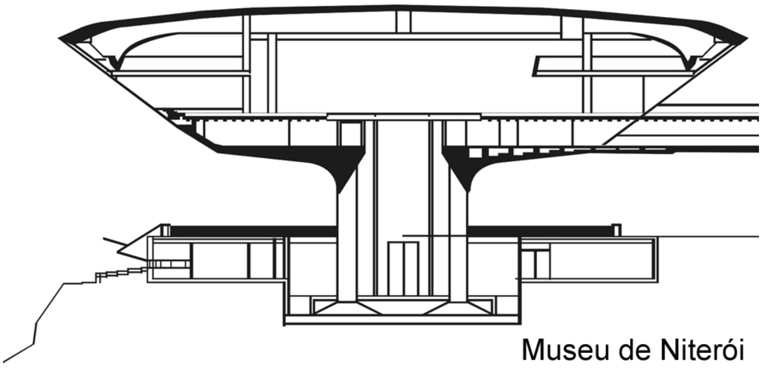

Figure 4: Surface of revolution

the symmetry and regularity of the static form predominated. From the sixties, he started to take advantage from surfaces of revolution, generated from straight or arched profiles. While the cylindrical form is ruled, the curvilinear geometry of the cupolas is not ruled, but there is a formal and clearly necessary principle, whose resulting aesthetics contains great plastic expressiveness.

The combination between those geometries (trapezoidal form + vaults + surfaces of revolution + cylindrical form) will propitiate the constitution of his formal curvilinear repertoire formation.

It is also important to emphasize that the curvilinear surfaces represented a way to alleviate the higher buildings visual impact in the landscape. Thus, the level of several cupolas was partially buried, in a clearly intention to open a determined visual field and not to obstruct it (see Fig. 3, *Bolsa de Bobigny*). The smoothness of these vaults reaching the ground shows the intention to attenuate the contact between structure and ground propitiating the slightness and gracefulness to the built set. This is what we call the *principle of lightness*.

In a similar way, this principle was used when the architect created some columns which seemed only "to touch" the ground, those roofs, with varied heights, were conceived with an intention to be naturally (and visually) supported by the ground.

There is another principle that we call the *principle of suspension*. The real structure, which support the building itself, does not clearly appear, remaining "hidden" or camouflaged, with an intention "to delude" the user of the space, promoting the idea of lightness and suspension.

It is possible to affirm that there are at least three types of metamorphoses in the formal proposals in Oscar Niemeyers architectural projects. The first one is the *trapezium*. In Fig. 1 we can notice that the introduction of the diagonal line was the first resource utilized by the architect, imposing dynamism in the composition, as well as a rupture with the orthogonal geometric of cubical form.

The second metamorphosis is the *parabolas*. Fig. 2 shows how the concrete shells are re-worked to shelter different functions.

The third metamorphosis is the *parabolic curves*, which we called in this research as "flexible". Fig. 3 shows the initial catenary being gradually bending until finally joined with the columns, constituting curvilinear porticos.

Those architectural form transformations were the "inventions" which Niemeyer proposed in each project during his whole architectural work. The formal repertoire was enriched in a way of re-using the previous formal principles such as in an introduction of new proposals.

The curved form geometry, which twists freely in the space, does not contain the severity and rigidity form/geometry from the main part of the buildings of Modern architects. The curved proposals by Niemeyer exceed the rationalism of Le Corbusier's works and as many of the called "organic" architects during the XX century. However, the conical sections have been presented in his works, especially parabola and circle. As Eero Saarinen, Jorn Utzon, Felix Candela and Eladio Dieste, for Niemeyer geometry of form was always based on parabolas. Although he knew the architectural works of Kenzo Tange and Píer Luigi Nervi, his projects do not contain neither the hyperbolic paraboloid nor the conoid. Although he always affirmed that in his projects the shapes were different from the others, Niemeyer did not apply another different geometrical knowledge in his projects, especially the ones which were being executed by contemporaries' architects and engineers of his age.

7. Digital models

Digital models allowed the examination of some important issues related to the form such as ruled, and developable surfaces, unfolding and Gaussian analysis.

7.1. Developable surfaces

As we know, a quadratic equation defines a second degree curve and a cubic equation a third degree curve. The difference between them is that the second degree one has two handles of control for each curve, while the third degree has three points of control. The great difference is that the quadratic curve will provide a less soft and continuous surface of the cubical curve. So, a curve is continuous unless it is derived from an algebraic equation of third degree.

The curves are considered unfoldable or *developable* if two dimensional surfaces generated from three-dimensional objects can be unfolded without the material strains, it wrinkles, it tears or deformed. Thus, it is a type of surface that can be formed by means of a leaf that is "rolled" without deformations. Examples of this type of surface are cylinders and cones (spheres are not developables). If the surface tears or wrinkles at the attempt to bend the surface into a planar shape, it is not developable.

The developable surfaces are those that allow a subdivision into portions which have a convenient curvature and are acceptable for this bending. The unfoldable surfaces derive from the four types of ruled surfaces described in the introduction.

7.2. Unfolding

In the last years, in some graphical computer programs (as Form-Z and Rhinoceros) it has become possible "to unfold" a three-dimensional shape into an only two-dimensional area. This technique is called *unfolding*.

After the form is unfolded sequentially side by side, we get the necessary information for its manufacturing, such as the real lengths of the entire edges, the measurement of angles and surfaces areas. The coordinates of each vertex can be geometrically identified. In this way, the digital three-dimensional model, guided for a technique-operational production of drawings executives for the work, allows to generate information as well in a way the detailed elaboration of budgets as for the construction.

Therefore, the unfolding technique, as a kind of computational descriptive geometry, is one of the basic resources that allowed the rationalization of constructive elements to be manufactured.

7.3. Gaussian analysis

The surfaces curvature analysis is essential to identify parts that may cause problems, e.g., where the bending of metallic plates becomes impossible. By the Gaussian curvature analysis we can verify and evaluate which are the regions that must be modified, in order to get acceptable curves. In critical regions, the surface must be divided, in order to introduce bending that has tangential continuity.

The Gaussian curvature is the product of the main curvatures, as much in transversal direction as in longitudinal one. In each point of one curve tangents are calculated in several directions. The average from those calculations is the *Gaussian curvature*.

The red color in Fig. 5 designates a positive value of the Gaussian curvature, i.e., where the surface is convex. The green color designates zero Gaussian curvature, that means the surface is plain in at least one direction. Finally, the blue color designates a negative value of the Gaussian curvature; here the surface is concave.

When the designed surface curvature exceeds an acceptable value for its production, the surface must be modified in a way to reduce the bending. The areas of concentration efforts are re-worked in order to reduce the bending to acceptable levels for metallic plate's curvature. The most appropriate technique to divide the surface into smaller parts is to introduce a curvature that has tangential continuity.

The Gaussian curvature analysis can be applied to determine which areas of a surface are (or not) developable. A surface is developable if and only if the Gaussian curvature is zero. Therefore, a developable surface is a ruled surface (i.e., "linear in one direction"), but there are plenty of ruled surfaces which are not developable.

During the research the three-dimensional digital models of some vaults, from different Niemeyer's projects, were submitted to the Gaussian analysis. The results proved that the Gaussian curvature is zero in each point of surface (Fig. 5). Therefore those surfaces are developable and ruled surfaces. In Niemeyer's works the developable surfaces are the simplest and the more appropriate solutions for the design of curved roofs.

8. Conclusions

The geometry used by the architect depends almost exclusively on parabolas. More elaborated geometry is applied to the columns drawings. In this way, the formal simplicity of buildings contrasts with the complexity of some columns.

The rationally geometrized curves are idealized, drawn and defined through section drawing, since they are normally utilized to generate roofs, either for extrusion, in the case of vaults, either for revolution surfaces, in the case of cupolas or spherical shells.

The "free" curves are idealized, drawn and defined in plane view, since they are utilized normally to generate marquees (except the *Casa do Arquiteto* in Canoas).

For high buildings the curves are defined in a plane view, parallel to the ground, that are extruded vertical and perpendicularly to the ground. While this, in low buildings, with curved roofs, they are extruded horizontal and parallel to the ground.

The "free" curves are conceived to demarcate undulating circulations, as much for coverings, as in the case of marquees, as for floors, as in the case of the ramp and slabs that "undulating" between *pilotis*.

There is a trend in his architecture to oppose regularity of external form from the internal spaces complexity. While the external perimeter is geometrically pure, the interior space is

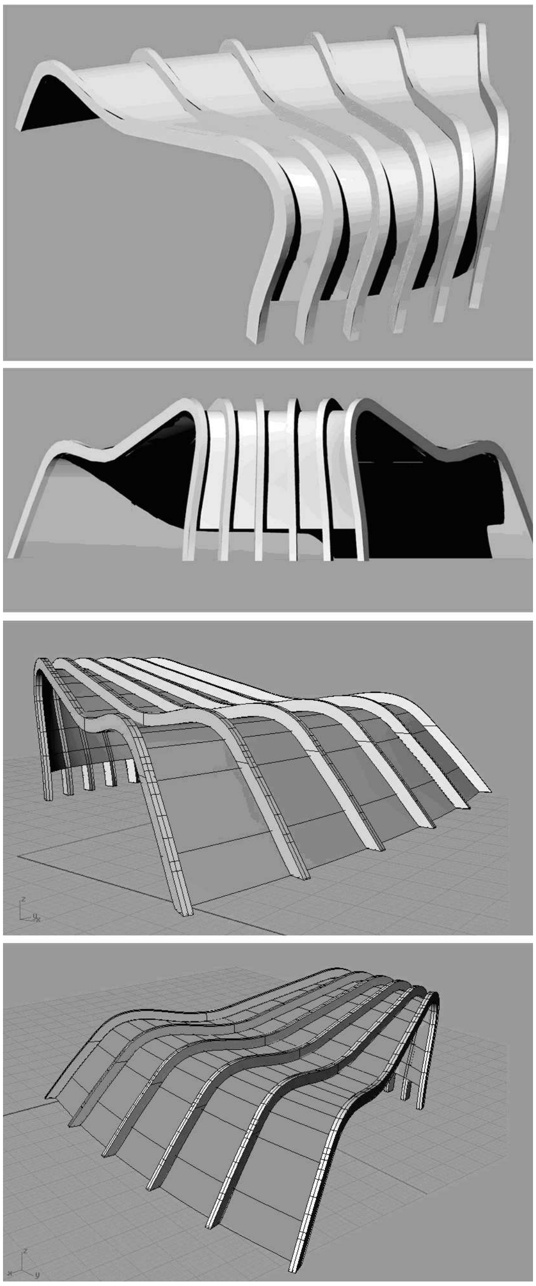

Figure 5: MESP Auditorium

cut by levels with different perimeters for the floors (see Fig. 4).

The results show that the curvilinear shapes in Niemeyer's buildings can be classified in three categories of surfaces: revolved (in turn of an axis), extruded (in one direction) and interpolated (between two curves).

For Niemeyer geometry is basic to accent the optics illusion character of the organization of form in space. To accent the sculptural character of the form proposals, the architect takes special care for the creation of openings (doors, windows and gaps). This situation occurs in such a way during selection of materials, as the glass, that allows disguising structure, as the geometry of the openings that are geometrically pure and give rhythm not to visually intervening with the form, in the same time that preserves its sculptural character.

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