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Traditional Descriptive Geometry Education in the 3D-CAD/CG Era

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Abstract. Graphic science is the subject which teaches geometry and graphics. It is taught in early undergraduate curricula at many Japanese universities as a liberal arts subject or as a basic subject for design and drawing. In traditional graphic science courses, descriptive geometry based on manual drawings was taught, but in recent years, the spread throughout society of 3D-CAD/CG has been accompanied by the need for education to respond to these. Therefore in the graphic science education at the College of Arts and Sciences of the University of Tokyo, where the author taught, 3D-CAD/CG has been implemented since 2007. The descriptive geometry education is given before 3D-CAD/CG education for the following reasons; 1) Traditional descriptive geometry is an excellent method in teaching and learning geometry of projection and of three-dimensional objects, and concepts and/or procedures in descriptive geometry can be applied in solving geometric design problems by the use of 3D-CAD/CG. 2) Even in the age of 3D-CAD/CG, hand drawing is still being used (especially for free-hand sketches). 3) Hand drawing is an effective method of developing spatial ability of students.
However, with the spread of 3D-CAD/CG, the descriptive geometry techniques in analyzing shapes and forms of three-dimensional objects are now losing their earlier practical importance. So emphasis is placed on teaching the theory behind the techniques, i.e., geometry of projection and of three-dimensional objects. In this paper, characteristics features of traditional descriptive geometry together with specific examples of classes will be discussed in order to describe the importance of descriptive geometry education, and the need to switch from the education focused on techniques to the education on the theory behind the techniques.

Key Words: Descriptive geometry, graphics education, graphics literacy

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1. Introduction

Graphic science is the subject which teaches geometry and graphics, and is taught in early undergraduate curricula at many Japanese universities as a liberal arts subject or as a basic subject for design and drawing. In traditional graphic science courses, descriptive geometry based on manual drawings was taught, but in recent years, the spread throughout society of 3D-CAD/CG has been accompanied by the need for education to respond to these, and graphic science education in the College of Arts and Sciences of the University of Tokyo, where the author taught, 3D-CAD/CG has been implemented to graphic science education since 2007 (Course name: *'Graphic Science II'*) [5, 7, 8]. The significance of traditional descriptive geometry, its educational significance in particular, is considered to be great, and descriptive geometry education (Course name: *'Graphic Science I'*) is given before 3D-CAD/CG education. But, with the spread of 3D-CAD/CG, the practicality of descriptive geometry as an analysis technique has been declining, and it is necessary to shift educational priority from education focused on practical techniques to the theory behind the techniques, i.e., geometry of projection and of three-dimensional objects [8, 9].

In this paper, characteristics features of traditional descriptive geometry, together with specific examples of classes, will be discussed in order to describe the importance of descriptive geometry education, and the need to switch from education focused on techniques to education on the theory behind the techniques.

2. What is descriptive geometry ?

Before beginning the discussions on specific examples of classes, the characteristic features of traditional descriptive geometry will be clarified, especially from an educational point of view.

2.1. Educational contents of traditional descriptive geometry

Educational contents of traditional descriptive geometry have been divided broadly into two parts. The first is the graphic representation of three-dimensional objects, i.e., projection, and another is the analytical and constitutional method of three-dimensional shapes by the use of their graphic representation.

The term, descriptive geometry, includes the word "geometry", and as shown by the fact that in Europe it is occasionally called "Constructive Geometry", it has been developed into and has been taught as geometry for analysis and construction of three-dimensional objects. Before the emergence of computers, descriptive geometry was the only practical way for analysis and construction of three-dimensional shapes, so education prioritized these techniques.

It is thought that as a result of the long history of technique-centered education, even among people working in descriptive geometry, more have begun to think that descriptive geometry simply deals with techniques. In the background to the techniques of descriptive geometry, there is "Solid Geometry", and it is necessary to be reminded that traditional descriptive geometry education also played an important role as solid geometry education, which is needless to say, important even in 3D-CAD/CG era.

2.2. Characteristics of descriptive geometry

The major characteristic of descriptive geometry is that it solves problems concerning three-dimensional shapes by reducing them to problems concerning their two-dimensional representations, i.e., "Graphics". A typical method for performing the conversion from three-dimensions to two-dimensions is to use projection. H. Stachel pointed out that a benefit of graphical solution by projection is that preparing a projection from a specified direction simplifies solving the problem, and also mentioned that it aids intuitive grasping of the geometric structure of the problem [2].

We, human beings, live in a three-dimensional world, and capture most of spatial information through our visual system, an "Eye". It is noted here that the eye is, basically, a two-dimensional detector. It uses the two-dimensional projected image on the retina, i.e., graphic representation (Figure 1). It can be said that when we are looking at an actual three-dimensional solid shape, we are seeing it through graphics. It is not too much to say that we recognize three-dimensional world only through graphics. Graphic system is naturally built in human beings. That is why graphics are so natural and, hence, useful for us.

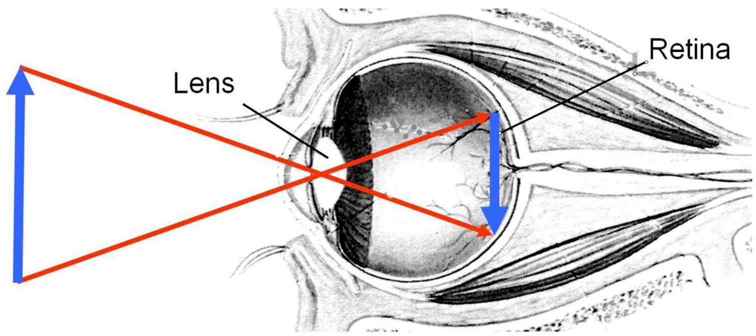


Figure 1: An eye — a two-dimensional detector

Graphics have, however, a fatal defect. This is the fact that because they represent three dimensions in two dimensions, they cannot represent depth direction (projection line direction) information.

It can be said that the essential defect of graphics described above is common to looking at actual three-dimensional objects with our eyes. Let's now imagine a village blacksmith who makes horseshoes using ancient methods. A blacksmith makes horseshoes by hammering the material over and over with a hammer. In this way, he makes three-dimensional shapes. During the work, the blacksmith probably does not vaguely look at the horseshoe. He occasionally rests his hands and looks at the horseshoe from the "front" to make sure that its U-shaped curved state is gradually approaching the shape which conforms with his image, and also, looks at it from the "side" to make sure that is becoming flatter. As mentioned before, the eye is an essentially two-dimensional detector, and it has sharp sensitivity to the two dimensions on the retina surface, but its sensitivity to the depth direction is quite low. To recognize a solid shape correctly, it is not enough to just look at it vaguely, but to look at the shape from a proper direction it can be seen sensitively. It is also necessary to select proper projection direction when we are solving spatial problems through the use of their graphic representations.

Finally, I would like to add one word about the importance of line drawings, which are mainly used in traditional descriptive geometry. Real three-dimensional objects have various attributes — a shape, use, materials, textures, color and so on. It should be noted here that some kind of abstraction is always necessary when we recognize something. Line drawings are the abstracted representation of the shape of actual three-dimensional objects, in which other attributes such as materials, textures and so on are abstracted. Only geometric constitutional relationship of the solid shape, i.e., points, lines and planes are shown in line drawings, which are indispensable for recognizing the shape.

The second characteristic is that the drawings in descriptive geometry are made "manually". H. Stachel has stated that even G. Monge, who established descriptive geometry, did not refer to manual drawing in any way, and that this is not the essence of descriptive geometry [2]. But even now, when 3D-CAD/CG has been implemented — not only at our college, but at many other educational institutions —, descriptive geometry is taught with manual drawings. It is widely known that not only in geometry, but in other types of learning, the concurrent use of sense organs typified by "eyes" and motor organs such as "hands" is a useful way to promote understanding and firmly fix memories. Manual drawing is considered to be important from an educational point of view.

2.3. Why is descriptive geometry taught ?

As stated in the introduction, graphic science education at the College of Arts and Sciences of the University of Tokyo includes descriptive geometry education (Course name: *Graphics Science I*). This is done for the following three reasons.

1) Descriptive geometry is a superior method of teaching and learning geometry of projection and of three-dimensional objects, in particular, for construction and manufacturing, i.e., "Constructive Geometry", since it enables intuitive grasping and rigorous logical reasoning of three-dimensional shapes, and since concepts and/or procedures in descriptive geometry can be applied even in solving geometric design problems by the use of 3D-CAD/CG.
2) Even in the age of 3D-CAD/CG, manual drawing (sketching in particular) is still used and is still practical.
3) Manual drawing is an effective way to enhance spatial ability of students [3, 5].

But with the spread of 3D-CAD/CG, its practicality as an analysis technique based on descriptive geometry (for example, finding the true length of a straight line, finding lines of intersection between two solids) has been declining, and it is now necessary to make a paradigm shift from education with priority on practical techniques to teaching the theory behind the techniques, i.e., geometry of projection and of three-dimensional objects.

3. Specific examples of education

Table 1 shows the typical curriculum of *Graphic Science I*. It is not very different from that for traditional descriptive geometry with priority on techniques. The differences between lessons centered on techniques and those centered on the theory behind the techniques are in the details. So this chapter explains differences between the two, based on examples of specific teaching of several items.

Table 1: Educational contents of *GS I* (One 90-minutes time slot per week for 15 weeks)

Week	*Contents*
1	Overall guidance
2	Projection (principle, various methods)
	⟨ *analysis and construction of solids by orthographic projection* ⟩
3	Principal projection
4	Auxiliary projection
5–7	Points, lines, planes
8	Polyhedra (regular polyhedra / semi-regular polyhedra)
9–10	Curved surfaces (categorization / helical convolute, hyperbolic paraboloid, etc.)
11–13	Intersections
	⟨ *pictorial projection* ⟩
14	Isometric projection / oblique projection
15	Perspective projection

3.1. Skew lines

Figure 2a shows a descriptive geometric solution which obtains a straight line connecting two skew lines AB and CD at the shortest distance. The minimum distance line of two skew lines is the common perpendicular of the two lines. This is done based on a characteristic of perpendicular projection: "projections of two straight lines intersecting perpendicularly in space intersect perpendicularly, if one of these projections is represented in true length". It is considered to be possible to teach the characteristics of perpendicular projection through these drawings, and at the same time, to let students inversely understand the geometric characteristics of two skew lines in space.

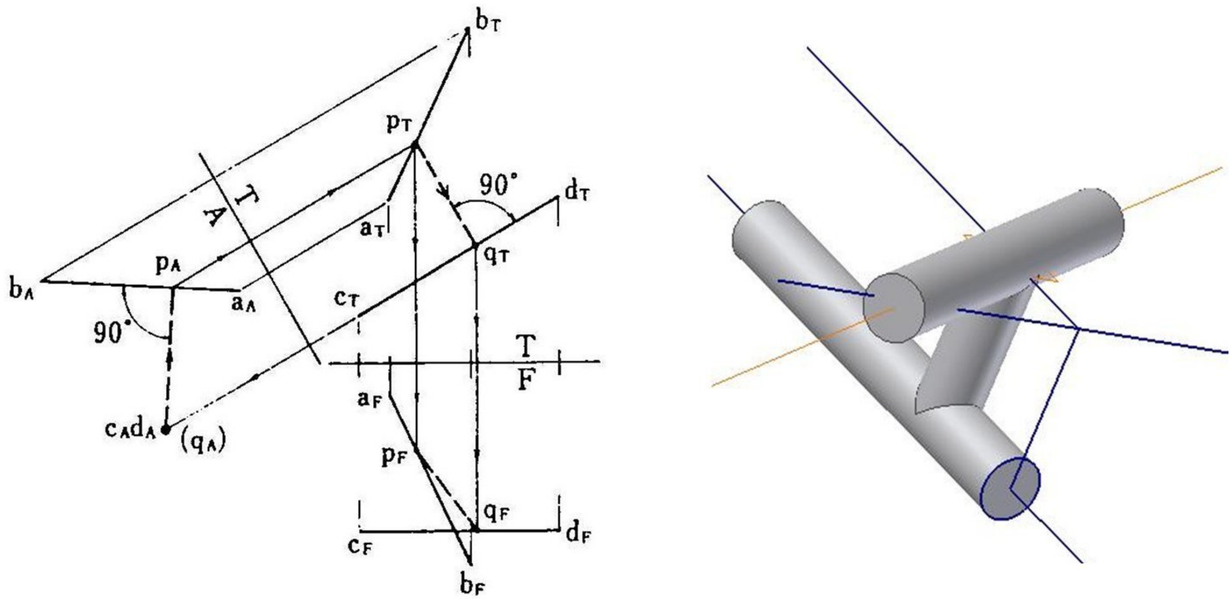
Figure 2a uses a point view and a true length view of straight lines. As already mentioned in Section 2.2, it is only possible to understand the geometric construction of two skew lines, when "they are viewed from a specified direction" such as this.

It should be noted here that the problem solving procedures in descriptive geometry, i.e., solving problems concerning three-dimensional shapes by reducing them to problems concerning two-dimensional figures, is also useful even in 3D-CAD/CG (see Figure 2b), since 3D-CAD/CG uses projective views as an interface with the user, and this is similar to descriptive geometry [5, 6, 7].

Not only the skew lines shown in this example, but in this course, the explanation is given using only the auxiliary projection. It does not deal with the "Rabatment" which is a unique technique for manual drawing developed to save space when drawing on paper, and it is presumed to have already become impractical. Also, for similar reasons, it does not deal with "traces" (of a line and a plane).

3.2. Intersections

When we create shapes of any kind, we start with a basic solid and remove unnecessary parts — cutting —, or inversely, form the desired shape by combining a number of basic solids — set



(a) Descriptive geometry (b) 3D-CAD

Figure 2: The shortest line which connects two lines in a skew position

intersection —. Cutting and set intersection are, geometrically, nothing more than applying the mutual relationships of points, lines, and planes, but they are important as methods for construction and manufacturing.

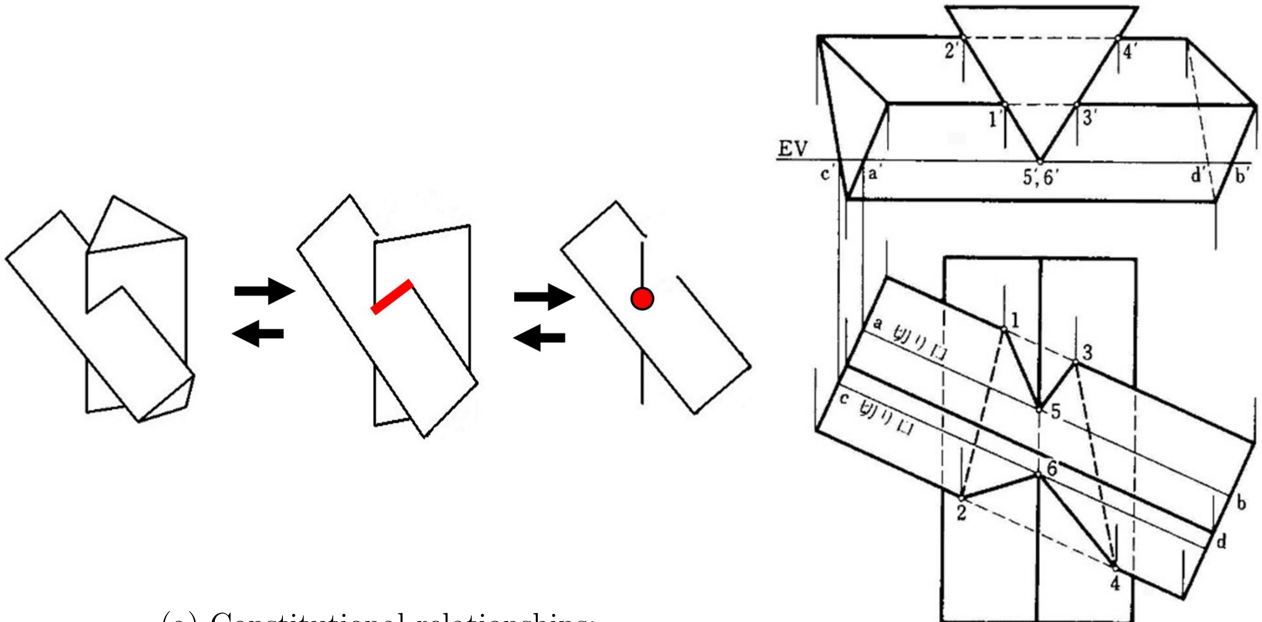
Figure 3b shows a solution method for intersections in descriptive geometry, taking two triangular prisms A and B as an example. The solution is as follows;

1) The points of intersection of the ridge lines of the triangular prism A and the planes of the triangular prism B are obtained. The triangular prisms are substituted and the same operation performed, i.e., the intersections of the ridge lines of triangular prism B and the planes of triangular prism A are obtained. Here, in the example in Figure 3b, the intersection points $1, 2, 3, 4$ are obtained by applying the fact that the side of triangular prism A is represented by an edge view in the top view.

Both triangular prisms are cut on the vertical plane which includes ridge line 5-6 and their sections are used to obtain the intersection points $5, 6$. It is also possible to obtain intersection points 5 and 6 by creating the edge view of the sides of prism B through an auxiliary projection. But the method of resolving a three-dimensional problem into two dimensions by cutting can be also used in 3D-CAD/CG [6, 7], so the cutting method is taught in this descriptive geometry course.

2) The line of intersection is obtained by drawing a line connecting the intersection points obtained in 1), and an intersection drawing is completed by making a "visible-invisible" judgment. In many descriptive geometry textbooks, the intersection point connecting procedure and/ or the "visible-invisible" judgment method are not explained, but are considered to be "imaging a solid". But, there are fixed rules for inducing these from geometric relations [4], and these rules are taught since similar rules are used in 3D-CAD/CG.

It is advocated that before teaching the above solution method, as shown in Figure 3a, the line of intersection of two solids is formed from intersection lines of "plane and plane" which form each solid, and that the problem of obtaining the intersection lines of plane and



(a) Constitutional relationships: “point ⊂ line ⊂ plane ⊂ solid body”

(b) Intersection between two solids

Figure 3: Intersection

plane can be resolved into the problem of finding the point of intersection of “plane and line”, and efforts are made to teach the importance of the geometric constitution: “point ⊂ line ⊂ plane ⊂ solid body”. Needless to say, these constitutional relationships are also important in 3D-CAD/CG.

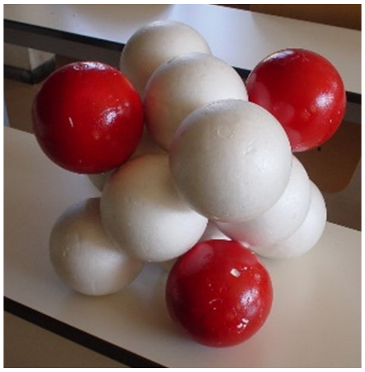
In actual design sites, 3D-CAD/CG is now usually used to practically obtain lines of intersection. But, in order to study the geometric constitutional relationships which are its background, it is important to first manually draw it step by step. It is widely known that not only in geometry, but in other types of learning, the concurrent use of sense organs typified by “eyes” and motor organs such as “hands” is a useful way to promote understanding and firmly fix memories, so it can be said that descriptive geometry is a superior way to teach solid geometry.

Regarding intersections, in addition to intersections of polyhedra cited in this example, intersections of two circular cylinders are taught. A circular cylinder is a curved surface with straight line elements, and considering it in the same way as ridge lines of a polyhedron permits the intersection of curved surfaces to be handled in the same way as polyhedrons. And the intersection of two circular cylinders is also practically important, so this is taught.

In traditional descriptive geometry courses, intersection drawing methods concerning combinations of polyhedrons, cylinders, cones, spheres, and tori in addition to the above examples are also considered. But when 3D-CAD/CG is used, it is possible to obtain solutions far more precisely in a much shorter time, so these solution methods have already lost their practical importance and are no longer taught.

3.3. Regular polyhedra

The construction of regular and semi-regular polyhedra is taught. Not only in this session, but also in other sessions, application examples are shown as many as possible in order to give the students an understanding of the fact that what they have just studied is not merely



(a) FCC-lattice
(tetrahedron + octahedron)

(b) Nuclear detection satellite
(icosahedron)

Figure 4: Application of regular polyhedra

a mathematical problem, but is intimately related to real solids (Figure 4).

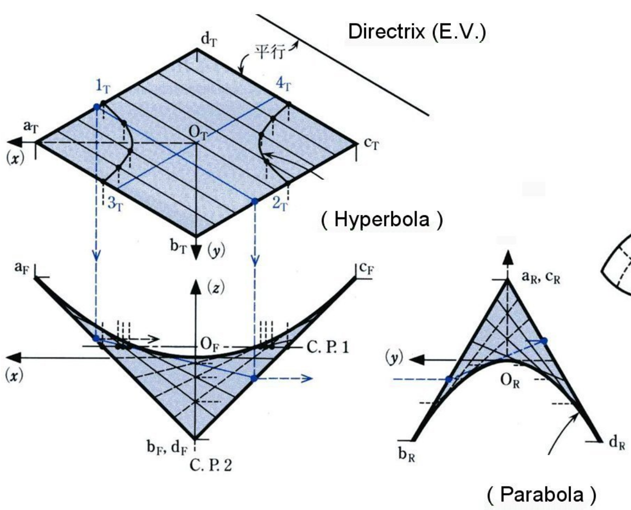
Showing real application examples may be also useful to let the students find "Geometry" from real objects, which have many attributes such as colors, textures and so on. It is no exaggeration to say that nurturing eyes that abstract only shapes from real objects is the ultimate purpose of geometry education.

3.4. Curved surfaces

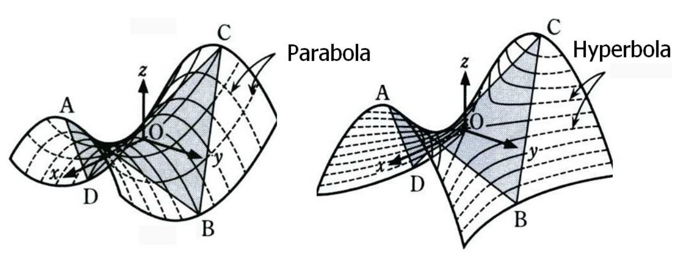
Regarding curved surfaces, the categorization of curved surfaces and some curved surfaces which are important in engineering and architecture, are taught. Categorization of curved surfaces in descriptive geometry is partly based on differential geometry, and it is important from the perspective of construction and manufacturing.

The contents of the handling of curved surfaces by descriptive geometry are discussed, taking the hyperbolic paraboloid as an example.

A hyperbolic paraboloid is a curved surface formed when straight line groups parallel to a



(a) Straight line elements



(b) Parabolic and hyperbolic elements

Figure 5: Hyperbolic paraboloid

Figure 6: An application of hyperbolic paraboloid (H-P shell) [1]

fixed plane (diretrix) connect two skew lines. As shown in Figure 5a, a hyperbolic paraboloid has straight line elements and adjoining straight line elements are at the “skew position”. In other words, they are not on the same plane. Therefore, this curved surface (theoretically) cannot be developed, and this fact, inversely, means that it is impossible to form a hyperbolic paraboloid from the development diagram without making cuts or wrinkling it. Preparing from a development diagram is often used to process metal sheets, and developability (or non-developability) of a curved surface is important from the perspective of construction and manufacturing.

A hyperbolic paraboloid has straight line elements. This fact means that it is possible for this curved surface to “be drawn using rulers”. But the fact that it has straight line elements means that it not only can be drawn using rulers, but also it can be made using linear members, lumbers, for example.

In recent years, the advance of 3D-CAD and manufacturing technologies has enabled design and construction of free-form curved surfaces, and these are now used to make automobile and aircraft bodies etc. But, when actually making something, if it is possible to make it with a simpler method, a complex method should not be adopted. Taking the hyperbolic paraboloid described above as an example, it has parabolic or hyperbolic elements (Figure 5b), so it is, in principle, possible to construct it by cutting out parabolic shaped or hyperbolic shaped members from a plate and superimposing them. But processing is very difficult, and straight line members, lumbers, are used to construct a hyperbolic paraboloid shell (Figure 6).

The discussion in this section has been limited to a hyperbolic paraboloid, but the importance of curved surface education in descriptive geometry, such as helicoid which are widely used in construction of spiral staircase and screws, and helical-convolute surfaces which are applied to gears and to embankments of roads, together with basic concepts and categorizations concerning curved lines or curved surfaces, has not declined at all, even now that 3D-CAD/CG has come into wide use.

4. Summary and conclusions

This paper discusses characteristics features of traditional descriptive geometry, together with specific examples of classes, in order to describe the importance of descriptive geometry education, and the need to switch from education focused on techniques to education on the theory behind the techniques, i.e., geometry of projection and of three-dimensional objects, especially for construction and manufacturing.

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