Journal for Geometry and Graphics
Volume 16 (2012), No. 1, 69–79.

Parametric Reconstruction of the Space in Vermeer's Painting "Girl Reading a Letter at an Open Window"

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Abstract. This paper presents a new approach to the reconstruction of architectural space depicted in paintings. Instead of a mere graphical or numerical method, it uses a tool for parametric design, to create a direct visual feedback during the reconstruction process. An interactive perspective drawing which overlays a precise digital image of the painting is adjusted by moving data sliders. The sliders at the same time provide the essential measurements of the depicted space. Thus we meet the task to extract dimensions with the greatest accuracy possible although the underlying work of art is characterized by blurred brushstrokes. Like any reconstruction attempt this is based on the assumption that the inspected picture obeys the rules of linear perspective. We at first discuss why this assumption is justified although there are no geometric construction aids detectable as e.g. in later works of Vermeer [2]. The approach has been developed on the occasion of an educational project launched by the *Old Masters Picture Gallery* in Dresden. For a special exhibition in 2010 dedicated to the work of a young Vermeer a life-sized room similar to the one shown in the painting *"Girl Reading a Letter at an Open Window"* was constructed.

Key Words: linear perspective, camera obscura, Johannes Vermeer, Grasshopper, parametric reconstruction, interactive optimisation

MSC 2010: 51N05

1. Introduction

In fall 2010 the *Old Masters Picture Gallery* in Dresden showed an exhibition on the early work of Johannes Vermeer (1632–1675). An interactive life-sized scene similar to the one shown in the painting *"Girl Reading a Letter at an Open Window"* (see Fig. 1) was part of this exhibition [1]. Several institutions have been involved in this project. The scene has

ISSN 1433-8157/$ 2.50

been put together in the workshops of the *University of Fine Arts* of Dresden. Students and craftsmen have built window, walls, furniture and even a puppet with costume and hair. All parts have been made movable and the visitors of the exhibition were able to investigate the correlation between the mock-up and the painting while looking through a ring placed at the assumed position of the painter's eye. For this mock-up the dimensions of the room and the position of the viewpoint were needed. This paper describes how the measurements were retrieved from the painting. The reconstruction was undertaken at request of Dr. Uta Neidhardt (art historian) and Christoph Schölzel (conservator), both working at the *Old Masters Picture Gallery*. They supported the geometrical investigation with insights into art history and with physical analysis of the painting.

Figure 1: Johannes Vermeer,
Girl Reading a Letter at an Open Window,
c. 1657, oil on canvas (83 × 64.5 cm).

Figure 2: Johannes Vermeer,
Officer and Laughing Girl,
c. 1655–1660, oil on canvas (50.5 × 46 cm).

2. Mapping quality of the painting

The reconstruction attempt can only succeed, if the painting is based on linear perspective. Thus we at first have to examine if it is likely that Johannes Vermeer used a method of central projection for *"Girl Reading a Letter at an Open Window"*. The main problem is that we do not have any written evidence or drawing of Vermeer handed down, which could prove or disprove the thesis. The only witness is the painting itself. But this painting possesses only a few geometrically defined features and its contours and details are not very precise.

From intuition many people looking at Vermeer's famous interiors suggest that the paintings depict real rooms in a photorealistic manner. This may be the reason why some

of VERMEER's paintings have already been subject to reconstruction attempts. One book on this topic is "Vermeer's Camera — Uncovering the Truth Behind the Masterpieces"by the architect Philip STEADMAN [6].

While the rules of linear perspective were well known at VERMEER's time, the first idea is that he could have used an underlying drawing to construct this painting. If so, we must be able to detect traces of the construction with technical means as it has been done in other paintings of that time [5]. But no trace of any construction can be discovered either under a microscope, or in an X-ray picture, or on an infrared reflectogram. Furthermore, only one of the vanishing points is situated on the picture — the others are far out of reach (see Fig. 4). That, however, does not necessarily imply that VERMEER did not draw the perspective onto the canvas before painting. But taking other drawings of that time into account it is unlikely. Finally, a slight inaccuracy with respect to the window points to the hypothesis that he did not construct.

So, if he did not construct the perspective, how did he produce this highly realistic image? Following the studies of Philip STEADMAN it seems to be evident that VERMEER used a *camera obscura* [6]. This is not the only possible answer but observations in VERMEER's paintings plus a glance at the historical background of the painting support STEADMAN's thesis.

Looking at the painting *"Officer and Laughing Girl"* (see Fig. 2) one argument for the *camera obscura* is the remarkable discrepancy in scale between the two figures. Any other painter of his time would have played down this snapshot-like effect. VERMEER in contrast accentuates the radical perspective and seems to be more interested in visual phenomena than drawing the individuals in proportion to one another.

Also, several commentators have suggested that the minutely detailed topographic maps in the background of many VERMEER-paintings might have been reproduced with technical means. If VERMEER used a *camera obscura* it is almost certain he used one with a lens. Those without lenses are only suitable for observing solar eclipses.

We can take into account that VERMEER lived in a century that was marked by discoveries only possible through the aid of rapidly developing optics. In 1610 Galileo GALILEI discovered the four largest satellites of Jupiter through his telescope. This discovery is connected to the Netherlands because GALILEI's telescope was developed following an invention by Hans LIPPERSHEY, a Dutch maker of eyeglasses. Antoni VAN LEEUWENHOEK, like VERMEER born in October 1632 in Delft, was the first to describe bacteria he observed through his microscopes. It is very likely that VERMEER was acquainted with him. VERMEER must have had the experience of viewing through a lens and most likely of projecting sceneries through a lens onto any kind of screen. The blurring effects and dispersed highlights in *"Girl with a Red Hat"* (see Fig. 3) could be a testimonial of this special optical experience.

STEADMAN's most striking find however, results from the comparison of ten paintings in which VERMEER apparently pictures the same room with changing scenes. STEADMAN draws reconstructed plans of the room, marks the viewpoints and draws the *angles of view* as limited by the frames of the paintings. For the reconstruction of six paintings the resulting *pyramids of view*, when extended through the viewpoint, intersect the back wall of the room in rectangles that have almost the same size as the actual canvas. Hence VERMEER could have separated the room into a large and light part with the scenery, and a small and dark part, a *camera obscura* in its literal sense. The divider, a curtain or a wall, must have contained a lens through which the scene was projected onto the back wall — bright enough to be traced, albeit upside down and mirrored.

We summarize: the Dresden painting with the girl reading a letter is one of the first paintings with VERMEER's characteristic domestic interior scenes. With its strong spatial effect it differs significantly from earlier works. The use of a camera obscura is likely. In any case VERMEER had a sound passion for questions of projection and we therefore suppose the following: the picture in substance possesses the geometrical quality of a photographic image and follows the rules of linear perspective. Thus it is a suitable candidate for an attempt to reconstruct the depicted space.

Figure 3: VERMEER, *Girl with a Red Hat,* c. 1665–1667, oil on panel (23.2 × 18.1 cm).

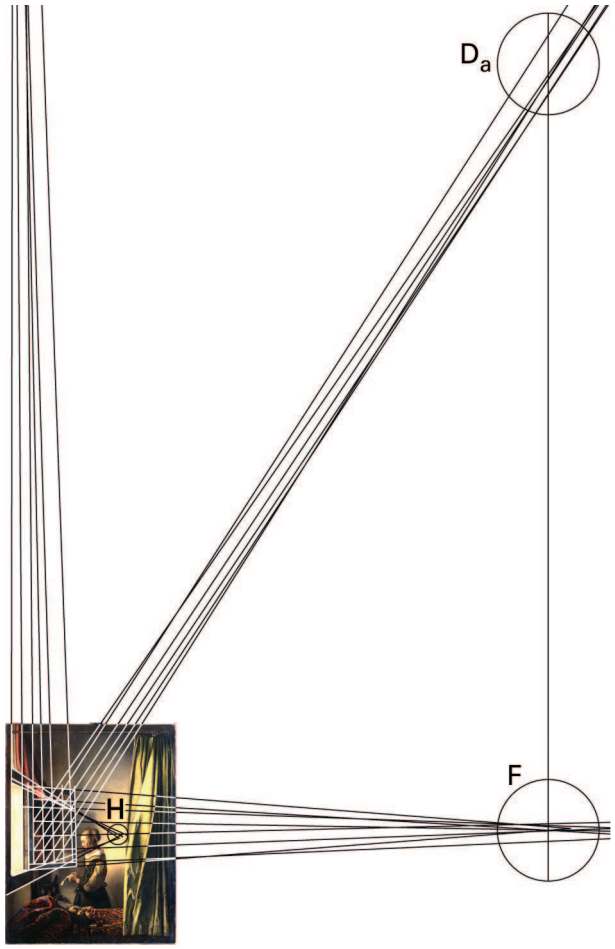

Figure 4: Conventional reconstruction.

3. Assumptions

In general the non-ambiguous reconstruction of a space is only possible from at least two images taken from two different viewpoints. If only one picture is available, additional assumptions derived from our empirical knowledge of space are required to compensate for the missing second image. We subconsciously use this strategy any time we look at pictures. Only special optical illusions with certain contradictions between visual recognition and physical feasibility, so called *impossible objects*, let us become aware of the fundamental ambiguity of two-dimensional pictures. The assumptions we make here derive not only from our intuitive experience with space but also from observations in other paintings by VERMEER.

The first assumption is: we deal with a rectangular room in a *frontal perspective*. A short excursion: We do not use the term "one-point perspective"because, even in a frontal perspective the use of several vanishing points is common (already made so by the work of Leon Battista ALBERTI). Our example with VERMEER also shows how this is very helpful. Not the number of vanishing points is significant for the description of the perspective, but the orientation of the main object's coordinate system in relation to the picture plane. Thus: when a rectangular object like the chamber in this picture is depicted in a way that two facets (walls) are parallel to the picture plane, the perspective is called *frontal*. Then in total eight edges are parallel to the *picture plane* and four are perpendicular. The vanishing point of the latter coincides with the point of the picture plane next to the viewpoint, called *main point*. We observe that the term "one-point perspective"(etc.) is often blocking deeper insight into the concepts of linear perspective only by the infelicitous choice of the name. But back to our subject: we assume the wall behind the girl is parallel and the wall with the window is orthogonal to the *picture plane*. Thus the horizontal lines in the window's wall are *lines of depth* and align to the *main point* H of the perspective. The bright embrasure of the window is also parallel to the picture plane. This kind of setup, a frontal perspective, is common and useful for a graphically constructed perspective. For the use of the *camera obscura* this setup is not required. Hence VERMEER forced it to be frontal to achieve a strong formal composition.

Our second assumption concerns the casement. Its subdivision is regular and the horizontal lines align at a *vanishing point* F at the horizon while the diagonals of the small rectangular glasses have vanishing points D_a and D_b at the same distance above and below F (see Fig. 4). We suggest the casement fits as usual into the window frame when closed.

A table stands in the foreground covered by a carpet, in keeping with the spirit of the times. On the table is a bowl of fruit. The dimension of the table is assumed to be common, the fruit in a natural size.

The chair in the corner is only partially visible, but is nevertheless the best reference for the rest of the measurements. This chair appears on several of VERMEER's paintings and similar furniture is preserved in the *Rijksmuseum Amsterdam*.

The intimate ambience of the painting originates partially from the green curtain at the right side. This is a *trompe-l'œil* playing with the habit of those times to cover a painting with a curtain. The curtain is not part of the space the girl is in but belongs to us, the curious observers.

4. Classical reconstruction

As STEADMAN writes, the reconstruction of the space in one of VERMEER's paintings will usually be attempted only if at least a part of a tiled floor is visible [8]. Hence at first glance the architectural details in the picture seem to be insufficient and the operation has no prospect of success. But actually only a slightly deeper insight into linear perspective in combination with the previously outlined set of assumptions brings the solution: if the casement is moved around its pivot, the vanishing points D_a and D_b will draw a hyperbola. This hyperbola has to contain the vanishing points of the diagonals of the window frame (see Fig. 5, only the upper part of the hyperbola is depicted).

The true difficulty for the reconstruction is the weak accuracy of the painting. No painting has the precision of a photographic mapping. During the lengthy process of painting a lot of changes were made, which can partly be revealed by X-ray. Single lines are unreliable. Thus

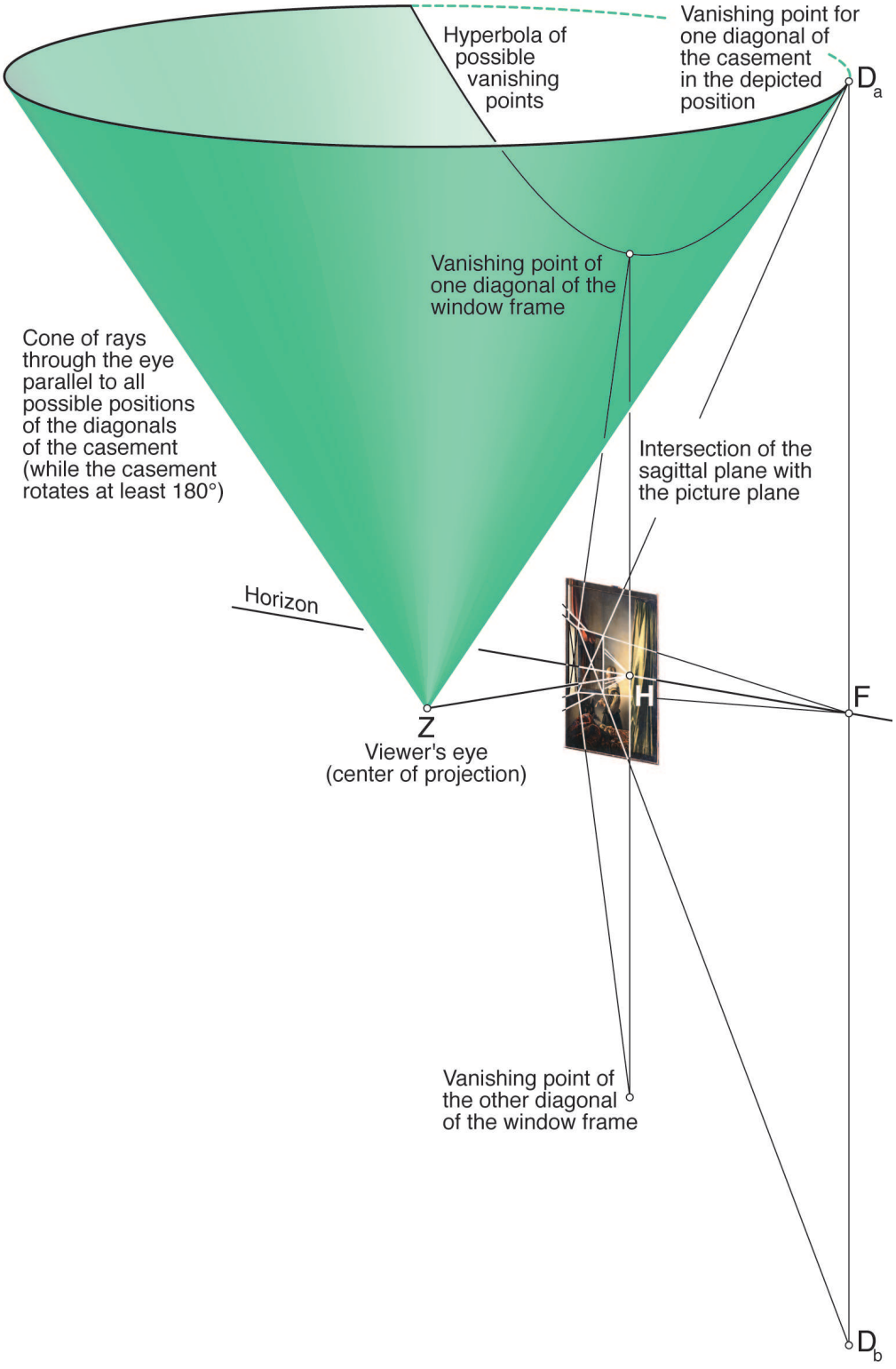

Figure 5: Spatial configuration of the determination of the center of projection from the dependency of casement and window frame.

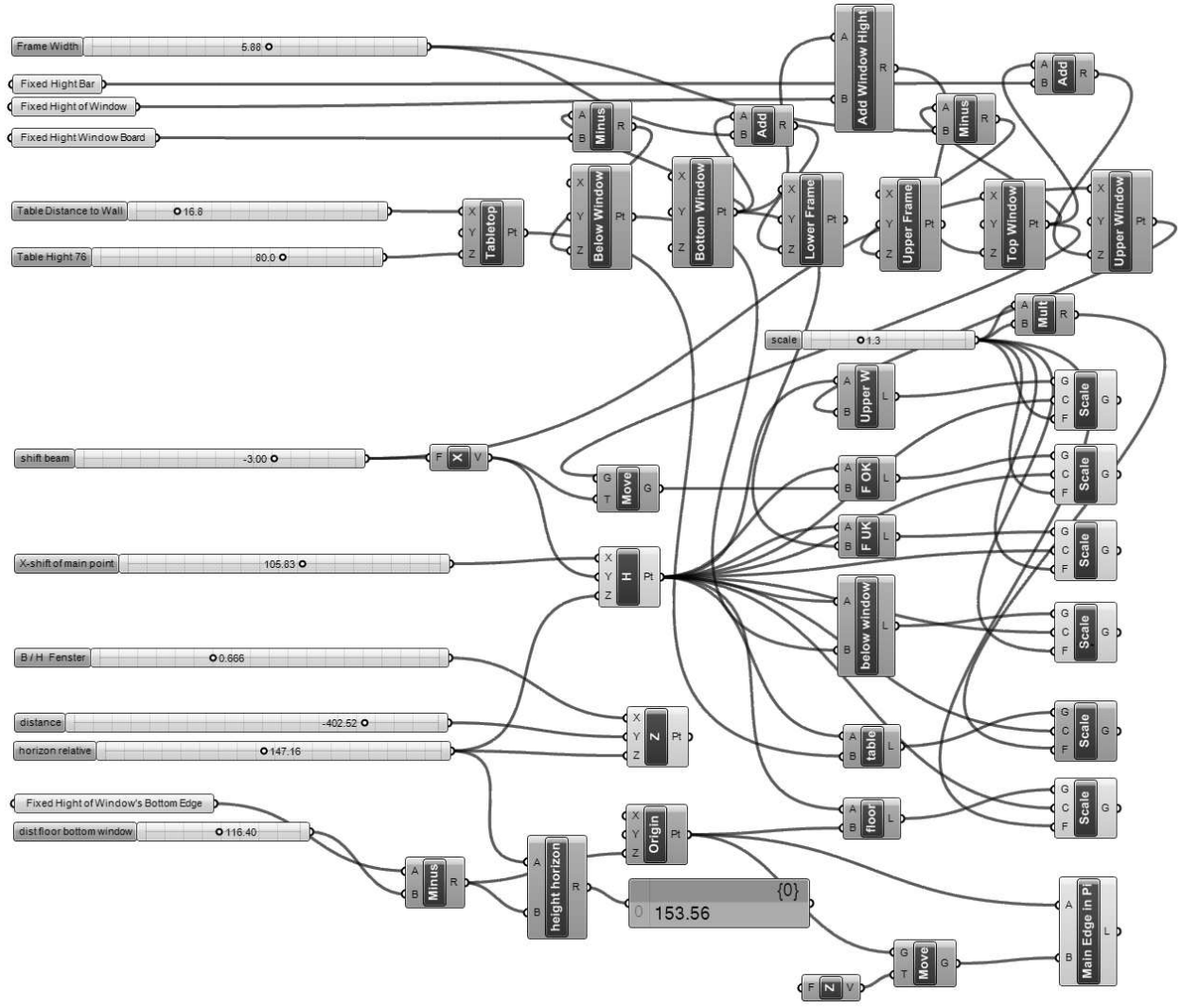

Figure 6: Screenshot of the Grasshopper canvas with data-sliders and components (extract).

only a holistic approach can be successful. It is especially arduous when you try to find a solution by tracing one line after the other. The extended lines, envisioned to be parallel in space, do not necessarily intersect in single points. Rather an area of possible locations for each vanishing point can be stated (see Fig. 4). By repeating this process and by adapting and redrawing, the solution could be approximated iteratively. But the problem is, any decision that might appear meaningful locally can damage the overall accuracy.

5. Interactive reconstruction

In place of the traditional approach of tracing line by line we set up one idealised linear perspective drawing according to the complete semantic recognition of the painting. The idealised drawing has perfect intersections and covers all obviously existing connections in the painting but leaves the actual dimensions undetermined (see Figs. 5 and 7). For example, the lines of depth are made to intersect, but the point at which they do so on the canvas is not yet fixed. This topologically true perspective is then manipulated with data-sliders to adjust the dimensions.

The idealised and interactive drawing has been created with the plug-in *Grasshopper* for the 3D modelling software *Rhinoceros*. *Grasshopper* is an easy to learn tool for parametric design. It provides a visual programming interface where symbolic instances of geometric objects can be connected with virtual wires (see Fig. 6). In the Rhino-viewport the idealised drawing is overlaying a digital image of VERMEER's painting that provides a resolution of about 5 px/mm. To obtain an optimal fit, the quality of the correspondence between the perspective drawing and the actual painting is visually evaluated in real-time. Thus it is possible to rate single points in relation to the overall appearance and compensate weak details.

The idealised drawing becomes adjusted to the painting in several well-defined steps. By following a logical sequence from the most important settings to the tiny details the manipulation of the data sliders gives perfect control over the result at any time. Methods of automatic image recognition are not applied.

The picture plane for the reconstruction is conveniently set through the hinges of the window. In the beginning the *main point H* has to be localised. Therefore the values "distance to the window wall" and "height of the horizon" are tuned. Thus the *main point* adjusts to a position a little bit above the height of the girl's eyes and behind her head, close to the green curtain. That means the observer is slightly taller than the girl. The next step is moving the vanishing point F of the horizontal lead glazing bars back and forth along the horizon until all bars are obviously covered best by the lines of the drawing. This action can be compared to manually focusing the lens of a camera.

Now the crucial step of the reconstruction follows: what is the *distance d* of the viewpoint to the picture plane? This can be answered if we take for granted that the casement fits into the window frame. Hence the proportions of casement and frame have to be the same. This can be controlled with the vanishing points of their diagonals (see Fig. 5). Under this condition the values "proportion of the window's dimensions" and *distance d* have to be equilibrated. A convincing conformity of drawing and painting arises at a ratio of 2 : 3 for the window (see Fig. 7). With respect to common window constructions this can be rated as a very convincing result. At the same time the aperture angle of the window is about 132 degrees.

Until now, we did not mind the actual size of the objects but only their spatial arrangement. The guess of the scale starts with the girl's height. She may be 155 cm tall and may stand in the axis of the window frame. Because her feet are not visible, their location has been estimated using the girl in VERMEER's *"The Art of Painting"* for comparison. These settings produce a first statement about the height of the window. All other dimensions refer to this. The corresponding distance d is about 4 meters, the distance to the girl a little less.

All other dimensions follow in a similar manner. The complete reconstruction procedure is outlined as follows:

1. Locate the *main point H*
2. Set the casement's vanishing point F at the horizon
3. Equilibrate the windows proportion with the distance of the viewpoint
4. Get the ornament of the lead glazing
5. Measure the window's frame
6. Adapt to the back of the chair and correlate with the former assumptions
7. Redefine the body height of the girl
8. Determine the position of the table
9. Set the position of the picture frame according to the size of the canvas

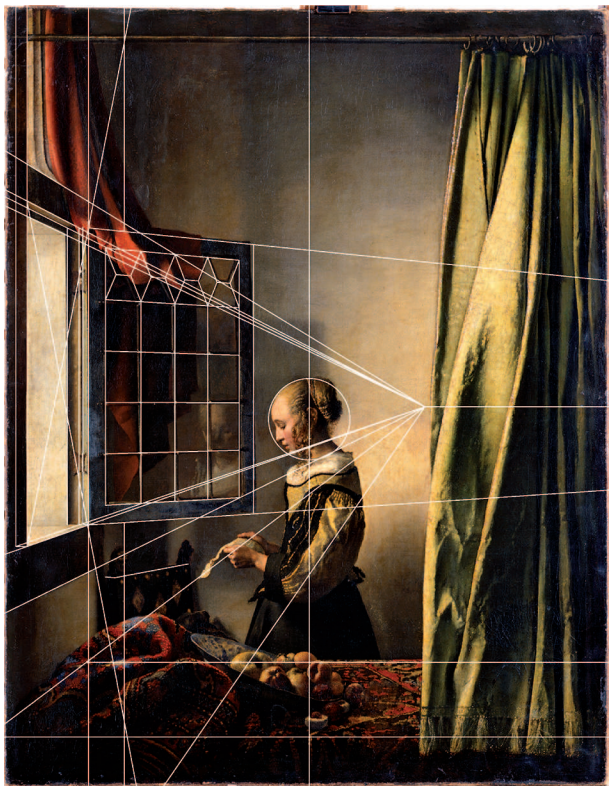

Figure 7: Idealised perspective drawing overlaying the picture.

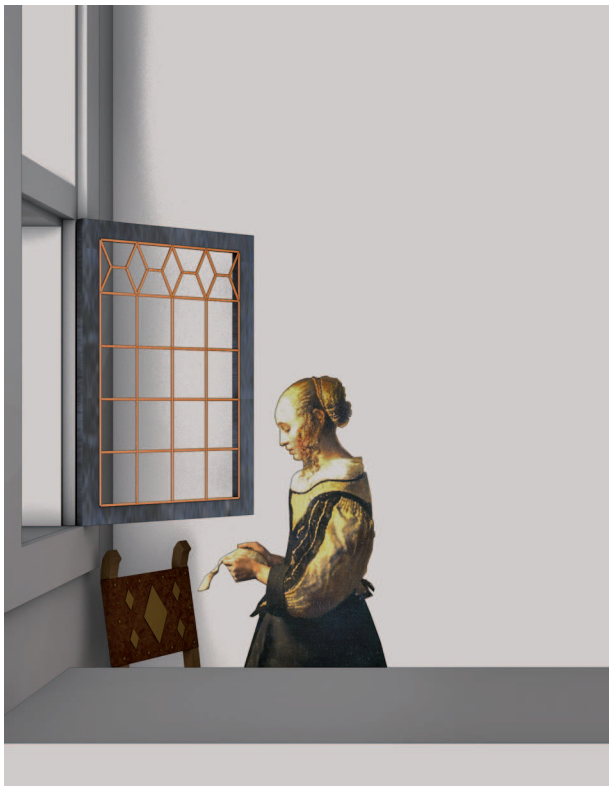

Figure 8: Test rendering of the 3D model with a virtual camera in the viewer's eye.

All measurements are put into a table and eventually corrected with the appropriate scale for the life-size model. Then a simple 3D model of the scene is constructed and rendered from the viewpoint for an easy verification of the result (see Fig. 8). In the end, the plans for the craftsmen are drawn (see Fig. 9).

6. Remarks

During the involvement with the painting some remarkable results cropped up. The *horizon* divides the canvas almost exactly into two halves. The *main point H* divides the horizon almost exactly at the ratio 1 : 2. The perpendicular line through the girl's center of gravity (the perpendicular line through her ear) divides the canvas into two halves.

The girl's position in the chamber can only be guessed. But the space left between table and casement is rather narrow. Probably the girl stands right between both. But this assumption conflicts with the reflection of the girl's head in the glazing. If the ray from the viewpoint through the reflection is traced, as in Fig. 9, it obviously cannot hit the girl. But as we know from the infrared reflectogram, the girl had been in a different position in an earlier version. The girl had been shown more from the back, as if the observer had looked over her shoulder, and she appeared to be smaller. This hints at an explanation for the reflection. She probably stood closer to the wall. After Vermeer corrected the position of the girl he left her reflection untouched.

In the X-ray picture the reproduction of a painting on the back wall can be identified. It is a Cupid with his bow, as also depicted in Vermeer's *"Girl Interrupted at her Music"* and *"Lady Standing at a Virginal"*.

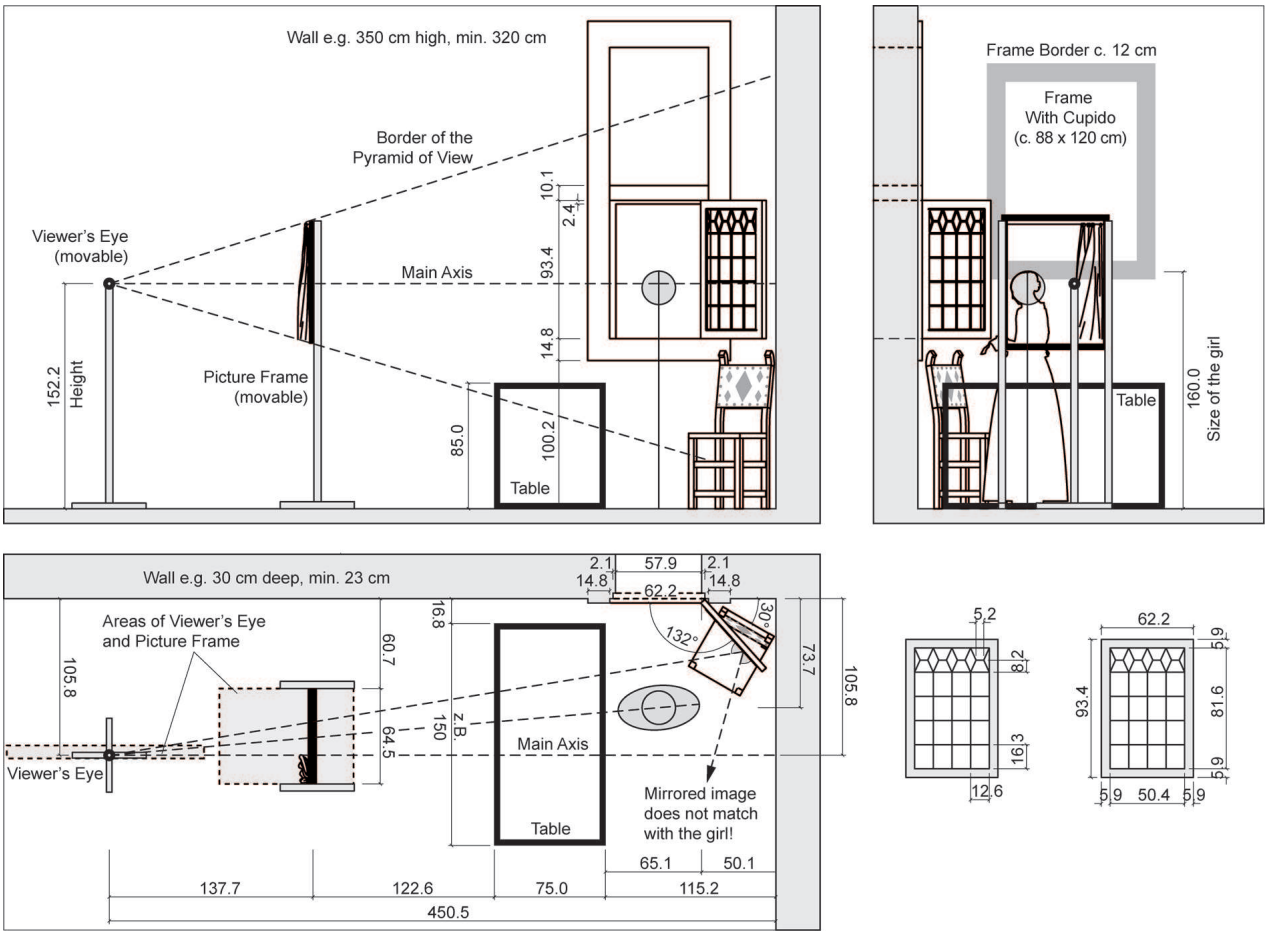

Figure 9: Plans of the tableau.

Figure 10: Uta Neidhardt (right) and her assistent are adjusting the scenery at the Old Masters Picture Gallery. The reading girl is a puppet.

Figure 11: Peeping through the ring at the calculated position of the artist's eye.

The conspicuous shadow of the casement cast onto the back wall behind the girl's head can not be explained with respect to the natural light from the window. It has to be considered as a pictorial invention with the goal to plastically model the profile of the girl out of light and shade with maximum contrast.

In comparison with the window in the painting *"Officer and Laughing Girl"* (see Fig. 2) it can be supposed that the chamber had a window with a pair of casements. But the incidence of light suggests the second casement had been covered to achieve a more intimate atmosphere. Another difference between the paintings concerns the upper part of the window. At the right side with the red curtain the frame is obviously curved, but at the left side it is simply straight, although there should be a curved profile visible as well. This manipulation might be due to the wish to frame the border of the painting in a more relaxed way. But on the other hand this variation makes us aware of the fact that the interpretation of the room by VERMEER could have been loose. Only the very good interlocking of the detected dimensions and proportions of the room ultimately grants a rather good certainty that the reconstruction was successful. At least for the mock-up, the plans proved to be feasible (see Figs. 10 and 11).

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Received August 6, 2010; final form May 30, 2012