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A Study on the Roof Curve of Japanese Pagodas

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Abstract. The curves of roofs are characteristics of Japanese traditional wooden architecture. However, the designs of these curves are decided by master builders who contract the construction of buildings. Their aesthetic characteristics have been changed through ages and they depend on *Kikujutu*, or highly developed Japanese skilled carpenters' techniques. In this study, the curves of first stage in five-storied or three-storied pagodas, which are on the list of Japanese important cultural properties, are selected as sample objects. Then we show the changes of master builders' techniques in each era and the characteristics of proportion of *Kayaoi* with using their real dimensions. Usually we measure *Kayaoi* by elevations, but they do not show the real shapes of each part of the roof, because we see them from ground level and look up roofs, not to see them horizontally. So we measure the heights, widths and inclinations of *Kayaoi* with sections and elevations from the authorized survey of overhaul properties, and make their figures with real dimensions. The design of Japanese traditional architecture is divided in the *Heian era* (8th c. – 12th c.), the *Kamakura era* (12th c. – 14th c.), the *Muromachi era* (14th c. – 16th c.), the *Edo era* (17th c. - 19th c.), and reflects the presence of technical and cultural exchanges with China and Korea in each era. Specially, it is not exaggeration to say that the roofs' curves are decided by their ethnic aesthetic viewpoint. Then we show how these roofs' curves are corrected visual image by *Kayaoi*. Their widths of both ends are wider than that of middle area, and these sizes are different in the construction ages or the techniques of *Kikujutu*. The main reason of their different size resulted from the correction of distortion in the visual image of *Kayaoi*. The beauties of Pagodas are depending on master builders' decision on the roofs' curves.

Key Words: Pagoda, roof curve, visual image, Japan

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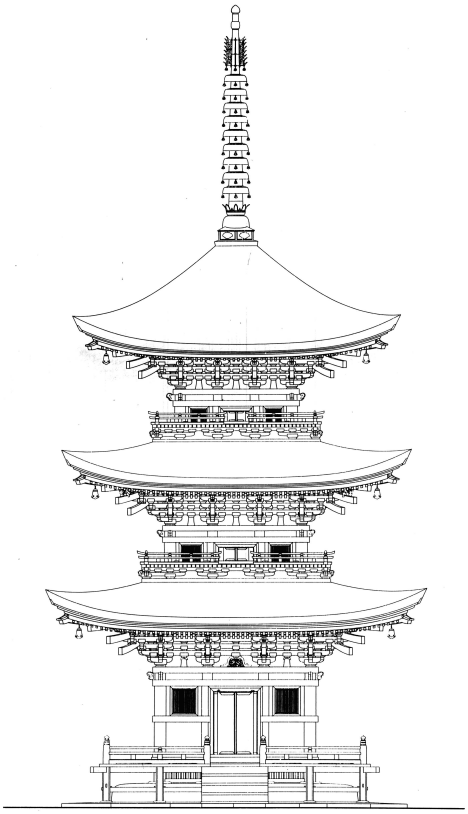

Figure 1: Three storied Pagoda [1]

1. Introduction

Japanese pagodas look so lightly with the stages of roofs, and the anchorage of peoples' religious mind from early date. But these figures are supported by master builders' experiences and their skills. In Japan, wooden architecture had to stand against the tremors of earthquakes, strong blow of typhoon and weight of snowfall, which amounts over 1 m in the northern area. With these handicaps, Japanese pagodas have beautiful curves on the roof eaves (Fig. 1).

Skilled carpenters have developed the method of design and construction systems with wild lumbers from different sources and characters of nature. We call this high quality traditional method of design *Kikujutu*. There are still many carpenters working in Japan, few of them have mastered *Kikujutu*, and the restoration of old temples and shrines has been owed by their skills.

The origin of *Kikujutu* stems from Mainland China in 538, when Buddhism introduced to Japan. Carpenters with the skills also came over to Japan at the same time and introduced the techniques. After long isolation between Japan and China in late *Heian era* (11th c. – 12th c.), the original type of Japanese architecture has been developed based on Chinese architecture. Even carpenters' tools have been developed in different manners, such as planers and saws. In Europe, China and most countries, planers and saws work when you push forward, but Japanese ones work when you draw backward.

From the works of origins of Japanese pagodas, which were built in 6th c. – 7th c., the carpenters began to work with Japanese manners and converted design systems and tools. Now we see the oldest wooden architecture in the world at *Horyuji* in Japan, after several

repairs.

In this study, we took five- and three-storied Buddhism pagodas, which are on the list of Japanese important cultural properties [1]. These are several studies about *Kikujutu*, or design method of curves of eaves lines of roofs in traditional temples (Fig. 2). In *Edo era* (17th c. – 19th c.), design textbooks of *Kikujutu* were written but none of them was written about the curves of roofs. However, at the end of *Edo era* in 1864, just before *Meiji* Restoration, a master builder Tousai Kiko described the "Bansyo-ke *Kayaoi* sori mitugousinnri" [5] as the completed textbook of *Kikujutu*, which was closed among the builder's family. After *Meiji* Revolution in 1868, European ideas of architectural design were imported to Japan, and a few researchers tried to describe these textbook with graphics and geometry.

After some disasters of earthquakes and war damages, the Architectural Institute of Japan and most universities stopped to study and teach wooden structures, except historians. Now we begin to look at the traditional architecture again and some researchers tried to describe these skills with CAD.

The purpose of this study is to inspect the roof curves of five-storied or three-storied pagodas, which are on the list of Japanese important cultural properties. We select sample objects and show the correction of distortion in the visual image of traditional roofs.

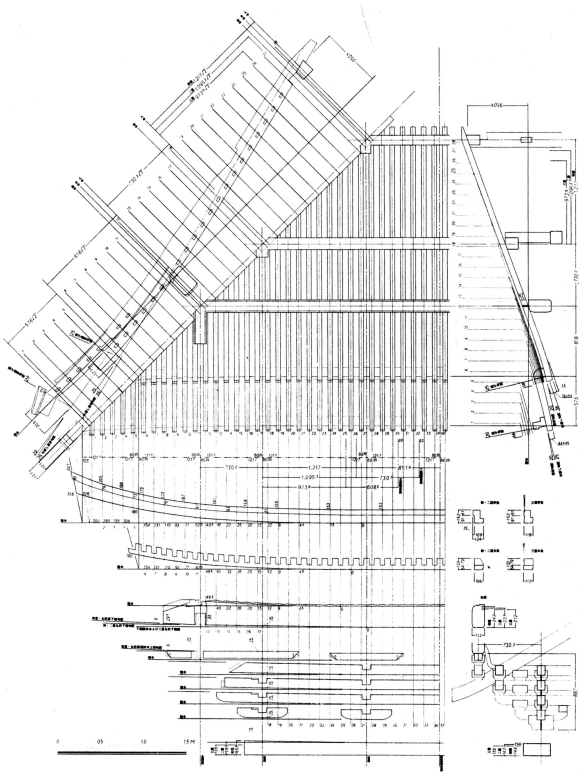

Figure 2: Drawing of the roof [1]

2. Method and objects

There are so many temples and shrines in Japan. The textbooks of *Kikujutu* not only focused on roofs but on all parts of buildings. We selected the part of roofs and checked the list of Japanese important cultural properties edited by the Agency for Cultural Affairs.

Table 1: List of Pagodas

No.	*name*	*type*	*built year*	*location*	*era*
1	Horyu-ji	five	7 c	Nara	1
2	Hokki-ji	three	706	Nara	1
3	Yakushi-ji	three	730	Nara	1
4	Ichijo-ji	three	1171	Hyougo	1
5	Joururi-ji	three	12 c	Kyoto	1
6	Kaijuuzann-ji	five	1214	Kyoto	1
7	Myoutu-ji	three	1270	Fukui	2
8	Koufuku-ji	three	13 c	Nara	2
9	Saiou-ji	three	13 c	Shiga	2
10	Meiou-in	five	1348	Hiroshima	2
11	Ryousen-ji	three	1356	Nara	3
12	Houfuku-ji	three	1376	Okayama	3
13	Hagurosan	five	1377	Yamagata	3
14	Nyoi-ji	three	1385	Hyougo	3
15	Choufuku-ji	three	14 c	Okayama	3
16	Kongourin-ji	three	14 c	Shiga	3
17	Hyakusai-ji	three	14 c	Shiga	3
18	Onjou-ji	three	14 c	Shiga	3
19	Anraku-ji	three	14 c	Nagano	3
20	Ishite-ji	three	14 c	Ehime	3
21	Jouraku-ji	three	1400	Shiga	3
22	Henjou-in	three	1416	Okayama	3
23	Koufuku-ji	five	1426	Nara	3
24	Saikoku-ji	three	1429	Hiroshima	3
25	Koujou-ji	three	1432	Hiroshima	3
26	Rurikou-ji	five	1442	Yagaguchi	3
27	Shinchoukoku-ji	three	1463	Gifu	3
28	Yahata-jinja	three	1466	Hyougo	3
29	Minamihokke-ji	three	1497	Nara	3
30	Syakubu-ji	three	15 c	Hyougo	3
31	Kokubun-ji	three	15 c	Nagano	3
32	Old.Toumyou-ji	three	15 c	Kanagawa	3
33	Shingaisan-sanjajinja	three	1515	Nagano	3
34	Nagusa-jinja	three	1527	Hyougo	3
35	Sanmyou-ji	three	1531	Aichi	3
36	Saimyou-ji	three	1538	Tochigi	3
37	Choumei-ji	three	1597	Shiga	3
38	Zensan-ji	three	16 c	Nagano	3
39	Housyaku-ji	three	16 c	Kyoto	3
40	Kongou-in	three	16 c	Kyoto	3
41	Hioshi-jinja	three	16 c	Gifu	3
42	Shinko-ji	three	16 c	Okayama	3
43	Honmon-ji	five	1607	Tokyo	4
44	Yusan-ji	three	1611	Shizuoka	4
45	Oppou-ji	three	1626	Nigata	4
46	Jimoku-ji	three	1627	Aichi	4
47	Kitymizu-dera	three	1632	Kyoto	4
48	Nata-dera	three	1642	Ishikawa	4
49	Shinzen-ji	three	1643	Gifu	4
50	Ninna-ji	five	1644	Kyoto	4
51	Kyouougokoku-ji	five	1644	Kyoto	4
52	Honzan-ji	three	1652	Okayama	4
53	Saisyou-in	five	1666	Aomori	4
54	Shinsyou-ji	three	1712	Chiba	4
55	Tousyou-gu	five	1818	Tochigi	4
56	Bittyukokubunn-ji	five	1844	Okayama	4
57	Daigo-ji	five	unkown	Kyoto	5
58	Kairyuou-ji	five	unkown	Nara	5
59	Gankou-ji Gokurakubo	five	unkown	Nara	5

Era 1: Heian and before, 2: Kamakura, 3: Muromachi, 4: Edo, 5: unkown

2.1. Data

After examining the list, there are 80 five- and three-storied pagodas as national treasure of Japan. We could find the restoration reports of pagodas and got 15 five-storied pagodas and 44 three-storied ones. So we collected about 70% of these pagodas (Table 1). Then the elevation and section drawings are extracted. As the shape of pagodas' roofs are different in each stage, we selected the first stage of roofs as data for this study.

2.2. *Kayaoi*

There are many parts for the formation of roofs, such as purlin, base-rafter, ornament-rafter, end-rafter, *Kioi*, *Kayaoi* and so on. So we had to select a specific member to study the curve of roof. *Kayaoi* was selected as the best member for studying the curve of roof, because *Kayaoi* is a surface member and its range covers both ends of the roof. And the most important thing is that *Kayaoi* is inclined against the slope of the roof (Figs. 3, 4).

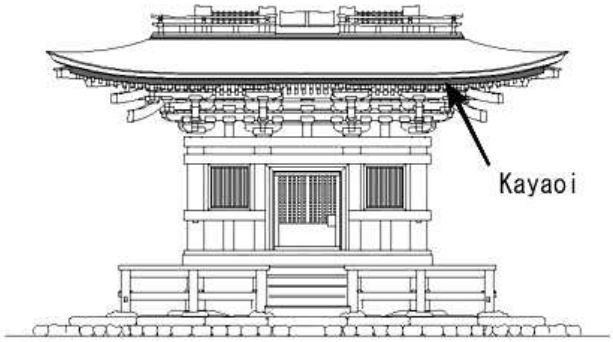

Figure 3: *Kayaoi* at elevation

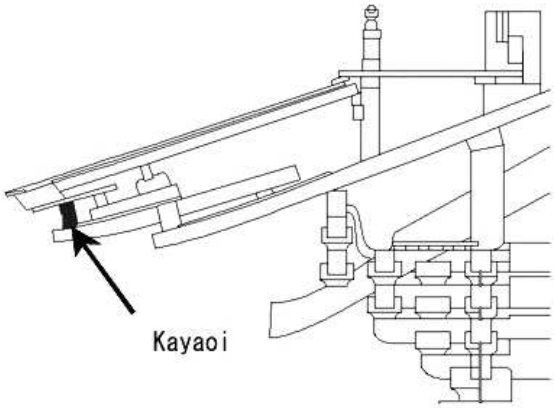

Figure 4: *Kayaoi* at section

2.3. Drawing of data

With the selected data the lines of each *Kayaoi* were traced, and different scale and width were equated to the same length for comparison.

3. Curve of the roof

There are several studies about curves of roofs for Japanese temples. Most of them are to find out the formula of curves of master builders' decision. Here we check the reliability of these features.

3.1. Typical curve lines

According to the previous report about the characteristics of curves in each era, *Kikujutu* had been developed with a geometrical method in *Edo era*, and made structural members refinement. However, the curves begin near both ends of *Kayaoi*, they affect their monotonous designs and the central part of *Kayaoi* looks cambered by optical illusion. On the other hand, the curves in *Muromachi era* and before look appropriate and like aesthetic lines, where the curves begin near the center.

Table 2: List ordered according to the ratio of curves

No.	*name*	*type*	*built year*	*era*	*ratio of curve*
36	Saimyou-ji	three	1538	3	0.0480
12	Houfuku-ji	three	1376	3	0.0453
4	Ichijo-ji	three	1171	1	0.0449
14	Nyoi-ji	three	1385	3	0.0445
38	Zensan-ji	three	16 c	3	0.0415
30	Syakubu-ji	three	15 c	3	0.0387
15	Choufuku-ji	three	14 c	3	0.0368
27	Shinchoukoku-ji	three	1463	3	0.0368
1	Horyu-ji	five	7 c	1	0.0365
31	Kokubun-ji	three	15 c	3	0.0361
16	Kongourin-ji	three	14 c	3	0.0352
17	Hyakusai-ji	three	14 c	3	0.0352
56	Bittyukokubunn-ji	five	1844	4	0.0349
33	Shingaisan-sanjajinja	three	1515	3	0.0344
18	Onjou-ji	three	14 c	3	0.0341
19	Anraku-ji	three	14 c	3	0.0328
57	Daigo-ji	five	unkown	5	0.0310
46	Jimoku-ji	three	1627	4	0.0308
22	Henjou-in	three	1416	3	0.0307
52	Honzan-ji	three	1652	4	0.0304
25	Koujou-ji	three	1432	3	0.0290
20	Ishite-ji	three	14 c	3	0.0290
29	Minamihokke-ji	three	1497	3	0.0278
5	Joururi-ji	three	12 c	1	0.0276
44	Yusan-ji	three	1611	4	0.0274
10	Meiou-in	five	1348	2	0.0272
28	Yahata-jinja	three	1466	3	0.0261
54	Shinsyou-ji	three	1712	4	0.0260
24	Saikoku-ji	three	1429	3	0.0254
13	Hagurosan	five	1377	3	0.0247
3	Yakushi-ji	three	730	1	0.0245
49	Shinzen-in	three	1643	4	0.0239
39	Housyaku-ji	three	16 c	3	0.0235
7	Myoutu-ji	three	1270	2	0.0231
50	Ninna-ji	five	1644	4	0.0224
2	Hokki-ji	three	706	1	0.0224
11	Ryousen-ji	three	1356	3	0.0218
8	Koufuku-ji	three	13 c	2	0.0216
9	Saimyou-ji	three	13 c	2	0.0215
35	Sanmyou-ji	three	1531	3	0.0215
26	Rurikou-ji	five	1442	3	0.0214
51	Kyouougokoku-ji	five	1644	4	0.0209
43	Honmon-ji	five	1607	4	0.0208
40	Kongou-in	three	16 c	3	0.0206
6	Kaijuzan-ji	five	1214	1	0.0202
45	Oppou-ji	three	1626	4	0.0200
41	Hiyoshi-jinja	three	16 c	3	0.0198
47	Kiyomizu-dera	three	1632	4	0.0184
21	Jouraku-ji	three	1400	3	0.0179
58	Kairyuou-ji	five	unkown	5	0.0166
42	Shinko-ji	three	16 c	3	0.0154
32	Old.Toumyou-ji	three	15 c	3	0.0154
37	Choumei-ji	three	1597	3	0.0145
34	Nagusa-jinja	three	1527	3	0.0123
48	Nata-dera	three	1642	4	0.0111
23	Koufuku-ji	five	1426	3	0.0110
53	Saisyou-in	five	1666	4	0.0091
55	Tousyou-gu	five	1818	4	0.0085
59	Gankou-ji Gokurakubo	five	unkown	5	0.0063

Era 1: Heian and before, 2: Kamakura, 3: Muromachi, 4: Edo, 5: unkown

3.2. Curves in each era

In this study, we subdivided the bottom line of *Kayaoi* into ten sections, from the center to the end, and marked the beginning of the curved part by dots. Figs. 5–8 show the curve lines and the starting points of the curves. Table 2 shows the ratio of curves when the width is 1.

The result of our analysis shows that the characteristics of curves are not so obvious in each era, but that of *Edo era* is apparently different from other eras. Compared with former eras, the points in *Edo era* have a variety. The initial points of the curves in the *Edo era* are not so close to the end of *Kayaoi* in every collected data. As a result of this study, there were developed *Kikujutu* with specific manner in *Edo era*, but the beautiful and well evaluated curve lines of pagodas roofs depend on the master builders' aesthetic decision, not on the *Kikujutu* textbook.

Kikujutu is a kind of design method, and master builders have to apply the skills for their design of architecture, otherwise they fell into mannerism.

4. Correction of the visual image

Kayaoi is fixed on rafters, so it is sloped along the inclination of rafters or roofs (Fig. 4). When we see the roof lines in elevation drawings, we look at *Kayaoi* from the horizontal direction. However, we can't see the real shape of *Kayaoi* from that direction, and we also never look at pagodas in this way. Here we check how *Kayaoi* look like. We set the view point at the crossing point of the orthogonal direction of the face of *Kayaoi* and 1.5 m height from ground level (Fig. 9).

4.1. Section and elevation

Then we measured the heights, widths and inclination of *Kayaoi* by sections and elevations, and made their real dimensions of shapes.

Fig. 10 shows the difference of height between section and elevation. The height "A" shows that from horizontal direction, and the height "B" shows that from view point, which is longer than "A". Covered by the other structural members of roof, such as *urakou*, *hiendaruki* and others, *Kayaoi* cannot show the real shape if seen from horizontal direction.

When we see *Kayaoi* by elevation, heights of both ends are not so different from that of the center. But we can note that heights are not the same, and the heights of both ends are longer than that of the center. This is because of the correction of the visual image. In

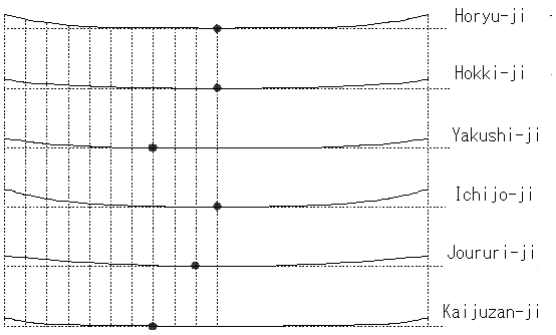

Figure 5: Curve lines of *Heian era* and before

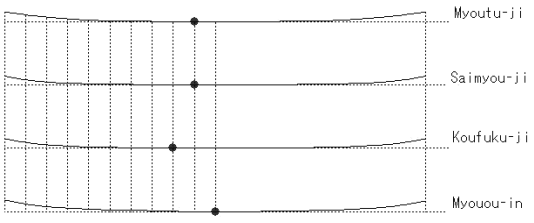

Figure 6: Curve lines of *Kamakura era*

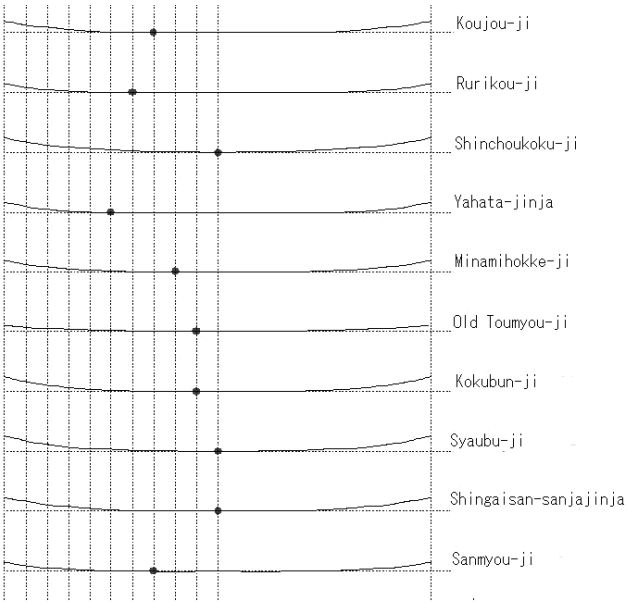

Figure 7: Curve lines of *Muromachi era*

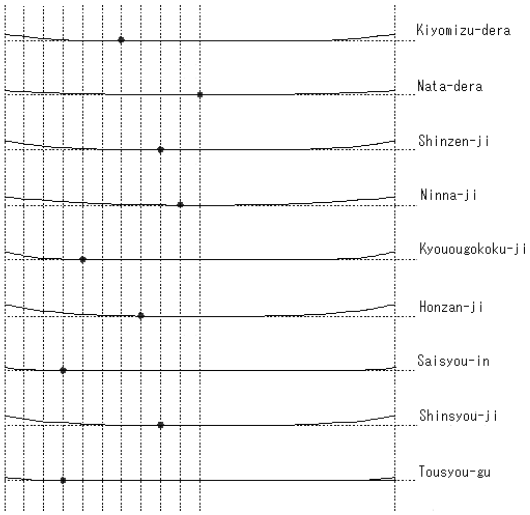

Figure 8: Curve lines of *Edo era*

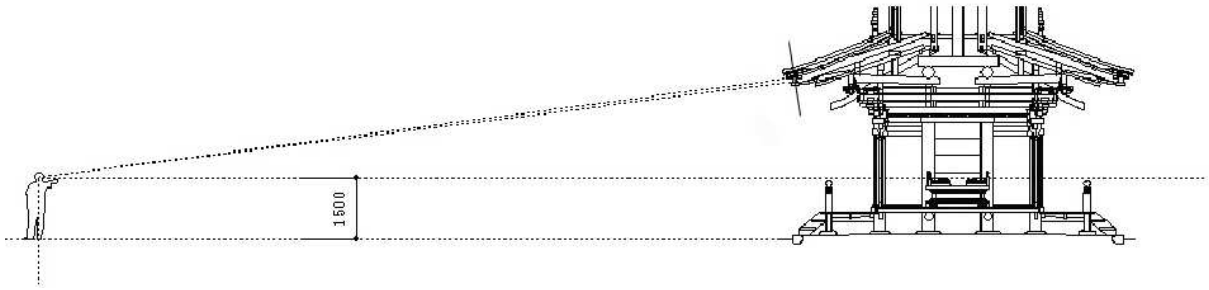

Figure 9: View point for the face of *Kayaoi*

Fig. 11, "b" is larger than "a", and "c" is almost the same as "d". The shape of elevation looks very thin, but that of real dimension is wider. This means, the elevation doesn't express the real image of the pagoda. The average ratio of "b" to "a" is 1.2 to 1.0, which seems to be achieved by a lot of master builders' and Japanese aesthetics.

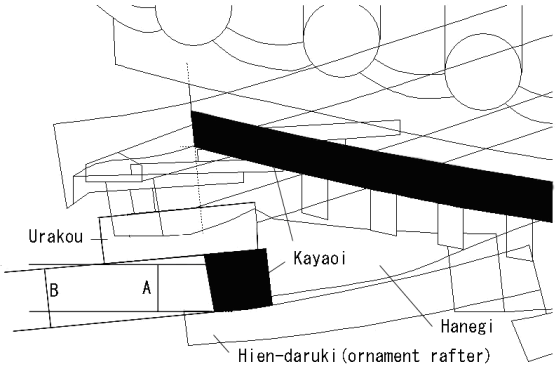

Figure 10: *Kayaoi* in section and elevation

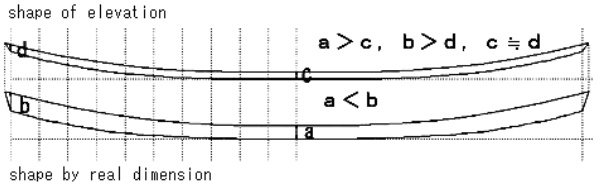

Figure 11: Dimension of *Kayaoi*

4.2. Inclination, curve and concavity

Kayaoi is warped to upside, and it draws the inclination toward the view point. As the result, the eaves are shaped concave, not just square-like, as shown in Fig. 12. The roofs whose warping points close to the center of *Kayaoi* tend to have a large proportion of curves and these are seen in the pagodas of elder era. But there is no relationship between inclination and shape of curves (Figs. 13, 14).

4.3. Visual image of *Kayaoi*

The curve of roof eaves draws the optical illusion of roof shapes. Because of the concavity of roofs, the distance from the view point of the end part and central part of *Kayaoi* differs with the curves and inclinations (Fig. 15). As the average concavities are not so large, we see roofs at the case of view point 2. That means the end part is far from the center and looks smaller. This is one of the reasons why the different size of *Kayaoi* carries a function for the correction of the optical illusion.

5. Conclusions

Chinese pagodas have been changing their design of shape, but Japanese ones have been keeping similar appearances and almost no change of design through ages. However, the inside structure has been dramatically changed with the invention of *Hanegi* (see Fig. 10), which support members of the roof with the idea of leverage and other structural technique. They enabled the shape of roof design to be independent from structural design. The techniques of

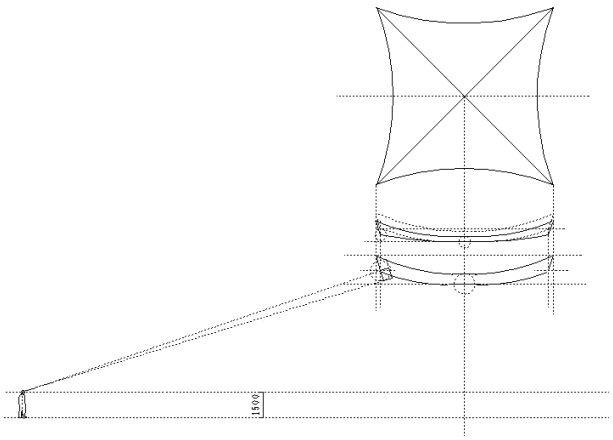

Figure 12: Concaved eaves by inclination

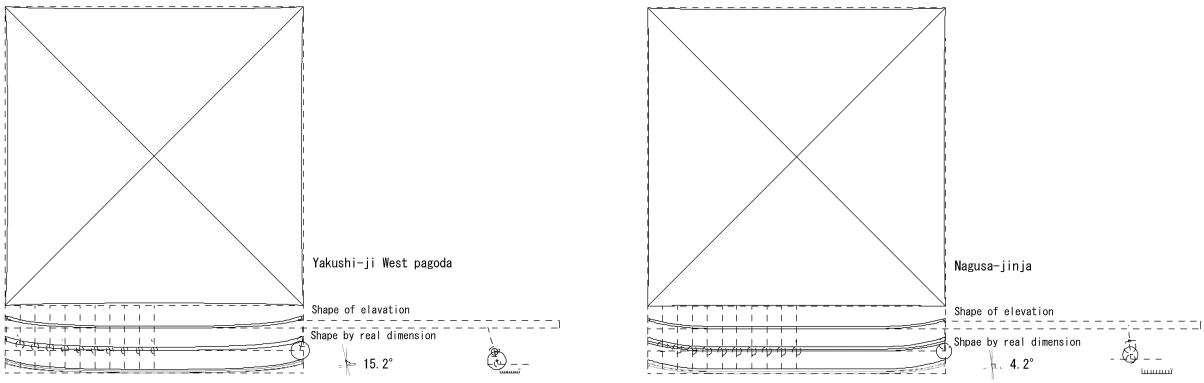

Figure 13: Curve and concavity of eaves

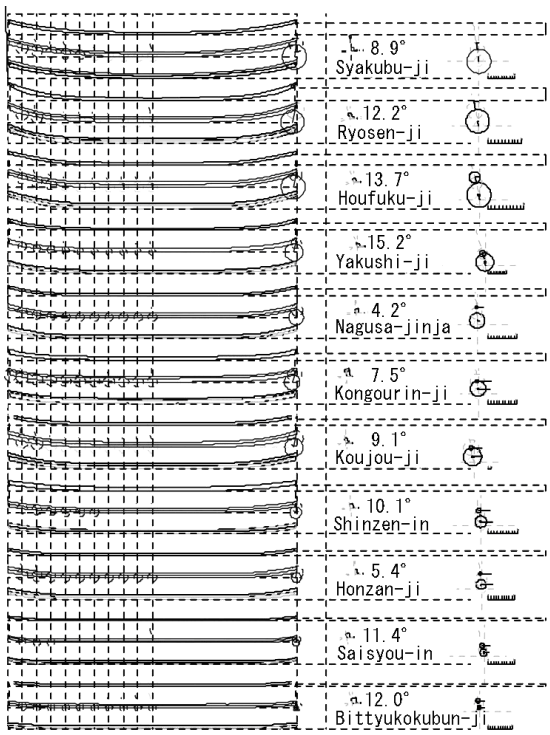

Figure 14: Variety of inclination and curves

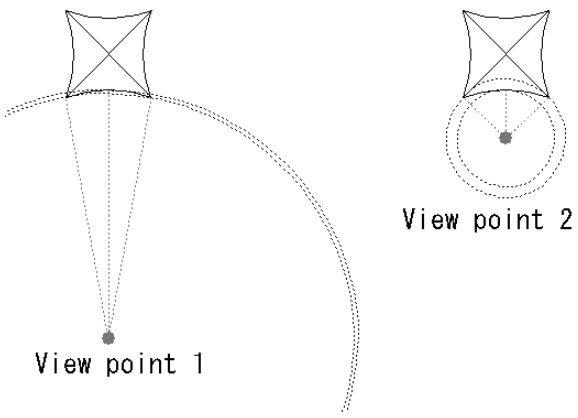

Figure 15: View point and distance to *Kayaoi*

traditional wooden architecture were completed in *Edo era*. At the same time, many master carpenters could easily build pagodas with *Kikujutu*. However, their qualities were not so good compared with other eras. They depend too much on the techniques and fell into a kind of mannerism. So the roof curves of *Edo era* were different from other eras, while characteristics of curves are not so obvious by each era. As conclusions we can report:

- Roof curves are not so much characteristic as mentioned in previous reports, while some different characters are found in *Edo era*. The ratio of curves and shape lines vary through the ages. The structural design had trends in each era, but the design of curves had superiority in master builders' aesthetics.
- The shape of *Kayaoi* is different from that of elevation because of its inclination. So the shape of real *Kayaoi* can bee seen from the point of visitors on ground level. And curves and inclinations make concavities of roofs and need to have a correction of optical illusion with the wide end of *Kayaoi*.

Acknowledgments

Our interest of *Kikujutu* and curves of roofs was raised by the master builder Yoji Takahashi, who has excellent skills for building masterpieces of temples and shrines. We had a *Kikujutu* workshop with students, and many suggestions and knowledge were given by him while working together. We express our special thanks to him.

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