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Analysis of Causes of Errors in the Mental Cutting Test – Effects of View Rotation

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Abstract. Since 1990, the Mental Cutting Test (hereafter MCT) has been widely used to measure spatial abilities in relation to graphics curricula. It became clear that the evaluation by MCT mainly reflects the ability to recognize the solid from the perspective drawing and the ability to consider the problems analytically. Most of the subjects who made low scores on the MCT, however, could not imagine the space itself, when they observed projection drawings. They seemed to have rather vague criteria for judging depth. In this respect, if the process of rotating the test solids is added to the MCT and given subjects discretionary views of the test solid, it might be possible to promote a better sense of recognizing a 3-D object and forming correct images. In this study, the Rotated-View MCT (hereafter RV-MCT) using a computer system in which subjects were able to rotate the problem solid on the horizontal plane within the limits $+180°$ and $-180°$ using a horizontal scroll bar was devised. The RV-MCT consists of 9 problems together with 2 exercises all of them were extracted from standard MCT, including both easy and difficult problems. The RV-MCT was conducted to twenty-nine female students. None of the subjects had taken any previous course on graphics. The standard MCT was conducted to another 31 subjects in the same period in order to provide reference data. The main results were as follows; (1) the mean score of the RV-MCT was significantly high compared to that of standard MCT; (2) subjects did not always make correct answers even when they could observe the

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hidden key surfaces by rotating the solid; (3) the relation among the cutting plane, the key surface and the visual direction was important for recognizing the cross section; (4) in case the visual line was close to the specific position with which one could observe both the cutting plane and the key surface almost as edge views, it became easier to recognize the intersection accurately; (5) thus the accomplishment of recognizing solids were considered to depend on 'taking an appropriate view' in the process of forming a correct image of the solid from the drawing.

Keywords: Graphics education, spatial abilities, rotated view, Mental Cutting Test

MSC 2007: 51N05

1. Introduction

In recent years, 3-D spatial abilities have received a great deal of attention. The *Mental Cutting Test* (a sub-set of the CEEB Special Aptitude Test in Spatial Relations [1], hereafter MCT) was used by Suzuki et al. [7] to measure spatial abilities in relation to graphics curricula.

Since 1990, the MCT has been widely used and various aspects of the test, such as causes of errors and strategies to get correct answers, have become clearer using not only standard MCT [2, 3] results, but also through various MCT systems, such as *Stereographic MCT* [11, 9, 10], *Real Solid Model MCT* [4], *Photo-stereographic Real Solid Model MCT* [5], etc. As for the results, it became clear that the evaluation by MCT mainly reflects the ability to recognize the solid from the perspective drawing and the ability to consider the problems analytically, i.e., determining the important quantity of the section in order to solve the problems.

Low scoring subjects, however, often failed to recognize the relative location of the cutting plane vis-á-vis the solid, and were unable to form correct images of 3-D objects.

Tsutsumi et al. [8] reported in the paper on *'A Mental Cutting Test using drawing of intersections'* that most of the inaccurate schemes included steps in which the subjects tried to draw intersections of MCT problems by using distinctive marks on the solids, such as contours, edges, pseudo points or parallel lines, and many such subjects were lacking a sense of depth, i.e., they could not envisage a three-dimensional object hidden in a solid, for example. From this point of view, it appeared that most of the subjects who made low scores on the MCT could not imagine the space itself that contained the three-dimensional objects when they observed projection drawings, and they seemed to have rather vague criteria for judging depth.

In this respect, one apparently effective method of analyzing the errors in the solving process is to allow the test subjects to rotate the test solid. If the process of rotating the test solids is added to the MCT and given subjects discretionary views of the test solid, it might be possible to promote a better sense of recognizing 3-D object and forming correct images of 3-D objects.

In this study, the *Rotated-View MCT* in which subjects were able to rotate the problem solid on the horizontal plane was devised and conducted to female students.

2. Method

2.1. Standard MCT

The standard MCT consists of 25 problems. In each problem, subjects were given a perspective drawing of a test solid, which was to be cut with a hypothetical cutting plane. Subjects were then asked to choose one correct cross section from among five alternatives. Full score was 25 and time limit was 20 minutes (Fig. 1).

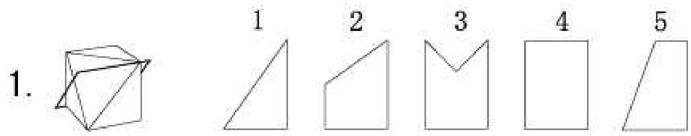

Figure 1: An example of MCT

2.2. Design of the Rotated-View MCT

The Rotated-View MCT (hereafter RV-MCT) using a computer system was devised for the purpose of making subjects detect a distinctive visual direction with which each subject was able to judge a cross section most successfully. In this test, subjects were able to rotate the problem solid on the horizontal plane within the limits $+180°$ and $-180°$ using a horizontal scroll bar (Fig. 2). The test was provided in HTML form and each rotatory solid was described using Java Applet [6]. The RV-MCT consists of 9 problems together with 2 exercises all of them were extracted from standard MCT (Fig. 3: the numbers of these 9 problems were 1, 3, 7, 12, 14, 15, 18, 22, 24 in the standard MCT).

In each problem, the subject was instructed to stop rotating the solid after free rotation when they could judge a cross section most successfully. Thus, the answer and the distinctive angle at which the subject stopped the rotation were collected. We did not set the time limit.

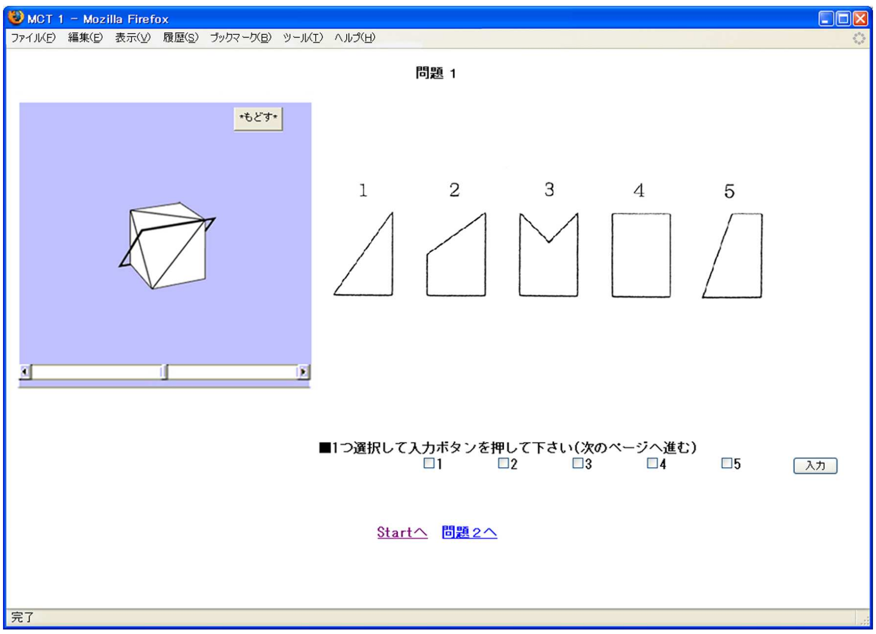

Figure 2: Rotated-View MCT

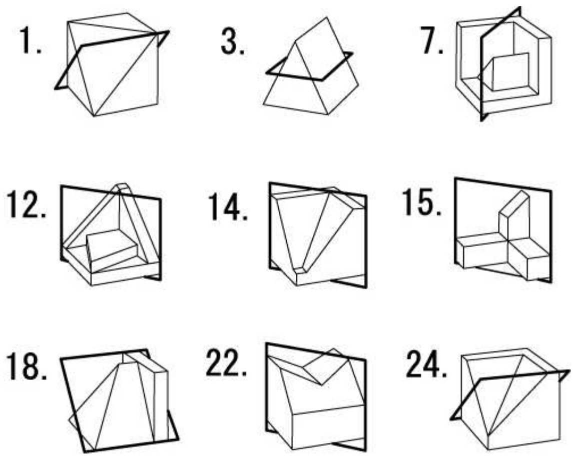

Figure 3: Test problems extracted from Standard MCT

2.3. Subjects

Twenty-nine subjects participated in the RV-MCT in November 2002. RV-MCT was performed using a computer system in a private laboratory. All participants were female students at Otsuma Women's University. They were selected randomly from a class. None of the subjects had taken any previous course on graphics.

2.4. Reference data

The paper-pencil standard MCT was conducted to another 31 female students in the same class as the subjects of RV-MCT who were also selected randomly in the same period in order to provide reference data.

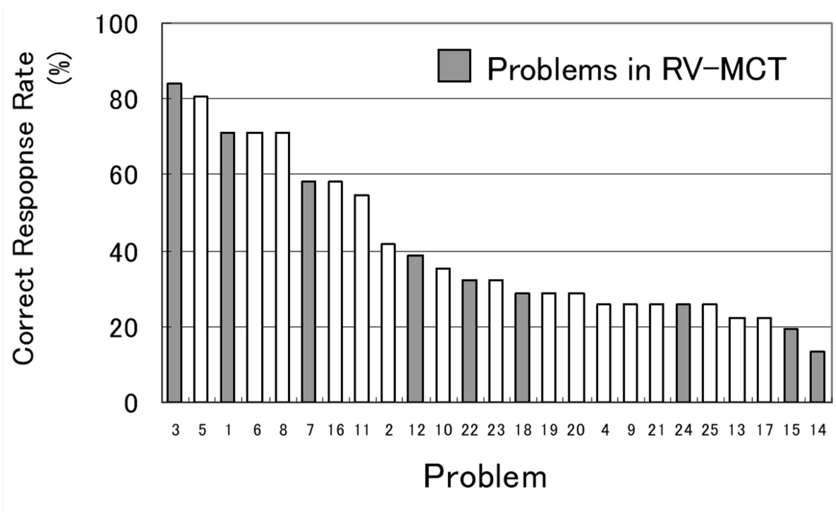

Figure 4: Correct response rates of 25 problems in Standard MCT

3. Results

3.1. Mean scores

The mean scores for the RV-MCT and the standard MCT were 5.39 ($\sigma = 2.06$, $N = 29$) and 3.71 ($\sigma = 1.77$, $N = 31$), respectively, when calculated out of 9. There was a significant difference ($P < 0.05$) between the mean scores of the two tests. Either of the scores were 14.97 and 10.30 respectively, when calculated out of 25. These values show that the present subjects were grouped into a low score class.

Fig. 4 shows the correct response rates of the standard MCT. It was shown that the problems in RV-MCT include both high rate and low rate problems concerning correct response rates of the standard MCT.

3.2. Correct response rates for each of problems

Fig. 5 shows the correct response rates of RV-MCT and standard MCT concerning 9 problems (nos. 1, 3, 7, 12, 14, 15, 18, 22, 24). The correct response rates of all the problems in RV-MCT were higher than those of the standard MCT. Significant differences were found in four low correct response rate problems, i.e., nos. 18, 24, 15, 14. The increases in the four problems were, however, not in proportion to the rates of standard MCT. Thus we can recognize that the increase in the correct response rate of RV-MCT is due to the nature of each problem.

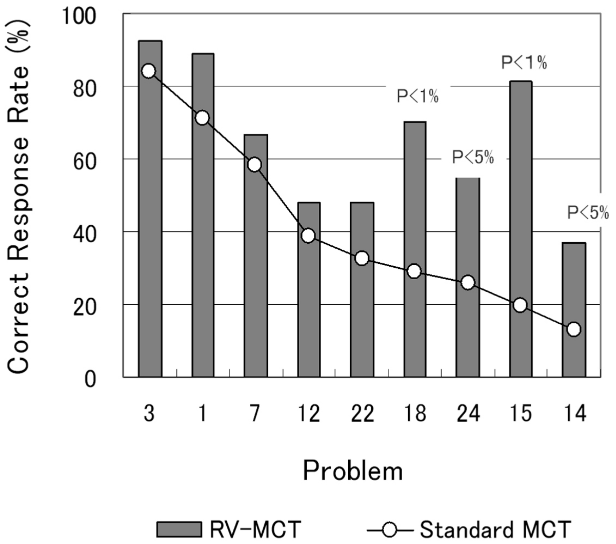

Figure 5: Correct response rates of 9 problems

3.3. Problems with significant differences between correct response rates of RV-MCT and standard MCT

(1) Problem no. 18:

Fig. 6 shows the selection rates of alternatives of problem no. 18, and Fig. 7 shows distinctive views where numbers of subjects stopped rotating the solid.

The correct response rate of RV-MCT (alternative “I”) increased about 40 points compared to that of standard MCT. Meanwhile, the wrong cross sections “II” and “III” largely decreased

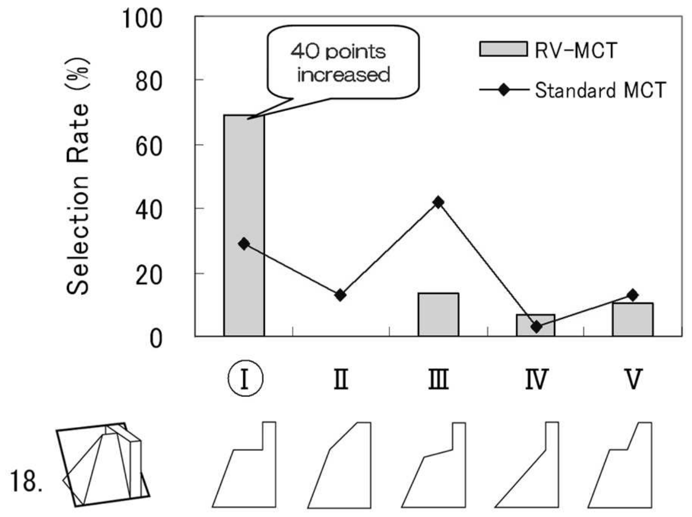

Figure 6: Selection rates of alternatives of problem no. 18

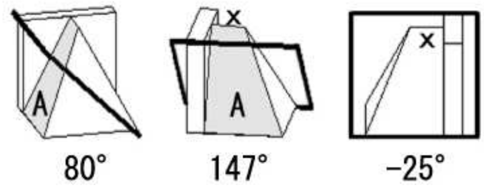

Figure 7: Distinctive views in no. 18

their selection rates. This problem has a portion hidden by the solid itself which in turn were required to be imaged. These alternatives have incorrect parts at the hidden surface (“A”).

Alternatives that were selected at each distinctive angle indicated that subjects could select the correct alternative when they got to see hidden surface “A” from rotated views such as at angles of 80° and 147° (Fig. 7). In case of a view at an angle of −25°, subjects might select the correct alternative because the contour was similar to the correct cross section, i.e., alternative “I”.

(2) Problem no. 24:

Fig. 8 shows the selection rates of alternatives of the problem no. 24, and Fig. 9 shows distinctive views where numbers of subjects stopped rotating the solid.

The correct response rate of RV-MCT (alternative “I”) increased about 30 points compared to standard MCT. Meanwhile, the wrong cross-section “II” decreased its selection rate. This problem solid is a concave polyhedron and has two triangular surfaces (“A” and “B” in Fig. 9) that were faced at an obtuse angle. The angle is, however, often taken as a right angle because the upper edges of the surfaces intersect each other at a right angle. The wrong cross section “II” has such an incorrect angle in the intersection with regard to the two surfaces.

Alternatives which were selected at each distinctive angle indicated that the subjects could select the correct alternative when they got to grasp the parallelism between surface “A” and an upper edge of the given cutting plane using a view at an angle of 70° (Fig. 9). In case of selecting the wrong alternative “II”, most of the subjects stopped rotating solid at angles of 0° or 80° when they could not see surface “A” any more.

(3) Problem nos. 14 and 15:

Figs. 10 and 12 show the selection rates of alternatives of the problem nos. 14 and 15, respectively. Figs. 11 and 13 show distinctive views where numbers of subjects stopped rotating the solids.

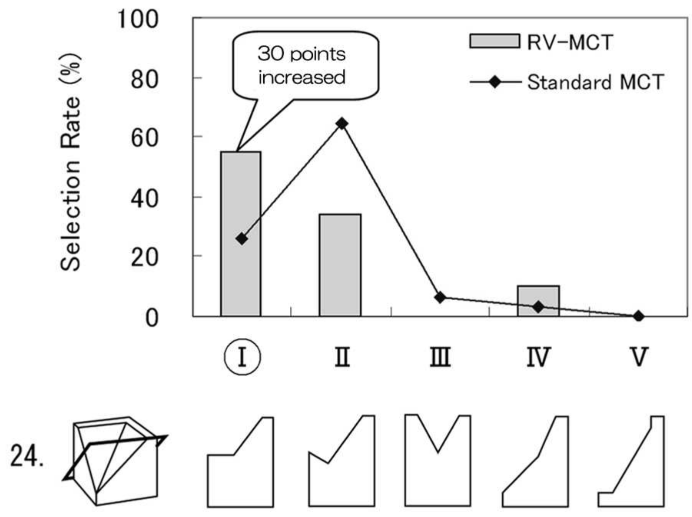

Figure 8: Selection rates of alternatives of problem no. 24

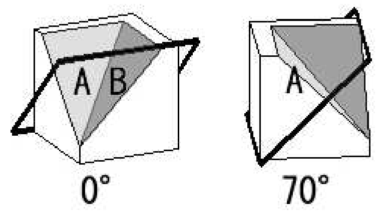

Figure 9: Distinctive views in no. 24

The correct response rate of RV-MCT (alternative "II") increased about 25 points compared to the standard MCT in problem no. 14 and about 60% in problem no. 15.

It was considered that the subjects got to grasp the relative location of the cutting plane with respect to the test solid in proportion as the angles increase (a view at angle of 77° in Fig. 11 and a view at an angle of −63° in Fig. 13).

It was also observed that numbers of subjects have repeated to rotate the solid between the two angles 37° and 77° (Fig. 11). Thus, the subjects might recognize the relative location of the cutting plane through successive changes of visual directions.

In problem no. 14 the wrong cross section "I" was also selected as much as "II". Most of the subjects who selected "I" stopped the solid at the angle of 20°. In this case, it seems difficult

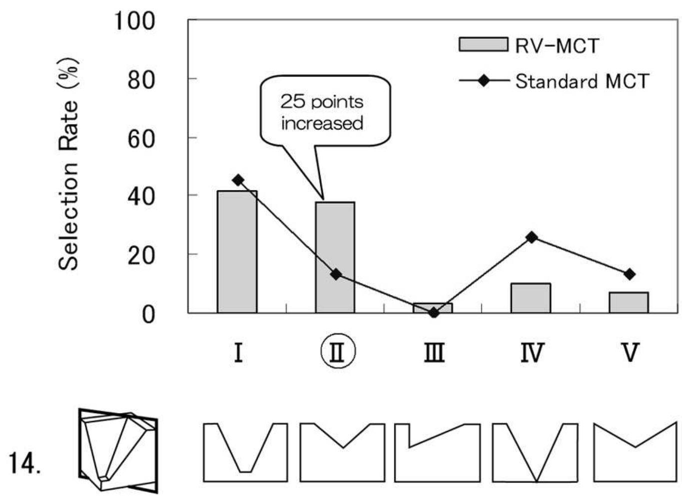

Figure 10: Selection rates of alternatives of problem no. 14

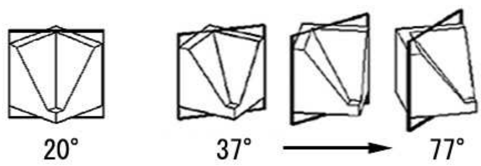

Figure 11: Distinctive views in no. 14

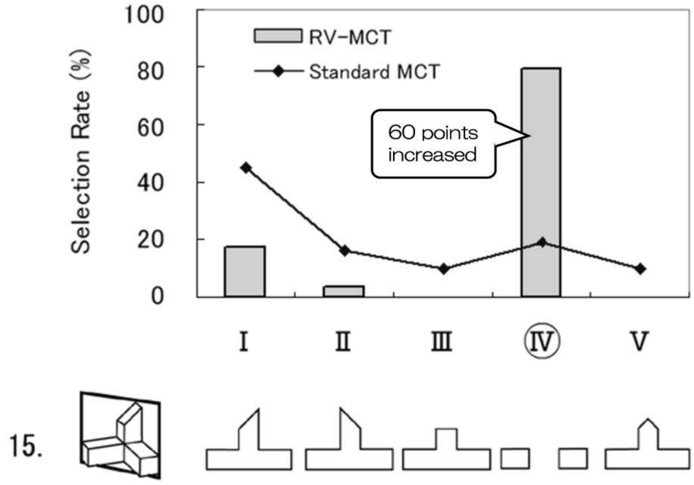

Figure 12: Selection rates of alternatives of problem no. 15

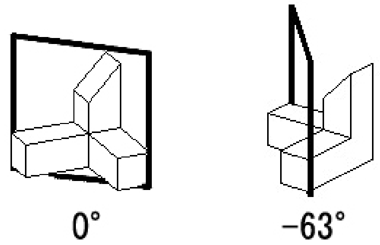

Figure 13: Distinctive views in no. 15

to grasp the relative location of the cutting plane with respect to the test solid because this figure looks rather flat caused by its symmetric feature and also because the cutting plane is often taken as intersecting with the prominent frontal part.

3.4. Problems with no significant difference between the correct response rates of RV-MCT and standard MCT

(1) Problem no. 7:

Fig. 14 shows the selection rates of alternatives of problem no. 7, and Fig. 15 shows distinctive views where numbers of subjects stopped rotating the solid.

This problem showed a high correct response rate in the standard MCT, thus the correct

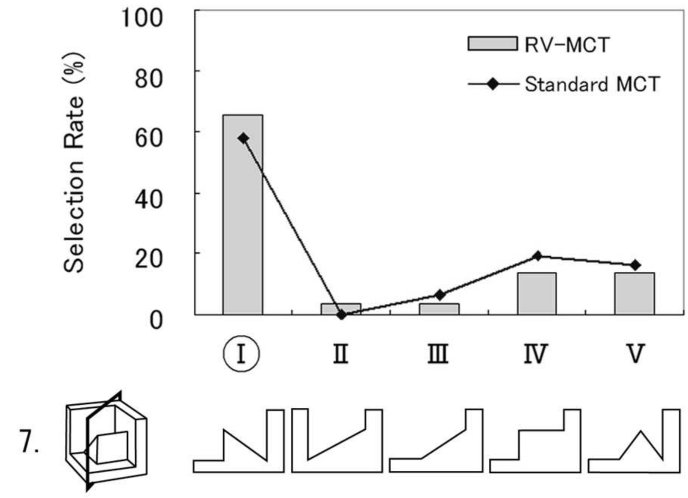

Figure 14: Selection rates of alternatives of problem no. 7

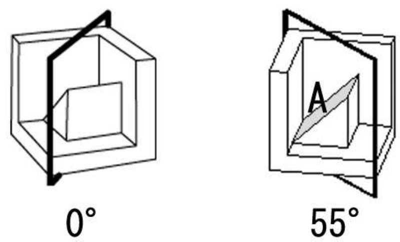

Figure 15: Distinctive views in no. 7

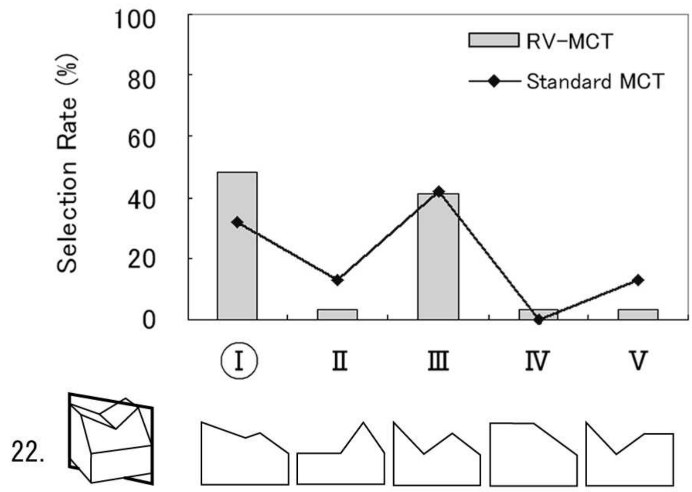

Figure 16: Selection rates of alternatives of problem no. 22

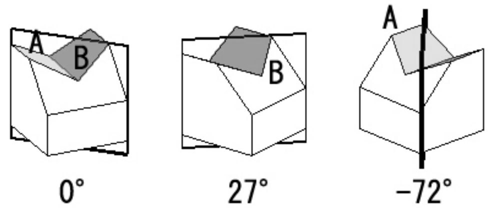

Figure 17: Distinctive views in no. 22

response rate did not change even when subjects rotated the problem solid. In the RV-MCT, more than 60 percent of the subjects who selected the correct answer stopped the solid at the angle of 55° (Fig. 15), where the hidden surface "A" could be observed.

Meanwhile, nearly 20 percent of the subjects who stopped solid at the same angle selected the wrong alternative "V" in spite of observing the hidden key surface "A".

This result indicates that subjects do not always make correct answers even when they observe hidden key surfaces by rotating the solids.

(2) Problem no. 22:

Fig. 16 shows the selection rates of alternatives of problem no. 22, and Fig. 17 shows distinctive views where numbers of subjects stopped rotating the solid.

More than 40 percent of the subjects in standard MCT selected the wrong cross section "III" mainly affected by the frontal "V"-shaped edges of the solid. Thus no. 22 was originally considered to be a difficult problem. Even in RV-MCT, almost a similar percent of subjects selected "III", in which they stopped the solid at angles of around 0°. The other subjects stopped the solid at 27° and −72°, where subjects could not observe one of the two surfaces "A" or "B" (Fig. 17). In the former case all subjects failed to select the correct answer affected by the characteristic edges of the solid.

4. Discussion

4.1. Relative location of the cutting plane with respect to the solid

It is often observed that low scoring subjects are unable to construct a sufficient sense of depth against the location of the cutting plane designed intentionally in the standard MCT.

By rotating the test solid, subjects tried to observe the test solid from the viewpoint with which one could see the cutting plane nearly as an edge view or from a view direction perpendicular to the cutting plane. As a result, the subjects were able to recognize the relative location of the cutting plane vis-á-vis the solid. The result may be caused by the fact that in these problems, the cutting plane was definitely separated from the focal portion of the test solid, where subjects could select proper view directions (see view from the angle of 77° in Fig. 11 and view from the angle of −63° in Fig. 13).

4.2. Hidden surface

It is often discussed that 'disability of envisaging a three-dimensional object hidden in a solid' is one of the main causes for error response. In other words, the disability is one of the major reasons for difficulties in forming correct images of 3-D objects. Thus, in case a hidden surface in a standard MCT problem is the clue for solving, it will be easy to construct a correct image of cross section if the subjects are able to see the hidden surface by rotating the solid in RV-MCT.

In RV-MCT the correct response rates of problem nos. 18 and 24 increased considerably, when rotating the solids to the views with which subjects could recognize the relation between cutting plane and key surfaces (see view from the angle of 80° in Fig. 7 and view from the angle of 70° in Fig. 9). Particularly, the problem solid no. 18 includes the hidden surface "A" as a key to solving the problem. The correct response rate of problem solid no. 7, however, did not increase so much even after rotating the solid to appropriate views (see view from 55° in Fig. 15).

The facts as mentioned above may indicate that the relation between the cutting plane, the key surface and the visual direction is important for recognizing the cross section.

4.3. The relation among cutting plane, key surface and visual direction

We have observed that there were two different cases in solving problems using RV-MCT. The first case was that even when the subject observed the key surfaces, the situation did not lead the subject to the correct answer. The second was the reverse that the observation of the key surface of the solid leads the subject to the correct answer.

It was considered that these differences may be connected with the positional relation between the cutting plane, the key surfaces and the visual direction. In case that the visual line is closer to that of the specific position with which one can observe both the cutting plane and the key surface as nearly edge views, the subjects will be able to recognize the intersection well (no. 18 in Fig. 18). While, when the above condition is not realized, it may be difficult to make images of the intersection (no. 7 in Fig. 18).

The results that correct response rates in nos. 22 and 12 did not increase in RV-MCT may also be due to the same reason as in no. 7.

Consequently, it was concluded that accomplishment of recognizing the solid depends on 'taking an appropriate view' in the process of forming a correct image of the solid from the drawing.

5. Summary and conclusion

The Rotated-View MCT in which the subject was able to rotate the problem solid on the horizontal plane was devised and conducted together with the standard MCT to female students. As a result, the process of rotating the test solids gave the subjects discrete views of the test solid, and made it possible to promote a better sense of recognizing and forming correct images of 3-D objects.

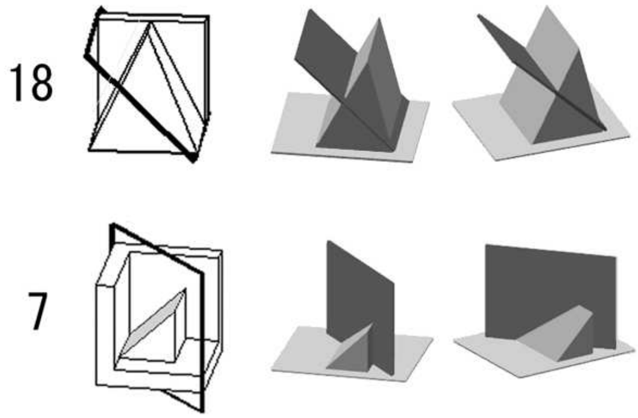

Figure 18: Two different cases of intersection and visual position

We could recognize that in case the visual line is closer to the specific position with which one can observe both the cutting plane and the key surface almost as edge views, it became easier to recognize the intersection accurately. It was concluded that the accomplishment of recognizing the solid depends on 'taking an appropriate view' in the process of forming a correct image of the solid from the drawing. Thus, learning Descriptive Geometry, especially to learn 'reading drawings' must have meanings.

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