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Three-Dimensional Ornamental Structures Based On The Wallpaper Groups In Architecture

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Abstract. The aim of this work is the application of the theoretical knowledge of the seventeen wallpaper groups for extension of two-dimensional ornaments into three-dimensional architectural parametrical ornament structures. The geometrical definition of three-dimensional ornaments will be related to digital fabrication by a robotic arm and constraints conditioned by fabrication and assembling methods of modular tiles will be shown. Within the work we establish a digital flow from design to fabrication using 3D NURBS modeling and visual programming software for code generation. Finally we fabricate 3D ornamental parts by a robotic arm and present achieved results.

Key Words: Wallpaper groups, 3D ornament, parametric design, robotic arm, digital work flow

MSC 2010: 51M20, 00A67

1. Introduction

The complexity of the subject of ornamentation as it occurs in art, mathematics, geometry, architecture, religion and psychology has inspired architects and designers to reconsider this field again and again and has also represented inexhaustible motivation for new design [8, 9]. The possibility to use digital tools to generate 3D models and realize architectural building elements has led, among other things, to the re-establishment of ornaments in architectural practice. The re-establishment of ornaments in architectural practice raises the issue of how ornaments may be given new quality and new value through the use of contemporary materials and fabrication processes [7, 12].

Our approach to this topic is multifaceted wherein we connect the geometrical rules of 2D ornaments with the parametrical design and material efficient fabrication of 3D ornaments by a robotic industry arm. The function of 3D ornaments is reflected in making the formwork for concrete modular parts that can be used as tectonic elements or façade panels.

We defined two different approaches how to extend 2D ornaments into 3D ones. The first method consists of shape analysis of 2D ornaments and their extension using plane and space curves with NURBS shapes in between. 2D ornamental motifs and colors on the individual cells are not subject of the analysis but it will be used as starting point for developing spatial forms. The second approach is mapping of 2D ornaments onto double curved surfaces and the generation of 3D modular tiles. The geometrical development of single parts is limited by constrains of the selected fabrication process and used tools.

Considering the complexity of our approach in this paper we will try to give some answers to the following questions: Is it possible to apply the potential and property of the modularity an ornament possesses for non-standard architectural design, while avoiding an easily recognizable repetition of basic geometric shapes? In which way is it possible to transform 2D ornamental wallpaper elements into parametric 3D ornaments by means of mathematical and geometric laws? Which constraints exist if a specific method is selected for the fabrication of elements using the robotic arm? How can individual digital design be transformed into digital robotic code? Which limitations exist in design with respect to the capacities of the robot?

The first project that marked the beginning of the use of robotics in architectural research was a project by F. Gramazio and M. Kohler from ETH Zurich. The project was titled *"Gantenbein Vineyard Façade", Flasch (Switzerland), Non-Standardised Brick Façade* [4]. It was aimed a precise positioning of 20.000 standard bricks that were used to make up the façade of a winery in Switzerland with the help of a robot. The positioning of the bricks was parametrically pre-defined with a precise set type of rotation for each individual brick, which in the overall outlook gives a unique, non-standard tectonic look. Parts of the façade were assembled in the laboratory at the ETH Zurich and then transported and positioned on the site. This project was followed by other projects ([2, 5, 13]) that used the robotic method of fabrication with different materials (brick, XPS and wood) and tools (cutting and milling), defining the design with various purposes (acoustic wall, spatial structures, etc). Apart from the ETH Zurich, other leading schools of architecture, such as Harvard and MIT, also participated in research projects on the use of robots and many PhD students have been involved in the existing projects. For example, a current project *Design Robotic Group* at Harvard University is focused on the production of parametric bricks ([3, 14]).

2. Fabrication

It is a known fact that the fabrication of double-curved elements, which are used in architecture for façades or supporting elements, is very expensive and requires extreme precision in the fabrication process. Materials used for such elements are glass, concrete, polymers and wood. Each of these materials has very different physical characteristics and requires a completely different fabrication strategy. Standards for the installation of these elements vary from country to country, which subsequently makes it difficult to standardize such architectural structures. The result of all of this is that there are very few derived objects today with double-curved elements in their structure. All this leads to the question of how and in which way the production of these elements could be made more rational and cheaper to standardize the use of such elements and to make its application in architecture more acceptable. Our project was focused on the fabrication of elements that can serve as elements for concrete formwork. The modern technique of making formwork elements out of EPS (expanded polystyrene) and XPS (extruded polystyrene) materials is basically CNC milling. A disadvantage of this approach is a very high consumption of material, as well as time inefficiency in the fabrication process. Namely, in order to make the desired form out of one XPS block, it is necessary in the first phase of milling to use a thicker profile drill, to remove a part of the material to approximate the height of the desired form. Depending on the thickness of the block it is sometimes necessary to remove the unnecessary part of the XPS material in several stages. In the second stage thinner drills are used (also in several stages) and another part of the material is removed to the level of the desired form, which gives a smooth final surface of the XPS element. Before starting the concrete casting process, it is necessary, depending on the type of XPS, to apply an appropriate protective layer, so that the formwork elements could be easily removed once the concrete has hardened. An important fact is that the costs of a formwork made for non-standard concrete parts exceed the costs of standard forms by a multiple. More precisely, it is necessary to produce a new and unique formwork for each element, whereas when standardized elements are used, formwork can be used for the fabrication of more than 1000 elements. Our approach to this problem is to use a hot wire for cutting EPS blocks. This method makes it possible to get the desired curved form of the future formwork element in only one cutting process. The downside to this type of cutting is that only certain forms can be cut limited by the geometry of the wire - curved or flat - and the possible surfaces which are generated by moving it through space. In our approach we are also using specific positions and geometries for the tool, so that after the cutting we get two congruent ESP elements that both can be used for formwork.

3. The geometry of an industrial robot arm and the tool

An industrial robot arm is a serial robot. By definition a serial robot is a kinematic chain system with an amount of $(n+1)$ systems which are linearly connected by n joints (L). Every joint L can either be rotational or translational. We used the robot arm IRB 140, produced by ABB (Fig. 1). The given robot arm has seven parts (systems) which are connected by six rotational axes and six rotational joints.

Since we wanted to design non-typical architectural shapes with the robot we had to program the whole geometry for the moving paths and visualizations. With the help of in-

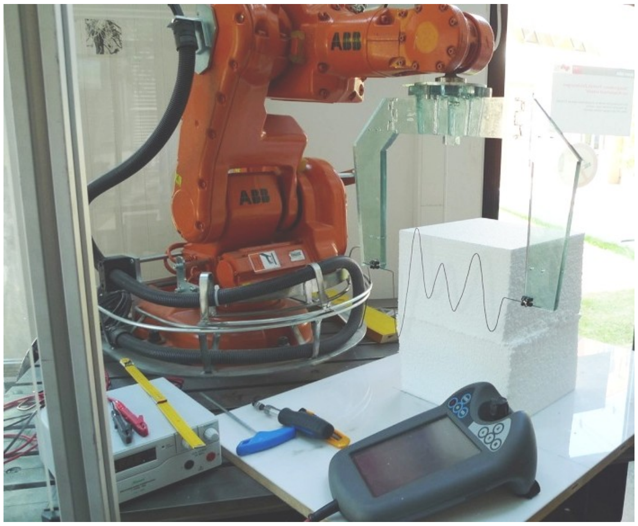

Figure 1: Six axes robot with a hot wire tool

verse kinematics, it is possible to calculate all necessary angles of movement of the individual axes, while with the help of the direct kinematics it is possible to assign all the angles of rotation in order to move the robot from the basic position into the designated position ([1, 6, 11]). The inverse kinematic calculation is based on geometric principles and was implemented in Grasshopper, a visual programming plug-in for the CAD-Software Rhinoceros (www.rhino3d.com). The movement visualization was also defined in Grasshopper. On the one hand, we had enabled an excellent visualization of the individual phases of programming the robot. On the other hand, this approach allowed a direct connection between the parameter design and the code of the robot. This permitted a great flexibility of working with the robot and quick changes in the design and the robots code. By simulating the robot's movements through every given task in Grasshopper, it was possible to remove singularities and possible collisions in the robot's movement. As tool we used a hot wire which we shaped in form of a straight line or flat curves (Fig. 1). Generally the hot wire can also be shaped as a space curve which we have not implemented yet.

4. Geometry of used surfaces

In order to setup a virtual model for designing the shapes we have to understand the geometry which our tools produce. Our tools consist of "hot wires curves" which keep their profile and are moved in space. So the wire generates a generalized sweeping surface (Fig. 2c). In special cases we get translational or rotational surfaces (Fig. 2a, 2b) [15]. If one uses a hot wire you have to take care that the wire does not stay too long at the same position at one of his points. Otherwise the material will burn there. So we also had to adjust the speed of the tool movement depending on the material and the heat of the wire.

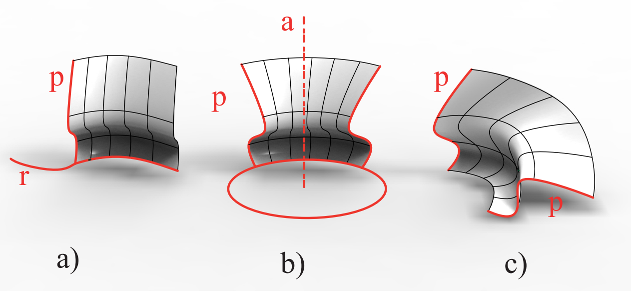

Figure 2: Translational surface (profile curve p, rail curve r), rotational surface (profile curve p, axis a) and a generalized sweeping surfaces generated by the profile curve p, right.

5. Modular patterns

We used EPS blocks of maximum size $250 \times 250 \times 160\,\mathrm{mm}^3$ in the fabrication process. Out of an EPS block of standard dimensions $1000 \times 500\,\mathrm{mm}^2$, we made eight blocks of smaller dimensions. The size of the blocks was determined by the size of the tools and the limitations of the size of the robot. We were using a hot wire $d = 1.5\,\mathrm{mm}$, $0.277\,\mathrm{Ohm/m}$. The thickness of the wire allowed stability, which was necessary when a curved wire was used for cutting the EPS elements.

We addressed two main objectives in our project, depending on the fabrication method and the geometry:

- modular fabrication of elements and
- efficient use of materials.

Modular elements allow us to divide the desired forms into smaller parts that can be fabricated individually and efficiently. Modular parts can additionally be incorporated into new structures following certain principles. The efficient use of material is reflected in the fact that each box block is cut with a certain plane of symmetry, which allows us to get congruent parts and equal use of both parts of the block. In order to achieve a variety of elements, blocks are not only cut along the planes of symmetry. Also special cutting surfaces were used to get congruent parts.

The starting point for the box cutting is the analysis of its plane symmetries. A plane S of symmetry divides a three dimensional shape into two (in general: oppositely) congruent parts which are mirrored images of each other (Figs. 3a, 3b). In our case we did not use only this kind of symmetry but also sections with cylindrical surfaces C orthogonal to the symmetry planes S whose profile curves p lie in S to get congruent parts. To generate these congruent parts we have to ensure that the profile curves p possess point symmetry with the box mid-point M as center. Figures 3e and f show examples with a cube.

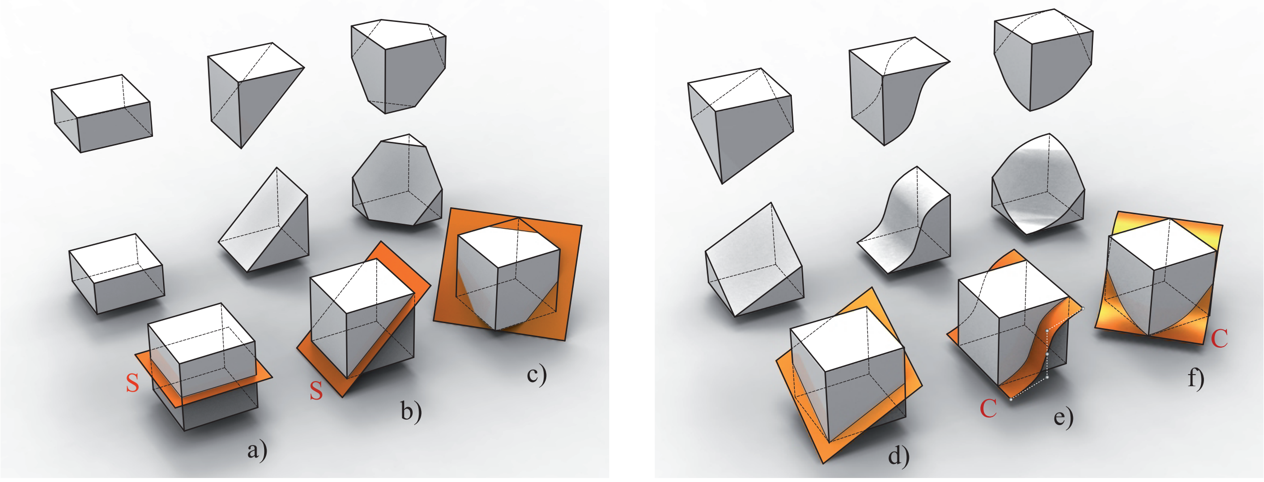

Figure 3: Congruent parts of a cube generated by symmetry planes S (a, b) and by plane sections through the cubes mid-point and orthogonal to one of the symmetry planes (c, d). Congruent cube parts derived by cylindrical sections C (e, f). Their profile curves p lie in one of the symmetry planes and they have the cube mid-point as symmetry center.

If an object is line symmetric about a line g one can get multiple variations of congruent parts by means of cutting surfaces which are also line symmetric about g (Fig. 4). The two congruent parts can be transformed into each other by rotation around g.

Figure 4 shows variations of congruent cube sections by use of a translational and two ruled surfaces. The cube in Fig. 4a is cut by a translational surface generated by two congruent curves with point symmetry in their mid-points. The cubes in Figs. 4b and 4c are cut by hyperbolic paraboloids. Variations of different paraboloids can be used to make a structure as shown in Fig. 5 left. Figure 5 right shows parts with geometrical continuity.

To define the geometry of the modular parts, the starting point consists of analyzing the wallpaper groups and extendibility of two-dimensional ornaments into the three-dimensional architectural parametric ornament structures, which is also the topic of the subsequent section of the paper.

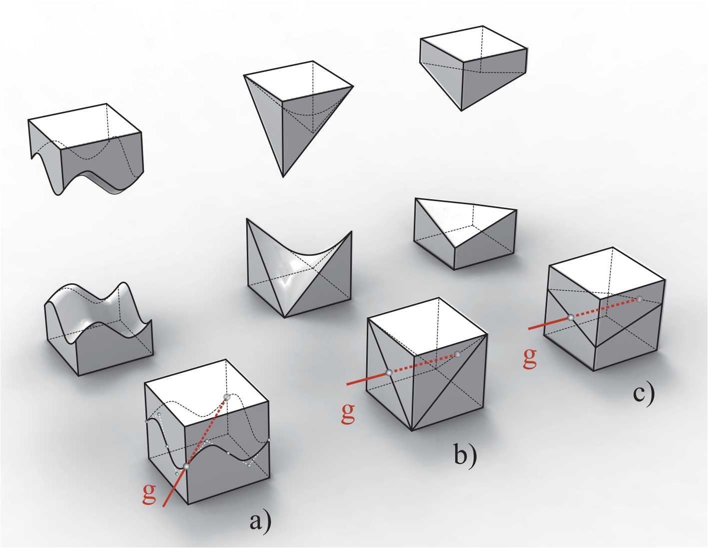

Figure 4: Congruent parts cut out of a cube. The cutting surfaces are line symmetric about a line g which is the intersection of two symmetry planes of the cube.

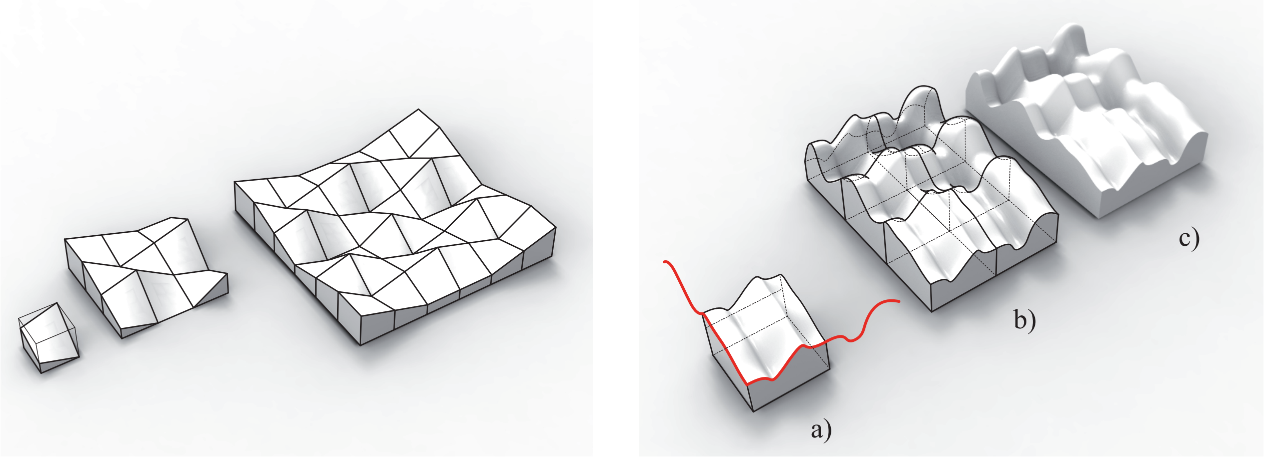

Figure 5: Cubes are cut with different hyperbolic paraboloids. They are arranged in an irregular pattern (left). If the cutting surfaces have special profile curves the adjacent parts can offer geometrical continuity (right).

6. Wall paper groups

The 17 different and well known wallpaper groups are the starting point in our design of the modular parametric elements ([10, 16]). In our approach, 2D ornamental motifs and colors are not the subject of the analysis but will be used as the starting point for developing spatial ornaments. Figures 6 and 7 show a selection of the 17 wallpaper groups with possible transformations of border curves and examples of possible generations of 2D patterns. Each figure includes letters to denote the procedures required to generate the given patterns. The letter denotes the generating curves, the primary cells, the prototype patterns and bigger parts of the pattern.

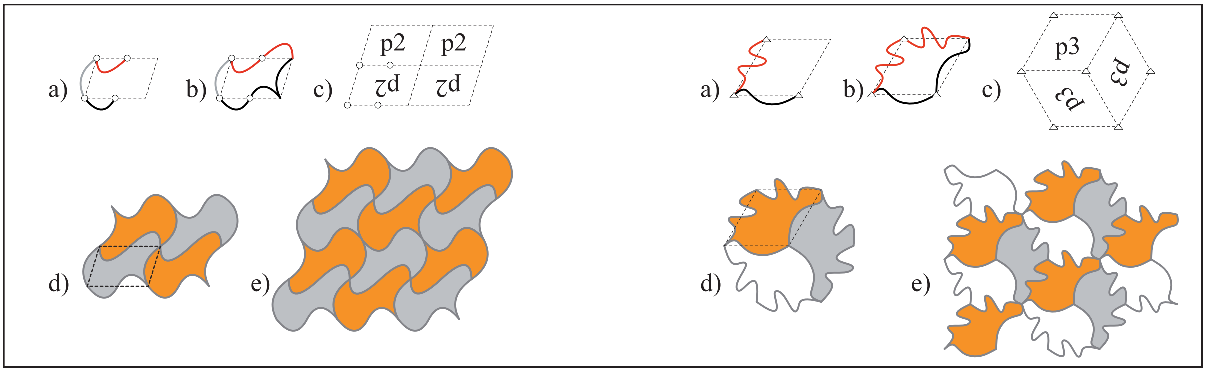

Figure 6: The group $p2$ contains four rotation centers of order two (180°), left. The group $p3$ has three different rotation centers of order three (120°), right.

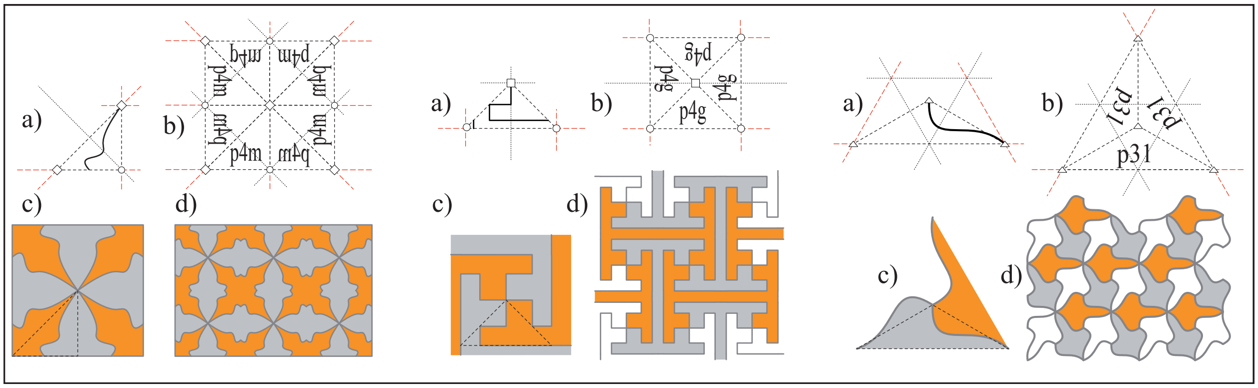

Figure 7: The group $p4m$ has rotation centers (90°, 180°), glide reflections and reflections (left). The group $p4g$ has centers of rotation (90°, 180°), reflections and glide reflections (central). The group $p3m1$ has rotation centers (120°), reflections and glide reflections (right).

7. Spatializing 2D ornaments

There are different possibilities to spatialize a 2D ornament. We consider two approaches. The first one is mapping the ornament onto a double curved surface. The second one is to produce single or different modular blocks and to assembly them into the complex structure. In Fig. 8 left we show the generation of a special pattern which belongs to different wall paper groups. So it can be employed and assembled in different ways (Fig. 8 right).

7.1. Mapping

The mapping of the pattern or ornament onto a double curved surface can be accomplished by means of the uv-plane. The mapping of the pattern of Fig. 8 can be seen in Fig. 9 left. To get a material thickness on the surface we extruded the spatial ornamental border curves along their surface normals. We additionally cut the different parts in various heights to get a certain structure (Fig. 9 right). We must point out that this kind of generation provides unique parts and is contradictory to a cost efficient production.

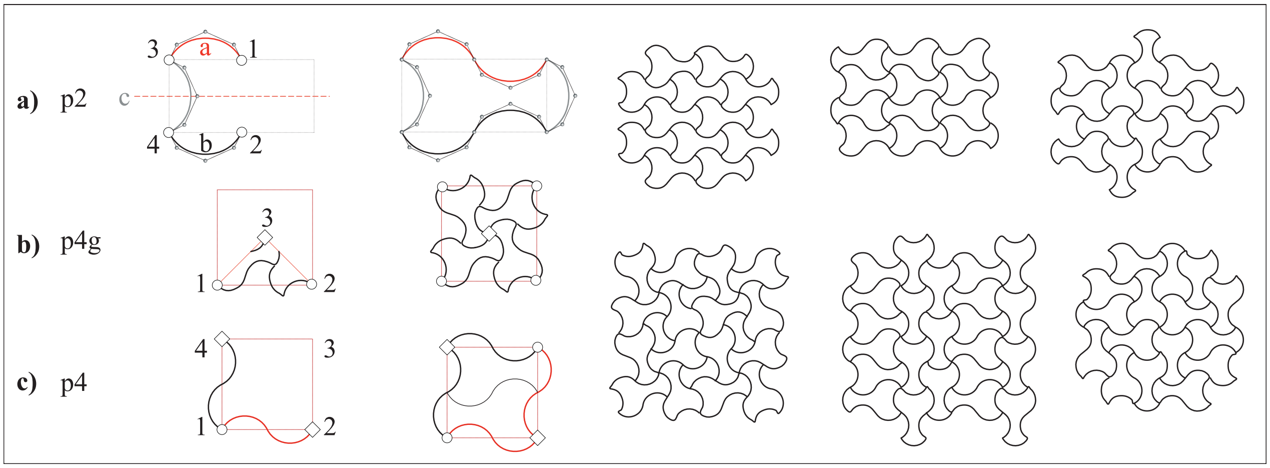

Figure 8: The generation of a special pattern cell which can be assembled in different ways (left). The pattern cell of the left side assembled in different ways (right).

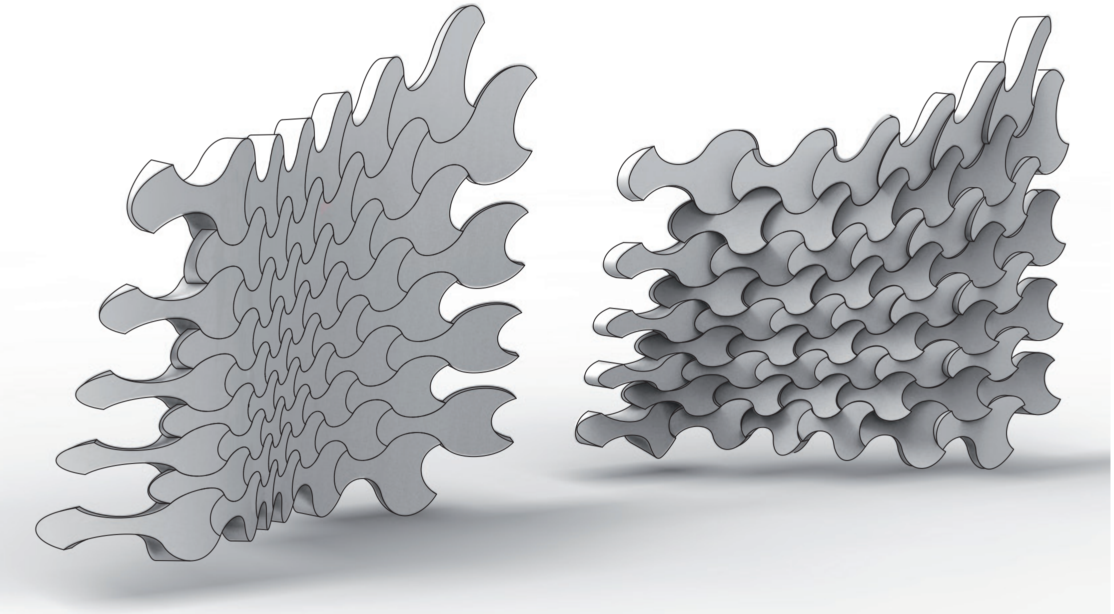

Figure 9: The pattern of Fig. 8 mapped onto a double curved surface.

7.2. Blocks

This way of generating modular elements uses the theory of the Sections 5 and 6. As an example one pattern cell of Fig. 8 is designed as a surface of extrusion in order to cut it out of a block. As this object has two symmetry planes we get line symmetry with respect to the intersection line g. If we intersect the block with a cylindrical surface which is also symmetric about g we get two congruent parts which both can be used. The cutting can be achieved by the robot and a straight hot wire as tool (Fig. 10).

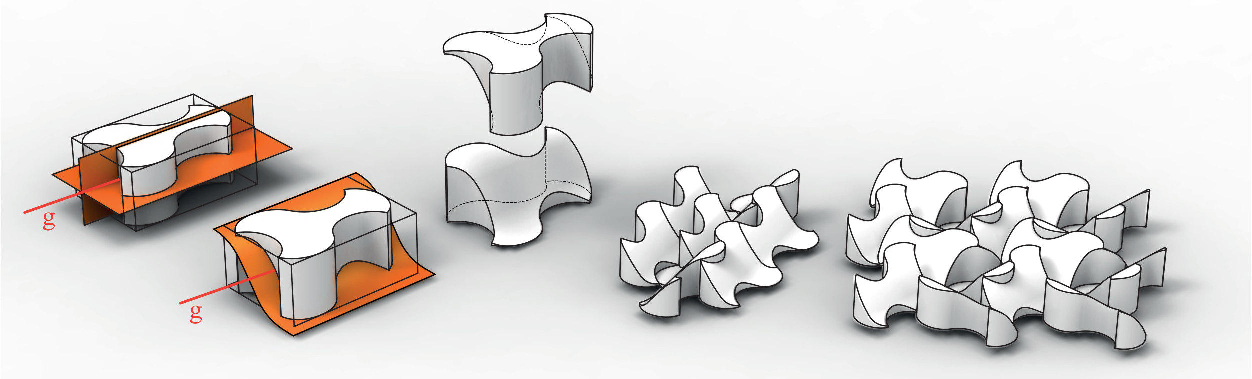

Figure 10: One pattern cell of Fig. 8 spatialized by extrusion. It has two symmetry planes which intersect in g (left). So it can be divided into two congruent parts by a cylindric surface (central). Assembling of the pattern parts in different manner (right).

8. Production and examples

Figures 11 and 12 show examples which were produced by students of our faculty using EPS as material and a robot arm for the production.

9. Conclusion

In our approach we tried to implement geometrical rules of wallpaper groups for generation of non-standard architectural elements. Our approach is based on geometrical principles and the suggested digital work flow showed huge flexibility in design and real-time feedback to the designer. We tried to present a way of generating "new" non-standard architecture by using identical or similar elements which are constructed in a geometric and intelligent way. In our opinion this could be a way of keeping the costs down and offering simultaneously a big variation in design. Nevertheless a profound geometric knowledge is required to understand the theoretical basics of ornamental design, to implement the digital work flow and to use an industry robotic arm in a proper way.

Acknowledgments

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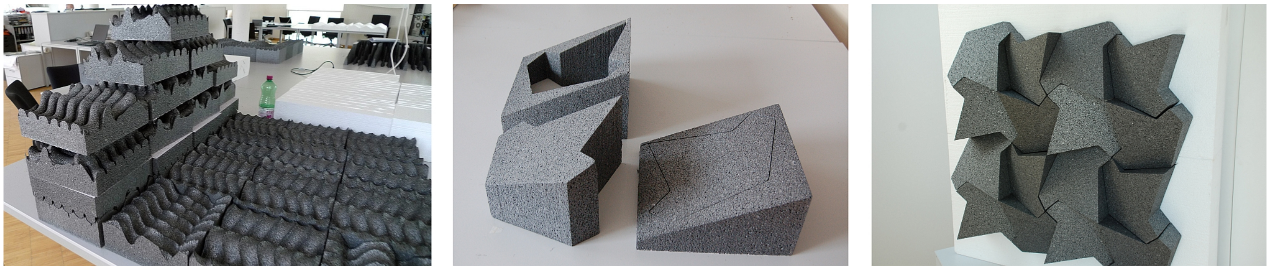

Figure 11: EPS blocks using translational surfaces (left). Basic pattern for one $p4$ ornamental group. With only two cuts 4 parts are obtained, two positive and two negative parts for formwork (central). The ornament $p4$ as a 3D structure (right).

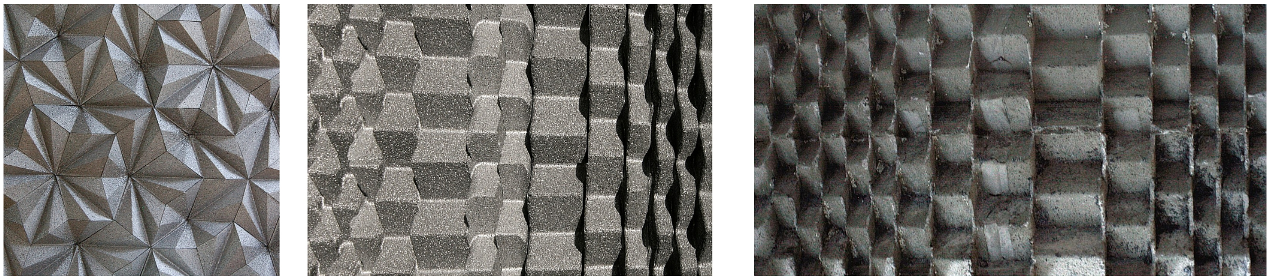

Figure 12: Penrose patterns as an inspiration for 3D formwork (left). Formwork consisting of several EPS blocks (central). The 3D pattern of the formwork fabricated in concrete (right).

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