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Relationship Between Strategies Used to Solve Spatial Orientation Problems and Examination Scores in Descriptive Geometry

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Abstract. A spatial orientation problem (SO-problem), which was developed to evaluate non-curriculum-based ability, was administered to freshmen prior to a course of descriptive geometry (DG course) together with the Mental Cutting Test (MCT). In the SO-problem, several statements concerning the direction of walking and the angle of turning are given, and examinees are asked to identify the direction the person was or was not walking in at a certain point in the sequence. This paper focuses on the relationship between the SO-problem scores from a term-end test of the DG course. There was no significant relationship between the scores of the SO-problem and those of any problems of the DG term-end test. However, the correlation between scores from the MCT and those from the DG term-end test was significant. This suggests that abilities evaluated by the SO-problem differ from those evaluated by the MCT. Students who used a highly abstracted strategy earned higher scores from a difficult problem on the construction of an intersection of two solids in the DG term-end test than those who did not use the strategy. Further investigation is necessary to confirm the reproducibility of tendencies in the relationship between strategy preference for the SO-problem and understanding of descriptive geometry.

Key Words: spatial ability, solving strategy, graphic education

MSC 2010: 97G80

1. Introduction

A prototype multiple-choice test which intends to evaluate non-curriculum-based ability was developed as part of a joint research project conducted by the National Center for University Entrance Examinations in fiscal years (FYs) 2003–2005 [3]. The prototype test does not require knowledge of a specific subject but the ability to understand new rules and/or information obtained from given materials and to apply them to a new situation or to infer a new idea. Sufficient information to reach the correct answer is provided in the test booklet.

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Since FY 2005, the validity of the test — whether the prototype test measures what it purports to measure — has been examined using various approaches [3, 4, 5, 6, 7]. Analysis of the response process is one of the validation methods together with relationships with other variables (tests, tasks, or inventories) [1].

The prototype test contains a spatial orientation problem, in which several statements about the walking direction and turning angle are given, and examinees are required to identify in what direction a person was or was not walking in at a certain point in a sequence. Shiina [6, 7] focused on the spatial orientation problem (SO-problem) and deduced the strategies used to solve the problem from the notations examinees made in the blank spaces in their test booklets. She revealed that the use of more complicated strategies has weak but significant correlations with scores from science and mathematics tests [6].

The SO-problem deals with understanding of movement in two dimensions, whereas, in descriptive geometry, three-dimensional objects are represented in two dimensions using specific procedures on treatment of points, lines, and objects. The SO-problem does not postulate trained abilities to handle three-dimentional problems. Whether the solving strategies or scores with the SO-problem have any relations to the understanding of descriptive geometry is of interest. This paper focuses on not only the relationship between scores from the SO-problem and those from a term-end test of the DG course but also on the relationship between solving strategies for the SO-problem and examination scores of a term-end test of descriptive geometry course.

2. Investigation

2.1. SO-problem and MCT

Figure 1 shows the SO-problem [3]. The problem is composed of two items (scoring units). In previous investigations [3, 4, 5, 6, 7], the entire prototype test was given to examinees, and the solving time for each problem was left to the individual's discretion within the entire time limit for the prototype test. In this investigation, the SO-problem alone was administered to students taking the DG course and the time limit for solving the SO-problem was same for all students.

The Mental Cutting Test (MCT) is a sub-set of the CEEB Special Aptitude Test in Spatial Relations [2], which has been given to university students in the DG course in Japan for many years to evaluate their spatial ability [8], was also administrated to the students for comparison. The MCT consists of 25 problems. In each problem, examinees are given a perspective drawing of an object with a hypothetical cutting plane and required to choose the true cross section among five alternatives.

2.2. Research design

The investigation was conducted in the descriptive geometry course (DG course) for freshmen at the University of Tokyo in fiscal years (FYs) 2010 and 2011. Treatment of points, lines, and objects through orthogonal projections was taught in a traditional manner. Perspective drawing and properties of regular polyhedra were also taught. The duration of the course was one semester (13 weeks, 90 min./week) in both FYs.

On the first day of the DG course, an introduction of the course contents was given to the students. After that, the MCT and SO-problem were administered to the students. The time limit for solving the MCT was 20 minutes in each FY; however the time limit for solving

Question

Read the following instructions and indicate the correct answer of for each item. The turning angle is defined as the angle between the heading direction after the turn and the previous heading direction.

Item 1. A person initially walked straight in a certain direction. Then s/he walked in one of three particular ways.

Way (1) After walking for a while in a certain direction, s/he turned to the left by 90 degrees and walked straight. Then s/he turned to the left by 45 degrees. At that time s/he was walking South.

Way (2) After walking for a while in a certain direction, s/he turned to the right by 45 degrees and walked straight. Then s/he turned to the right by 90 degrees and walked straight. After that s/he turned to the left by 90 degrees. At that time s/he was walking West.

Way (3) After walking for a while in a certain direction, s/he turned to the right in by 90 degrees and walked straight. Then s/he turned to the right by 45 degrees and walked straight. After that s/he turned to the left by 45 degrees. At that time s/he was walking North.

Choose the ***one correct combination*** of directions that correspond to the initial heading directions for the three ways.

(a) (1) SE (2) NW (3) W (b) (1) NE (2) SE (3) W (c) (1) SE (2) NW (3) E
(d) (1) NW (2) SW (3) E (e) (1) NE (2) SE (3) E (f) (1) NW (2) SW (3) W

Item 2. A person initially walked North. S/he then turned to the right by 90 degrees two times, turned to the left by 90 degrees, and turned to the right by 45 degrees. The turning sequence is unknown. S/he walked straight for a while between each turn. Among the possible turning sequences, which two heading directions were impossible when s/he turned to the left by 90 degrees ?

(a) E (b) W (c) S (d) N (e) NE (f) SE (g) NW (h) SW

Figure 1: Spatial orientation problem (translated from [3])
(Correct answers are (f) for Item 1 and (c)(h) (marked as a pair) for Item 2)

the SO-problem was 15 minutes and 10 minutes in FY 2010 and FY 2011, respectively. In FY 2011, the time limit for the SO-problem was shortened empirically because 15 minutes seemed to be too long for the students who took the course in FY 2010. After the course, the term-end test was given to the students. The students were not allowed to refer to textbooks and notebooks. The time limit was 90 minutes.

2.3. Contents of DG term-end test

Table 1 lists the contents of the DG term-end test in each FY. In each FY, the DG term-end test was composed of four problems (Q1, Q2-1, Q2-2, Q2-3). These problems are classified into two types. One is a fill-in-the-blank format problem on basic knowledge of descriptive geometry (Q1). The contents of Q1 were almost the same in both FYs. The other is a problem for geometric construction. In each FY, three problems (Q2-1, Q2-2, Q2-3) were classified as this type. Among these three problems, Q2-2 was exactly the same in both FYs. In the DG term-end test in FY 2010, Q2-1 and Q2-3 were similar to homework given during the DG course.

3. Results and discussion

Analysis was conducted using data from students who took the MCT, SO-problem and DG term-end test.

Table 1: The contents of the DG term-end test in each FY

(a) FY 2010

	Type	*Contents*	*Note*
Q1	F	Properties of auxiliary views, properties of regular polyhedra, shape of section of circular cylindrical surface and circular conical surface	
Q2-1	C	Construction of piercing point of given line and given plane, construction of the perpendicular line from given point to given plane	Similar
Q2-2	C	Construction of right circular cone with given axis, apex, and length radius of base circle (Only front view drawing required)	Common
Q2-3	C	Construction of intersection of given vertical triangular prism and horizontal quadrangular prism (Only front view drawing required)	Similar

(b) FY 2011

	Type	*Contents*	*Note*
Q1	F	Properties of auxiliary views, properties of regular polyhedra, shape of section of circular cylindrical surface and circular conical surface	
Q2-1	C	Construction of true length of given line, construction of piercing point of given line and given plane, construction of intersection of given two planes	
Q2-2	C	Construction of right circular cone with given axis, apex, and length radius of base circle (Only front view drawing required)	Common
Q2-3	C	Construction of intersection of given vertical triangular pyramid and horizontal triangular prism	

(Abbrev. of Type: F = Fill-in-the-blank format, C = Problem for construction)
(Abbrev. of Note: Similar = Similar problem to homework, Common = Common problem)

3.1. Statistics on SO-problem, MCT, and DG term-end test

Table 2 lists the number of students and statistics on the SO-problem, MCT and DG term-end test in each FY. "Prop. Correct" in the SO-problem denotes the proportion of students answering the item correctly.

There was a significant difference in the mean score in the MCT between the FYs ($t = 1.990$, $p < 0.05$), although attributes such as age and academic ability of the students in each FY were almost the same. As for the SO-problem, the independence between the score for Item 2 (correct: 1 or wrong: 0) and FY was rejected using a chi-square test ($\chi^2 = 10.703$, $p < 0.01$). It is possible that the decrease in the proportion of answering Item 2 correctly was due to the shortened time limit.

Table 2: Statistics on SO-problem, MCT and DG term-end test in each FY

	FY2010			FY2011		
	Total	*Male*	*Female*	*Total*	*Male*	*Female*
No of subjects	98	93	5	108	103	5
	Time limit (min.)	*Prop. Correct*		*Time limit (min.)*	*Prop. Correct*	
		Item 1	*Item 2*		*Item 1*	*Item 2*
SO-problem	15	0.93	0.94	10	0.88	0.78
	Time limit (min.)	*Mean*	*S.D.*	*Time limit (min.)*	*Mean*	*S.D.*
MCT	20	21.37	2.98	20	20.49	3.31
	Time limit (min.)	*Mean*	*S.D.*	*Time limit (min.)*	*Mean*	*S.D.*
Term-end test	90	79.39	16.75	90	50.02	18.40

The mean score in the DG term-end test in FY 2011 was much lower than that in FY 2010. The distribution shapes of the total scores also differed between the FYs. The total score in FY 2010 had a bulge to the high area (skewness $= -1.032$), and that in FY 2011 had a bulge to the low area (skewness $= 0.650$).

Table 3 lists the statistics on each problem in the DG term-end test in each FY. The means of scores for three problems of geometric construction in FY 2011 were much lower than those in FY 2010. As shown in Table 1, most of the problems in FY 2011 required more complicated work on the part of the students, although the contents of each problem were similar between FYs. In addition, problems similar to homework were not included in FY 2011. These differences might have incurred a heavier load for the students in FY 2011.

Table 3: Statistics on problems in DG term-end test in each FY

	FY2010 ($n = 98$)			FY2011 ($n = 108$)		
	Full score	*Mean*	*S.D.*	*Full score*	*Mean*	*S.D.*
Q1	23	20.11	2.21	26	20.94	2.98
Q2-1	26	22.41	5.49	22	13.28	6.31
Q2-2	18	11.01	6.66	18	8.38	6.95
Q2-3	33	25.86	9.91	34	7.42	9.27
Total	100	79.39	16.75	100	50.02	18.40

3.2. Relation between scores from SO-problem and DG term-end test

For each FY, the means of the total scores in the DG term-end test were calculated for the following two groups: students who obtained the correct answer in each item of the SO-problem and those who did not.

In Items 1 and 2 in both FYs, there was no significant difference between the means of the total scores in the DG term-end test for the two groups. The same results were obtained for mean score for in each problem in the DG term-end test.

3.3. Relation between MCT and DG term-end test

Table 4 lists the correlation coefficients between scores from the MCT and DG term-end test. In both FYs, the correlation between scores from the MCT and total scores from the DG term-end test were significant. In FY 2011, scores of all problems from the DG term-end test significantly correlated with those from the MCT. Although the scores from two problems were not significantly correlated in FY 2010, it is possible that the correlations were weakened by the bulge of the MCT score to the higher area.

Table 4: Correlation coefficients between scores in MCT and DG term-end test

	FY2010	FY2011
	MCT score	MCT score
Q1	0.208*	0.143*
Q2-1	0.025	0.192*
Q2-2	0.121	0.290**
Q2-3	0.373**	0.347**
Total	0.284**	0.373**

(* : $p < 0.05$, ** : $p < 0.01$)

In contrast, there was no significant correlation between scores (correct / wrong) in the SO-problem and those in the DG term-end test, as mentioned Section 3.2. The difference in the relation to scores from the DG term-end test suggests that abilities evaluated by the SO-problem differ from those evaluated by the MCT.

3.4. Strategies used to solve each item in SO-problem

Most of the students made notations in the blank spaces on their sheets of the SO-problem, although they were not required to show their solving process. Shiina [6] deduced the strategies used to solve each item in the SO-problem from these notations and classified the strategies into several categories. She then made a few modifications to these categories [7]. In this paper, the modified categories [7] were used.

For Item 1, the deduced strategies were classified into the following categories:
(1a) "Arrows in sequence" strategy,
(1b) "Arrows in reverse" strategy,
(1c) Combined use of (1a) and (1b) strategies, and
(1d) Other strategies (including "unclassified").

In FYs 2010 and 2011, 94.9 and 93.5 % of students made notations for Item 2, respectively. Their deduced strategies were classified into one of the above four categories.

For Item 2, the deduced strategies were classified into the following four categories:
(2a) "Using arrows" strategy,
(2b) "A list of combinations" strategy,

(2c) "Coordinate system" strategy, and
(2d) "Calculation" strategy.

Table 5 is the frequency distribution table listing the number of the above categorized strategies used to solve Item 2 for each student. The percentage of students who used more than one strategy in FY 2010 was larger than that in FY 2011. In FY 2010, the time limit for solving the SO-problem was longer than that in FY 2011. This suggests that sufficient time encourages students to use a variety of strategies.

Table 5: Numbers of strategy categories used to solve Item 2 in each FY

	FY2010		FY2011	
	Freq.	*Percentage* (%)	*Freq.*	*Percentage* (%)
0	22	22.4	16	14.8
1	45	45.9	73	67.6
2	28	28.6	16	14.8
3	3	3.1	3	2.8
Total	98	100.0	108	100.0

3.5. Relation between strategies used to solve Item 1 and scores from the DG term-end test

For Item 1, a one-way analysis of variance (ANOVA), in which the total score in the DG term-end test in each FY test was a dependent variable and the strategy used to solve Item 1 was an independent variable (four levels: (1a) – (1d)), revealed no significant associations in either FY 2010 or FY 2011.

Almost 90 % of the students solved Item 1 by using one of the three strategies: (1a), (1b), or (1c). These three strategies are very similar in terms of using arrows, although the drawing order differed. It is possible that this high homogeneity of strategies made it difficult to detect the association between solving strategies for Item 1 and understanding of descriptive geometry.

3.6. Relation between strategies used to solve Item 2 and scores from DG term-end test

For each categorized strategy used to solve Item 2, the means of total scores in the DG term-end test for students who used the strategy and who did not were calculated, and the means of the two groups were t-tested.

Table 6 lists the statistics for the total scores in the DG term-end test in each FY, together with the results of the t-test. In the DG term-end test in FY 2011, there was significant difference between the means of the total scores for the two groups on the use of the "calculation" strategy, although there was no significant difference between the two groups on the use of any strategies in FY 2010.

Considering that the difficulty in each problem is quite different within a FY, the analysis that focused on the score in each problem might be useful to gain insights on the relation between the use of strategies and scores from the DG term-end test.

Table 6: Statistics of total score in the term-end test for use of each strategy

		FY2010				FY2011			
Category of strategies		*N*	*Mean*	*S.D.*	*t-value*	*N*	*Mean*	*S.D.*	*t-value*
Using arrows	Used	55	79.71	18.38	0.214	70	47.65	17.56	−1.836
	Did not use	43	78.98	14.59		38	54.38	19.33	
A list of combinations	Used	27	76.94	19.59	−0.890	18	55.64	24.09	1.133
	Did not use	71	80.32	15.59		90	48.89	16.99	
Coordinate system	Used	13	79.00	14.76	−0.089	15	54.17	21.06	0.940
	Did not use	85	79.45	17.11		93	49.35	17.97	
Calculation	Used	15	78.50	17.21	−0.222	11	63.50	16.75	2.634**
	Did not use	83	79.55	16.76		97	48.49	18.03	

(** : $p < 0.01$)

For each problem in the DG term-end test in FY 2011, the means of scores for students who used each strategy and who did not were calculated, and the means of the two groups were *t*-tested.

Among the problems in FY 2011, the means of scores in Q2-3 showed significant differences on the use of the "using arrow" and "calculation" strategies (Table 7). The tendency for using the "calculation" strategy in Q2-3 was consistent with the total scores in FY 2011.

Q2-3 is a problem for constructing an intersection of two solids. In FY 2011, students were required to construct both a front view and top view in a limited time. In addition, Q2-3 was not similar to homework. The rote memorization of construction procedures would be insufficient to deal with this problem. As shown in Table 3, the means of the Q2-3 scores in FY 2011 were very low compared with the full score.

Although the "using arrows" strategy was used by many students to solve Item 2, the strategy is straightforward and lacks ingeniousness. It is hardly surprising that the students who used the straightforward strategy earned lower scores from a difficult construction prob-

Table 7: Statistics of Q2-3 score in the DG term-end test in FY 2011 for use of each strategy (Full score in Q2-3 is 34)

Category of strategies		*N*	*Mean*	*S.D.*	*t-value*
Using arrows	Used	70	5.95	7.67	−2.044*
	Did not use	38	10.13	11.27	
A list of combinations	Used	18	12.00	13.06	1.720
	Did not use	90	6.51	8.10	
Coordinate system	Used	15	10.73	9.74	1.500
	Did not use	93	6.89	9.13	
Calculation	Used	11	16.27	10.79	3.514**
	Did not use	97	6.42	8.58	

(* : $p < 0.05$, ** : $p < 0.01$)

Table 8: Statistics of Q2-1 score in the DG term-end test in FY 2010 for use of each strategy (Full score in Q2-1 is 26)

Category of strategies		*N*	*Mean*	*S.D.*	*t-value*
Using arrows	Used	55	22.18	5.62	−0.460
	Did not use	43	22.70	5.37	
A list of combinations	Used	27	22.63	6.05	0.245
	Did not use	71	22.32	5.31	
Coordinate system	Used	13	24.54	2.85	2.444*
	Did not use	85	22.08	5.73	
Calculation	Used	15	21.73	7.06	−0.515
	Did not use	83	22.53	5.20	

(* : $p < 0.05$)

lem in descriptive geometry.

The "calculation" strategy is highly abstracted in that the turning actions are converted into angles and that the possible heading directions are identified by re-converting the calculated values of angles into directions [6]. The use of such a highly abstracted strategy suggests the students had an advanced ability to think abstractly. It is interesting that the students who used this strategy to solve Item 2 earned higher scores from a difficult construction problem in descriptive geometry.

As for each problem in the DG term-end test in FY 2010, the means of scores in Q2-1 showed significant differences on the use of the "coordinate system" strategy (Table 8), although the tendency was not consistent with the total scores in FY 2010. Q2-1 is a basic construction problem of a line and a plane and very similar to homework, and the means of Q2-1 scores in FY 2010 were very high compared with full scores. In the "coordinate system" strategy, the turning actions corresponded to rotation in the coordinate system. The use of this strategy with a little ingenuity might result in higher scores, even in a basic construction problem such as Q2-1 in FY 2010.

Unfortunately, the relation between the use of strategies and scores from the DG term-end test were not consistent between FYs. There were many differences between the investigations in FYs 2010 and 2011. First, the time limit for solving the SO-problem in FY 2010 was longer than that in FY 2011. It is possible that there were too many strategies for solving the SO-problem, which was caused by too much time, resulting in a vague relation between the use of strategies and the scores from the DG term-end test. Second, the difficulty of the DG term-end test differed between FYs. Further investigation using the DG term-end test with the same difficulty as FY 2011 is necessary to confirm the reproducibility of the tendencies found in FY 2011.

4. Conclusions

The relationship between the SO-problem, which was developed to evaluate non-curriculum-based ability, and scores from the term-end test of the DG course was analyzed. The MCT was also given to the students for comparison to the spatial orientation problem.

There was no significant relationship between the scores from the SO-problem and scores

of any problems of the DG term-end test. The correlation between scores with the MCT and those with the DG term-end test were significant. This suggests that abilities evaluated by the SO-problem differ from those evaluated by the MCT.

The students who used a straightforward strategy earned lower scores in a difficult construction problem of constructing an intersection of two solids in descriptive geometry than those who did not use the strategy. In the problem, the students who used a highly abstracted strategy earned higher scores than those who did not use the strategy. The ability to find an efficient strategy seems to be related more to the ability to deal with complicated construction problems in descriptive geometry rather than the ability to use intuitive strategy.

Further investigation is necessary to confirm the reproducibility of tendencies in the relationship between strategy preference for the SO-problem and understanding of descriptive geometry.

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