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Perspective: Theories and Experiments on the "Veduta Vincolata" (Restricted Sight)

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Abstract. The modern perspective admits the free rotation of the eye situated in the projection centre (restricted sight) and with this, and with the motion of the eye, it is capable of simulating even the impression of curvature of the visual field that PANOFSKY erroneously ascribes to the curvature of the retina.
The restricted sight admits an ample displacement of the observation point along the direction of the normal to the picture plane, whereas it is much less tolerant for a displacement parallel to the picture plane that emphasizes the 'marginal aberrations' and not only.
This limit has been passed, during the 17th century, by artists like Agostino TASSI through the repetition of the primary point (which is also the vanishing point of the normals to the picture plane).
Thus, there are two possible interpretive keys of an architectural perspective: on the one hand the geometrical key, which reveals itself by means of an inverse procedure, capable of describing the shapes that are represented in space; on the other the architectural key, which obtains the same result simply observing the typical characteristics of an architecture, like the symmetry, the horizontality of the architraves, the verticality of the pillars, the proportions of the Order.

Key Words: Perspective, PANOFSKY, restricted sight, *'veduta vincolata'*
MSC 2010: 00A66, 51N05

1. Introduction

Erwin PANOFSKY's work on *'Perspective as symbolic form'*, has had a powerful impact not only on the art critics, and consequently on the artistic-historical literature, but also on the studies that deal with the theme of perspective from the scientific point of view. The reflections stated by PANOFSKY in the incipit of his essay were, in fact, at times, assumed in an uncritical and superficial way, asserting that a perspective only describes an image of the space that it represents, similar to that of the human vision, if:

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- the eye of whoever is looking is positioned exactly in the projection centre that is used to generate the perspective image;
- the same eye remains motionless and therefore with the direction of the gaze perpendicular to the picture plane.

The observation condition of the perspective, in which the eye is placed exactly in the projection centre, though not necessarily motionless, but free to rotate, is known to the Italian scholars as *'veduta vincolata' (restricted sight)*.

The aforesaid interpretation of PANOFSKY's work, combined with the high esteem that its author deservedly is held in, has had weighty consequences, like those concerning a strictly axial and static reading of the architecture of the Renaissance period, along with a decline of perspective, as a planning verification tool, that has hit all the architecture of the modern movement[1]. Perspective appeared to be a misleading tool, culpable of misshaping the represented objects and unable to simulate the human perception of the space.

This prejudice towards perspective appears, today, to be totally anachronistic, particularly if we consider the continual use that our contemporaneous world makes of the perspective image in every bi-dimensional expression of space and in all its applications, both ludic and professional[2].

2. Panofsky and the *'restricted sight'*

We would therefore, first of all, like to suggest a different way of reading the part of PANOFSKY's text that is at the root of the above-mentioned misinterpretation.

In fact, in the passage where he mentions the characteristics of the 'Zentralperspektive', recalling the constraint of the sight, PANOFSKY explicitly refers to the two earliest centuries of perspective's history, namely the period from the Renaissance to DESARGUES[3], and he uses expressions that clearly are related to the aforesaid restrictions to the rules established at that time (particularly in the proposition XXX of *De Prospective Pingendi*). In other words, the passage in question, if carefully re-read, tells us that PANOFSKY, being a son of his time, did not agree with this idea of the founding fathers of the legitimate construction.

But why does PANOFSKY confine this conception of perspective to DESARGUES and not, earlier, for instance, to GUIDOUBALDO DEL MONTE, assuming that this last found the construction of the vanishing point?

Because, while GUIDOUBALDO only found a geometrical rule that mechanically justifies the convergence of the images of parallel straight-lines, DESARGUES defines the points and the straight-lines at infinity, as a support of classes of parallel straight-lines and planes, and thus explains what the vanishing points represent. With this attainment, perspective becomes

[1] See, in particular, the essays written by Alberto SARTORIS and Bruno ZEVI.

[2] We refer to the architectural rendering, but also to more elaborated applications which go from the introduction of virtual models in the film shooting (VFX) to the photogrammetric survey of last generation (IBM, Image Based Modelling).

[3] *"This correct construction was in fact invented in the Renaissance, and although later subjected to various technical improvements and simplifications, it nevertheless remained in its premises and goals unchanged to the time of Desargues."* This is not entirely true because, as far as the restricted sight is concerned, JACOPO BAROZZI DA VIGNOLA already admits the possibility of freely rotating the eye that is positioned in the projection centre. Indeed, he paints, in the Sala del Concilio of the Palazzo in Caprarola (from 1559), four columns which create a perspective depth scene in the four corners of the room, being each column painted one-half on one wall and the other half on the other wall of the corner. This is master piece that cannot be fully appreciated without freely rotating the eye.

Figure 1: The vault of the Sant'Ignazio Church in Rome, frescoed by Andrea Pozzo, is one of the several 'Glorys', a pictorial genre that were very common during the 17th and 18th century; in this fresco, thanks to the linear perspective, to the surface and the reduction of the apparent sizes of the figures, skilfully controlled, is induced the sense of ascending into the depth of the sky.

a tool capable to deal with infinity in finite terms. An epochal change took therefore place at that time, modifying profoundly on the one hand the conception of space and on the other the human ability to represent it. To experience this different conception and expressive capability, it is enough to visit one of the Roman Baroque churches and to turns one's eyes towards the vaults, where the figures precipitate attracted into the depths of the sky by an inversion of the force of gravity (Figure 1).

But let us get back to Panofsky and his *'restricted sight'*. Another limit of the legitimate construction, or better of the Renaissance perspective, is that of its relation to space, considered as an isotropic and homogenous continuum. Whereas on the contrary, admonish Panofsky, the perceptive space is anisotropic and not homogeneous.

To understand this idea, we imagined two different situations.

In the first there is the Cartesian space, in which is immersed a plane that becomes support to the perspective, generated by sectioning the visual pyramid: the perspective is thus a two-dimensional (Figure 2) geometrical structure.

In the second there are two spaces overlapping in a perspective collineation, one is isotropic, the other anisotropic (relief perspective, namely, in the Italian tradition, *prospettiva solida* — Figure 3)[4].

Well then: we identify the Renaissance perspective in the first case and, particularly, we recognize, in this first case, the second of the procedures proposed by Piero della Francesca, as well as by many other authors: those who use orthogonal projections to

[4]In the opinion of Panofsky the psycho-physiological space of the visual perception is unhomogeneous and anisotropic, like the space of the relief perspective is. The adjective 'solid' belongs to the Italian tradition of projective geometry: see, e.g., Ferdinando sc Aschieri (1895), when he defines the properties of a 'solid' homology.

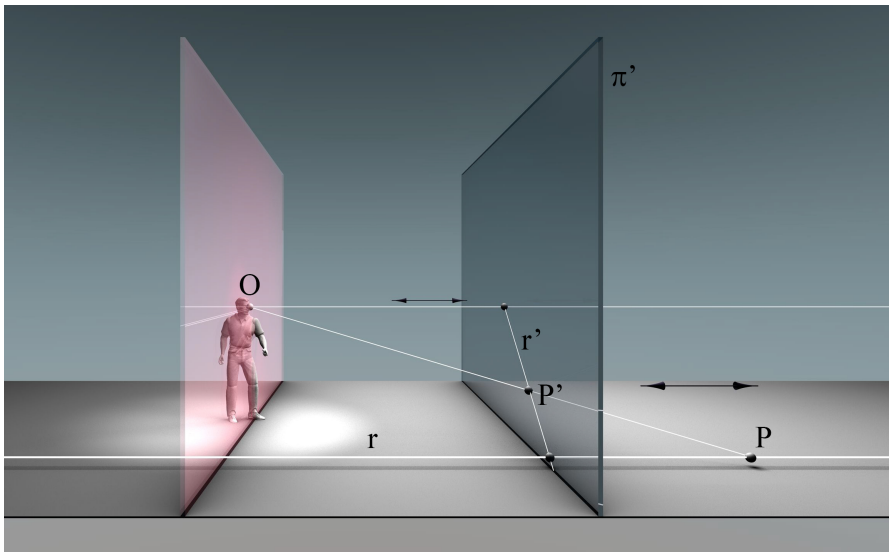

Figure 2: In the Renaissance conception, perspective is the intersection of the visual pyramid with a plane; the objects to be represented $(\mathbf{r}, \mathbf{P})$, the observer $(\mathbf{O})$, the picture plane $(\boldsymbol{\pi}')$ and the perspective itself $(\mathbf{r'}, \mathbf{P'})$ are immersed in a Cartesian space, homogeneous and isotropic.

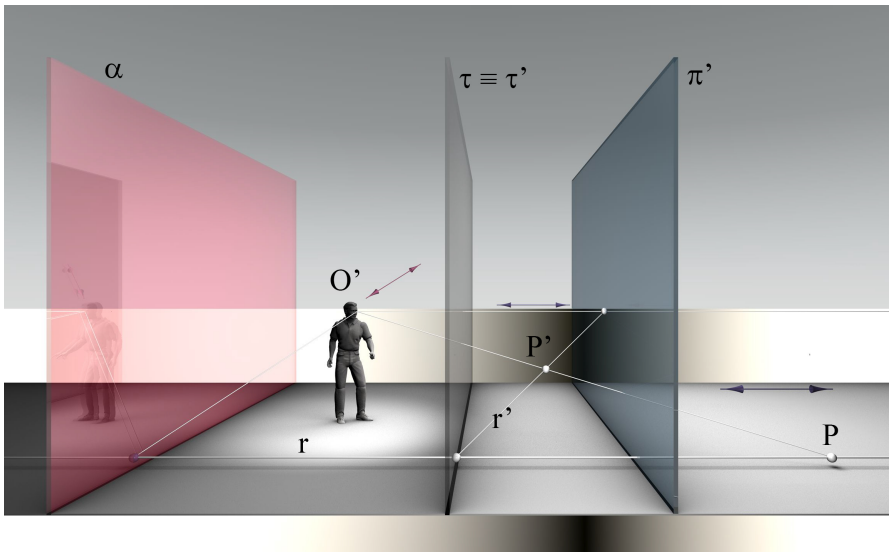

Figure 3: The relief perspective, which is the basis of the modern conception of perspective, puts into perspective relation two overlapping spaces: one, the object of the representation, is homogeneous and isotropic, the other which supports the representation is contracted and therefore unhomogeneous and anisotropic. The observer $(\mathbf{O'})$, the limit plane $(\boldsymbol{\alpha})$, the objects to be represented $(\mathbf{r}, \mathbf{P})$, are immersed in the isotropic space of the reality, which here is represented by an even background; the vanishing plane $(\boldsymbol{\pi}')$ and the perspectives of the represented objects $(\mathbf{r'}, \mathbf{P'})$ are immersed in the anisotropic space, here represented by a veiled background that alludes to its contraction. The plane of the traces or collineation plane $(\boldsymbol{\tau} \equiv \boldsymbol{\tau}')$, locus of united points and straight-lines, belongs to both spaces. The relief perspective of a straight-line is determined by the trace of the line on the plane of traces and by the vanishing point of the same line, on the plane of vanishing points, which is constructed like in the two-dimensional perspective.

generate a plane perspective. This procedure operates in the isotropic space and generates, therefore, a plane and static image.

In the second case, instead, we identify the relief perspective of Borromini and later the one, still three-dimensional, but theoretical, of Wilhelm Fiedler, who, thanks to the projective correspondences which come from the concept of infinity, transforms the isotropic space of the Renaissance conception into the anisotropic space of Baroque conception, in all its dynamism.

With Panofsky's words, we could say that relief perspective transforms the mathematical space into a psycho-physiological space, and vice versa.

It is finally interesting to note that, if Panofsky is able to measure the limits of the Renaissance conception, it is because Desargues enabled him to do it, as well as he enables all of us today. Thanks to the creation of a geometry capable to include infinity among its axioms.

Now let us get back to the Renaissance perspective eye, which is fixed and motionless. Panofsky observes that this perspective is unable to reproduce the phenomenon of the vision, exactly because it does not consider the mobility of the eye (besides of the binocular vision). Well, this is the perspective, the one in between Alberti and Guidoubaldo, which does not consider the mobility of the eye or, to be clearer, which is unable to consider it.

At this point of the essay it is reasonable to expect a surmounting, an answer to the question that arises in the reader, and namely: if not this, then, which perspective? Here comes the mobility of the eye into play, because the free rotation of the eye, around a projection centre, 'gives the field of vision a spheroid shape'. And up to this point we can only agree with all the Panofskyan reasoning.

In fact, if we observe a plane perspective from its projection centre, looking freely towards the right and the left hand side, and up and down, we will see that the straight-lines that at first appeared to be parallel and horizontal, being such with respect to the picture plane, appear to converge now at the right, now at the left; and those which appeared to be parallel and vertical, appear to converge now at the top, now at the bottom; and that, in the continuity of this transformation, the visual field seems to assume a spheroidal appearance (Figure 4).

As it is well-known, Panofsky uses considerations of a physiological nature to explain this phenomenon, but it can all easily be explained even within the rules of perspective as we

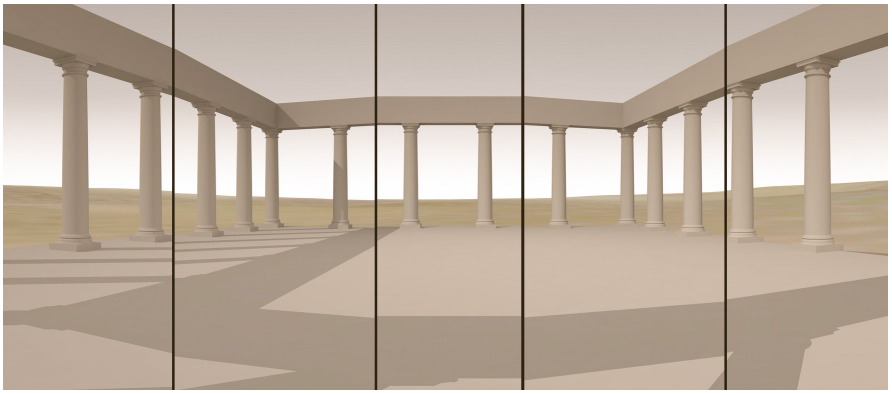

Figure 4: In the rotation of the eye, which pivots in the projection centre to look at the surrounding space, the visual field spontaneously assumes a spheroidal appearance, due to the convergence of the straight-lines that are parallel to the picture plane.

know it today, and following a path that seems to be not only simpler, but able to reconcile the legitimate Renaissance construction to the Baroque construction and the antique perspective to the modern one, extended to the projective space.

3. Perspective and Architecture: two ways to create illusion

Now we come to the crucial point that, in our opinion, constitutes the only misinterpretation of which PANOFSKY was a victim.

The points that stimulated his attention were two: the marginal aberrations and the curvature of the retina, which would cause the curvature of the visual space. A closer observation of these two reasoning may perhaps take us too far from our objectives and nevertheless, very briefly, it is necessary to mention the following.

As for what concerns the marginal aberrations: these will be seen only if the perspective is observed from a viewpoint different and very far from the projection centre. The marginal aberrations belong therefore to the rational and isotropic space, and not to the anisotropic and non-homogeneous space typical of the perception. And it is not just chance that this phenomenon can skilfully be used in the anamorphosis, exactly thanks to a displacement of the observation point, with respect to the projection centre.

PANOFSKY grounds his theories on the eighth proposition of Euclid's Optics, where Euclid affirms that straight-line segments, parallel to each other and placed at different distances from the eye, are seen under angles which are not proportional to the respective distances from the same eye. This means, for instance, that if the segment AB is situated at a distance from the eye which is twice the distance of segment CD, the angle subtended by CD in the eye is not twice the angle subtended by AB.

Beyond the debate that has heated up about this proposition and its historical significance[5], it is of interest to observe that this geometrical relation precedes the perspective, understood as section of the visual pyramid. Using a modern language, which is more familiar to us, we would say that the proposition describes the projection operation, but not yet the section operation. And when we carry out the section operation by means of any surface, leaving out the sphere centred in the eye, then the projections of the aforesaid segments will have lengths which do not represent the same ratios expressed by the angles subtended by them. All this, however, in an evaluation that is abstract and unrelated to the perspective. Because, if we restore the restricted sight, then those straight-line segments, whichever is the surface to which they belong, subtend in the eye the angles that Euclid talks about.

As for the curvature of the retina, the studies of the Transactionalist School[6] (KILPATRICK and others, 1961) have shown that the shape of the retina does not have any importance as far as the visual process is concerned; we refer, in particular, to the experiments conducted

[5] We refer, in particular, to the studies carried out by C.D. BROWNSON (1981) on the compatibility between Euclid's Optics and the linear perspective.

[6] With the term *'School'* we here refer to that group of American psychologists who, working during the 1950s around the idea that the perception of space is influenced by experience, enabled us to understand how perspective evokes, in our mind, a space. As KILPATRICK recalls in his foreword (KILPATRICK 1967), to the retinal image, that is so important in PANOFSKY's essay, correspond an infinite number of possible configurations, in space, of the perceived object. Then why do we only recognise and perceive one of these configurations? The transactionalist's experiments allowed demonstrating that we associate to the possible configurations the model, only one, which we elaborate and know through the experience. In this way, our brain builds a bridge between the world as it appears to be (projective space) and the world as it is (Euclidean space).

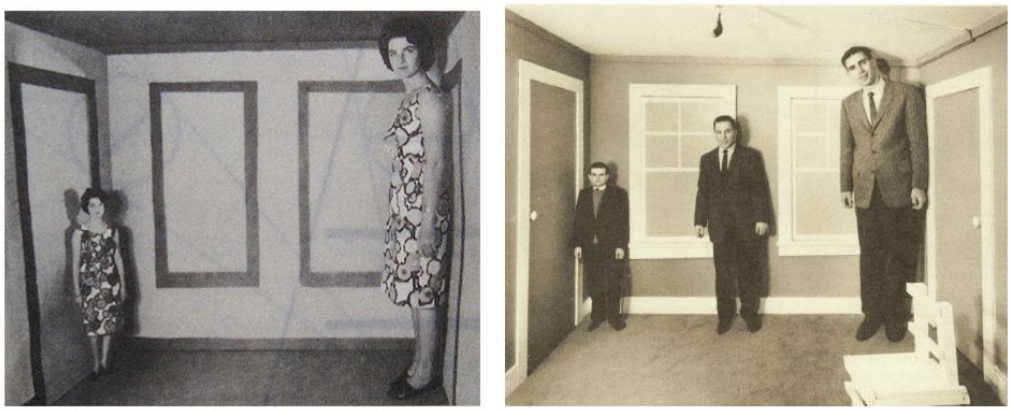

Figure 5: Two images of the original experiment carried out by Ames.

using aniseikonic glasses (Ames, 1946, see also Kilpatrick, 1967), but also to the famous distorted room (known as *'the Ames room'* — Figure 5), to the rotating trapezoid and many others (Ittelson, Kilpatrick, Ames, 1967).

As it is determined in a theory that stably endures since the 1950s, there exists a world, made of solid shapes and referable to geometrical models, which belongs to the experience of all the senses and of the reason. Our brain compares this Cartesian world with the visual experience, which is subjective, in order to chose the formal models that best respond to the same experience. Therefore, when we observe the *Ames room*, we perceive a space that has a parallelepiped shape, because this is our everyday and most common experience, even if the room is of a pseudo-pyramidal (Figure 6) form. It is not just chance: it is exactly the perspective that shapes the room in a skilful play of false correspondences, misleading the eye and the perception of the space.

We will focus further on this ambiguity, because it is present in the architectural perspectives, which are the main topic of this study. In fact, as in the experience of Ames a simulated and misleading architecture (the room of regular shape) prevail on the perspective (that is, on the real shape of that room), so in the large trompe l'oeil, from Renaissance to Baroque, there are always two interpretive keys: the geometrical key and the architectural key.

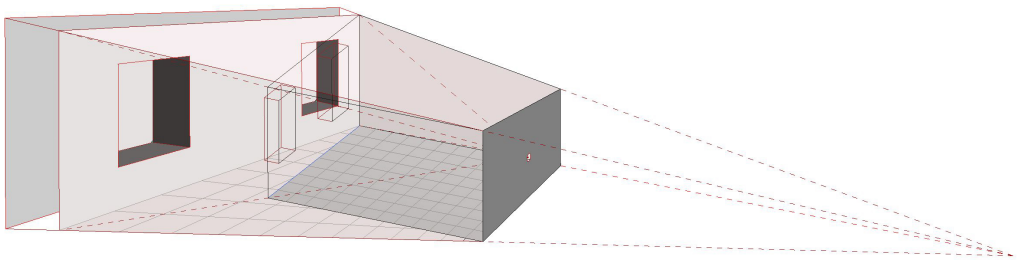

Figure 6: The true shape of the *Ames room* and its geometrical construction.

These two keys can be coherent or not, and it can therefore happen that one perception strengthens the other, or that one (and it is always the architecture) corrects the other. As we will see shortly, these two manners have, in the illusionistic perspective, their champions: on the one hand Andrea Pozzo, who uses perspective in full accordance with its geometrical code, on the other Agostino Tassi, who on the contrary does not respect that code, he just imitates it, deliberately introducing numerous errors or corrections, which nevertheless are not considerable because they are disguised by the dominant interpretative key, namely that of the architectural space.

In particular, to better understand, on the one hand Pozzo respects the uniqueness of the vanishing point of straight-lines that are image of parallel straight-lines in the illusory space, on the other Tassi skilfully bestows to those straight-lines a large number of vanishing points, as many as is needed to follow the onlooker in his movements inside the frescoed room.

These vanishing points are those relative to the straight-lines, perpendicular to the picture plane, that are nearest to the onlooker; he will thus always see a correct perspective on the portion of wall that he is seeing best, and he will not remark the incongruities that are found on the areas of the painting that are farther away from him. When the observer moves, in parallel with the wall, to enjoy the other parts of the fresco painting, then the parts of the perspective that were incorrect, with respect to his first standing position, become correct, and vice versa the parts of the perspective that at first were correct will become incorrect; but these parts grow farther away and they are therefore harder to see. The illusionistic effect of the painting as a whole is still there, or rather, it is intensified.

Therefore, on the one hand Pozzo takes advantage of the eye's rotation, pivoting in the projection centre and resorting to other expedients, mitigating the effects of a displacement of the observer; on the other hand Tassi disregards the code in order to nullify the effects of the displacement.

The perspectives painted by Tassi, thus, behave like the *Ames room*, because they appeal to the viewer's experience and to his mental models, in order to hide the derogations imposed to the perspective rules and the true shape of the space that these perspectives describe, if interpreted literally.

All this to say that, if we want to go more thoroughly into these aspects of perspective and its applications to the art of the trompe l'oeil, we do need to reason about the question that we posed at the beginning of this paper, and namely on what the *'restricted sight'* is and on what its limits are; on how, in certain cases, these limits can be overcome and on how the related effects can be experimentally reproduced and measured.

4. Restricted sight: a virtual experiment

We have thus created an interactive model, which permits us to reproduce in laboratory a condition that otherwise would have required much more effort. The model represents a bare natural environment in which a peristyle formed of twenty-four Tuscan-order columns is placed (Figure 7).

The peristyle is intersected by a wall, so that eight of the columns are located beyond it. On the wall is painted the perspective of the part of the peristyle that the wall is hiding. The onlooker can freely move about inside the scene, and should start standing exactly on the projection centre point. The following experiences are possible.

– The rotation of the eyes of the viewer: the perspective perfectly simulates the illusory space in every condition, making it appear to be the natural prosecution of the real

Figure 7: The model used in the restricted sight experiment, as it really is. The wall that cuts it at a fourth of its length, houses an illusory perspective that represents the part of the colonnade that is hided.

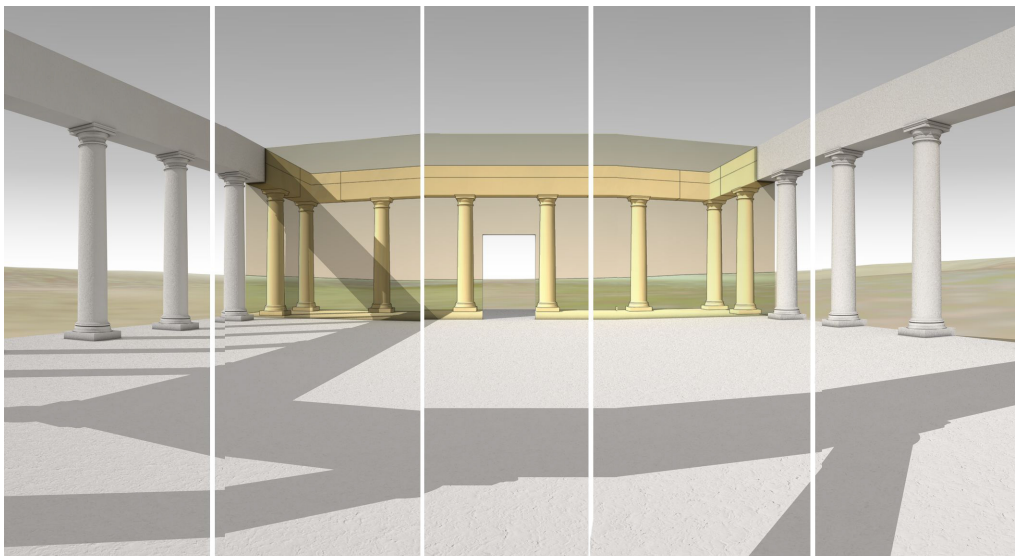

Figure 8: The perspective, if observed from the projection centre, rotating the gaze freely around, simulates the sensation of curvature of the visual field, described by Panofsky.

space; in particular, rotating the eyes to the left and to the right, up and down, we perceive the convergence of the images of the lines that were parallel in the normal observation, with a 'motionless eye', as Panofsky would have said, and we can see how, in the continuity of the vision, this mutable convergence gives a spheroidal shape to the visual field (Figure 8).

– The displacement of the viewer along the normal to the picture plane: the perspective continues to simulate a space that is perfectly contiguous to the real one, but it expands it or contracts it depending on whether the viewer is moving away from or nearer to the painting (Figure 9).

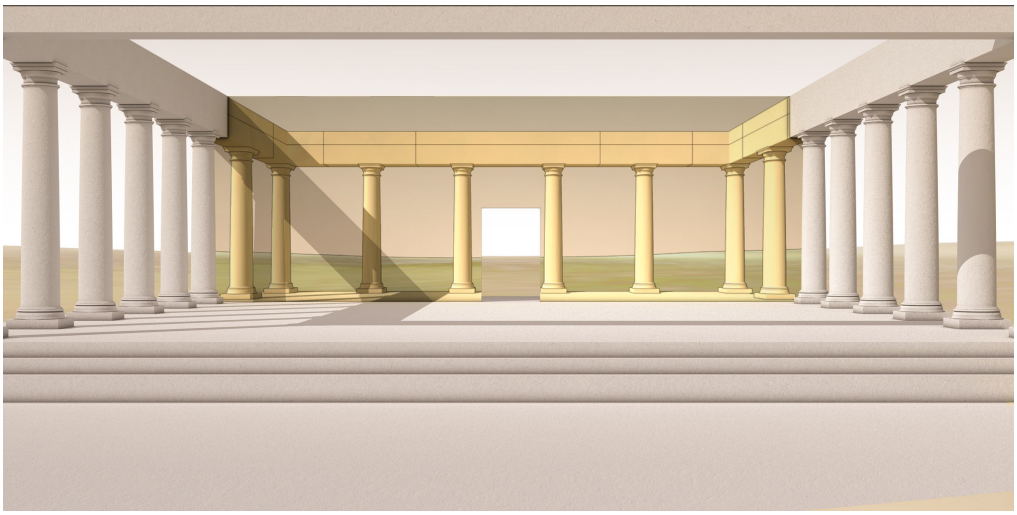

Figure 9: The displacement of the viewer along the normal to the picture plane produces a contraction or a dilatation of the illusory space.

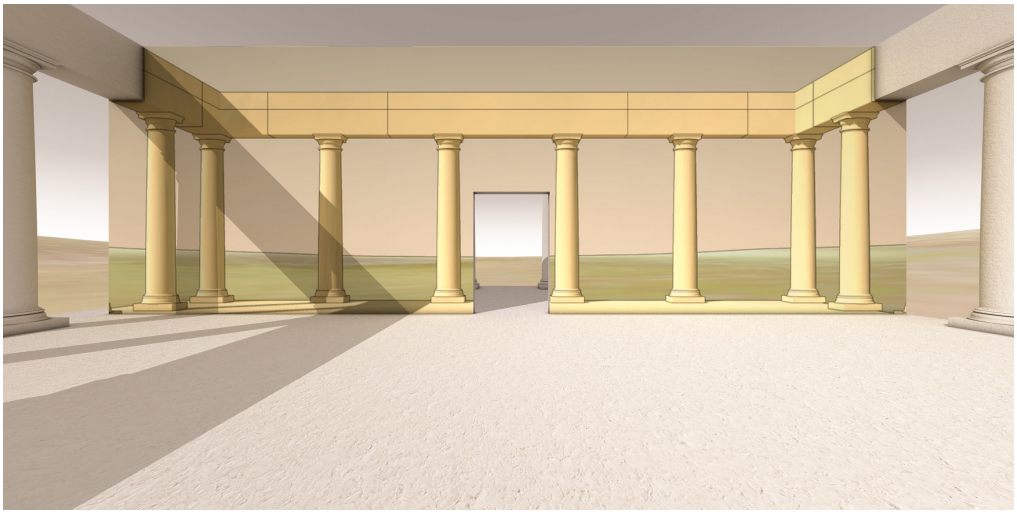

Figure 10: The contraction or the dilatation of the illusory space are proportional to the displacement of the viewer, as this scheme easily shows us.

- The extent of the effects of the displacement of the viewer is described by a linear proportion (Figure 10): if the viewer is moving away from the picture plane twice the primary distance, the space simulated by the perspective is twice deeper than the real one; if the viewer halves his distance from the picture plane, the perspective contracts, by half, the depth of the simulated space; if the viewer moves so close to the wall whereon the perspective is frescoed that he goes beyond the door opening that is located at the centre of the wall, then the perspective will collapse into the two dimensions of the plane.
- Finally, the displacement of the viewer along the parallel to the picture plane: the perspective rapidly loses the continuity with the simulated space and this is why the quadraturist painters increase the number of vanishing points of the straight-lines that

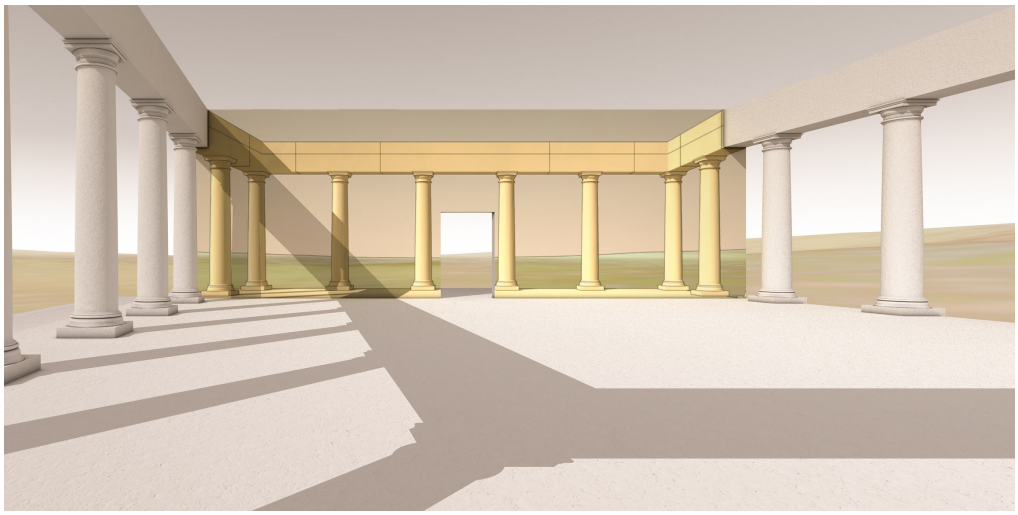

Figure 11: The collimation between the illusory space and the real space is lost when the onlooker translates parallel to the picture plane: this is the phenomenon, together with the marginal aberrations, which induces Agostino Tassi to increase the number of vanishing points of the straight-lines that are perpendicular to the picture plane.

are perpendicular to the picture plane, obtaining in this way an extension of the limits of the restricted sight (Figure 11). The multiplication of the above-mentioned vanishing points allows, moreover, to restrain the apparent deformations (the ones Panofsky calls 'marginal aberrations').

That said, we would like to apply the observations made so far to the investigation and analysis of the two emblematic cases that we have mentioned.

The first is a perspective, situated in the Casa Professa del Gesù in Rome, painted by Andrea Pozzo during the years 1681–1686; the second is a room in Palazzo Lancellotti, also in Rome, frescoed by Agostino Tassi during the years 1617–1621.

In both cases we deal with unitary perspectives, that is to say, they have in common the intention of simulating a single architectural space, even if this is painted onto various surfaces; as if the aforesaid illusory space were projected from the centre onto the whole host enveloping environment. This is why it would be helpful to coin a new term for this kind of perspectives, for instance poly-perspectives, or multi-faceted perspectives, considering the three-dimensional form of the support surface.

The only difference between these two perspectives is in the freedom of the onlooker who, in the first case, freely can move his eyes in all directions, paying, though, the effects of his displacement, as we will see in a while; whereas in the second case the viewer not only is free to look in all directions, but also to move about in the room.

The corridor in the Casa Professa del Gesù is a long and narrow space, totally devoid of decorations in relief, and with a low-profile barrel vault ceiling. The end wall is oblique, with respect to the axis of the corridor. The viewer is standing, approximately, at the centre of the corridor and he therefore sees the two long side walls and the barrel vault strongly foreshortened (Figure 12).

The study of this multi-faceted perspective, unitary as we said, have to be done analyzing, independently one from another, the parts projected on the different enveloping surfaces, since each of these parts of the perspective has its own characteristics.

Figure 12: Panorama of the corridor that leads to Sant'Ignazio's rooms in the Casa Professa del Gesù, in Rome.

In the side walls (Figure 13), the perspective goes much beyond the limits set by the application of the art of perspective to the visual field of the viewer, limits of which we can find explicit treatment already in Piero della Francesca and which has then been handed down from treatise to treatise until the 20th century.

As is known, some scholars attribute to this limit angle an aperture of 60 degrees; others attribute different values, each time explaining these statements in the most various ways (and the most inconsistent). Actually, the limit gains sense only in the case of the legitimate construction (from Renaissance to Desargues, as Panofsky recalls) because it makes sense to talk about it only in the case in which the eye is obliged to look at the perspective, keeping the gaze perpendicular to the picture plane. And in this case too, we should have to make

Figure 13: View of a side wall along the normal to the picture plane: to take the whole painting in at a glance, it is necessary to dilate the visual angle until it measures 150 degrees.

Figure 14: View of the same wall, with the direction of the gaze parallel to the picture plane.

many explanations, which we have not space for here. But Andrea Pozzo very well knows what the constraints of the restricted sight are, and what not, so he takes it for granted that the viewer can look at his perspective with strong angulations and even with the direction of the gaze rotated at an angle of 90 degrees with respect to the walls and therefore parallel to these (Figure 14). This leads to the presence of considerable apparent deformations in the most peripheral areas, deformations that, however, we call 'apparent' because they can only be seen if the onlooker moves away from the restricted-sight position.

The 'lesson' Pozzo teaches us also helps to show, were it needed, that perspective and anamorphosis is the same thing: both obey the same rules.

The entrance wall (Figure 15) houses the only part of this perspective that does not have particular characteristics, unlike the end wall which is, as already mentioned, strongly oblique - Figure 16). Pozzo here paints an illusory space that widens and regularises the

Figure 15: View of the wall, with the entrance that gives access to the corridor.

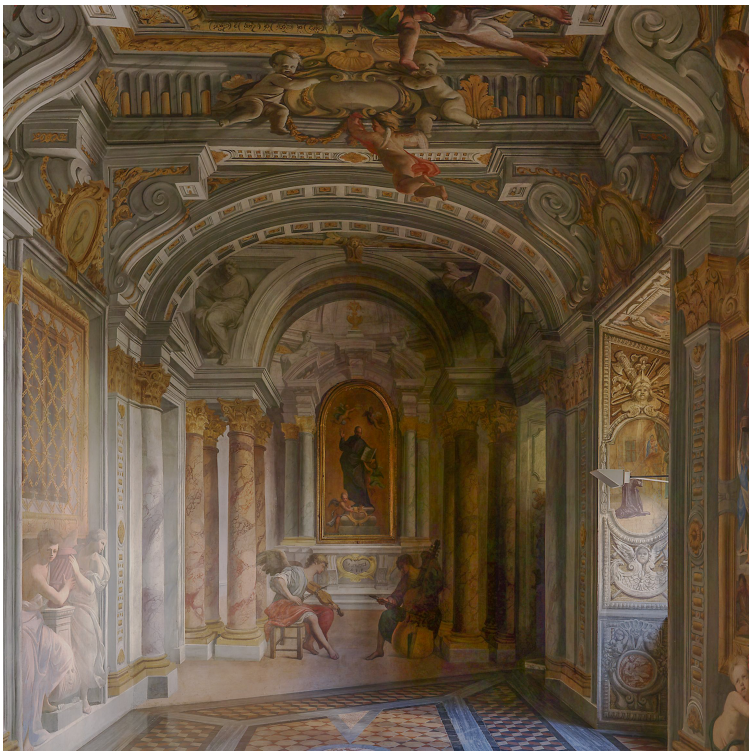

Figure 16: View of the end wall, with the direction of the gaze parallel to the axis of the corridor, and therefore oblique with respect to the same wall.

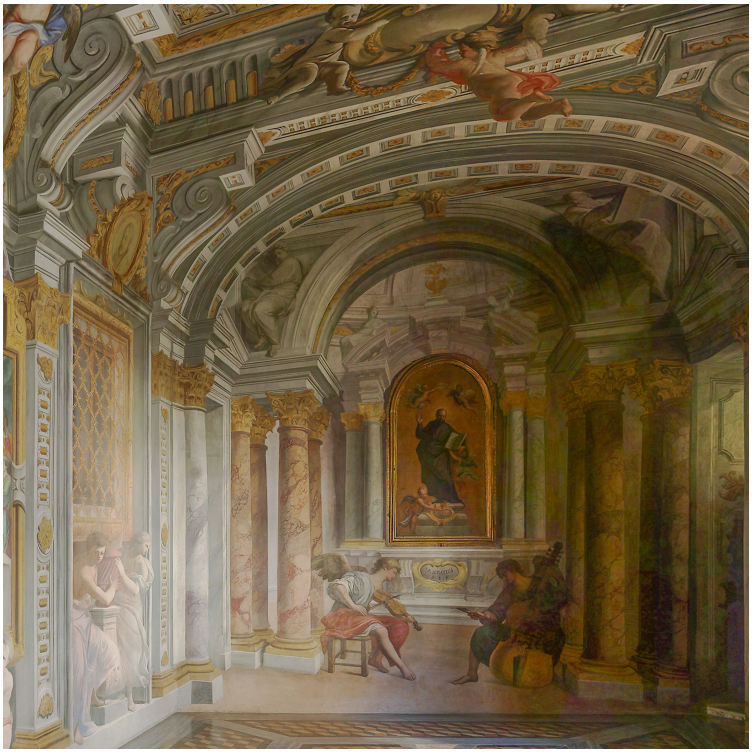

Figure 17: Frontal view of the end wall, where the presence of two vanishing points is evident. This image is taken from the projection centre.

corridor, insomuch that the viewer gets the feeling of standing in front of a perspective that only has one vanishing point, and precisely that of the images of the parallel straight-lines of the side walls of the corridor, which ideally proceed into the depth of the scene. Actually the perspective, being painted on a support that is oblique with respect to the sides of the corridor, uses two vanishing points (Figure 17).

Here we deal with another 'lesson' of Andrea Pozzo who shows us how conventional the classification of the perspectives in frontal, at an angle and of an inclined picture plane is, since a perspective of one kind transforms itself into the other when it is observed by freely rotating eyes: the experience of the colonnade very well emphasized this.

Finally, the barrel vault. Here the illusory effect reaches its highest level, because the perspective frescoed by Pozzo goes as far as to destroy and dematerialise the support, substituting the smooth surface of plaster with a rich framework of corbels and beams perfectly rectilinear, which evenly divide the deepest spaces in which angelic figures are moving (Figure 18).

How has this miracle of geometry been achieved? Very simply intersecting the vault surface with the planes that project the edges of the beams, as immediately will understand the viewer who has severed himself from the constraints of the sight (Figure 19).

Now, without lingering further on the geometric peculiarities of this work, and keeping to the point, we will consider the solutions utilized by Pozzo for the decorations of the corridor, to make them usable even in their most common use, that is, the visitors crossing of the corridor, which therefore is subject to a continuous and strong displacement of the viewer.

Among the lacunars of the ceiling there are angels that hold some medallions, on which are painted several portraits. This detail encourages a reading, bi-dimensional too, of the illusory space, almost as if it were the representation of a portrait gallery. And indeed, the sides of the corridor are also hosting some sacred representations that are in two dimensions and that therefore can be read in two ways:

- standing on the projection centre these representations look as if they were paintings hanging in the illusory space of the gallery that the perspective represents;
- moving about in the corridor, the illusion effect collapses, but not the paintings, which can be seen one by one as if they were part of the real space.

In this way the dynamic passage of the corridor retains an interest, even if the perception of depth vanishes, and the perspective, no longer readable, becomes a decoration of abstract beauty (Figure 19).

Different, but not less efficacious, the expedient used by Agostino Tassi at Palazzo Lancellotti (Figure 20). Here the artist does not want and does not allow the perspective to lose its effectiveness because of the onlooker's movement inside the space of the corridor. The perspective therefore follows the viewer in his movements displaying the same number of images, skilfully merging them together the one with the other.

The study of the fresco shows, in fact, a multiplication of the main point, or better, of the point which is the foot of the perpendicular drawn from the eye to the picture plane, and that coincides with the vanishing point of the straight-lines that are perpendicular to the picture plane[7].

The effect that arises from this is amazing, but it can fully be understood only in a direct inspection. In fact, the dimensions of the room (fourteen by nine metres and nine metres

[7]Reconstruction of the vanishing points in the perspective mentioned in Figure 20, created by Agostino Tassi to dilate the limits of the restricted sight.

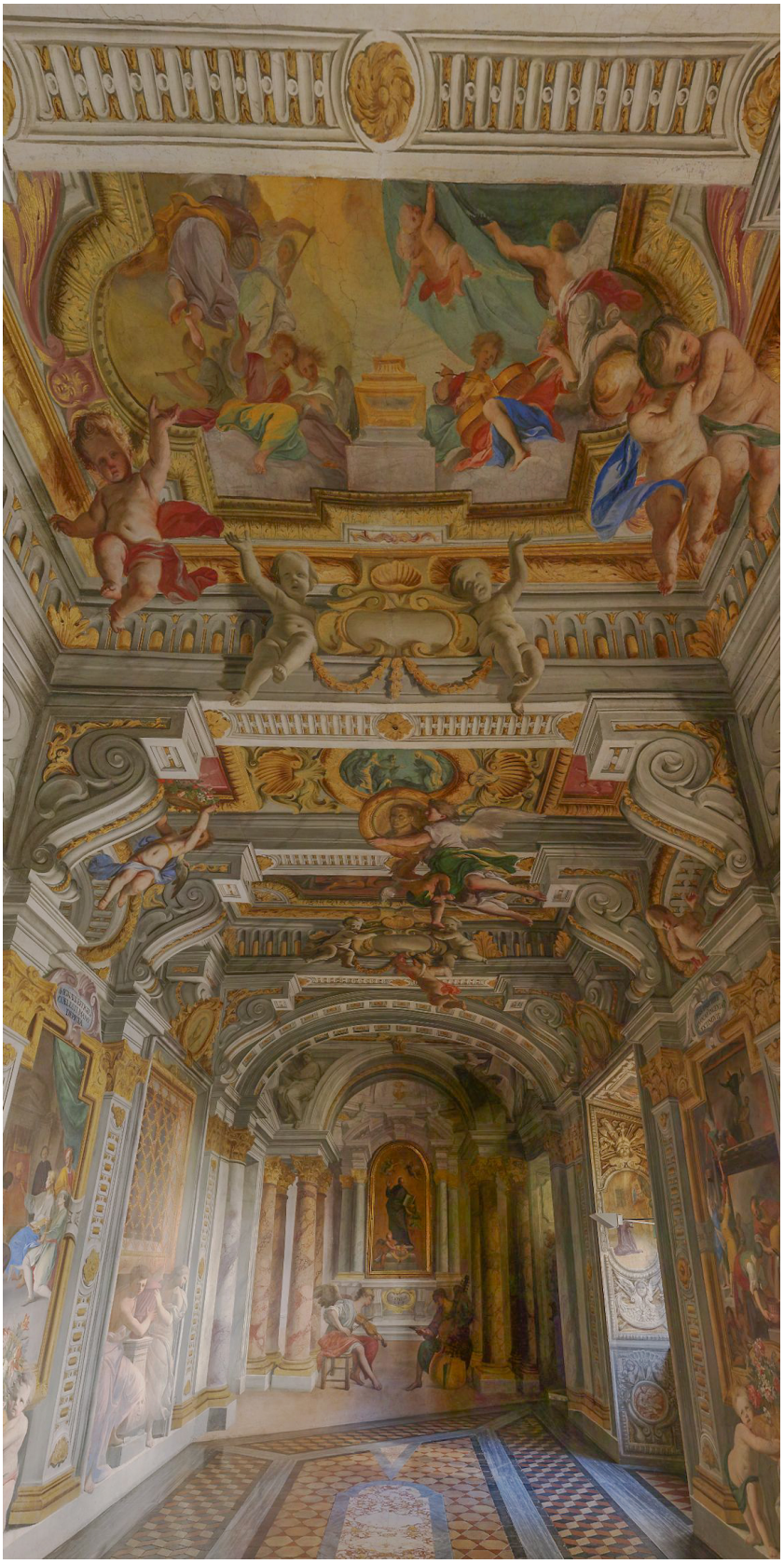

Figure 18: View of the vault; the perspective simulates corbels and rectilinear beams which evenly divide the spaces.

Figure 19: Detail of the anamorphosis that decorates the vault, seen standing on a spot that is different from the restricted sight position.

and a half in height) are such that it is impossible to appreciate the perspective as a whole, looking at it from a single point of observation. The viewer, therefore, is induced to linger on the detail that is nearest to him and there he finds, unfailingly, a correct perspective; whereas the farther parts, which are not as much present from that observation point, perfectly hide the derogations that that point of observation, and only that one, opposes to the perspective rules.

5. Conclusions

To recapitulate, we have defined the following:

1. The modern perspective (with this term we allude to the perspective from Desargues onwards, to remain at Panofsky's scheme) admits the free rotation of the eye situated in the projection centre (restricted sight) and with this, and with the motion of the eye, it is capable of simulating even the impression of curvature of the visual field that Panofsky erroneously ascribes to the curvature of the retina.
2. We can ascribe the aforesaid error to the fact that Panofsky writes in 1927 and thus

Figure 20: One of the walls of the room, situated in Palazzo Lancellotti, frescoed by Agostino Tassi.

a quarter of century before the scientific achievements of the transactionalist school. Furthermore he is influenced by the discovery of the optical corrections adopted in the classical architecture and he believes that the ancients did apply the same corrections, with the same effects, to the perspective. In this supposed analogy, nevertheless, he does not consider the different sizes of the two artefacts, nor does he consider the visual fruition of the ones and of the others.

3. The restricted sight admits an ample displacement of the observation point along the direction of the normal to the picture plane, whereas it is much less tolerant for a displacement parallel to the picture plane, that emphasizes the 'marginal aberrations' and not only.
   This limit has been passed, during the 17th century, by artists like Agostino Tassi through the repetition of the primary point (which is also the vanishing point of the normals to the picture plane). The perspective, thus revised, transforms itself from one single perspective geometrically coherent into a whole of perspectives, each of which is coherent in itself, merged into one, like the single frames of the tracking shot in a cinematographic filming. This produces a multiplicity of vanishing points, like those observed by Panofsky in some of the Roman paintings of Second Style. It is interesting to note that the straight-lines that Panofsky interrupts on the vertical axis of the Roman perspective construction can instead be interrupted on the horizon[8], which is

[8] An example of perspective in a painting of Second Style, where a use of the vanishing points that is similar to the one adopted by Agostino Tassi can be seen.

in line with what we have said above. Furthermore, PANOFSKY limits his verification efforts to the research of the vanishing points, whilst the research of a rule, in perspective, should always be extended to the reduction of the apparent sizes. Now, if the perspective is correct, from a projective point of view (legitimate, as PANOFSKY would have said), the cross-ratio of four points which define, in the illusory space, three equal intervals, has to be equal to four-thirds. Well, this measurement, made on many Pompeian and Roman (Casa di Augusto) perspectives, returns exactly this value, showing that, at least as far as the depths and their illusionistic rendering is concerned, the perspective is the one that the Renaissance artists have re-discovered, perhaps with different procedures, but with the same result.

4. Finally, the study conducted by PANOFSKY does not consider the two possible interpretive keys of an architectural perspective: on the one hand the geometrical key, which reveals itself by means of an inverse procedure, capable of describing the shapes that are represented in space; on the other the architectural key, which obtains the same result simply observing the typical characteristics of an architecture, like the symmetry, the horizontality of the architraves, the verticality of the pillars, the proportions of the order. When we observe such a perspective we receive a strong impression of depth because of the sum of these two effects and because of the motives that the *Ames room* experiment has very well emphasized.
   Now, these two keys can be coherent, that is, aiming at achieving the same result (like in POZZO), or they can be independent of each other (like in TASSI), in which case the second prevails over the first because it appeals to a psychological mechanism, whereas the first can be verified only by means of reason.
   From this derives the great liberty that perspective gives the artists and all those who want to use perspective to simulate the space, 'bursting through the depths' of the wall surfaces, extending the architectural frameworks, projecting human figures into the depths of the sky. This power of the architectural illusion is so strong that, if we project a perspective onto a vault from below upwardly, not from a projection centre, but following the normal to the impost level, like TASSI does at Palazzo Pallavicini Rospigliosi, we still obtain a strong and happy effect of depth, totally in derogation to the restricted sight, sight that, in this case, simply is not possible.

In conclusion, we believe that a deeper study of the restricted sight, of its limits and its effects, more thorough than this, can eventually free perspective, and its history, from some false opinions that derive from an uncritical reading of PANOFSKY's famous work. At the same time, this study can unveil the parts of the Panofskyan philosophy that are still entirely valid and which parts have to be revised in the light of the most recent conquests of science.

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