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Perspective Concepts - Exploring Seeing and Representation of Space

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Abstract. There had been a long tradition in analyzing the human seeing and conditions of perception in order to represent space and spatial objects similar to the way, we are seeing. EUCLID tried already to describe the characteristics of perspective in his *'Optics'*. ALBERTI later in the Renaissance period referred to the ideas of Euclid and explained linear perspective as the section of the pyramid of vision. He described the image as practically received by the device *'velum'*, the image plane, set between the eye and the object to be represented. There had been various seeing or perspective machines, which are able to represent the concept of perspective as a practical way to produce images of spatial objects according to the process of seeing.
Our students of architecture rebuilt some of these machines. The approach to perspective by the seeing machines explains the origin of the perspective concept from simulating the seeing process. The knowledge of optics and physiology of the eye had been part of the foundations of perspective. In this tradition can be analyzed the work of Guido HAUCK, who tried to develop another concept of perspective, the *'subjective perspective'*, which he based on the new physiological optics, received primarily by Hermann VON HELMHOLTZ.
Following the geometric background of perspective we will observe the development of the comprehension of vanishing points. The clear concept of points of infinity and vanishing points initialized the development of projective geometry. Only then it was possible to develop the representation method *'Relief Perspective'* systematically, where half space is projected to a spatial layer. We will ask and discuss, how the comprehension and concept of perspective had been developed in geometry and what had been the influences between theory and practice from various disciplines like geometry, art, architecture, physiology and perception theory.

Key Words: Perspective, space, representation, seeing, perspective machine, relief perspective

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1. Basics by Euclid and Alberti

When we go back in the history of sciences, we meet the efforts to gain scientific knowledge and to understand the world around us. One instrument of our relation to the outer world is our seeing. Therefore it had been already an early question, how we see and what we can conclude from it.

Euclid developed a geometry of vision in his *'Optics'* [7] around 300 BC. The motivating force for his research had been the wish to derive statements about the distances of the planets and stars from the analysis of seeing, therefore to gain scientific knowledge. In Euclid's *'Optics'* we can read for example:

> *"Objects of equal size unequally distant appear unequal and the one lying nearer to the eye always appears larger. (...) Parallel lines, when seen from a distance, appear not to be equally distant from each other."* [7, p. 358]

These are already basic characteristics of perspective.

The reception of Euclid had been an important foundation in the Renaissance time. Alberti referred in *'De Pictura'* [1] to Euclid's *'Elements'*, when he started with the definitions of points and lines, and to Euclid's *'Optics'*, when he spoke about the characteristics of seeing, for example the changes of proportions in the image, and the angle of vision. Alberti wrote his *'De Pictura'* as a practical guide for artists, painters, although he described to pick up the basics from the mathematicians:

> *"To make clear my exposition in writing this brief commentary on painting, I will take first from the mathematicians those things with which my subject is concerned. (...) In all this discussion, I beg you to consider me not as a mathematician but as a painter writing of these things. (...) The painter is concerned solely with representing what can be seen."* [1, Book 1].

An important historical step had been, that Alberti explained clearly the practical creation of the perspective image by the section of the pyramid of vision with the help of the device *'velum'*, the image plane, set between the eye and the object to be represented.

> *"Nothing can be found, so I think, which is more useful than that veil which among my friends I call an intersection. (...) This veil I place between the eye and the thing seen, so the visual pyramid penetrates through the thinness of the veil."* [1, Book 2].

It is passed on, that Alberti made impressive demonstrations of his indicated way to produce perspective images.

Whereas Brunelleschi verified the perspective paintings with his demonstrations by comparing them with the spectators view on the object, Alberti gave the initial point for developing methods to create a perspective image as a representation of a spatial object according to seeing the object.

2. Perspective machines

To create images of spatial objects according to a concept for the process of seeing leads us to several historical perspective machines, which bring again optics and the geometry of perspective close together. By analyzing these drawing and optical devices we grasp the origin of perspective as a representation of space from simulating the seeing process. We studied some of these perspective machines with our students of architecture to rebuild them for exploring their use during the 'Science Night' at our university for a public audience in April 2014. The audience had been surprisingly highly interested to test and explore the machines. The students prepared models of El Lissitzky's *"Wolkenbügel"* ("cloud-irons", 1923–25) as objects for the demonstration of the perspective machines (Figure 1).

2.1. Brunelleschi's mirror device

There exist only descriptions of Brunelleschi's experiment [6, 13]. It had been told that Brunelleschi presented 1425 a perspective demonstration of the *Florentine Baptistery*. The aim of the demonstration had been, to show that his perspective paintings delivered the same image as looking at the real building from the specific viewpoint. A spectator had to stand in front of the *'Baptistery* with his painting of the building on a panel with a small hole. The spectator had *"to peek through this hole from the back of the panel at a mirror held in such a way as to reflect the painted surface"* [13, p. 13]. The device does not help to draw a perspective, but has the aim to verify the perspective painting or drawing and to convince the spectators of the accuracy of the perspective image when compared with the view to the real object. This mirror experiment only worked with an ax-symmetric building.

It is told that Brunelleschi worked for the demonstration of his *'Palazzo de' Signori'* without a mirror and the hole, but cut away the area of the sky above the building in his drawing to enhance the illusion of the perspective drawing by merging image and reality.

Figure 1: Model of the *"Wolkenbügel"* during the 'Science Night'

Figure 2: Replication of Brunelleschi's mirror device

2.2. Dürer's perspective machines

Dürer's perspective machines however appear as active devices to draw perspectives. The historical researches came to the result, that Dürer experimented lately since 1514 with perspective machines. More details are presented in the paper about Dürer's contribution to perspective [15]. In his *'Underweysung'* Dürer described on the last pages four devices to

Figure 3: Replication of Dürer's perspective machine

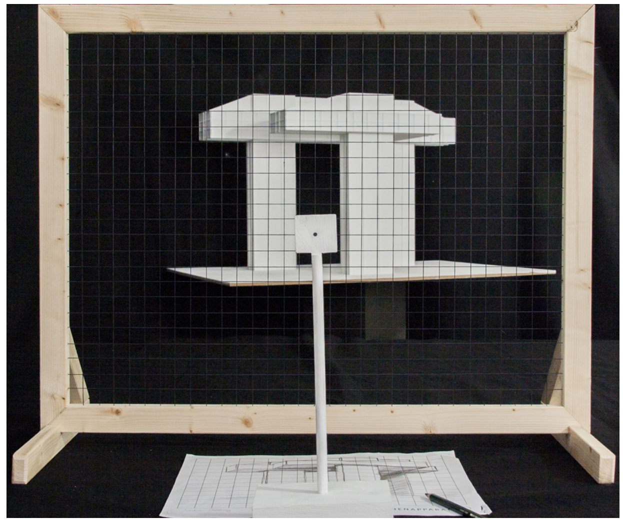

Figure 4: View through the grid of the rebuilt Dürer's drawing device

draw perspectives, which have some similarities but also differences. The first two are already presented in the first edition of 1525, the other two additionally in the second edition of 1538 [5]. We picked up two to rebuild them for the 'Science Night'. One of these machines can be understood as the materialization of the drawing method (Figure 3). The drawing method, which Dürer developed in two assigned views, was transferred to a spatial interpretation in the perspective machine. The string represents the straight drawn line. The original object gets scanned point by point with the help of the string and its intersection point with the image plane, like the *'velum'* of Alberti. The machine helps to understand the concept of perspective but is not really applicable as a drawing instrument, because it takes a long time to get the image point by point.

The students rebuilt and used a second well-known drawing device of Dürer, where a grid is placed in front of the object (Figure 4) and the squared drawing surface corresponds with this grid, but has not to be in the same scale as the image frame. The eye position is fixed with the help of a stick or hole. The device has the aim supporting to draw what you see. The device is a drawing tool with the advantage of working with different scales, so that the image can be drawn according to the seen object in a bigger or smaller scale. By using this device you do not feel comfortable to compare always the seen object through the grid with the grid on the drawing paper. The resulting wish to superpose the viewed and the drawn object is realized in the *camera obscura* and the *camera lucida*.

2.3. *Camera obscura*

With the *camera obscura* we make again a step in the history of optics. It is an optical device, where the spatial surrounding is projected on an image medium. The device can be a box or a walk-in spatial installation where light is falling through a small hole on the opposite wall, the prototype of a photo camera. We built and used both, small pinhole cameras and a walk-in *camera obscura* (Figure 7) in a workshop with Marek Pozniak, photographer and artist.

It is surprising what we can get by a simple paper box with a pinhole (Figure 5). Instead of a box we used also an old film can. The image then is projected on a cylinder and we receive

Figure 5: Photo, made by a small pinhole cardboard box camera

Figure 6: Panoramic photo, made by a small pinhole film can camera

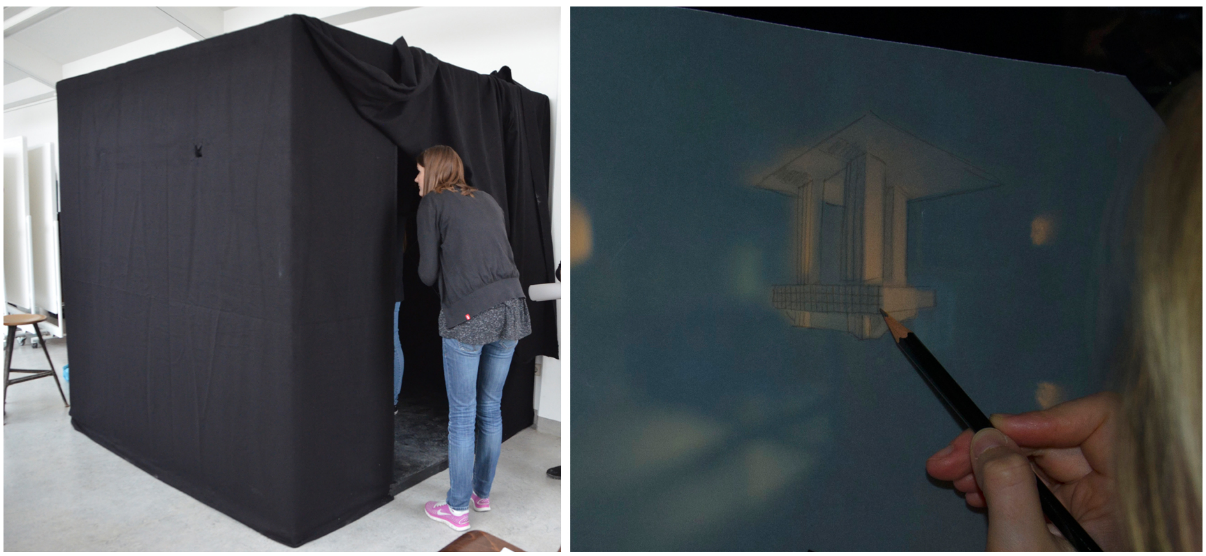

Figure 7: Walk-in camera obscura from outside and drawing in the walk-in *camera obscura*

a panoramic photo (Figure 6). The *camera obscura* had been significantly improved in the history of photo cameras by integrating a lens in the hole of the *camera obscura*. Depending on the lens we receive deformations of the spatial objects. It had been interesting to realize that the image in the *camera obscura* (Figure 7) on a transparent paper is upside down with the effect that you are forced to draw what you see and not what you think, because you do not understand the object in all details when seen upside down.

By using the *camera obscura* we comprehend the received image by the method of projection, by light, which gives us another fundamental concept of perspective.

2.4. *Camera lucida*

The term *'camera lucida'* (lat., light chamber) is opposed to the term *'camera obscura'* (lat., dark chamber) with reference to the older device. The English physician and chemist William Hyde Wollaston [13, 19] developed the first *camera lucida* in 1807. He used a prism with four optical faces in order to produce two successive reflections, so that the image is not inverted or reversed. The seen image is superimposed with the drawing surface. Our students prepared their own prism for a *camera lucida* (Figure 8). Additionally, we used the *'NeoLucida'* (Figure 9) by Pablo Garcia and Golan Levin [9] as well as a webcam version.

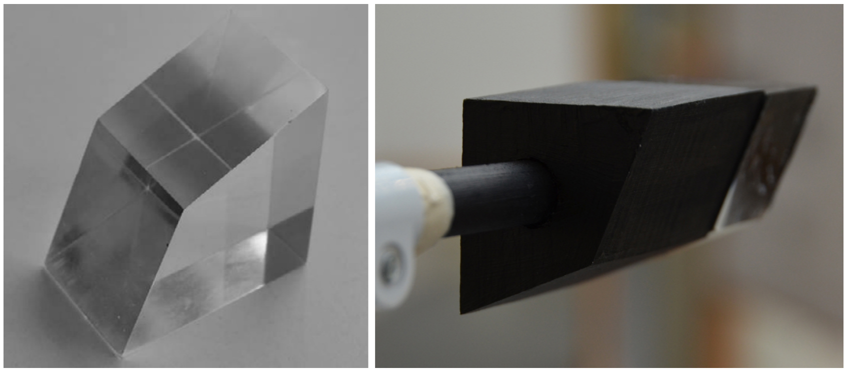

Figure 8: Prism of a self-made *camera lucida*

Figure 9: Drawing with the help of a *'NeoLucida'* [9]

2.5. Lambert's Perspectograph

With the *'Perspectograph'* of Johann Heinrich Lambert of 1752 we are focusing on the perspective transformation of a plane figure. It is the mechanical transfer of the relations between a plane figure and its perspective image figure. Lambert showed for example in his drawing (Figure 10) the perspective transformation of the ground plan of a garden. Collineation describes the relationship between the original figure and the image figure. With the *'Perspectograph'* we shift away from an artistic approach of rebuilding the seeing process to a mathematical understanding of perspective as a transformation. The basis can be found in Girard Desargues' famous theorem of 1639, fundament of projective geometry, where the relationship between two images of a figure is observed instead of the relationship between original and image [19].

Precise explanation, high quality replication and digital animation of the *'Perspectograph'* can be found in the extensive material of the *Perspectiva Artificialis Project* of the University of Modena and Reggio Emilia [4].

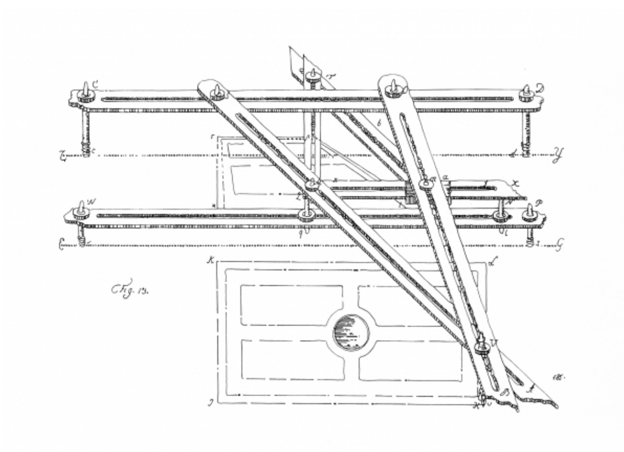

Figure 10: Lambert's drawing of the *'Perspectograph'* [14, p. 161ff.]

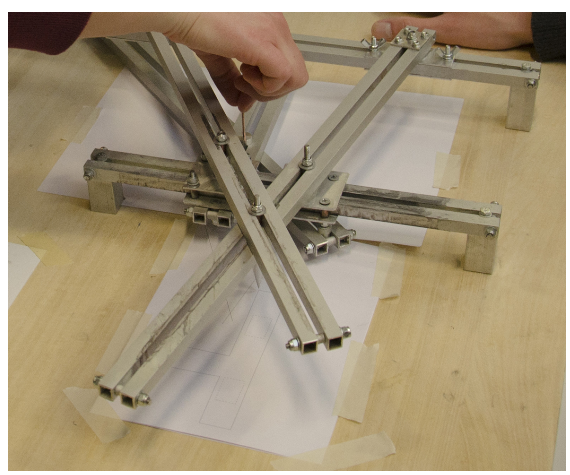

Figure 11: Replication of Lambert's *'Perspectograph'*

3. Subjective perspective

Erwin Panofsky critizised in his highly noticed article *"Perspective as Symbolic Form"* [18] (1924/25) the perspective theory as a rational theory, boldly abstracted from reality, far away from an actual subjective visual impression with the assumptions that we see with one single unmoving eye and that our seen image would be adequately represented as the section of the pyramid of vision. The structure of an infinite, continuous and homogeneous mathematical space would be opposite to the structure of the psychophysiological space. He states:

> *"In a sense, perspective transforms psychophysiological space into mathematical space. (...) It takes no account of the enormous difference between the psychologically conditioned 'visual image' through which the visible world is brought to our consciousness, and the mechanically conditioned 'retinal image', which paints itself upon our physical eye. (...) Finally, perspectival construction ignores the crucial circumstance that this retinal image — entirely apart from the fact that the eyes move — is a projection not on a flat but on a concave surface."* [18, p. 31].

But what could be the geometric solution?

Panofsky refers to the work of Guido Hauck *"Die subjektive Perspektive und die horizontalen Curvaturen des dorischen Stils"* [11] (1879), where Hauck tries to develop a *'subjective perspective'*, combining the mathematical and the aesthetic viewpoint based on the modern physiological optics. Hauck criticizes that the new achievements of physiological optics did not effect the progression of perspective because of the apparent confirmation of the *camera-obscura*-images by the development of photography.

Hermann von Helmholtz provided as physiologist a new basis for the science of seeing. He dealt with all human types of sensation as basis for cognition. In refer to seeing he concluded in his *"Handbuch der physiologischen Optik"* [12] that although the retina receives the optical image like a *camera obscura*, the nerve cells, connected with the retina, effect seeing not the eye itself. Perceptions of external objects were seen as acts of our ability of imagination, as psychic activity. He emphasized that we learn through experiences, we perceive with various sensations and perceptions, we make images of an object, if we move our eyes or body and view the object from various sides or touch, etc. The perception of the object is the epitome of all these possible sensations. The perceived images are automatically connected with our imaginations and experiences. Thus an active instead of a passive viewer is assumed.

Hauck looks back in the history of the Renaissance perspective and analyzes that many of the strong perspective construction rules were broken in the paintings, like in Raffael's *'School of Athens'* for example the representation the persons and objects at the sides, in order to achieve a satisfactory image. His method of subjective perspective directs towards a satisfactory image according to the perception of the object. He sets the following conditions:

1. *Principle of collinearity:* Each line perceived as straight line should be also represented straight.
2. *Principle of verticality:* Perceived vertical lines should appear as verticals.
3. *Principle of conformity:* The apparent length of a line segment is proportional to the angle of vision.

But not all conditions could be completely fulfilled at the same time. Collinearity and conformity are contradictory. He demands the conformity only for the most important lines in the image and the collinearity only for the vertical lines and the horizon. Various perspective systems are his results. Perspective for him teaches the creation of compromises in the conflict of the condition of collinearity and conformity. The collinear-perspective system with conformity only around the main point of the perspective corresponds with the traditional central perspective. As a second system he suggests the conform-perspective system corresponding with the subjective image of perception.

The construction of such a *'subjective perspective'* is based on the idea of the development of the retina as part of a sphere. Hauck follows the idea of using cylindrical mapping of mathematical cartography. Georg Glaeser calls those perspectives derived from a spherical image surface *'transformed spherical perspectives'* [10].

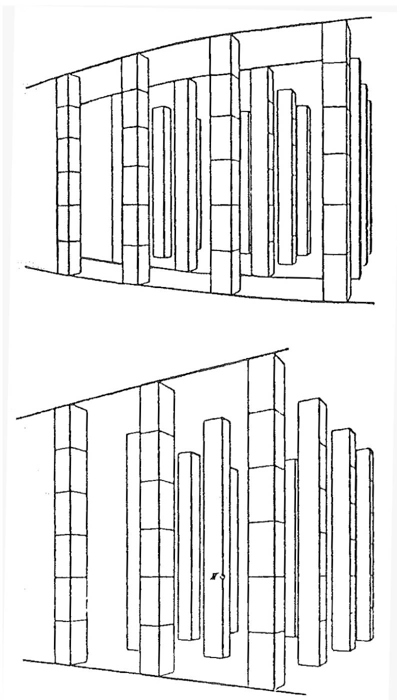

Figure 12: *Piers hall* in conform- (above) and collinear- (below) perspective [11, 18]

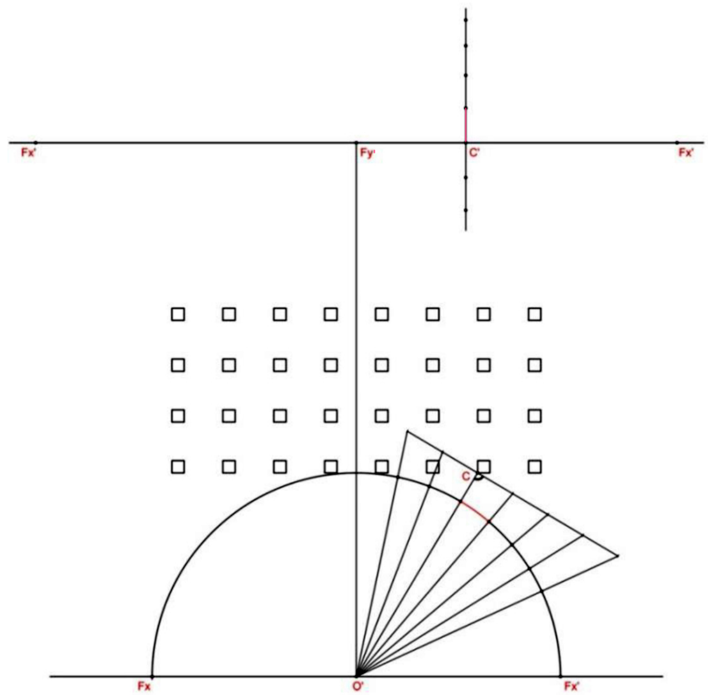

Figure 13: Construction method of the *'subjective perspective'* according to Hauck [11]

The semicircle represents the semispherical retina in top view and front view. The eye O is located in the centre. Hauck starts with the rectified semicircle as the horizon in the conform-perspective. The perspective image point is received by the trace point method according to traditional perspective but with the semisphere as image surface and with the difference that the determined distorted heights are transferred on a vertical through the image location on the horizon.

It was a remarkable attempt developing a new concept of perspective according to the new psychophysiological knowledge, but the mixture of geometric method and arbitrary decisions remains unsatisfying. This may be also the reason that Hauck had attracted a particular attention by art experts but not by mathematicians. Geometry is not an empirical science. Henri Poincaré described this difference between geometrical and representative space in the triple form — visual, tactile, and motor — in his work *"Science and Hypothesis"* in a very clear way:

"Representative space is only an image of geometrical space, an image deformed by a kind of perspective, and we can only represent to ourselves objects by making them obey the laws of this perspective." [20, p. 57]

4. Perspective transformation

For the further development of perspective in geometrical space the comprehension of vanishing points had been an important step. Guidobaldo del Monte [17] had been the first to introduce the term *'punctum concursus'*. Guidobaldo characterized it as the point in which the images of parallel lines converge. But the most important step had been that he described to find the *'punctum concursus'* of the images of parallel lines with the help of the intersection point of a parallel line through the point of view with the image plane. Then the image of a line not parallel to the image plane can be constructed with the help of its trace point and vanishing point, although he did not use the term vanishing point. Using perspective representations for his drawings to explain the spatial concept had complicated the comprehension of his explanations. In his drawing (Figure 14) point A is the point of view and X is received as the *'punctum concursus'* of the parallel lines BC, DE, FG by a parallel line through the view point A. The points B, D and F are the trace points of the lines.

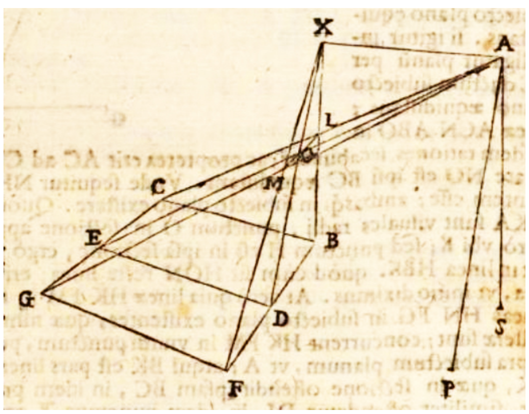

Figure 14: Perspective images of parallel lines through *'punctum concursus'* [17, p. 42]

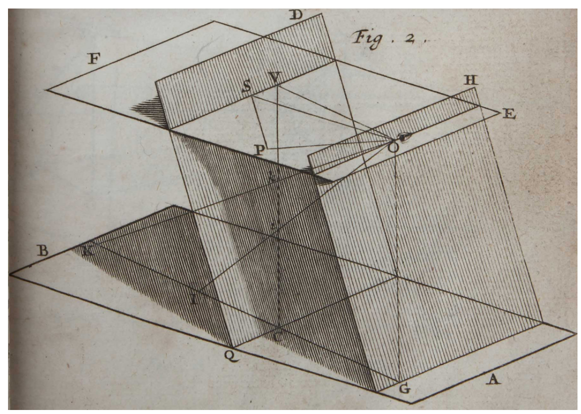

Figure 15: Definitions of perspective terms by Taylor [23, Fig. 13, Plate I]

Brook Taylor, who handled the vanishing point in full general terms in his book *"Linear perspective"* (1715), had done the next step. Although he characterized linear perspective as the art of describing exactly the representations of any given objects, as they would appear from any given point, he worked out a mathematical approach to perspective. He defined the most important used terms. The vanishing point had been defined as *"the point where the visual ray which is parallel to any original line cuts the plane of projection"* [23, p. 18]. And then he indicated all special cases and defined also a vanishing line. For his definitions and explanations he used the general case, where the plane of projection is oblique (Figure 15).

Taylor wrote down his theory of perspective in a sequence of definitions, theorems and proofs. The most important step had been the general definition of vanishing points and lines. The concept of vanishing points and the deeper comprehension of the relation between the original figure and its perspective image had initialized the development of projective geometry.

LAMBERT's studies with reference to his *'Perspectograph'*, the analysis of Jean Victor PONCELET in *"Traité des propriétés projectives des figures"* (1822), and Jakob STEINER's thoughts in *"Systematische Entwicklung der Abhängigkeit geometrischer Gestalten von einander"* (1832) lead finally to the system of projective geometry, as Karl Georg Christian VON STAUDT worked it out in *"Geometrie der Lage"* (1847). He made a dedicated difference between a *'geometry of position'* and a *'geometry of measure'*. In relation to our topic the introduction of points and lines at infinity had been the most important step for the final clear understanding of vanishing points. VON STAUDT wrote:

> *"Two straight lines, lying in one plane, have either one common point or a common direction. Two different planes have either a common straight line or a common position."* [22, p. 23] (translated by C.L.)

And he called his approach to geometry *'perspektivisch'* (perspectival).

This understanding gave finally the background for analyzing the projections themselves as transformations by studying geometric properties that are invariant under affine or projective transformations.

5. Relief perspective

Relief perspectives had been used for theater stages, for example *'Teatro Olimpico'* in Vicenza (1585), or as built relief perspectives already in the 15th/16th century like *'Chiesa di Santa Maria presso San Satiro'* in Milan (1479–99), but a systematic geometric approach to relief perspective could be achieved only due to the comprehension of perspective as a transformation of spatial objects. Then the perspective transformation or collineation is applied to the spatial figure. The half space behind a front plane is transformed to the layer between the front plane and a parallel vanishing plane (Figure 16). The infinite half space is transformed in a finite space. The idea of the perspective transformation of a plane figure like in LAMBERT's *'Perspectograph'* is applied to a spatial object or the half space.

The fundamental works for relief perspective *"Traité de Perspective Linéaire"* by Jules DE LA GOURNERIE, *"Grundzüge der Reliefperspektive"* by Rudolf STAUDIGL [21], and *"Grundzüge der Reliefperspective nebst Anwendung zur Herstellung reliefperspectivischer Modelle"* by Ludwig BURMESTER [3] had been all published in the 19th century. The amazing examples of relief perspective models by BURMESTER (Figure 18), rebuilt by Daniel LORDICK with the help of a 3D printer [16], show the perspective transformation of typical solids.

STAUDIGL pointed out the importance of relief perspective in refer to a systematic approach to perspective transformation:

> *"Apart from the values which have such studies for the sculptor, the same should also be of interest for those who devote themselves to the study of Descriptive Geometry, because the relief perspective is the most general method of projection, from which the orthogonal, the oblique and the perspective projection arise as special cases"* [21, p. IIf.] (translated by C.L.)

The *relief depth*, the space layer between front plane and vanishing plane, determines substantially the relief perspective. If the relief depth is zero, we get the usual perspective (Figure 17).

We explored the creation of relief perspective models with our students of architecture in the so-called *'All School Charrette'*, where students of all semesters work together in small

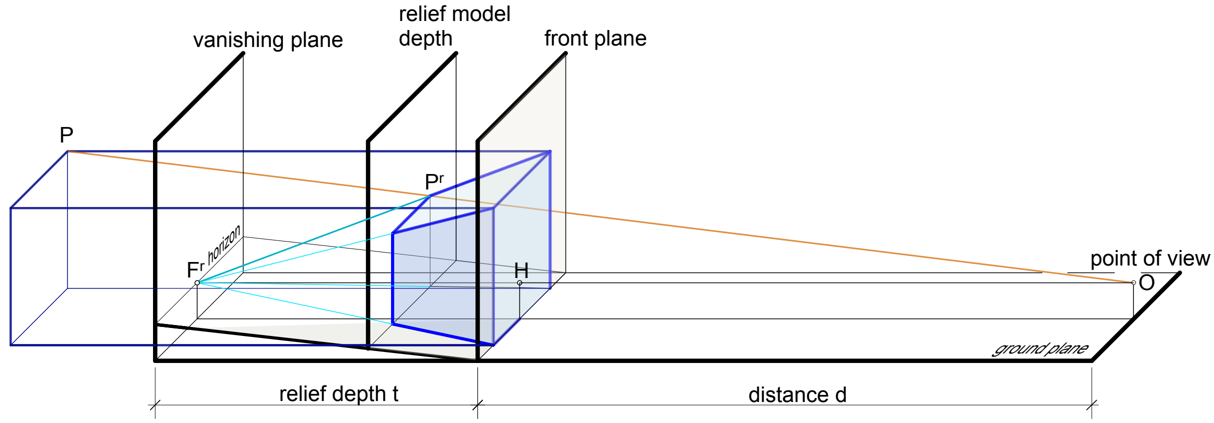

Figure 16: Concept of relief perspective transforming half space in a space layer [8]

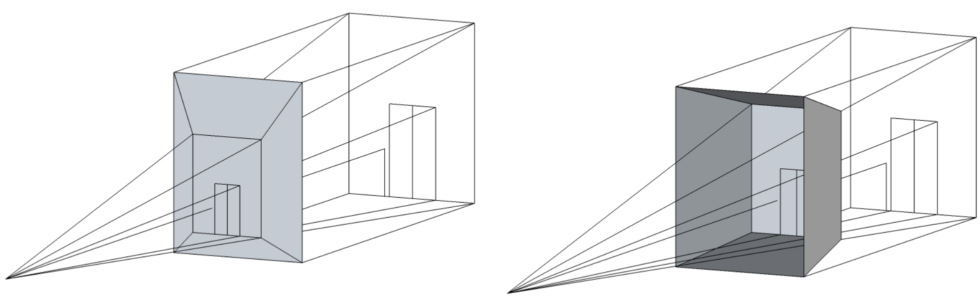

Figure 17: Perspective and relief perspective

groups in a one-week project, introduced and supported by all professors of our faculty. James Frazer Stirling's *'Clore Gallery'*, designed and built 1980–85 in London, had been the topic for the architectural analysis. Two perspectives of the entrance hall by Stirling formed the basis of the idea to build a relief perspective model according to these perspective drawings. The two perspectives represent the entrance hall from two opposite view directions. Stirling's design approach by the two perspectives was supposed to get adequately represented by the relief perspective models.

Examples of the students' works are presented in Figures 19–21. The relief perspective

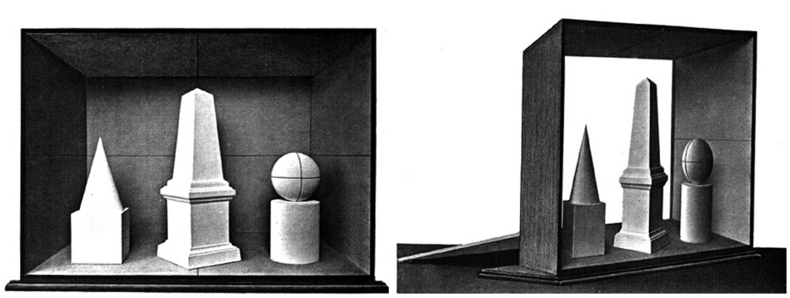

Figure 18: Typical solids in a relief perspective model by Burmester [3, plate IV, no. 1]

Figure 19: Detail of the relief perspective model by our students

model represents space from a specific point of view. The spatial model itself does not remain an independent object; it becomes related to the spatial perspective transformation. Therefore we come back to our topic from the beginning, where we followed historical research efforts to explore seeing and find representations according to our seeing procedure. When we look more in detail in Figure 19, we can comprehend the perspective deformation of the spatial objects, although it can be hardly shown in plane images.

Figure 20: Students' relief perspective models (photos: Bernhard FRIESE) [9]

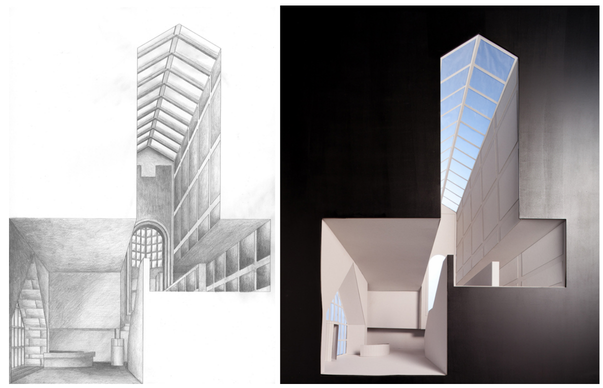

Figure 21: Perspective drawing and relief perspective model (photo: Bernhard FRIESE)

Figures 20 and 21 show the views in opposite directions in the entrance hall of the *'Clore Gallery'*. Additionally to the relief perspective models Figure 21 shows a corresponding perspective drawing. The relief perspective models had been built by using the same position of the viewpoint as in the perspective drawing. The interest and stimulus in relief perspective can be found again in art, especially in applications for stage design. But only the geometric-mathematical development of vanishing elements, projective geometry and a transformational approach, systematically worked out in the 19th century, enabled a theoretical and applicable concept of relief perspective.

6. Conclusions

By studying the perspective concepts and the representation of space according to seeing, we get aware of the fruitful interrelations between theory and practice as well as the interactions between various disciplines like geometry, art, architecture, optics, physiology and perception theory. The optical and artistic approach to the perspective image entered a dialogue with the transformational and geometric-mathematical approach.

In the Renaissance time the epistemological conception of the world refers to the rediscovery and development of perspective theory. The philosopher Max BENSE pointed to the relationship between art and philosophy:

> *"Both, the perspective theory of the painter as well as the epistemology of the thinker, assume explicitly the subject-object relation thematically."* [2, p. 79] (translated by C.L.)

And he concluded, that such a universal relation had been first established in the field of aesthetics and the philosophical treatment of the same problem lagged behind. Perspective theory and practice had been the motor for philosophical and epistemological questions.

It is interesting to see, that studying visual representation in art and science gave new impetus for our relationship to the world and the sciences as well as for the development of geometry in direction towards projective geometry as an important historical step in mathematics. Going back to these diverse roots of perspective in optics, art, geometry and mathematics remains an important foundation in our actual visual dominated culture.

Acknowledgements

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