

# Computer-Aided Teaching of Descriptive Geometry\*

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**Abstract.** In this paper we discuss teaching methods in an undergraduate level course of Descriptive Geometry. Rapid development of CAD/CAM software has made classical methods of Descriptive Geometry entirely obsolete. However, this discipline is still very important for strengthening one's spatial intuition. In order to integrate the classical and modern approaches, we have developed several electronic accessories that we use in the teaching process. In this paper we discuss some didactic issues, as well as technical problems that we faced. Finally, the outcome of this “integrated” approach is evaluated in a student questionnaire.

*Key Words:* teaching, Descriptive Geometry, CAD

*MSC 2000:* 51N05

## 1. Introduction

Descriptive Geometry is a classical geometry discipline that allows the representation of three-dimensional objects in two dimensions by using a specific set of procedures. The resulting techniques are important for engineering, architecture, design, art, etc.

Teaching techniques and their improvements that we discuss in this article are closely related to the course of Descriptive Geometry at the Faculty of Mathematics, University of Belgrade. This course is held in the third year of undergraduate studies for all students (including students of Computer Science).

In the last decade, CAD (Computer Aided Design) and CAM (Computer Aided Manufacturing) software have become a standard for engineers and designers. Since that software has built in procedures for Descriptive Geometry, practical knowledge of Descriptive Geometry has become less important. However, studying Descriptive Geometry significantly improves spatial intuition, and hence we are of the opinion that it should not be removed from the curriculum.

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STACHEL (see [15]) has also discussed the role of Descriptive Geometry in the teaching of geometry. The way to conciliate the classical and the modern is to integrate contemporary CAD/CAM software into the teaching of Descriptive Geometry. Many authors have been interested in methods of teaching Descriptive Geometry and its role in education as well as the corresponding modern tools that they use (see [13, 14, 15, 2, 3, 16, 5, 11, 4]). Of course CAD/CAM software is important not only for teaching Descriptive Geometry, but for many other areas (see [7, 9, 18]).

In this paper we discuss facilities and difficulties of the classical and modern approaches to teaching of Descriptive Geometry. Our paper is organized in six sections plus an appendix and references. Section 1 is of an introductory nature and deals with the motivation to study the problem of teaching methods in Descriptive Geometry. Section 2 is devoted to the classical approach to teaching Descriptive Geometry and its difficulties. Section 3 deals with computer-assisted teaching of Descriptive Geometry. We explain some reasons to use AutoCAD and present some student's work. In Section 4 we discuss requirements and technologies for creating interactive electronic notes on Descriptive Geometry which are related to orthogonal and central projections. Section 5 summarizes our experience in computer-aided teaching of Descriptive Geometry. We support our discussion with some statistical data derived from a student questionnaire. Section 6 contains our conclusions and some directions for further improvement of our electronic accessories. Appendix A contains the questionnaire form and Appendix B contains statistics from our survey.

## 2. The classical approach and its difficulties

The classical approach to the teaching of Descriptive Geometry means that a professor uses blackboard, compass, ruler and books as the main media. In planar geometry courses these teaching accessories are adequate to help the professor to unequivocally present geometrical content. However, these media are not sufficient for presenting 3D objects and their relations.

A static, paper drawing of a 3D object does not carry the full information that describes the object. The correct understanding of the object has to be supported by the reader's intuition and additional text. Therefore, various misunderstandings are possible. For example, a careless student may think that some lines are coplanar even though they are not. Interactive 3D models show the spatial relations more clearly, since they allow the reader to look at the object from different viewpoints.

Also, by using dynamic 3D geometry one may emphasize important details in the construction. In Fig. 1 the first picture shows the rotation of a plane, while the second one shows relations between a spatial object and its projection. Both images are simply different views of the same 3D problem.

Furthermore, a common drawing in Descriptive Geometry is usually very complex and shrouded by a huge number of overlapping lines. Hence, the task of understanding and copying these drawings may be quite difficult. When studying at home, students first have to decipher and then learn and understand what their drawings mean and represent. The deciphering process usually takes more time and effort than the learning part, and we should aim to reduce this time. One way to address the complexity of the drawing is to use layers to split the construction process into geometrical steps. Nowadays, there are many computer software and file formats that support the principle of layering. In the classical approach, one way to deal with complexity is to use different colors on the blackboard and on paper.

Problem solving in Descriptive Geometry is usually divided into several geometrical steps

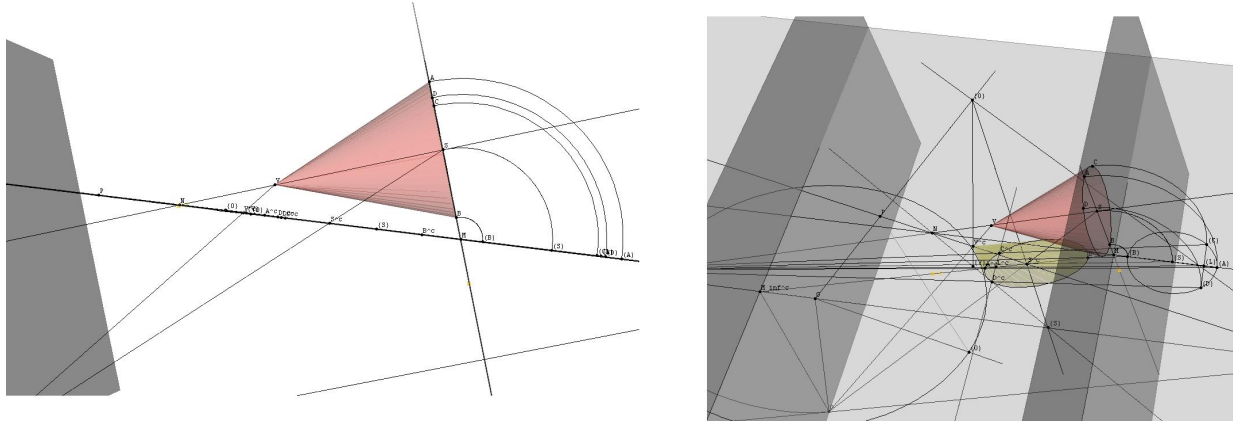


Figure 1: Two views of an interactive 3D object in [6]

that are usually simple but require a good understanding of space. Again, layers help us to repeat the construction step-by-step and also to hide the parts of the drawing that have less importance.

In order to develop a student's spatial intuition, it is useful to have an interactive tool for 3D representation of spatial objects. In the classical approach, solid models of 3D objects are used.

A problem with classical media is also their inaccuracy. For example, if it happens that for a geometrical reason some lines should meet at a single point, but due to a drawing inaccuracy they actually don't meet. With the use of computer software this problem is eliminated.

To address these issues, the teacher needs to find an appropriate balance between the "compass and ruler approach" and the use of computer accessories. This gives us the space for developing various computer-teaching accessories that are briefly introduced in the next section.

### 3. Computer-assisted teaching of Descriptive Geometry

Some fifteen years ago we introduced computerized accessories in the teaching of Descriptive Geometry. After analyzing others experiences, we settled on AutoCAD software that we have used for the last ten years. The choice of AutoCAD proved itself a wise one since this software was, and still is, a standard in CAD. The biggest drawback of this software is its high price, but after some educational discounts the expense becomes bearable. The contents of the course remained unchanged, but the students first have a short course in 2D AutoCAD. After that, they use AutoCAD, instead of a compass and ruler to get more accurate drawings. Another advantage is that they can split their drawings into layers. The content of layers corresponds to geometrical procedures and, therefore, the layering improves the organization of the drawing as well as the reasoning. More details on this topic we refer to in our forthcoming paper.

The overall result of the use of AutoCAD is positive. The students become more interested, since they like to use something more contemporary and learn the basics of AutoCAD, which is a useful reference in job seeking. Moreover, some students attack more serious projects that are later used as teaching accessories. As a consequence, we improve the efficiency and quality of the studies. This is confirmed by the results of a student questionnaire

presented in the Appendix. This approach is compatible with those at other universities. Consequently, it improves the mobility of students and teachers to other university centers, which is a principle in European higher education nowadays. We emphasize that other authors (see [3]) have discussed how modern media support modern education.

We also present and discuss the most important student work so far. Since every student had to solve a problem in Descriptive Geometry using AutoCAD, a natural outcome was the *Online Problem Book* in Descriptive Geometry (see [17]). This is a collection of solved problems including drawings and explanations. The drawings are DWF (Drawing Web Format) files, made using AutoCAD. This format is compact and platform-independent and, therefore, very popular in the engineering community. The files can be opened in most Web browsers without an additional plug-in. Most of the problems are produced by students. Moreover, the problem book is very popular among students. Its main advantage over classical books is the presence of layers.

So far, we had used only the 2D functionality of AutoCAD. The next step was *AutoCAD Toolbox for Reversing the Projection*, created by a student (see [12]). This can take a projection of a point (which lies in the plane) back to the original point in 3D. Therefore, we can obtain a 3D model of an object from its projection, which is beyond the capabilities of classical Descriptive Geometry. This works for orthogonal projection as well as for perspective projection with a given center of the projection. The 3D model of our geometrical object then can be viewed in AutoCAD and also with some free 3D viewers. The one we use (the reasons are explained later in this article) is JavaView. Examples of a 3D object derived from their projection can be found in [12]. The next example of how the 3D functionality of AutoCAD and JavaView can be used to make documents with interactive 3D objects is in Descriptive Geometry electronic notes, produced by two students (see [8, 6]). The production of an electronic book with interactive 3D contents offers some special issues which are discussed in the next section.

#### 4. Requirements and technologies for creating interactive electronic notes

One of the projects undertaken by students was digitizing existing written notes in Descriptive Geometry. The two main requirements of this project were:

- Keeping the structure of existing written notes, but replacing the static 2D images with equivalent interactive 3D models. This exercise should support and improve a student's spatial intuition. Also this task addresses the problem of misinterpretation that occurs with static 2D drawing.
- Making the geometrical relation between 3D model and its 2D projection as clear as possible, which itself is the task of Descriptive Geometry. Fig. 2 describes the problem of orthogonality of a line and a plane: the first drawing is a 3D model and the second is the corresponding construction.

We also found that electronic notes should meet the following technical criteria.

- Easy to use: because we have students who have different levels of knowledge in the use of computers.
- Small in size: so that they can be distributed easily among students and professors.
- Platform independent: so that they do not require additional software or a specific platform to run on.

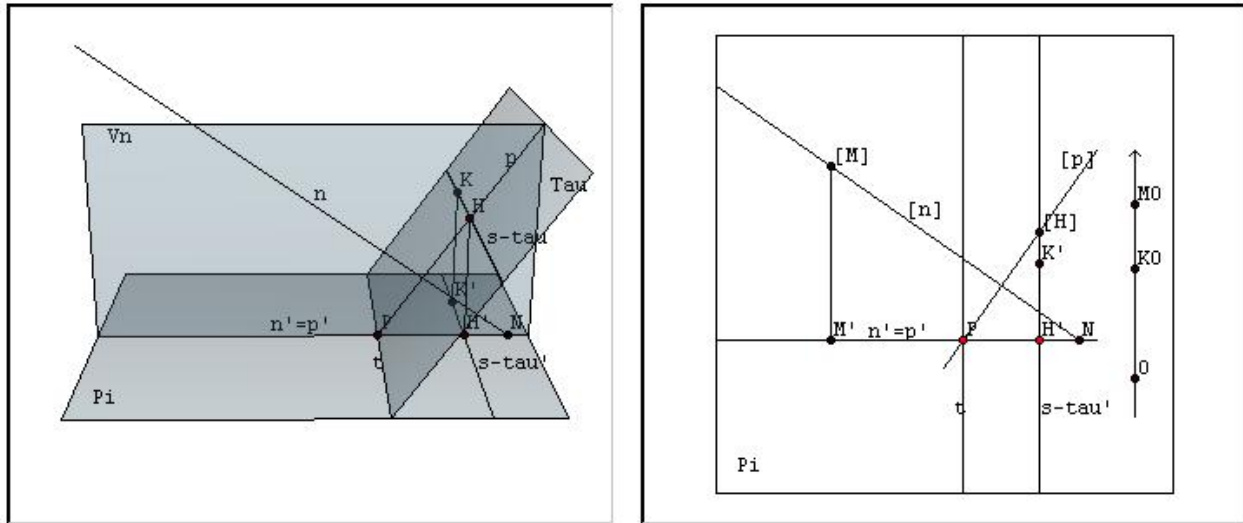


Figure 2: Orthogonality of a line and a plane in [8]

- Their structure should be as simple as possible to allow easy changes and expansion.
- The expense of creating and maintaining should be in sound correlation with the actual gains and benefits of the accessory.

In order to produce such an accessory we decided to write it as a simple HTML document. Then, we would choose the proper tools for representation of its content. For content representation we have two major decisions to make. We need to decide how to write mathematical formulas in the web page and how to make interactive 3D images and import them into the HTML document.

For the mathematical formulas we consider two possibilities:

1. MathML, which is the current standard for writing mathematical formulas. But it requires additional software to be installed on the system in order to work. The MathML standard is not fully supported yet, and it depends heavily on the Web browser.
2. Use of gif images. To put a mathematical formula into a web page as a gif image has the disadvantage that if we change the font style or size the gif image remains the same. Also, gif images carry no mathematical meaning, and therefore no mathematical related search is possible. This second choice is used in [8].
3. In [6] a different approach is used: the mathematical text is written in LaTeX and then converted into HTML with gif images. The conversion is done with a free *LaTeX2html* translator (see [1]). The advantage of this approach is that the LaTeX text can also be converted into MathML once it becomes better supported.

For the representation of 3D images we use JavaView, which is an interactive 3D geometry and visualization tool that displays interactive 3D images as Java applets. Its only requirement is the Java runtime environment. It is free, light and geometrically oriented. A different possible approach is use of the PDF file format instead of HTML. Namely, starting from version 8.0 of Acrobat Reader, interactive 3D objects can be included in PDF files. The preferred format for 3D objects is U3D. Note that once we have a book written in LaTeX it is easy to convert it into PDF. The problem is the conversion of JavaView objects into U3D, which requires commercial software at the moment. e point out that other authors (see [16])

have also discussed the use of Internet tools in education.

## 5. Current experiences in the application of computer aided teaching

Some of fundamental principles of the Bologna process are efficiency and quality of studies and mobility of students and teachers. In that spirit, every teacher is confronted with the challenge of arranging a course in order to satisfy the above-mentioned concepts. In that context the challenges in a course of Descriptive Geometry include an additional problem: how to incorporate new technologies in order to fulfill additional expectations. Because of this need, we have noticed that the interactive work of students and professors throughout the year in preparing students seminar work in AutoCAD improves the efficiency and quality of studying. By analyzing the results of exams we see that the results of courses with interactive work are significantly better than of those without interactive work by up to 80 %. In addition, it is a general attitude of students at the University of Belgrade that interactive work should be more intense. With the introduction of the AutoCAD software package we noticed the following:

- The interactive work is more intense because students come to tuition sessions during the school year in order to complete small course-based projects.
- Students are very dedicated to the completion of these projects, despite the fact that their grade does not depend on the quality of the completed project.

Students are especially interested when they are engaged in a multinational project because the results of their projects are the basic references for their career. Also we noticed that students with these projects accept obligations that can serve them as a substitute for a part of the classical exam. We also noticed that the students who have used AutoCAD were more motivated to learn Descriptive Geometry, which became one of the more popular courses. Also, the opportunity to participate in creation of new accessories is a strong driving force for some students. In order to measure the impact of computer-aided teaching of Descriptive Geometry we produced a student questionnaire (see Appendix A). It mainly addresses the following questions:

- What optional computer-teaching accessories do students prefer?
- How do these accessories compare to classical media?
- How do these opinions relate to the computer literacy of a particular student?

When we analyzed the data we received the following result:

1. A surprisingly high percentage of the students use some of the optional teaching accessories for individual work. Moreover, most of the students use more than one accessory.
2. The majority of the students who use some teaching accessory claim that the interactive content encourages better understanding of drawings and improves spatial intuition.
3. The constant update of the data shows an increase in the popularity of the accessories.

The above results offer proof that the new teaching materials are well accepted by the students and that the students are of the opinion that these accessories help them to understand the content of the course more easily. Certainly, one should not underestimate the effect of natural curiosity that comes into play when introducing a new and modern tool into the classroom.

## 6. Conclusion and future development

We understand that the information revolution of our time can easily render classical courses archaic. We show that by introducing appropriate accessories it is possible to make a course more contemporary while preserving the contents of the course. So we get classical contents in modern packaging.

Introduction of modern software packages (e.g. AutoCad) in Descriptive Geometry improves the quality of studies; the students become more involved and interested. They also gain some practical skills that are useful in the job market. It is encouraging to learn that students can produce high quality and very useful teaching accessories.

The world of Internet, computer technologies and their standards are constantly changing. Therefore, the software that is used to support teaching and file formats of electronic books has to be carefully chosen. Our experience is that the software has to be either an industry standard or written completely from scratch. It is useful to convert and store the mathematical data in some meta-language (e.g. LaTeX) that can be easily converted into any file format that is popular at a particular moment.

We consider two directions to further improve our electronic accessories for teaching Descriptive Geometry. One is to create new resources similar to the electronic notes [8, 6], for example dynamic test sheets. The other direction would require a more radical change in a classical course of Descriptive Geometry. The idea is not to teach students particular methods of Descriptive Geometry but to solve the same spatial problems using 3D features of AutoCAD. In such a way, the geometrical solution of a problem stays the same, but the technique is no longer classical. The solution is immediately a 3D object that can be easily projected in many ways, each of which would require a separate drawing in classical Descriptive Geometry. This approach is also useful for strengthening a student's spatial intuition. It is a kind of geometrical modeling, but applied to classical problems of Descriptive Geometry.

## References

- [1] *LaTeX to HTML translator*. <http://www.latex2html.org/>.
- [2] A. ASPERL: *How to teach CAD*. Computer Aided Design & Applications **1–4**, 459–468 (2005).
- [3] A. ASPERL, M. HOFER: *How modern media support geometry education*. Proc. 3rd Conf. on Education in Computer Aided architectural Design in Europe, Geometry and Graphics, TU Dresden (2008).
- [4] K.H. BRAKHAGE: *WinCAG – Education Software for Geometry*. The 11th Internat. Conf. on Geometry and Graphics, Guangzhou/China (2004)
- [5] M. CONTERO, F. NAYA, P. COMPANY, J.L. SAORIN, J. CONESA: *Improving visualization skills in engineering education*. Computer Graphics and Applications, IEEE **25(5)**, 24–31 (2005).
- [6] M. DJORDJEVIĆ: *Online Book in Descriptive Geometry (perspective projection)*. <http://alas.matf.bg.ac.rs/~mr04208/ng/> (2008).
- [7] D. LEUTNER: *Double-fading support – a training approach to complex software systems*. Journal of Computer Assisted Learning **16(4)**, 347–357 (2008).
- [8] M. LJUCOVIĆ: *Online Book in Descriptive Geometry (orthogonal projection)*. <http://www.matf.bg.ac.rs/~daad/people/MarkoLjucovic/index.html> (2008).

- [9] S. MARTIN-MICHELLOT, P. MENDELSON: *Cognitive Load While Learning with a Graphical Computer Interface*. Journal of Computer Assisted Learning **16**(4), 284–293 (2001).
- [10] K. POLTHIER: *JavaView – Interactive 3D Geometry and Visualization software*. [www.javaview.de](http://www.javaview.de) (1999).
- [11] C. PÜTZ: *Topics Suitable for Descriptive Geometry Courses for University Students of Architecture*. J. Geometry Graphics **4**(2), 209–222 (2000).
- [12] D. SEGAN: *Reverse projection using AutoCAD*. <http://www.kvota.net/ng/> (2000).
- [13] H. STACHEL: *The status of today's Descriptive Geometry related education (CAD/CG/DG) in Europe*. Proc. Annual Meeting of JSGS 2007, 40th anniversary of Japan Society for Graphic Science, Tokyo, Japan, pp. 15–20 (2007).
- [14] H. STACHEL: *Educational Software for Descriptive Geometry*, In V.O. THOMAS, W.P. LEWIS (eds), Proc. 5th Internat. Conf. on Engineering Computer Graphics and Descriptive Geometry, Melbourne/Australia: vol. 1, pp. 305–307 (1992).
- [15] H. STACHEL: *What is Descriptive Geometry for?* DSG-CK Dresden Symposium Geometrie: konstruktiv & kinematisch, TU Dresden, pp. 327–336 (2003).
- [16] H. SUEOKA, S. SHIMIZU, H. YOKOSAWA: *The Use of Internet Technology for the Development of 3-D Spatial Skills*, Proc. 2nd Internat. Conf. on Information Technology Based Higher Education and Training, Kumamoto, Japan (2001).
- [17] S. VUKMIROVIĆ: *Electronic Problem Book in Descriptive Geometry*. <http://alas.matf.bg.ac.rs/~vsrdjan/files/zbirka/proba.htm> (1998).
- [18] W.T. SUNG, S.C. OU: *Web-based Learning in the Computer-Aided Design Curriculum*. Journal of Computer Assisted Learning **18**(2), 175–187 (2002).

## Appendix A: Questionnaire

1. Are you a student of Computer Science or Pure Mathematics?
2. How would you grade your computer skills (1–5)? (1 - lowest, 5 - highest)
3. What aspects of the Internet do you use:
  - a) e-mail
  - b) data mining
  - c) blog
  - d) chat
  - e) educational content
4. Do you use any computer accessories for the learning of Descriptive Geometry (yes/no)?
5. If yes, which one?
  - a) Electronic Problem Book
  - b) Electronic notes
  - c) Solved problems for class work
6. How would you compare interactive 3D images to classical drawings
  - a) They are clearer and easier to understand
  - b) There is no big difference
  - c) They are harder to understand



7. What media do you prefer for drawing Descriptive Geometry pictures:

- a) AutoCAD
- b) Ruler, compass and paper

## Appendix B: Results of the Questionnaire

There were 48 students involved in the Questionnaire, 18 students of Computer science and 30 students of Pure Mathematics.

<i>answer (Q2)</i>	<i># of students</i>	<i>percentage</i>
1	1	2.08 %
2	1	2.08 %
3	21	43.75 %
4	14	29.17 %
5	11	22.92 %
<i>answer (Q3)</i>	<i># of students</i>	<i>percentage</i>
a	47	97.92 %
b	35	72.92 %
c	12	25.00 %
d	18	37.50 %
e	40	83.33 %
<i>answer (Q4)</i>	<i># of students</i>	<i>percentage</i>
Yes	42	87.50 %
No	6	12.50 %
<i>answer (Q5)</i>	<i># of students</i>	<i>percentage</i>
a	16	33.33 %
b	19	39.58 %
c	39	81.25 %
<i>answer (Q6)</i>	<i># of students</i>	<i>percentage</i>
a	38	79.17 %
b	9	18.75 %
c	1	2.08 %
<i>answer (Q7)</i>	<i># of students</i>	<i>percentage</i>
a	28	58.33 %
b	20	41.67 %

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