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The Effects of High School Geometry Instruction on the Performance in Spatial Tasks

George Kospentaris, Panagiotis Spyrou

Department of Mathematics, University of Athens
Panepistimioupolis, Athens, Greece
email: 25aris@math.uoa.gr

Abstract. The present study explores the effect of school geometry instruction on spatial ability. We tested 282 students of varying mathematical background looking for differences in their performance on a variety of spatial abilities. The spatial tasks were confined to those relevant to basic concepts of formal geometry, in order to investigate whether these differences would be more conspicuous. Moreover, we interviewed 36 students to examine the relationship between the strategies they employed and the geometry instruction they had experienced in school. The results of our analysis suggest that the school geometry experience positively contributes to the students' spatial abilities.

Key Words: High School Geometry, Spatial abilities

MSC 2010: 51N05

1. Introduction

In the spatial abilities research has been considered crucial to identify educational factors that reinforce some possible innate or early acquired predispositions. Battista, Wheatley and Talma (1982) [1] considered geometry instruction as an independent variable and explicitly mentioned the significance of the geometry course in the students' development of spatial abilities. They found that a semester course in informal geometry improved students' spatial visualization ability to perform 3D rotations.

Gittler and Glück (1998) [10] considered geometry instruction in high school as one main factor of improving the students' spatial abilities. They focused, however, like Battista et al., on only one aspect of the spatial abilities: namely 3D rotation. Moreover Tsutsumi, Schröcker, Stachel and Weiss (2005) [19] in their study with university students attending a descriptive geometry course, they focused again on a single spatial ability: sections of solids by planes (Mental Cutting Test). Their results are similar to those that Suzuki (2002) [18], using the same test instrument, had also reported about comparative studies

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concerning university students and using the same test instrument. Leopold, Górska and Sorby (2001) [14] had already tested a broader range of spatial abilities (mental rotation, mental cut and visualization of 3D object from 2D pattern) with university students relative to a descriptive geometry course. Overall, the aforementioned research findings suggest that a semester course in descriptive geometry improved spatial visualization skills in students as measured by these particular tests.

In our study we tested a broader variety of spatial abilities relative to formal geometry instruction in secondary level education (including the facets of spatial abilities mentioned above). Moreover, we looked for particular geometrical components (terms, concepts, methods, etc.) of the strategies used by solvers for these spatial tasks, in order to ground the effect of geometry instruction on empirical data.

2. Some specific theoretical points related to the methodology

2.1. On the nature of "spatial abilities" and their relevance to geometry

In an attempt to preclude the interference of factors irrelevant to geometry, we first discarded the types of tasks that appeared to be indirectly related or not related at all to geometry. We analysed a great number of test items that have been used in the spatial abilies research on the basis of the geometrical idea that underlies each task. We identified five basic geometrical areas and accordingly we selected the types of spatial tasks used in our study. These areas can be classified as follows:

- *Analysis of a form or a configuration*: distinguishing the basic parts of a form or configuration and their spatial (topological and metric) relations is a substantial prerequisite for many tasks.
- *Rotation transformation*: a figure is rotated on the plane or in space.
- *Handedness* (or sometimes *"chirality"*) of a form or configuration: this property is inversed when the reflection transformation is applied to an object. For example, one of our hands is the mirror image of the other, and thus the right hand glove does not fit the left because the handedness has been changed.
- *Reflection transformation*: a 2D or 3D form is reflected with respect to a line or plane.
- *Projection transformation*: faces of solids are projected on different planes (usually parallel to the primary directions in space).

The above classification does not of course exhaust the whole diversity of the tasks that one can encounter in the literature, but we argue that it sufficiently captures the rationale shaped after almost 80 years of research (for an overview see: Smith (1964) [17], Caroll (1993) [5], Hegarty and Waller (2005) [12]).

2.2. On the strategies adopted by solvers

The role of the strategies employed by the solvers in spatial tasks has been already stressed by Tsutsumi et al. [19] and Peña, Contreras, Shih, Santacreu (2008) [15]. One significant question is whether or not the effect of geometry can be discerned in the development of different types of strategies. For instance, Kyllonen, Lohman and Woltz (1984) [13], report that individuals sometimes used *analytic strategies* to solve a spatial visualization task: in the paper-folding test they often applied the analytical principle that when the paper is folded down the middle before the hole is punched, the pattern of the holes should be symmetrical around the fold. Symmetry is a fundamental geometry topic.

More generally, we can note that an alternative strategy seems not to be solely a matter of some particular 'spatial knowledge', as in the case of symmetry above. According to Van Hiele (1986) [20] the attainment of Level 1 in geometry is characterized by the knowledge of the properties of the shapes and the analytic relation of their parts, which is a quite different mode of looking at a visual form than that of Level 0. Tsutsumi et al. [19] suggested that through Descriptive Geometry education may be progressed some kind of logical thinking ability combined with the intuitive spatial recognition ability. Furtherly, as Hershkowitz, Parzysz and Van Dormolen (1996) [11] support, the development of an appropriate "visual *language*" and "visual concepts and processes" helps the solver to organize and work out the spatial relations.

2.3. On the studies focused on mathematics-spatial ability relation

Friedman's meta-analysis of 75 studies (1995) [8] suggested that the correlations between spatial ability and mathematical performance ranged from 0.30 to 0.45, which is rather low. Narrowing the mathematics domain to geometry allows us to investigate whether or not the previoulsy observed spatial ability-mathematics correlation would increase, as a common sense hypothesis may imply.

2.4. On the geometry instruction and its assessment

The term "geometry instruction" has been used alternatively for: a number of specially programmed class sessions, a semester or year course, or a more systematic and thorough occupation characterizing a whole educational level (the complete geometry syllabus in elementary or secondary school). We choose, like Gittler and Glück [10], the last option as the more substantial one.

Secondly, we considered that the geometry syllabus and the work at school sufficed to differentiate the individuals for the aims of our study.

Thirdly, to differentiate between the individuals of the same educational level we measured their geometric attainment. This could be done by using a specially designed test or by relying on teachers' marks. Smith [17] has stressed the complexity of the problem of finding an adequate criterion of geometric achievement. Moreover, Gittler and Glück [10] did not attempt to assess each student's involvement in geometry lesson, while Leopold et al. [14] relied on the students' subjective impressions about it. In this study we opted for the school marks, considering that this may be viewed as a broader index that includes a wide range of the aspects of the students' geometrical/mathematical educational experience.

3. Method

The stages of the study were:

1) Administering a questionnaire of spatial tasks to student groups of different educational levels
2) Interviews with a number of these students and
3) Readministering the questionnaire to a number of students after two years to test longitudinal effects (pre-test and post-test control).

3.1. Sample

Our sample was chosen with the purpose to test whether or not different levels of geometry educational level would produce different spatial performances. The characteristics of our sample are summarized in Table 1.

Almost all Greek adolescents falling in the 15–20 age range have received some basic geometry instruction in elementary and lower secondary school. The lower secondary geometry syllabus in Greece consists of: basic geometrical concepts and terminology; measurement, congruence relation and sum/difference between line segments and angles, kinds of quadrilaterals and their basic properties, the Pythagorean Theorem, symmetry and reflection transformation, polygons and measurements in circle (lengths and areas), congruence and similarity of triangles and the basics of trigonometry. All the above topics are taught with the least possible degree of formality.

The first subgroup of our sample with no extensive geometrical instruction consisted of 14 students attending a public Technical Vocational School having completed only the lower secondary school (Gymnasio in Greece). These students (labelled group *'Nu'*) were prospective nurses (5 males, 9 females) and their mean age was 27 years old. They had finished school a long time ago, did various jobs and attended the school to acquire some extra qualification. This age range is particularly interesting assuming that these individuals are in a mature cognitive level from a developmental viewpoint.

Moreover, we administered the questionnaire to another subgroup with minimal geometrical instruction, but of different age. This subgroup (labelled *'Gy'* for Gymnasio) consisted of 81 seventh graders (12–13 years of age), having only the basic geometrical knowledge provided in elementary school.

The remaining three subgroups, more educationally advanced, were: 95 tenth graders (first year in high school, at the beginning of their course prior to any instruction, labelled *'H10'*), 51 twelfth graders (labelled *'H12'*) and 41 students of Mathematics (Mathematicians, labelled *'Ma'*).

In Greece, high school Geometry focuses on plane Euclidean Geometry (as an axiomatic system and its basic theorems) and is taught in a two years course (10^{th} and 11^{th} grade). Furthermore, the students opting for sciences and polytechnic institutions are expected to attend a year's course in Analytic Geometry (11^{th} grade). Elements of transformations (reflection with respect to a line or a point) are taught in a rather perfunctory manner (and usually not examined by teachers' tests). The teaching is rather formal and is based on the solving of geometrical problems that require a number of different techniques.

The last group Ma, the majority of which were in their second year of their course, includes plausibly a higher percentage of mathematically competent individuals than the rest

Table 1: The sample of this study

	Group Nu (Prospective Nurses)	Group Gy (7th grade)	Group H10 (10th grade)	Group H12 (12th grade)	Group Ma (Mathematics students)
Written test	14	81	95	86 (51 + 35[1])	41
Interviewed	0	13	10	3	10

[1]35 students of H10 were re-tested as H12, see Sect. 3.3.4 below

of groups. These students, to be accepted in the university, had to pass examinations on national level and they had been prepared intensively studying mathematics and sciences. Therefore, this group includes a great portion of early "mathematically oriented" students.

3.2. The questionnaire

Our questionnaire consisted of 25 items dealing exclusively with the five basic areas we identified in Sect. 2.1. To cover adequately those ideas either we selected and modified test items from already existing questionnaires or we constructed new ones. Note that some tasks can combine the application of more than one of these geometrical areas. For example, the mental rotation task requires rotation and reflection, resulting in a change of handedness, but before the application of the transformations, an analysis of the form has to be done to assess its handedness. Specifically:

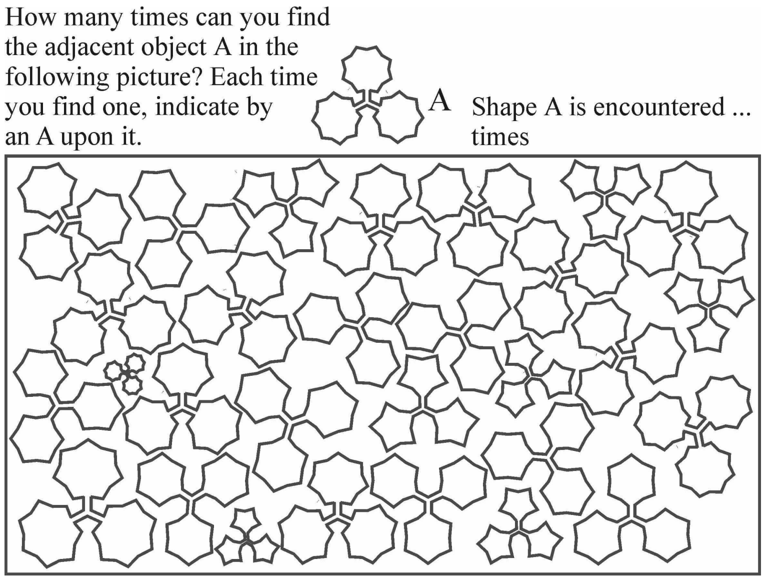

Figure 1: Form Analysis task FA1

- *Analysis of form* (*Form analysis*). Task FA1 is an experimenter-constructed spatial problem aimed to test the student's ability to analyse a complex form. In this task an analytic method would be the counting of the sides or vertices of the polygon (Fig. 1). Task FA2 is a variation of a typical spatial task aiming at configuration analysis (see Fig. 2, left); Task FA3 concerns hidden figures (requires the student to identify a given geometrical figure into a complex configuration). Tasks FA4, FA5, FA6 are three variations of the typical Form- Board Task (which requires the student to pick the correct combination of parts which assembled together give a particular shape).
- *Rotation transformation*. Task R1 is a simple rotation task (see Fig. 2, right). Task R2 is similar to R1 with a more familiar concrete object (e.g. a hand), in order to test whether the embodied familiarity had any effect (see Fig. 3). The task R3 deals with a 3D abstract, geometrical object rotating around an axis in 3D (see Fig. 3, the required rotation has as its axis the line (E), direction to the right of the viewer and angle 90°).
- *Mental rotation* (the concept of handedness). Task MR1 is a 2D variation of a mental rotation task (see Fig. 4). In task MR2 an everyday object (a saw) is rotated and reflected on the plane, whereas, in MR3 the object is a picture of a hand.

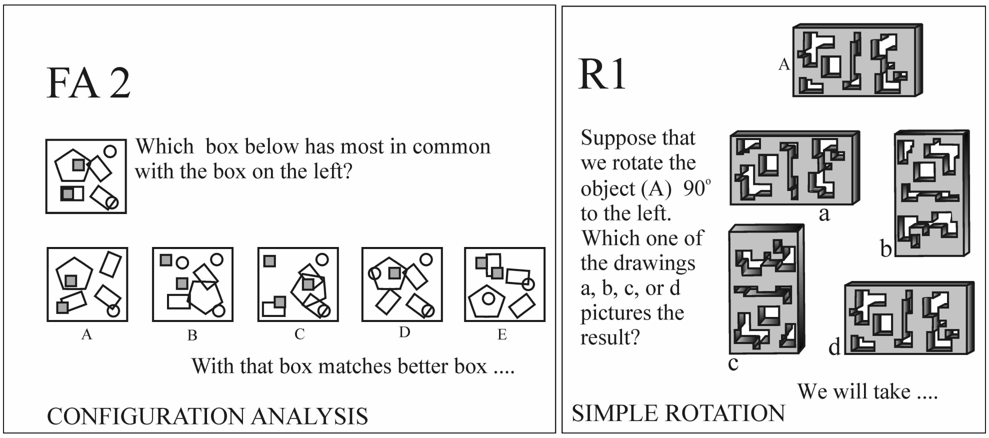

Figure 2: Configuration Analysis (FA2, left) and simple Rotation (R1, right) tasks

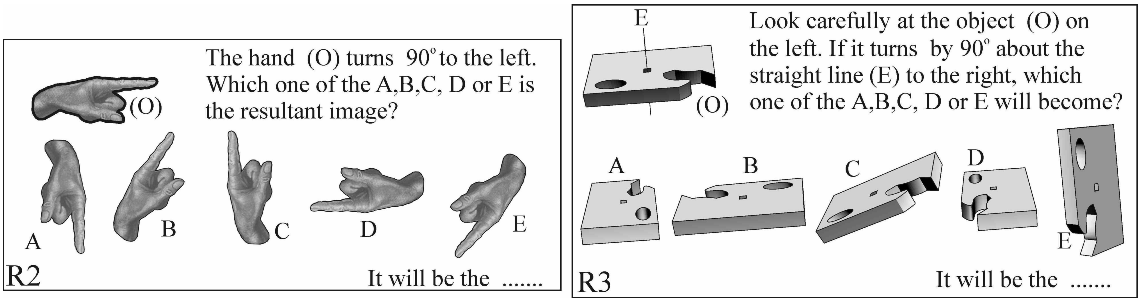

Figure 3: Examples of Rotation R2 (left) and R3 (right) tasks

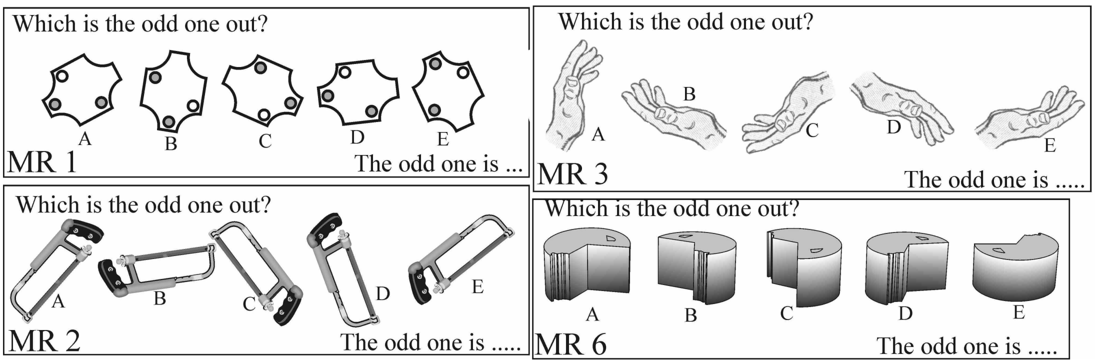

Figure 4: Examples of Mental Rotation MR1 (upper left), MR2 (a saw), MR3 (a hand) and MR6 (below right) tasks

Task MR4 derives from a typical Shepard-Metzler figure. MR5 is a cube having letters on its faces and been rotated or reflected (relative to the plane of a face). Finally, in task MR6 we designed a 3D abstract, geometrical object rotated (relative to a perpendicular axis) or reflected (relative to a plane) (see Fig. 4)

- *Reflection Transformation.* The task Ref1 is a modification of the typical Paper Folding task with punched holes. In task Ref2 the student has to discriminate a translated figure among some other reflected figures when the axis of reflection is horizontal (see Fig. 5). Tasks Ref3, Ref4 have similar structure as Ref2, but the axis of reflection is

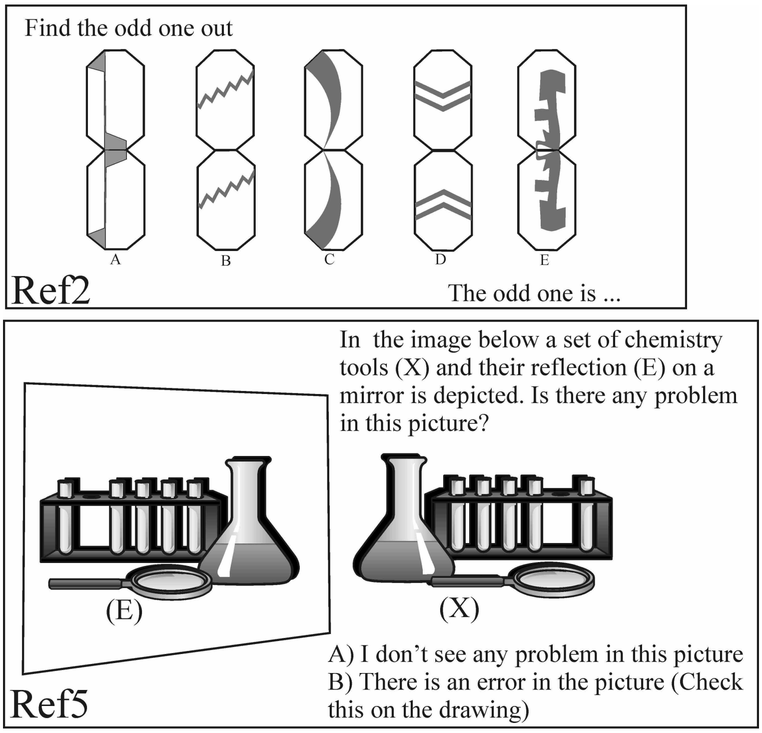

Figure 5: Examples of Reflection Transformation Ref2 (above) and Ref5 (below)

slanted (45°) or vertical. In task Ref5 the solver is asked to find an error in a picture of a reflection of a real object on a mirror (see Fig. 5).

- *Projection Transformation.* This area was examined by three types of tasks which are modifications of typical tasks that can be found in the literature.

  The first type included two View Point (VP) tasks. In task VP1 the participant is given a solid geometrical object and has to find how it would appear from a different viewpoint (projection on a plane). In task VP2 three different views of a solid object are given and the solver has to identify the object.

  The second type of tasks was represented by the task named ‘Cut’. This is a slice task, which requires the participant, given a solid object and a plane cutting it, to identify the shape of the cut (a Mental Cutting task).

  The third type of tasks included two ‘fold’ tasks. In task Fold1 a solid object is given and the solver has to ’unfold’ it and to find its flat development. This task combines projections on planes and rotation of the projections. Finally, in task Fold2 the ‘unfolded’ development of a cube with different signs on its faces is given and the solver has to identify the correct ‘folded’ cube and not one of the distractors.

3.3. Procedure

3.3.1. The pilot study

The development of the questionnaire was completed during a pilot study which involved students of different educational levels. In the pilot study we addressed various methodological issues including: the reliability of the questionnaire, whether the level of difficulty of the tasks ‘matched’ the sample (not too difficult or easy), the students’ understanding of the wording of the written questions, the identification of appropriate process time limits, etc.

3.3.2. The main study

The main study took place from 10/2006 to 10/2007. The questionnaire was administered to the students in their classrooms. The participants were given approximately 40 min to complete the questionnaire, which is roughly the net time of a class session in Greece.

3.3.3. The interviews

The aim of the interviews was the identification of particular elements of geometrical knowledge affecting solvers' strategies. Furthermore, we could search for misconceptions about spatial relations that the geometrical knowledge may help to overcome. We interviewed 36 students: 13 students of group Gy, 10 of H10, 3 of H12 and 10 of Ma. Twenty-one of the interviewees were selected as *high scorers* (raw score 20–25 in the spatial test) and the rest were *low scorers* (raw score 0–7 in the spatial test), in order to investigate whether the difference in the students' score level would be linked to different types of strategies. Seeking basically the geometrical aspects in these strategies we did not interview members of group Nu. The interview discussion was based on the written answers of each student, using their test paper for rough sketches or notes. We audio taped and transcribed the whole procedure.

The interview protocol was loosely structured, based mainly on a sequence of appropriate questions for each task, like: *"Why did you give this answer?"*, *"Can you remember how did you get at this?"* and *"Try to describe as accurately as you can the thoughts that led you to this answer"*.

We considered these questions to be easy, nonintrusive and appropriate to trigger a discussion, which in no way was restricted to the formal geometric aspect of the tasks. Finally, we explicitly asked whether any of the tasks reminded to them anything related to school knowledge.

3.3.4. The longitudinal study

In order to investigate the difference of the individuals' spatial ability relative to their involvement with school geometry before and after the instruction, we administered the questionnaire to 35 students from H10 group (10th grade) after the completion of their two year geometry course (post-test). These students were not included in the 51 individuals of H12 group (see Table 1).

3.4. Method of analysis

The written responses of the students were submitted to analysis according to Rasch Model of Item Response Theory (Bond and Fox (2001) [3]). One of Rasch Model's advantages compared to the Classical Test Theory's raw score is that it scales the difficulty of items and the ability of people on the same metric. Thus the difficulty of an item and the ability of a person can be meaningfully compared. According to this model each student's total score is computed as a probability function of his/her ability and the item difficulty and is displayed along a log odds unit (logit) scale. A student with 50% correct responses is set to zero in the logistic scale: i.e., the student's ability is zero logits.

After the Rasch score computation the mean scores of the subgroups were statistically compared, to test whether these groups were significantly different in their spatial abilities.

3.4.1. Index of geometrical and mathematical competence and correlation to spatial score

The average geometry school mark derives from the combined student's work assessment by the teacher in an everyday basis, his/her performance in written tests and his/her final examination mark in the end of the year. The average geometry school marks were chosen as the measure of students' 'geometrical knowledge', because they have been considered to depict more reliably each student's 'degree of involvement' with Geometry. This involvement is of special interest for us since we do not test particular subject matter knowledge. A specific geometry questionnaire would delimit the range of tested topics and would not take into account signifant aspects of the whole educational experience (homeworks, class interactions, examinations, etc.). This index was available only for the group of the twelfth graders H12, for whom we had the teachers' marks in the two years Euclidian Geometry course at our disposal. For groups Gy, H10 whatever maths marks we had access to, they were mixed with other mathematical topics. Students of mathematics did not attend some specific Geometry course. Therefore, we did not have any specific index of geometrical competence for them.

4. Results

4.1. Written responses

The Cronbach's Alpha for the 25-item test of the study was found to be 0.826, an accepted value as a measure of the reliability (internal consistency) of the test. The data found to fit the Rasch model well, so the application of the model was permissible. The reliability index for the persons' measure (the Rasch equivalent of KR-20 or Cronbach Alpha) was 0.83. The highest Rasch score attained among the students was 3.69 and corresponded to 24 correct items. In Table 2 the average and standard deviation of the Rasch score for each experimental subgroup is depicted.

Table 2: The mean Rasch score of each subgroup

	Group Nu (Prosp. Nurses)	Group Gy (7th grade)	Group H10 (10th grade)	Group H12 (12th grade)	Group Ma (Math. students)
Mean Rasch score	-1.3175	-0.528	0.0497	0.678	0.991
Standard Deviation	0.57	0.995	0.883	1.184	1.216

The differences between the mean rasch scores of the groups found statistically significant using t-test ($p < 0.001$ for groups Nu-Gy, Gy-H10, $p < 0.005$ for H10-H12 and $p < 0.05$ for H12-Ma). This indicates that the educational (including geometrical) differences produced significantly different spatial performances. Further evidence below will illuminate the specific role of geometry.

On the right side of Table 3 the measured difficulties of the items, together with the mean scores, against the same logits scale are shown. It can be observed, for example, that the rotation task R2, with hand images, proved to be quite easy, but this is not the case for the corresponding mental rotation included in the task MR3. Reflection tasks Ref1-5 appear to be adequately mastered by both High School and Mathematics students, who had been taught this transformation in their geometry course.

Table 3: The difficulties of each item presented against the same scale as the means of students' Rasch scores

Group	*Mean Rasch Score*	*Difficulty Measure*	*Task*
		-2.02	FA5
Nu	-1.3175	-0.98	R2
		-0.82	Ref4
		-0.71	Ref1
Gy	-0.528	-0.48	FA2,R1,MR1,Ref3
		-0.40	MR6,FA6
		-0.21	Ref2,FA1
		-0.07	MR4
H10	0.0497	0.04	Vp1
		0.16	Ref5
		0.30	MR2
		0.48	Vp1,FA4
H12	0.678	0.59	Fold1
		0.68	MR5
		0.74	R3
		0.90	Fold2,MR3
Ma	0.991	1.07	Cut
		1.80	FA3

4.2. Correlation with Maths teachers' marks

As already mentioned, for subgroup H12 we had at our disposal the students' average marks from the two years Euclidean Geometry course. The Pearson product moment correlation coefficient between those marks and the test Rash score was found to be $r = 0.455$ ($p = 0.001$). This value is slightly greater than those observed in the previous literature (Friedman [8]).

The corresponding correlations of the specific task subtests (raw) scores were for FA: 0.345 ($p = 0.014$), for R: 0.07 ($p = 0.62$), for MR: 0.499 ($p = 0.000$), for Ref: 0.432 ($p = 0.002$) and for Projection transformation tasks (VP, Cut and Fold): 0.237 ($p = 0.098$). The second greatest value is observed in the reflection tasks, which examine a type of transformation that is taught in the geometry course.

For subgroup H10 the same correlation coefficient between the three years lower secondary Maths average (including some geometrical topics: see Sect. 3.4.1) and the test score was found to be $r = 0.252$ ($p = 0.021$), which is rather low. For subgroup Gy, taking as Maths index the teacher's grades (average) for the first semester (not including substantial geometrical instruction), the corresponding correlation coefficient found $r = 0.4624$ ($p = 0.000$).

4.3. About the longitudinal study

Thirty five students of H10 group completed the two-year geometry course and were tested using the same questionnaire. Their pre-test mean Rasch was 0.155 and their post-test was 1.07. This increase was greater from that observed in the rest of the participants who answered

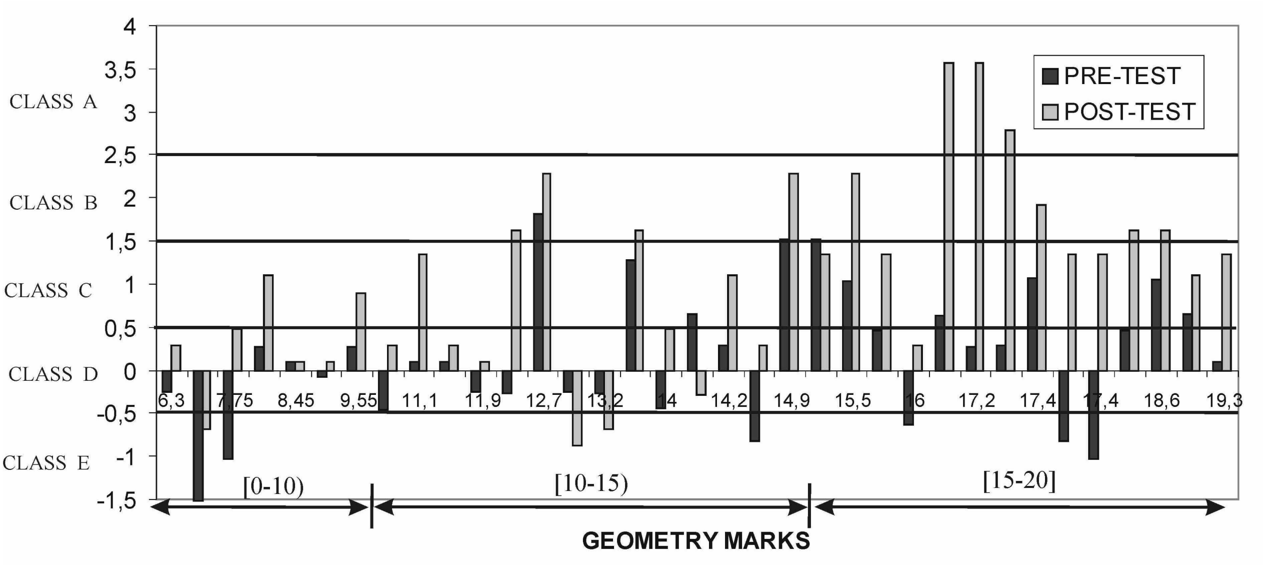

Figure 6: Pre-test and post-test Rasch scores

the test for a first time (group H10 had a mean 0.0497 and H12 0.678), and this difference is probably due to practice effect. In Fig. 6 are depicted the Rasch scores of the students ordered according to their geometry marks on the horizontal axis. It is evident that the students with greater geometry marks tended to improve more their spatial performance. If, for example, we group the Rasch scores in five classes (class E: < -0.5, class D: -0.5 to 0.5, class C: 0.5 to 1.5, etc.) we observe that of the students with geometry marks in the ranges [0–10), [10–15) and [15–20] showed different patterns of improvement. These results are in Table 4.

Table 4: Pre-test and post-test spatial ability differences of three geometry marks categories

Rasch score difference (pre-test/post-test)	*Geometry marks* [15–20]	*Geometry marks* [10–15)	*Geometry marks* [0–10)
Remained in the same class	14.2%	57.2%	57.2%
Ascended 1 class	42.8%	35.7%	42.8%
Ascended 2 classes	28.5%	7%	0%
Ascended 3 classes	14.2%	0%	0%

4.4. The Interviews

4.4.1. Some general observations

The students were probed by the interviewer to state explicitly the thoughts that led them to their written answers. The students' initial answer was frequently quite confused. When asked to elucidate, they tried to make this description correct and more exact. Thus, in many cases, they were led to the refutation of the wrong written answer and the discovery of the appropriate one.

In Gy group a tendency was observed to evoke situations from their experience with real objects: *"I imagined a dice and took its sides ... "*, *"If I see it as an envelope ... "*, *"It's like* *the inkblots on papers the psychologists give . . .* ", etc. In the Rotation Tasks some interviewees reported that they had tried to answer them by using their own hands (R2) or by physically turning the test paper.

Furthermore, the visual context and the picture details seemed to exert a more profound effect upon these students, causing confusion and difficulties in the formulation of an abstract description.

Four of the low score students showed the following characteristic: they appeared to have an adequate, yet separate, understanding of the notions of right angle ("90°") and rotation direction ("left of right"), being unable to combine them. Similarly, to solve R3 task three parameters of the transformation had to be taken into account: rotation angle (90°)-direction (right)-rotation axis (ε). Nevertheless, the students tended to take into account only one or two, disregarding the rest. In Piaget's (1971) terms [16] this characteristic could be labelled inability to "coordinate operations".

Among the low scorers of all groups, but especially in Gy group, the lack of geometrical instruction has been apparent. Some of the interviewees

1) claimed it was possible to superimpose noncongruent line segments (edges) or angles,
2) were unaware that the vertical and the sloping segment from a point to a line cannot be equal,
3) gave two or three mutually exclusive answers or changed their answers many times without justification (some of them even substituted their initial correct answer with an erroneous one during the interview), etc.

Twenty one of the 36 interviewed students were spatial high scorers. Twelve of them used geometrical terms (such as "rotation", "rectangle", "trapezium", "90° = right angle", etc.) in their explanations. One low scoring student recalled geometrical ideas that were not relevant to the given tasks.

The above general characteristics can be summarized in Fig. 7. Note that according to Bliss, Monk and Ogborn (1983) [2] the {-notation signifies that the categories are not mutually exclusive.

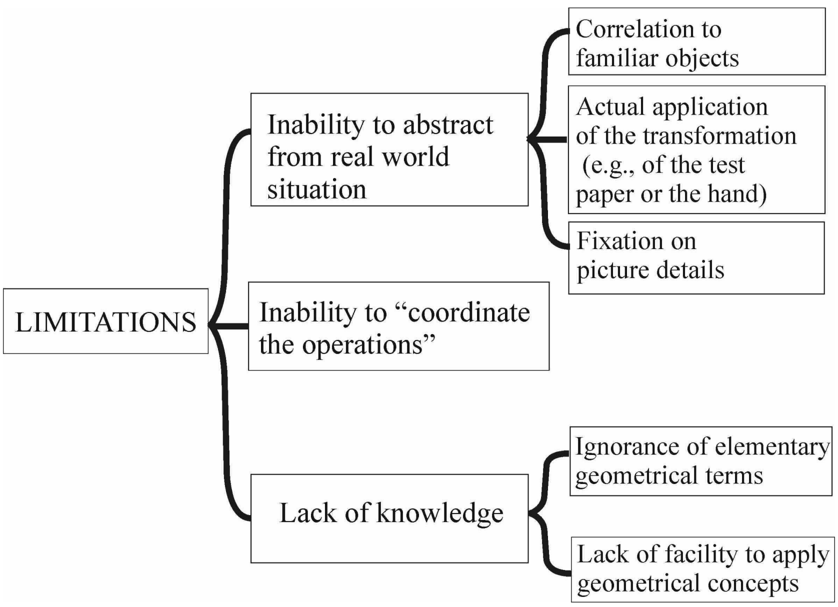

Figure 7: Limitations observed in low score students

4.4.2. Task specific observations

Form Analysis task FA1: there were students, especially in Gy group, that regarded the form as a "syncretic whole" (gestalt) (Piaget [16]). The majority applied counting as a means to identify the given shape but some of them were confused about what exactly they had to count. It emerged that they had difficulty to discriminate the convex angles of the polygon from the concave ones.

Rotation tasks R2, R3: many erroneous responses can be attributed to the above mentioned inability to "coordinate two or more operations" (Piaget [16]). When the students were at a loss the interviewer asked what "90° turn" meant and they all answered "to form a right angle". Subsequently, the interviewer asked them to sketch a right angle. They all did it correctly, with the exception of one student who sketched the right angle as a little curved scribble (something like a $\frown$). However, some of them were not able to apply this notion in the task, in contrast to all 12th graders and Mathematicians.

Mental Rotation tasks MR: the 2D, plain, abstract geometrical form in MR1 made easier the construction of the structural description (because the spatial relations were in one plane), resulting to numerous explicit and exact accounts of their strategies. For example: Student T.A., group H10, high scorer [on task MR1]

> T.A.: I imagined myself situated at the dark dot on this "nose" of the figure and look to the opposite "nose" then the white dot is on my left hand and the other dark dot is on my right hand this is reversed in B.

When the stimulus becomes more complicated (e.g., in 3D) the requirements of spatial analysis outweight the students' abilities for verbal description: Student A.Th., group Ha, low scorer [on task MR4, Shepard-Metzler rotation task]

> A.Th.: Here I remember that I had initially counted the little boxes [building blocks of the solids] but I can't remember what I had found ...
>
> R.: They all consist of the same number of little blocks ...
>
> A.Th.: Maybe it is C ... it is in a different position relative to the rest ... Yes, but the others ... how can I tell it ... if they were in a three dimensional over here ... they would lean on it normally, but C is standing on its tip ... If we imagine a plane ... C touches this plane with its tip ...
>
> R.: I see ... but in all of them you can imagine a plane touching their corresponding angle point, these planes are simply rotated ... for example, you can imagine a plane touching A at this corner tip ...
>
> A.Th.: Well, yes ... Maybe I saw the blocks...I mean I noticed how the shaded blocks appeared ...

In some cases geometrical notions seem to help: Student E.P. group M, high scorer

> E.P.: In each case we have a right angle ...
>
> R.: What do you mean?
>
> E.P.: The great bar with the middle one ...
>
> R.: Saying the great bar you mean that made of 6 blocks?
>
> E.P.: Yes ... and the other one with the 4 blocks, the second in size ... These form a right angle ... The little one [with 3 blocks] is directed towards us ... The right angle is rotated ... Here the second bar [4 blocks] in on the left, here again is on the left, and here ... However in D points to the right ... (see Fig. 8)

Or in another description: Student L.Z., group M, high scorer

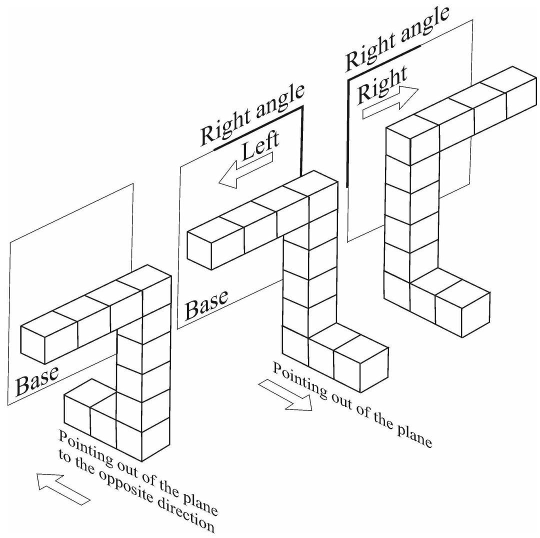

Figure 8: A diagram for the descriptions given by the interviewees E.P. and L.Z.

L.Z.: These two greater bars [the 6-block and the 4-block] they form something like a base ... The third one is directed upwards...

R.: Upwards? Which is the up direction and which one down?

L.Z.: The base is a plane ... They form a plane and the third one points out of the plane ... Take A and B, you can make their bases coincide, then the third bar points out of the plane ... If you make the base of D coincide with that of the others the little bar points to the opposite direction (see Fig. 8)

Reflection tasks Ref: Ten students of groups Gy and H10 had formulated some kind of intuitive reflection rule that was difficult to describe verbally. For five of them this rule functioned adequately in most cases. For the rest five this worked only when the axis was horizontal or vertical, not generalizing to slanted axes.

Two of the interviewees (group Gy) spoke about their experience with folding papers. One (in group Gy) explicitly referred to her experience with COREL DRAW computer software program, while three other interviewees mentioned mirrors and reversions. These were instances of out of school experiences leading to the formation of the concept of reflection transformation.

Two of the H10 group and *all* of H12, Ma students had a conscious and distinct reminisence of the geometrical symmetry concept, which we consider to be the outcome of schooling.

In the Ref1 task the majority of the wrong answers derived from the erroneous notion that the folding transformation amounts to a simple translation (and ignorance of the reversion of the spatial relations) (Fig. 9). This is corroborated by the fact that the participants' most frequent wrong answer in the written test (23% compared to 10%, 3%, 1.5% of the other wrong answers) corresponded to this type of translation.

The students who appeared unaware of the reflection transformation were not able to find the correct answer despite the clues given by the interviewer.

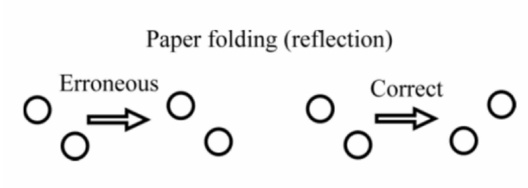

Figure 9: The reflection transformation (in the paper folding task) as mistakenly applied by some students (left)

5. Discussion

This study focuses on the effect: school geometry instruction $\longrightarrow$ enhancement of spatial abilies. Many researches have already explored educationally homogeneous groups and the correlations of spatial scores to mathematics. In our case the participants were students of quite different levels of geometrical competence.

The results of the written test answers presented in Tables 2 and 3 support the view that progressing through the educational levels is accompanied with a significant improvement in these particular spatial tasks performance. This gain could not be plausibly attributed to some general vague “maturity factor” because group Nu (Nurses) had an age average greater than that of Ma (Mathematicians).

The importance of the educational experience may be also inferred from the fact that, group Nu (Nurses), out of classroom for a long time interval, scored even lower than Gy (seventh graders), despite the latters’ three grades inferiority. Moreover, the students of Mathematics (Ma) scored higher than any other group, which can be attributed to their wider and more substantial geometrical knowledge (see Sect. 3.1).

Specific results of the study constitute evidence that this improvement could be attributed to a certain degree to geometry instruction. The first area of evidence is the correlations between the students’ spatial test score and their school marks. As already mentioned we considered that the school marks are the more appropriate index of geometrical aptitude. In our study we also narrowed the tasks domain to areas related to formal geometry and we took the school marks as index of mathematical aptitude. The correlation between test scores and mathematics performance did not exceed the range found in previous studies (Friedman [8]), except the specific case of the score in MR tasks, where it reached 0.499. It is noteworthy, however, that when the index of math aptitude included only the Euclidean Geometry course marks, the correlation reached its peak (0.45). According to some statisticians this is an almost large correlation for social sciences (Cohen (1988) [6]). From the task categories, that of Reflection Transformation, which is related explicitly to the formal geometry syllabus, showed the second greatest correlation (0.432) with Euclidean Geometry marks. We have to take into account here that this topic is taught in a quite perfunctory manner in secondary school in Greece. All students of mathematics (Ma) used the term “symmetry” dealing with Ref tasks during the interviews. Tasks R1 and R2 were quite easier compared to the task R3 that included a 3D rotation (see the Rasch difficulty scale in Table 3). This may account for their almost zero correlation to geometry (0.023 for R1, -0.008 for R2), in contrast to R3 (0.22).

The high school syllabus in Greece (for Euclidean Geometry, Analytic Geometry or Physics) does not include anything specifically related to right or left-handed systems of reference. Nevertheless, the MR tasks showed the greatest correlation to geometry marks.

To tackle efficiently the FA and MR tasks one has to adequately find the spatial relations between an object's parts. This requisite may be facilitated by school geometry experience, where these relations are discussed and analyzed systematically. Analytic properties of shapes, rather than gestalts, are mastered by the students beyond Van Hiele Level 1 (Van Hiele [20]). This ability may account for the FA and MR tasks increased correlation to geometry marks (0.345 and 0.499, respectively).

Note that the instruction in high school concerns exclusively Plane Geometry, which is in line with the fact that the overall 2D tasks score correlated more substantially (0.44) to geometry marks. Nevertheless, the corresponding correlation with the 3D tasks score was found to be even greater (0.48). We argue that this finding suggest that the capacities that geometrical instruction supports, can be transferred or generalized to 3D tasks, in accordance to the findings of Gittler and Glück [10] and Glen et al. (2009) [9].

The second area of evidence for the role of geometry comes from the longitudinal study results (Sect. 4.3). Despite the obvious difficulties in the comparison of an individual's performance between the two phases (see also Gittler and Glück [10]), these results constitute a significant improvement in their performance, whatever the practice effect may be. The different patterns of Rasch score increases between pre- and post-test (Table 4) suggest that the greater the involvement of a student with geometry the greater the possibility to improve spatial test performance.

The third area of evidence for the influence of geometry derives from the interview data in relation to the particular strategies of the students. The effect of factors like the "visual language" to which we have referred already (see Sect. 2.2), can be corroborated by the observed difficulty of members of groups Gy and H10 during the interviews in their endeavour to explicitly state the problem situation and the thoughts which led them to a particular answer choice (for example, see student A.Th. in Sect. 4.4.2). This process of linguistic reformulation bears some resemblance to the geometrical analysis where the role of language is crucial: the spatial objects and relations have to be conceptualized and 'transcribed' into a verbal expression in order to be processed and communicated. The geometrical instruction trains the individual to, among other things, verbalize and communicate data about forms (concrete or abstract). The geometrical terms, encapsulating the concepts in a compact form, help the students to organize the visual image, to apply a structure, which enables them to make the perceptual computations more efficiently (Hershkowitz et al. [11]. This role of language may thus be reinforced by geometric educational experience.

The capacity for abstraction is another feature that could be improved by geometrical knowledge, since geometry has an important abstracting component, even at the elementary level. From the interviews it was made apparent that the low scorers showed a tendency to focus on the concrete details of the image, rather than on appropriately abstracting the underlying spatial relationships that the task asked for. This tendency points to the characteristics of the pre-geometrical visual 'Level 0' of Van Hiele (Van Hiele [20]).

Conclusively, we argue that the findings in this study suggest an explicit improvement of the spatial performance in more geometrically educated individuals. These results combined with the characteristics of the experimental groups do not allow us to attribute this improvement to some general maturity factor. Particular aspects and qualitative elements of the research prompt us to consider geometry instruction as at least one of the factors that contribute to the improvement of some specific spatial abilities. These findings seem to reinforce furtherly the conclusions about the effect of mathematics and physics (Burnett and Lane (1980) [4]), in general, and about geometry (Battista et al. (1982) [1]; Gittler and Glück [10]; Leopold et al. [14]; Suzuki [18]; Tsutsumi et al. [19]), in specific on the students' spatial ability.

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