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Strategies for Assessing Spatial Ability Tasks

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Abstract. Even after almost exactly one hundred years, a clear-cut and valid analysis of spatial ability, acceptable to the academic community, does still not exist. The developmental concepts for factor-based psychometric spatial ability tests were designed with the intention of making the probands solve the tasks with homogeneous considerations. Research findings, however, indicate that the probands use many diverse strategies when solving spatial ability tasks. Such findings shift the emphasis of research among others to the use of the diverse strategies. Ever since Barrat [1] many strategies have been identified and examined. This paper attempts to group the various strategies and systemize and describe them as the model of "the four pairs of strategies to solve spatial ability tasks".

Key Words: strategies, pairs of strategies, factors of spatial ability, spatial ability

MSC 2010: 97U50

1. Introduction

> *"For a general definition of spatial abilities [...] the evaluation of strategies used in solving spatial tasks is more adequate".* (Souvignier [29]; citation translated)

At the beginning of the 20th century the concept of spatial intelligence was defined within other factors of intelligence (e.g., speech comprehension, speed of perception, conclusive thinking, language proficiency; [30, 31]). Between 1950 and 1994, during the factorial phase of spatial ability research (this term was introduced and explained in [21]) researchers examined spatial ability more closely and identified the constituent parts of this aspect of intelligence. Many models were designed, identifying diverse factors of spatial ability. The models contained between two and more than ten differing factors [2, 6, 12, 16, 17, 18, 20, 22, 25, 32].

2. The limitations of factor analysis

In order to strengthen the scientific sustainability of the factor-analytical models, the individual factors were examined by means of psychological testing procedures. This led to two findings:

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1. In spite of scientific research for more than a hundred years, attempts to establish **one** consistent and valid model of spatial ability have not been successful. A specific definition of factors could not be formulated in spite of the analysis of a great number of proposed models. Factor-analytical models of spatial intelligence were formulated — among others — by Thurstone [32], French [6], Guilford [12], Rost [25], Lohman [17], McGee [22], Linn & Petersen [16], Lohman [18], Carroll [2] and Maier [20].

2. Research results regarding spatial ability often differed widely, be it in studies on gender differences, in research projects on single factors of spatial ability, the neuro-psychological localization of spatial intelligence, or more generally in the various papers on differential or psychometric perspective (e.g., differing results in tests with or without a time limit) [9, 11, 15].

3. Strategies in focus

Studies in which test persons were interviewed or wrote down how they dealt with the tasks after having solved spatial ability tasks, indicate possible reasons for the two dissatisfying observations made above:

> *"The classic factor-analytical-psychometric research perspective requires implicitly that all spatail ability tasks can be solved by probands using the same solving strategy"* (Grüssing [11]; citation translated).

The assumption that there is a consistent and homogeneous strategy for finding solutions of tasks had to be abandoned because of inter-individual varying solving strategies and intra-individual change of strategies [29]. Because of the diverse strategies used by the probands there are highly reciprocal effects and dependencies between the diverse factors of spatial ability [20]. Such findings indicate that in some cases intended task solving strategies are hardly used at all [20]. To quote Lohman:

> *"One of the major problems is that tests are solved in different ways by different subjects. Subjects change their solution strategies with practice or when items difficulty increases"* [17].

Because of such findings the analysis of factors became of decreasing importance. Souvignier pointedly stated that the interpretation of factors was based solely on the description of test requirements with great emphasis on the factors, and that therefore their corresponding definitions represent only an abstract list of test procedures in the respective analyses [29]. The focus of spatial ability research now increasingly focuses on the identification and description of the solution strategies used. It is asserted that

> *"conventional alternative solution strategies by means of further cognitive qualifications or modified spatial-visual relations should be regarded with due attention"* (Maier [20, p. 55]; citation translated),

or that especially the strategies used should be the focus of interest [11], and it is also stated that the flexible use of strategies or the use of one adequate strategy — depending on the task — forms an important aspect in gaining optimal test results [10].

4. Strategies described in publications

The analysis of current studies on strategies used showed that four pairs of solution strategies could be identified. The four pairs of strategies, formulated and explained below, make no claim to be complete. The majority of publications, however, acknowledge these four pairs of strategies or parts thereof as **the** relevant strategies. Examples of spatial ability solution strategies in publications are the following:

Table 1: Examples of spatial ability solution strategies

Barrat [1]	Key features strategies Move object strategies Move self strategies
Just & Carpenter [14]:	Mental rotation around the global coordinate system Mental rotation around a user coordinate system Compare the characteristics of objects with one another Change of perspective
Dünser ([5, p. 159]):	Moving oneself or moving the object Concentration on details or on the whole Reflection and visualization
Schultz [26]:	Mental rotation strategy Perspective change strategy Analytical strategy

In addition to the four pairs of strategies which are described below, there are further terms frequently formulated: avoidance strategies, complementary strategies, mixed strategies, verbal-analytical strategies, and logical consequential thinking [11, 20, 29]. After close analysis these strategies can be regarded as parts of one of the pairs of strategies. K. Suzuki, E. Tsutsumi and others analyzed in very detail how subjects are solving the mental rotation test (MRT) [35] and the mental cutting test (MCT) [3] and described the used strategies for solving these tests [33, 34, 27, 28].

5. Four pairs of strategies

The individual pairs of spatial ability solution strategies form antagonistic pairs. In tests geometrical objects are generally comprehended either holistically or analytically. Test persons either construct a mental spatial model of the objects depicted (spatial strategy) or they start from the level image of the object (planar thinking). When solving spatial ability tasks probands are positioning themselves outside the scene or conversely, test persons — particularly in tasks of spatial orientation — put themselves into the proposed setting and mentally move around the objects. Test persons in general prefer verifying or falsifying procedures in solving the given tasks. If there are several acceptable solutions, they either try to find the right solution straight away or they exclude false solutions one by one until only one solution is left as the correct one.

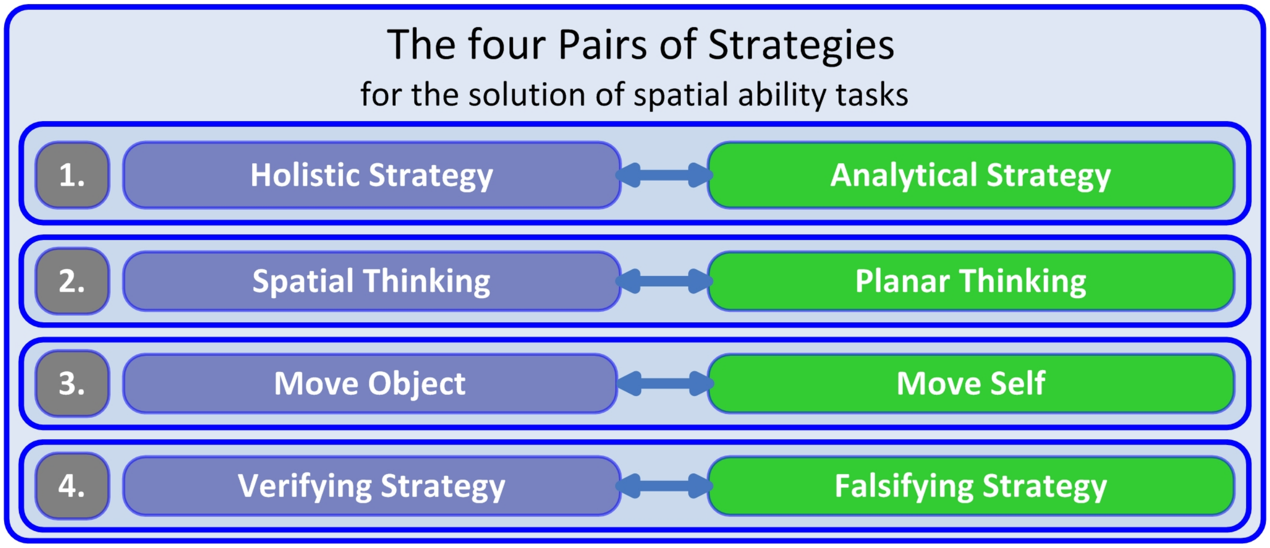

Figure 1: The four pairs of strategies for the solution of spatial ability tasks

5.1. Holistic strategy – analytical strategy

The most frequently identified pair of strategies in literature is the differentiation between a holistic or analytic approach. Barrat [1], Cooper [4], Schultz [26], Hosenfeld, Strauss and Köller [13], Glück [8] and Kaufmann [15] have given important information on this fact. When using a holistic strategy, the whole setting or the complete visual information is mentally generated and subsequently manipulated, noting the spatial relation of the objects to one another. The more capable a test person is in spatial ability and the simpler the given task, the more likely a holistic strategy will be applied. As early as 1953 Barrat examined a holistic procedure as "whole approach" as opposed to an analytical approach as "part approach". Using an analytical approach (also called "key feature strategy") the test persons focus either on single objects of the whole setting or geometrical objects and compare them with possible solutions, or apply analytical-verbal description or logical-consequential thinking. Compared to analytical strategy, a holistic strategy requires less time but more mental effort as information is presented in a more complex manner. This explains why in literature the use of holistic strategies is often referred to as "real" spatial ability [15], stating that the use of this *"strategy is more in accordance with the concept of space than the small steps of an analytic approach"* ([11]; citation translated).

5.2. Spatial thinking – planar thinking

In spatial thinking, test persons create a mental, three-dimensional model of the setting and solve the task by working on this mental model (transforming, rotating, cutting, folding, ...). The imagination moves within this three-dimensional space and in this way the solution is worked out. By contrast some spatial ability tasks can be solved by manipulating (mostly rotating) a two-dimensional picture of the setting. Thus there is no need to mentally create a three-dimensional picture for the right solution. In this way the task may be solved by considerably less complex deliberations than was originally intended (cf. [20, p. 64], [7]). In the intelligence structure test *IST-70*, Putz-Osterloh [23] and Putz-Osterloh and Lüer [24] identify tasks with a cube, which can be solved purely by planar strategy, and tasks which require spatial thinking. Gittler made similar observations with three dimensional cube tests. Some of the tasks of the original three dimensional cube tests could be solved by simply turning the plane images of the cube, reducing the complexity of the task considerably [7, p. 152].

5.3. Move object – move self

For the third pair of strategies the position of the proband is the relevant criterion. In solving spatial ability tasks, test persons can position themselves mentally as observers outside the setting and move the individual objects (move object) or can place themselves inside the setting and move around mentally (move self) [1, 20]. Studies show that with different types of tasks, the two possible test person's positions result in different efficiency. Schultz [26] shows that for solving the mental rotation test of Vanderberg and Kuse [35] the move object strategy is most efficient, and that on the other hand spatial orientation tasks are best solved with the move-self-strategy. So test persons can solve the tasks more efficiently if they transfer themselves into the scene and mentally move around in the setting.

5.4. Verifying strategy – falsifying strategy

In his "four-level-model to categorize solving strategies" Lüthje [19] identifies a further pair of strategies. In the process of finding solutions probands can either verify or falsify. In this context verifying means that test persons aim at the correct solution directly and look for it actively. In contrast, test persons can also work with the excluding strategy, identify incorrect solutions and exclude them step by step. This falsifying strategy selects the right solution by process of elimination.

6. Features of the strategies

The four pairs of strategies are not independent of one another. Numerous studies in literature identify crosslinks between the diverse eight strategies mentioned. Probands using the holistic approach tend to think spatially [15], women tend more frequently to use analytical solution processes, whereas men prefer to use holistic processes [10]. The strategies for solving spatial ability tasks depend on the following parameters [11, 15, 29]:

1. Intrapersonal preference
2. Size of the individual 'stratégie répertoire'
3. Type of task
4. Level of difficulty and complexity of the task
5. Individual experience in solving similar and related tasks.

With tasks of high complexity, strategies are often used to reduce their difficulty. With challenging tasks, "complementary and avoiding strategies" are used, requiring a less challenging spatial-visual qualification and thereby enabling a more successful handling of the task [20, p. 69]. Complimentary and avoiding strategies can be the following: consequential-logical thinking, verbal-analytical strategies, the use of several strategies in solving a task, change of strategies within parts of the task, concentrating on parts instead of the whole setting, or also the reduction from three to two dimensions, thereby reducing the task from three dimensional to two dimensional. Within one task often several strategies are used. Therefore it seems to be of particular importance that students have a wide range of strategies in order to be able to choose the optimal strategy suiting the situation. Lohman [18, p. 213] states that test persons use all the strategies at their disposal in spatial ability tasks. Glück and Vitouch [9, p. 325–326] found out that the range of strategies and the flexibility in adapting them to the requirements of the respective task is more relevant than basic cognitive processes. The phenomenon of strategy changes within a task occurs more often in complex than in simple tasks. Thinking about one or more changes of strategy within a task on the one hand requires the test person to have command of a broad spectrum of strategies, but it also compels the test person to adopt meta-cognitive processes. The choice of the best possible processes to solve a task in a specific situation requires reflection, calculation and decision-making at a higher level [15].

7. Summary and prospects

Ever since the beginning of the 20th century the scientific community has been searching for a precise and generally accepted factor-analytical model of spatial intelligence. So far it cannot be presented. Factor-based psychometric spatial ability tests have been designed with the notion that probands have to solve the tasks with preferably the same strategies. Research results show that opposed to this assumption, test persons consider various approaches to solve the tasks. Thus the exploration of the processes leading to the correct solution move to the centre of reflection and research. Since Barrat [1] a multitude of varying strategies have been detected and scrutinized. This paper trusses the various strategies and systemizes and describes them in the *"Four pairs of strategies for the solution of spatial ability tasks"*.

The research project *GeodiKon* (Development of a Didactic Concept for Teaching Geometry) from the University of Education of Salzburg, is designed to sharpen the awareness for differing strategies for the solution of spatial ability tasks. 46 school classes with 896 students in various counties in Austria had been supplied with specific learning material and information concerning the pairs of strategies. This will be evaluated in order to find out whether the awareness and training of strategies and factors of spatial ability brings about an improvement of spatial ability.

Further studies will be dedicated to the question of whether increased and deliberate work with haptic material and/or with new digital media (tablet, smartphone, ...) can contribute towards enhancing students' spatial ability.

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References

[1] E.S. Barratt: *An analysis of verbal reports of solving spatial problems as an aid in defining spatial factors*. Journal of Psychology **36**, 17–25 (1953).

[2] J.B. Carroll: *Human cognitive abilities. A survey of factor-analytical studies*. Cambridge University Press, New York 1993.

[3] *CEEB Special Aptitude Test in Spatial Relations*. Developed by the College Entrance Examination Board, USA, 1939.

[4] L.A. Cooper: *Individual differences in visual comparision processes*. Perception & Psychophysics **19**/5 (1976).

[5] A. Dünser: *Trainierbarkeit der Raumvorstellung mit Augmented Reality*. PhD-Thesis, Department of Psychology, University of Vienna 2005.

[6] J.W. French: *The description of aptitude and achievement tests in terms of rotated factors*. University of Chicago Press, Chicago 1951.

[7] G. Gittler: *Entwicklung und Erprobung eines neuen Testinstruments zur Messung des räumlichen Vorstellungsvermögens*. Zeitschrift für Differentielle und Diagnostische Psychologie **5**/2 (1984).

[8] J. Glück: *Spatial strategies – Strategy use in spatial cognition*. University of Vienna 1999.

[9] J. Glück, O. Vitouch: *Psychologie*. In S. Günzel (ed.): *Raumwissenschaften,* Suhrkamp, Frankfurt am Main 2009, pp. 324–337.

[10] J. Glück, H. Kaufmann, A. Dünser, K. Steinbügl: *Geometrie und Raumvorstellung — Psychologische Perspektiven*. Informationsblätter der Geometrie (IBDG) **24**/1, 4–11 (2005).

[11] M. Grüssing: *Wieviel Raumvorstellung braucht man für Raumvorstellungsaufgaben? Strategien von Grundschulkindern bei der Bewältigung räumlich-geometrischer Anforderungen*. Zentralblatt für Didaktik der Mathematik (ZDM) **34**/2, 37–45 (2002).

[12] J.P. Guilford: *The structure of intellect*. Psychological Bulletin **53**/4, 267–293 (1956).

[13] I. Hosenfeld, B. Strauss, O. Köller: *Geschlechtsdifferenzen bei Raumvorstellungsaufgaben — eine Frage der Strategie?* Zeitschrift für Pädagogische Psychologie **11**/2 (1997).

[14] M.A. Just, P.A. Carpenter: *Cognitive Coordinate Systems: Accounts of Mental Rotation and Individual Differences in Spatial Ability*. Psychological Review **92** (1985).

[15] H. Kaufmann: *Lösung- und Bearbeitungsstrategien bei Raumvorstellungsaufgaben*. In *Raumgeometrie — intuitiv und konstruktiv*, CD-ROM, vers. 2K.8, Okt. 2008, ISBN 978-3-200-01710-8 2008.

[16] M.C. Linn, A.C. Petersen: *Emergence and characterization of sex differences on spatial ability: a meta-analysis*. Child Development **56**, 1479–1498 (1985).

[17] D.F. Lohman: *Spatial abilities: A review and re-analysis of the correlational literature.* Technical Report No. 8, Standford University, Aptitude Research project, Stanford/CA. 1979.

[18] D.F. Lohman: *Spatial abilities as traits, processes, and knowledge*. In R.J. Sternberg (ed.): *Advances in the psychology of human intelligence*, **40**, 181–248, Erlbaum, Hillsdale/NJ 1988.

[19] T. Lüthje: *Das räumliche Vorstellungsvermögen von Kindern im Vorschulalter*. Franzbecker, Hildesheim 2010.

[20] H.P. Maier: *Räumliches Vorstellungsvermögen: Komponenten, geschlechtsspezifische Differenzen, Relevanz, Entwicklung und Realisierung in der Realschule*. Europäische Hochschulschriften: Reihe 6, *Psychologie*, Band 493, 1994.

[21] G.J. Maresch: *Spatial Ability — The Phases of Spatial Ability Research*. J. Geometry Graphics **17**/2, 237–250 (2013).

[22] M.G. McGee: *Human spatial abilities: psychometric studies and environmental, genetic, hormonal and neurological influences*. Psychological Bulletin **86**/5, 889–918 (1979).

[23] W. Putz-Osterloh: *Über Problemlöseprozesse bei dem Test Würfelaufgaben aus dem Intelligenztest IST und IST-70 von Amthauer*. Diagnostica **23** (1977).

[24] W. Putz-Osterloh, G. Lüer: *Wann produzieren Probanden räumliche Vorstellungen beim Lösen von Raumvorstellungsaufgaben?* Zeitschrift für experimentelle und angewandte Psychologie **26**/1 (1979).

[25] D.H. Rost: *Raumvorstellung. Psychologische und pädagogische Aspekte*. Beltz, Weinheim 1977.

[26] K. Schultz: *The contribution of solution strategy to spatial performance*. Canadian Journal of Psychology **45** (1991).

[27] K. Shiina, T. Saito, K. Suzuki: *Analysis of Problem Solving Process of a Mental Rotations Test – Performance in Shepard-Metzler Tasks*. J. Geometry Graphics **1**/2, 185–193 (1997).

[28] K. Shiina, K. Suzuki: *Design of Modified Mental Rotations Test and its Error Analysis*. J. Geometry Graphics **3**/2, 211–218 (1999).

[29] E. Souvignier: *Förderung räumlicher Fähigkeiten – Trainingsstudien mit lernbeeinträchtigten Schülern*. Pädagogische Psychologie und Entwicklungspsychologie **22** (2000).

[30] C. Spearman: *General Intelligence, objectively determined and measured*. American Journal of Psychology **15**, 201–293 (1904).

[31] L.L. Thurstone: *Primary Mental Abilities*. The University of Chicago Press, Chicago 1938.

[32] L.L. Thurstone: *Some primary abilities in visual thinking*. Psychometric Laboratory Research Report No. 59, Univ. of Chicago Press, Chicago 1950.

[33] E. Tsutsumi: *A Mental Cutting Test Using Drawings of Intersections*. J. Geometry Graphics **8**/1, 117–126 (2004).

[34] E. Tsutsumi, Ka. Shiina, A. Suzaki, K. Yamanouchi, T. Saito, K. Suzuki: *A Mental Cutting Test for Female Students Using a Stereographic System*. J. Geometry Graphics **3**/1, 111–119 (1999).

[35] S.G. Vandenberg, A.R. Kuse: *Mental Rotations, A Group Test of Three-Dimensional Spatial Visualization*. Perceptual and Motor Skills **47**, 599–604 (1978).

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