Journal for Geometry and Graphics
Volume 19 (2015), No. 2, 269–282.

Decomposition, Deformation, Dispersion and New Complexity in Architecture and Urban Planning

Paweł Rubinowicz

West Pomeranian University of Technology Szczecin, Poland
Al. Piastów 17, 70-310 Szczecin, Poland
email: pawel@rubinowicz.com.pl

Abstract. The article examines the concept of complexity in architecture and urban planning presenting a proposal for a systematics within the four methods describing complex forms: decomposition, deformation, dispersion, and new complexity. The principles of each method are defined on a general level. However, within each of the methods, more complex simulations can be formulated for an unequivocal geometric encoding. The methods can be applied to the creation and the analysis of both, abstract (2D and 3D composition) and real (specific building or city) forms. The article investigates also application of these methods to the interpretation and systematics of contemporary architectural creation, to the development of new tools in design, and to a better diagnosis of city development principles. In recent years new tools have appeared enabling an application of advanced digital techniques to the exploration of unique formal solutions in architecture and new analytic methods for urban structures. Geometry becomes more and more an universal language, important for establishing bases for new digital techniques and for the interpretation of the structure of complex forms in architecture and urban planning.

Key Words: contemporary architecture, urban analysis, fractal geometry, deterministic chaos, 3D virtual city models.

MSC 2010: 00A67, 51N05

1. Digital turn towards geometry

Contemporary digital tools create new possibilities for developing buildings and cities. They so broaden the aesthetic awareness. Using computers in architectural and urban planning is already a well-established process. It became popular in the 1980s and 90s. Initially, the application of new techniques boiled down to changing from analogue to digital tools. Although the new environment influenced the quality and efficiency of work, it did not bring

ISSN 1433-8157/$ 2.50

new values to designing, similarly to the fact that replacing a typewriter by a computer text editor did not influence the quality of content. Recent years witnessed a major change. New tools and possibilities of using more advanced digital techniques become popular which enabled a wider use of computers' capacities. This has been followed by a search for unique formal solutions in architecture and new methods of analyzing the city space.

Catalysts of the new development in architecture include parametric modeling, NURBS and new prefabrication techniques: the possibility of recording a digital form of a facility (CAD) as a sequence of instructions (CAM) recognized by CNC machines, the dynamic development of techniques and growing accessibility of 3D print, et cetera. New technologies enable the creation of complex forms and reduce the building cost. A sign of these changes is the growing interest among young architects in programming, creating scripts (e.g., Grasshopper) and, consequently, in a more advanced form coding. Although the interest in complexity has been present in architecture since deconstructivism (1980s) and it originated in its primary phase from philosophy and literature, digital tools are the driving force of the contemporary development.

In urban planning, the recent tool is a 3D virtual city model. The development in geoinformation research, airborne laser scanning techniques (LIDAR), aerial photography analytic techniques, which enable an automation and significant acceleration in the process of generating virtual city models, and new standards of encoding urban structures (CityGML) cause, that the accessibility and accuracy of 3D models is increasing in a geometric progression. While sitting at a desk and staring at the computer screen, we can easily browse through virtual landscapes of most of world's agglomerations, using common free software of Google Earth. The applied tool extends the possibilities of simple visualization. 3D city models are more often a medium for the application of advanced urban analyses, which couldn't be executed without computer support.

In both architecture and urban planning, the subject of creation or analysis is complexity. However, geometry is an universal code, necessary for establishing bases of new digital techniques and for the interpretation of complex forms. This article presents a proposal for a classification of this problem with respect to architecture and urban planning.

2. Decoding the complexity

The development of IT obviously influences different means of creation, including architectural design. Computer tools and digital fabrication techniques allow rising buildings of very complex shapes. Many architects use these opportunity. There is appearing a new style of digitally designed architecture and a related scientific research area. On the other hand, the tendency to use complex forms has outlined in architecture even before the digital revolution. One example among others is the 1980s deconstruction movement, where natural harmony in composition was replaced by transformations and reorganizations of the form. This article does not undertake a broader critique on architecture. It aims to recognize mechanisms behind the creation of complex forms, in extent possible to be described in geometric language.

This chapter presents the proposed classification of possible methods for developing complex forms. Their hierarchy was created to interpret contemporary architectural projects. The methods of *decomposition*, *deformation*, *dispersion*, and *new order* have been described by the author in his PhD thesis [16]. The description is general enough so that it can apply to various real or abstract spatial compositions, including urban structures. A form is understood as a specific shape, and the methods describe the construction process. Definitions

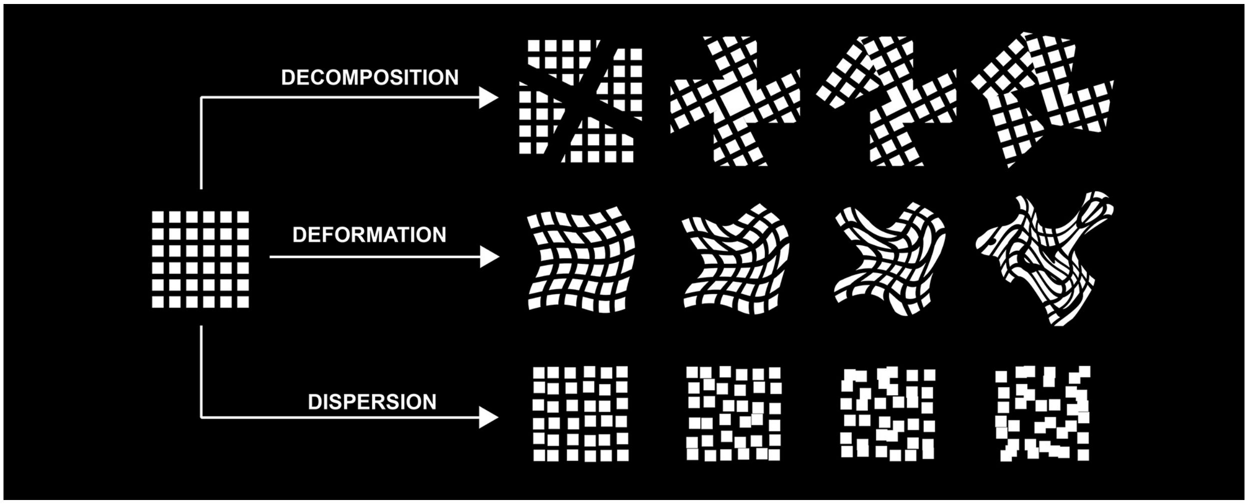

Figure 1: Schemes presenting methods of decomposition, deformation and dispersion

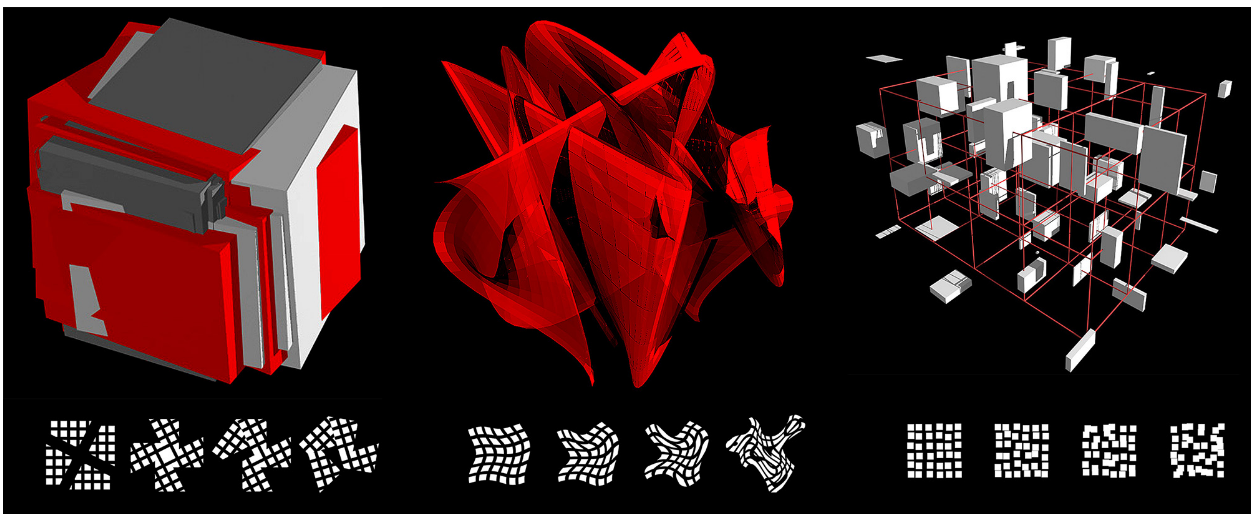

Figure 2: Compilation of schemes presenting creation of complex forms with examples of spatial arrangements. From left: methods of decomposition, deformation and dispersion

of the methods determine pattern and scope of possible geometric operations (measures). However, each method allows more detailed simulations that provide an unequivocal, strict geometric description. The presuppositions of these methods are presented in Figure 1. An example of developed compositions can be seen in Figure 2.

2.1. Decomposition

A set of possible operations within the mechanism of this method includes *break*, *crush*, *divide*, *separate*, and *fragmentation*. The goal is to break the initial form or the free compilation of various forms. The main feature of the method is the purposeful breaking and far reaching transformation of a structure leading to new composition and aesthetic values. It is based on two essential measures:

A) dividing the form and separating specific parts of it in a way which does not result from its construction (form organization logic);

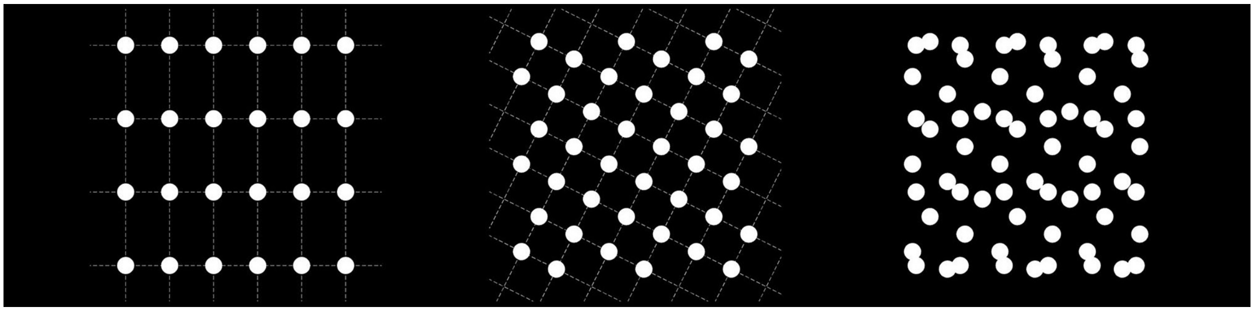

Figure 3: Example of an application of the decomposition method: simple overlapping of two regular grids leads to a much more complex form

B) composing a form using independent parts — not by matching their shapes but by clashing them and allowing mergers.

The decomposition can be used to rebuild the initial form (first A, and then B) or to develop a new form (only B). The aim is to highlight the heterogeneity of its construction, based on a strong articulation of diversity of its parts and their geometric independence. The measure A is presented in Figure 1. An example of B is the simple overlapping of two regular grids, which leads to a much more complex form (Figure 3).

2.2. Deformation

This method aims at plasticizing and transforming the structure of a form and at the same time preserving its indivisibility. Operations or terms typical for this method include the following: *link*, *connect*, *fold*, *unify*, et cetera. The presuppositions of this method are associated with mathematical topology. The composition measures concentrate on the form, treated as a whole, contained in one specific shape with all constituent parts subordinated to it. The construction process involves two measures:

A) the transformation of the initial form while preserving links between its parts;

B) making the form from its parts — by a fluid merger leading to their mutual unification.

This method can be used for rebuilding the initial form (measure A) or building a new form (measure B). The goal is to highlight the homogeneity of the form based on the unification of its parts and their complete subordination to the general geometric pattern of that form. The measure A is presented in a chart (Figure 1). An example of using the measure B is the simulation presented in Figure 7.

2.3. Dispersion

The main feature of this method is to use a random (stochastic) factor in the process of shaping the form. It is also necessary to distinguish a number of equal constituent parts. Their large number enhances the dispersion effect. Building a form involves independent transformations of its constituent parts, e.g., *moving*, *rotating*, *changing proportions or color*, et cetera. Although only one and the same type of transformation is applied to all constituent parts, the way how it is used (for each part) depends on a random factor. This method can be used for rebuilding the initial form of an orderly structure. By using dispersion, the structure undergoes a gradual transformation losing its original clarity. This principle is presented in Figure 1. The use of a random factor can be measurable as a pre-set simulation parameter.

The similarity of dispersed forms is an interesting issue. It turns out that two forms can be similar, although no pairs of their constituent parts are identical. This has been presented in the simulation in Figure 8d.

2.4. New complexity

This method aims at shaping complex forms by a simple process of formation. The principle of organizing (building) such forms is described as a system of higher order [16, 17]. The method refers to the mathematical theory of deterministic chaos, fractal geometry, and emergency. Rules can be expressed in different ways, for instance according to the mathematical, fractal and emergent models. Each of the models defines a separate category of form definition. In the mathematical model, the form is defined using various formulas and rules. Computer simulations enable to observe basic systems of higher order at a purely theoretical level, e.g., the Mandelbrot set, cellular automata simulations, IFS-fractals (Figure 4). In the emergent model, the process involves solely the determination of spatial relations and rules of mutual influence between particular elements of the form. An example is the simulation of cellular automaton CA (Figure 4b) [20]. The principle of the fractal model is based on transformations that can be described in the form of a simple geometric pattern. An example of the above is a simulation using IFS [12] (Figure 4c).

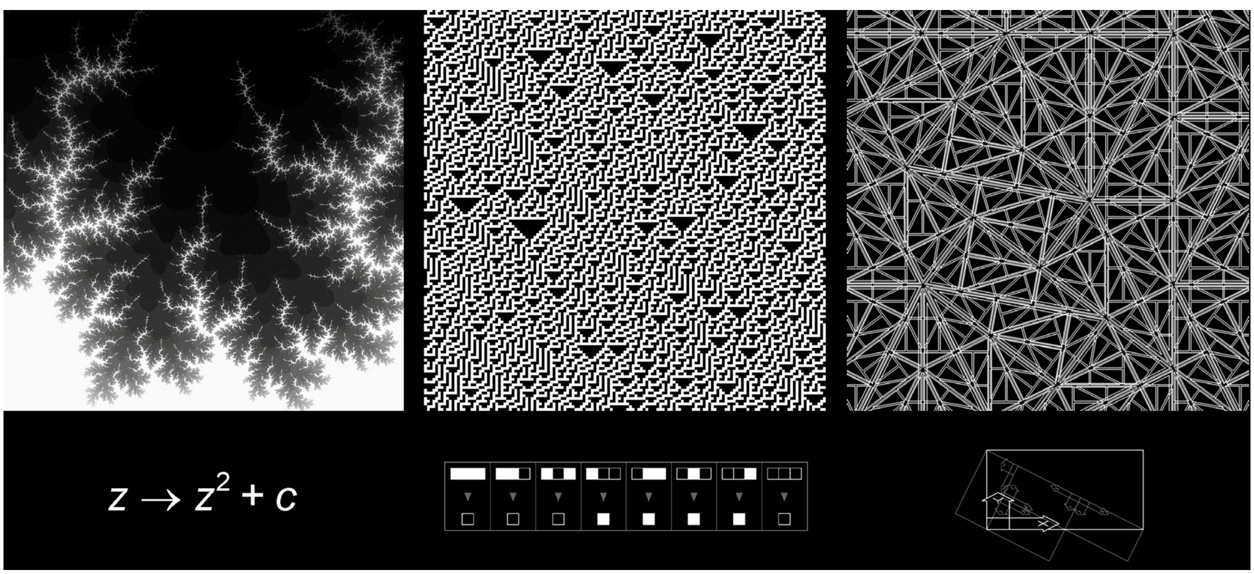

Figure 4: The method of new complexity: complex forms (upper part) are defined by simple rules; (lower part): M set, 1D CA and IFS simulations

3. Complexity in architectural creation

3.1. Decomposition in architecture

A clear example of the effort towards developing complex forms in architecture by using the method of decomposition is the postmodern architectural movement *'deconstructivism'*. It started in the early 1980s and matured in 1988 when the Museum of Modern Art in New York (MoMA) organized the exhibition of *Deconstructivist Architecture*. This exhibition presented works of at that time little known architects, such as Bernard Tschumi, Rem Koolhaas,

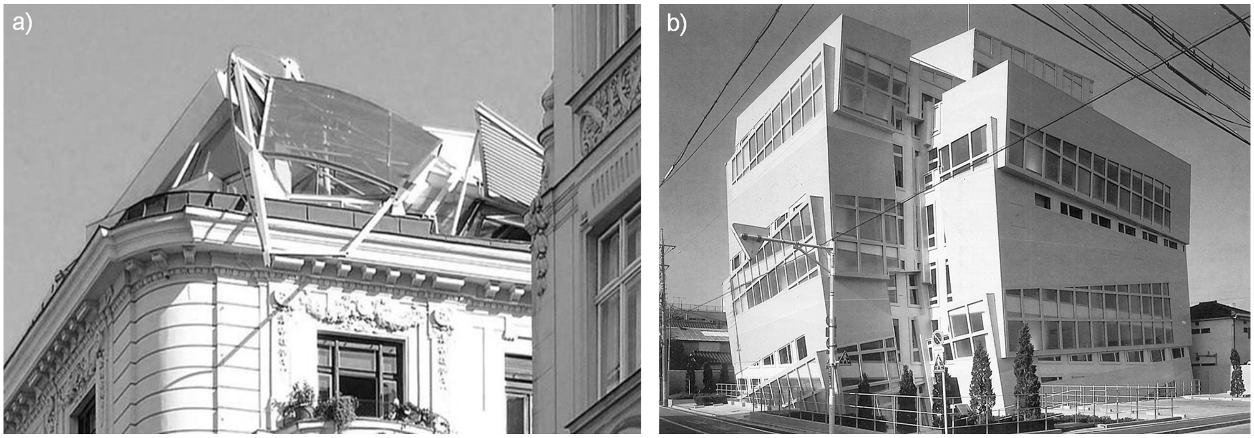

Figure 5: Decomposition method: a) rooftop remodeling above Falkestrasse in Vienna by Coop Himmelb(l)au, 1983; b) *Nunotani office building* in Tokyo by P. Eisenman, 1990

Peter Eisenman, Daniel Libeskind, Zaha Hadid, Frank O. Gehry, Helmut Swiczinsky and Wolf D. Prix. The prevailing trait of works by deconstructivists is the heterogeneity of architectural forms. In other words, they use a number of independent and intentionally unsuitable parts and divisions which do not result from a logical structure, as well as clashes and mergers of various elements. A rooftop remodeling above Falkestrasse in Vienna, designed by the Coop Himmelb(l)au Group in 1983 (Figure 5a) is a vivid example of the above with its small scale and significance for a general architectural arrangement. The other example can be office building of the Nunotani Corporation in Tokio by Peter Eisenman. The initial point of the geometric construction of the building commonly used by Eisenman is the so-called 'L-form'. The shape is multiplied and exposed to a number of transformations: minimal offsets and rotations. Further sequences of the process are respectively superposed creating the resulting form (Figure 5b).

On the one hand, Jacques Derrida's philosophy had a major influence on the development of the trend. On the other, geometry is the language of deconstruction. This can be seen in designs by Bernard Tschumi (in *Parc de la Villette*, a clash between orthogonal networks of pavilions and natural landscape and the deconstruction of cube) and Peter Eisenman (use of a geometric diagram). Deconstructivism continued developing until the end of the 20th century as an expression of vanguard. Although it has finished, it still influences the contemporary architectural thought being a source of cautious or non-cautious inspiration.

3.2. Deformation in architecture

The endeavor of contemporary architects, aimed at larger complexity of forms, also includes designs focusing on fluid, flexible and streamlined forms. Instead of straight lines, various curves are used. Planes are replaced by complicated non-ruled surfaces. In the middle of 1990s, folding stood in opposition to deconstructivism in the contemporary architecture. In 1993, a special edition of *Architectural Design* was entitled *Folding in architecture*. The editor was architect Greg Lynn. The article by the editor signified a new turn in architecture, and one chapter of the article was particularly telling: *Curving away from Deconstructivism* [9]. The new trend was developed under the influence of philosophical works by Gilles Deleuze. On the other hand, attempts made by architects to preserve the homogeneity of form and combine various elements of the composition, are deeply rooted in geometry and mathematical

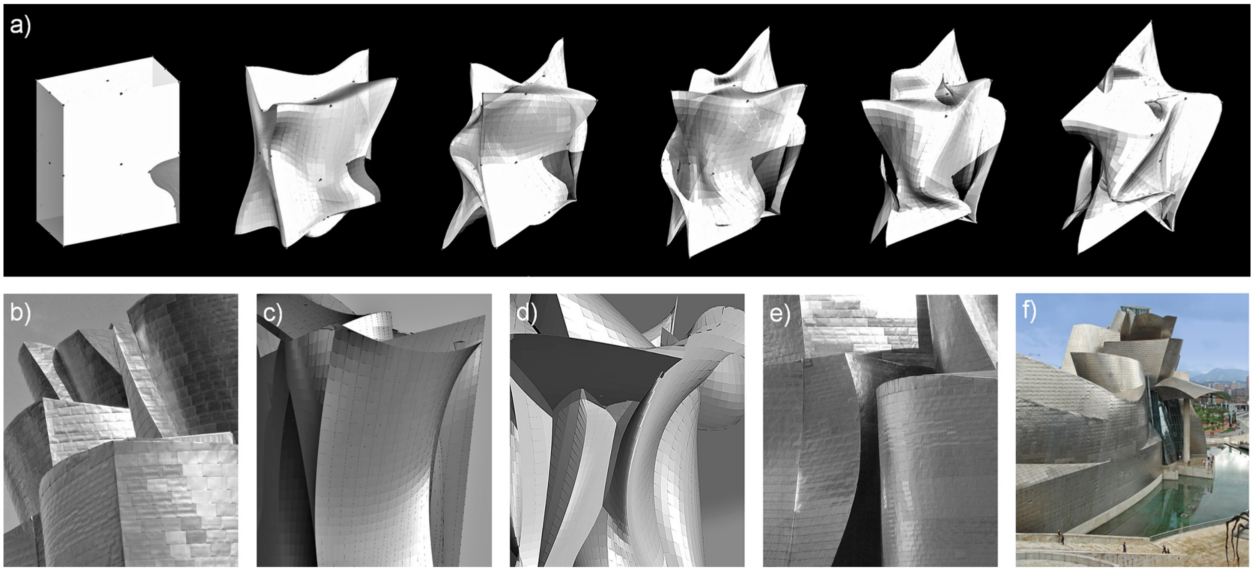

Figure 6: *Guggenheim Museum* in Bilbao (by F. Gehry, 1997) in comparison to topological simulations (by the author), as an example of the deformation method: a) transformation of single cuboid; b, e, f) photos of the building; c, d) transformation of three cuboids

topology. The *Möbius strip* and the *Klein bottle* are frequently referred symbols. Contemporary architects try to blur the boundaries between notions considered to stand in opposition to one another, such as: interior and exterior of a building (e.g., *Möbius House*, designed by Ben van Berkel), or material and virtual space (e.g., *Virtual Trading Floor*, designed by Asymptote).

The idea of folding fits into the presuppositions of the deformation method, presented in this article. New computer modelling tools (e.g., NURBS) and new building techniques (CNC prefabrication) became catalysers for the architectural development in the field. A pioneer example of using new technologies is the *Guggenheim Museum* in Bilbao by Frank O. Gehry. Figure 6 presents photographs of the museum (Figure 6b, e, f) and computer simulations by the author resulting from the topological transformation of three cuboids (Figure 6c,d). Applications of the deformation method also includes the *blob architecture*. An example of the above is Peter Cook's and Colin Fournier's *Kunsthaus* in Graz (Figure 7).

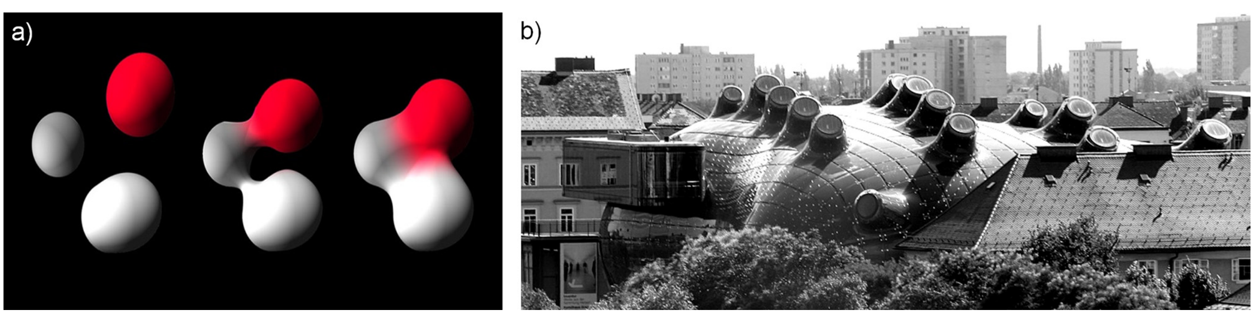

Figure 7: Computer modeling of blob forms, example of deformation. Left: simulation with the Bryce 5.5 program; right: *Kunsthaus* in Graz (by P. Cook, C. Fournier, 2003)

3.3. Dispersion in architecture

Although the dispersion method is used for designing many contemporary architectural facilities, it is hard to define any wider ideological background for it as well as any specific trend or style which would highlight the significance of stochasticity in shaping architectural forms. This issue was subject of a few research projects [7, 18]. An example of using these methods is the *Holocaust Monument* in Berlin by Peter Eisenman. Within an orderly orthogonal system of 2711 concrete blocks we may distinguish irregularities (slight deviations from the vertical plane and variable height of elements). These irregularities give a specific aesthetic dimension of the whole facility. Dispersion is associated with natural conditions, whose perception is closer to a man. We may compare, for example, a wall made of various size stones, free surface made of granite brick, and tectonics of medieval tenement houses with similar forms of regular and repeated structure.

The majority of applications of this method refers to the composition of facades. Randomness can be fixed in a building (e.g., the distribution of windows and colors of materials in the *Sharp Centre for Design* in Toronto by William Alsop) and may result from the way those facilities are used (e.g., mobile light breakers in the *National Library* in Paris by Dominique Perrault). Yet another example of the application of a random factor is the *Torre Agbar building* in Barcelona by Jean Nouvel. Its external concrete shell is perforated by 4400 random square openings. Additional layers are placed on the main construction, including corrugates steel sheet cladding of variable colors (from red to blue) and glass panels as light breakers (Figure 8).

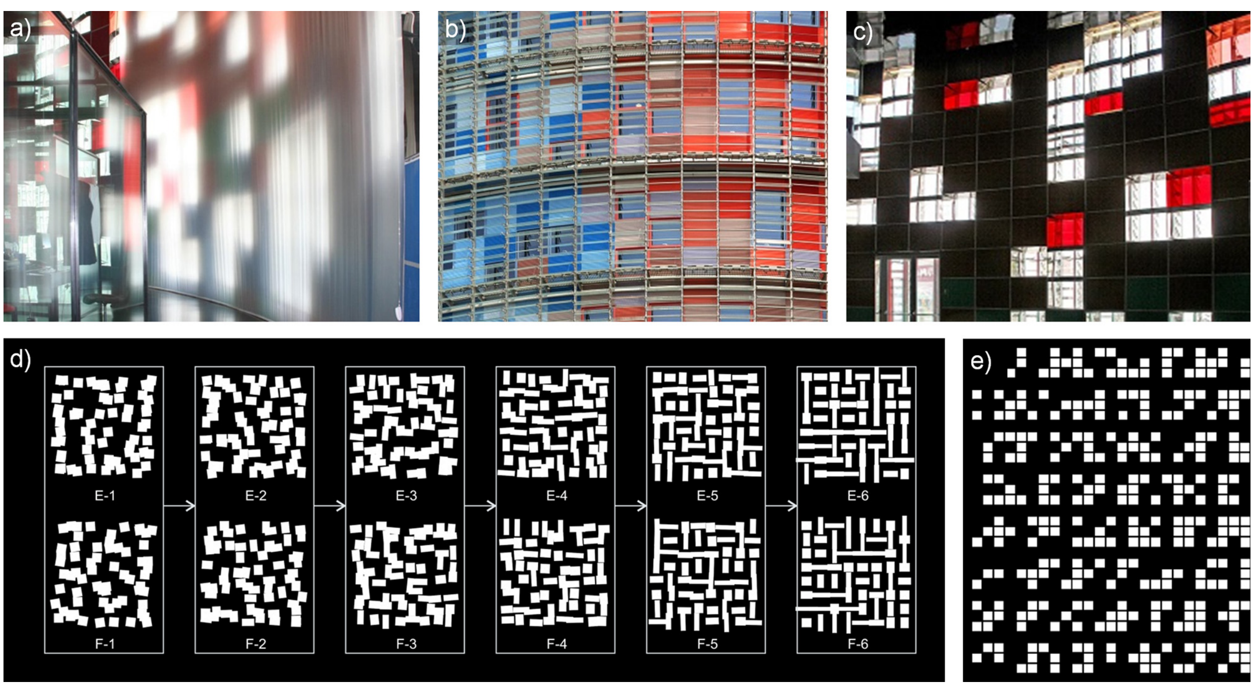

Figure 8: Application of dispersal method in architecture: a, b, c, e) facade of the *Torre Agbar* in Barcelona by J. Nouvel, 2006; d) similarity of dispersed forms

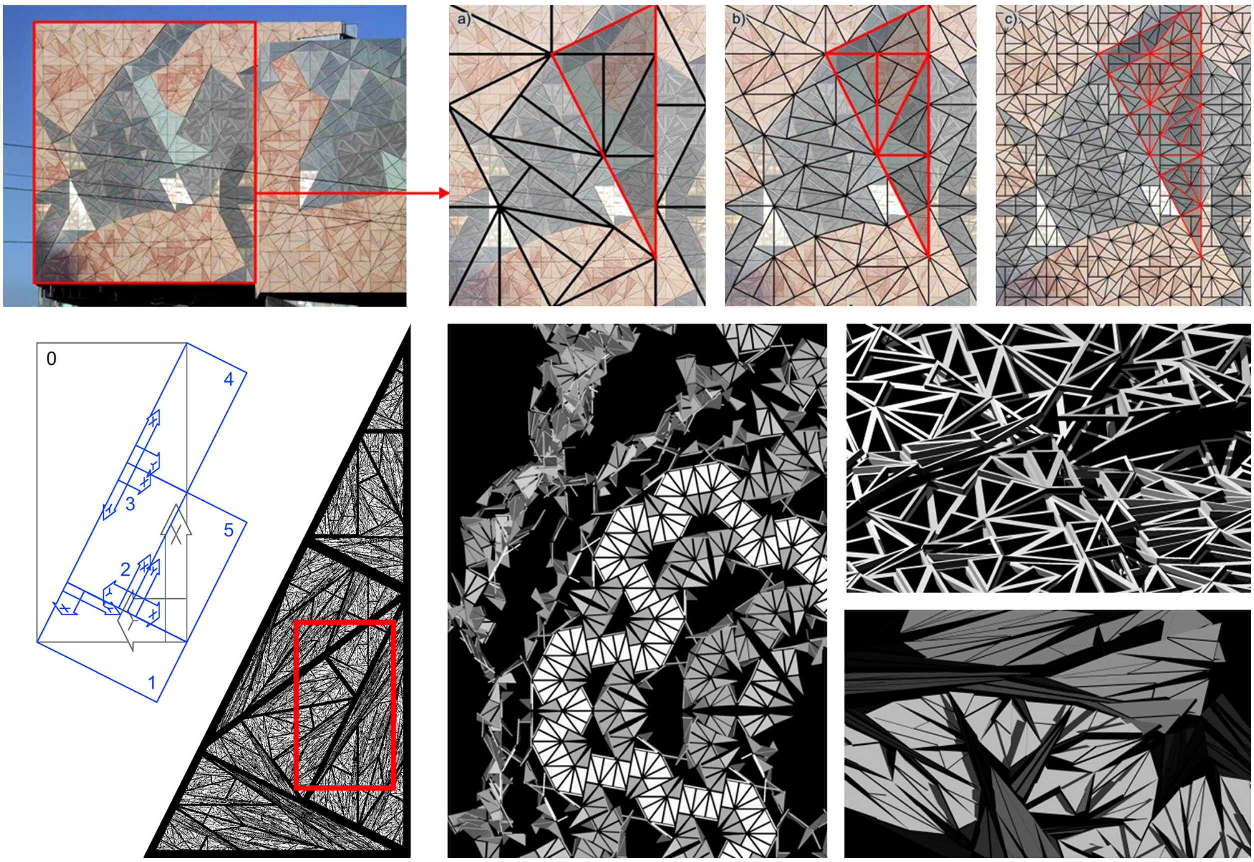

Figure 9: Composition of the facade of the *Federation Square* in Melbourne (by Lab A-S) as an example of the new complexity in architecture. View of facade and examples of radical reorganization of composition by micro modification of its construction scheme

3.4. New complexity in architecture

The development of forms according to the new complexity method is something new in architecture, and searching for possible applications in its early stage. However, inspirations derived from the mathematical theory of deterministic chaos and fractal geometry already appeared in the output of several architects as well as publications by architecture critics [6] in 1990s. For example, at the design of a new opera house in Cardiff of 1994, the American architect Greg Lynn used a method which he described as *branching*. Branching referred to fractal geometry [8]. However, there is a difference between inspiration and application. We are able to provide only a few examples of architectural facilities developed, where high complexity results from a deterministic process. It is also important that the specific nature of designing such processes and forms changes our understanding of creation in architecture.

One of preliminary examples of new complexity being applied to architecture is a multi-function complex of buildings at the *Federation Square* in Melbourne, designed by the Australian architects Donald Bates and Peter Davidson in 2003. The composition of all facades is determined by *pinwheel tiling*, an non-periodic tiling developed and described several years earlier by the American mathematician Charles Radin [14]. A simple and repeatable construction produces a complex result (Figure 9). Although the complex is a rather direct quotation from mathematics, the mechanism of developing its form can be planned. The rule of *pinwheel tiling* can be described using the IFS method [16, 12]. Minor modifications of the

IFS rule (including 3d transformations) comprise a specific encyclopedia of complex forms, and intuitive predicting the result extends beyond human imagination (Figure 9). Another later example of a facility is the center of water sports in Beijing developed for the Olympic Games (by PTW Architects), where the geometric form results from using the Weaire-Phelan model.

4. City complexity and urban planning

Methods of complex form classification presented in this article can also be used in urban planning as tools for interpreting the development of a city and its structures. However, it is not possible to extrapolate architectural observations directly to urban planning. It results from basic differences between the notion of architectural and urban creation. The goal of architecture is to develop building facilities from scratch. The immediate spatial context, neighborhood of other buildings, can be important for design decisions. However, by nature, creating architectural facilities is a one-off activity enclosed in time (from design to building). Obviously, the process of city development is much more complicated and has a different nature. Usually, a city already exists at the moment of *urban intervention*. It has been developed continuously. Thus, complexity is not a predetermined goal but a starting point for design. For these reasons, contrary to architectural creation, an analytical process is a basis for urban planning.

4.1. Application of the complexity classification to urban planning

The methods for developing complex forms, i.e., deconstruction, deformation, dispersion, and new complexity, have been formulated to support the methodical division and creation of architectural forms. In urban planning, these methods gain a new meaning and other applications.

The decomposition method in architecture is closely linked with ideas of deconstruction, and can be a tool for various geometric experiments and computer simulations, which may inspire the process of architectural design. In the urban scale, an equivalent of applying the method is imposing a new structure to a city, such as rebuilding Paris in the 19th century according to plans of Georges Eugene Haussmann (Figure 10). The decomposition method can be a mere tool for a methodical organization of such spatial transformations of a city. The potential of the method in creating new geometric simulations in urban planning is doubtful and may lead to controversies.

The deformation method in architecture is mainly expressed by the effort of creating fluid, liquid forms and using complex ruled or non-ruled structures. However, both presuppositions of the method and origin of architectural folding, are much broader. The first pioneering examples of the trend is an urban project of the *Rebstockpark Housing Estate* in Frankfurt on Main by Peter Eisenman (1990) [16]. The urban parametric modeling (UPM) [19, 11] fits into the formula of the method, which proves the possibility of recording and transforming the structure of the city as a set of combined elements from the scale of projection to facade details.

The dispersion method in architecture, which introduces a random factor to the composition, has the strongest influence on the facade or outer surface of a building. The significance of stochasticity in urban planning is much broader, naturally embedded into the process of planning and the structure of the city. Master plans usually define objectives at a general level while reserving space for individual architectural solutions (e.g., color, composition of facade,

Figure 10: Rebuilding Paris in the 19th century according to plans of Georges Eugene Haussmann — as an example of the decomposition method in urban planning

roofing, variable height of a building within a permissible scope, etc.). Possible applications of the dispersion method to urban planning are much broader than in architecture. New tools can be used, e.g., for analyzing plans and examining the morphology of an urban structure.

New complexity, as a method of creating or interpreting forms, is more developed in urban planning than in architecture. A city has a more complex structure than a single, even most complicated building. Shaping urban space is a process linking different and usually spread in time operations. Even with a very strictly defined and respected urban composition framework, the randomness and emergence are natural components of a city development process. The 'new complexity' in town planning is thus a natural element of the creation process, also reflected in scientific research. It is sufficient to refer to the research by Michael Batty presented in several publications and books: *Fractal Cities* (1994) [3] and *Cities and Complexity* (2007) [2]. Using emergence in urban planning is expressed in methods of simulating urban growth using cellular automata (CA) or simulations supported by the agent based modeling. The scope of searching for the system of higher order focuses on the area of analysis rather than creating new spatial solutions. However, nowadays the potential resulting from advanced urban analyses is crucial for the direction of any city development.

4.2. The nature of the city complexity

What is the source of complexity in contemporary cities? Is it the result of stochastic activities or phenomena (related to dispersion method)? Is the organization based on mechanisms described in the article as a part of the new complexity method? Trying to find answers to those questions is inspiring in itself. Although one cannot expect equivocal results, each examination of the boundaries between various categories of complexity of the urban tissue broadens our knowledge about the structure of a city and enhances our designing capacity. This issue has been a subject of various research [2, 13]. Self-similarity and a fractal organization can be found in urban systems [3]. The language of the new complexity method can be used to describe the African village *Ba-ila* [5] or plans of renaissance ideal cities. Figure 11 presents examples of computer simulations developed using IFS and cellular automata CA.

The degree of complexity of contemporary cities is much larger, and even if we find specific mechanisms of mathematical self-organization, undoubtedly the stochastic factor is an important component of their structure. Figure 12a–c includes an example of using the dispersion

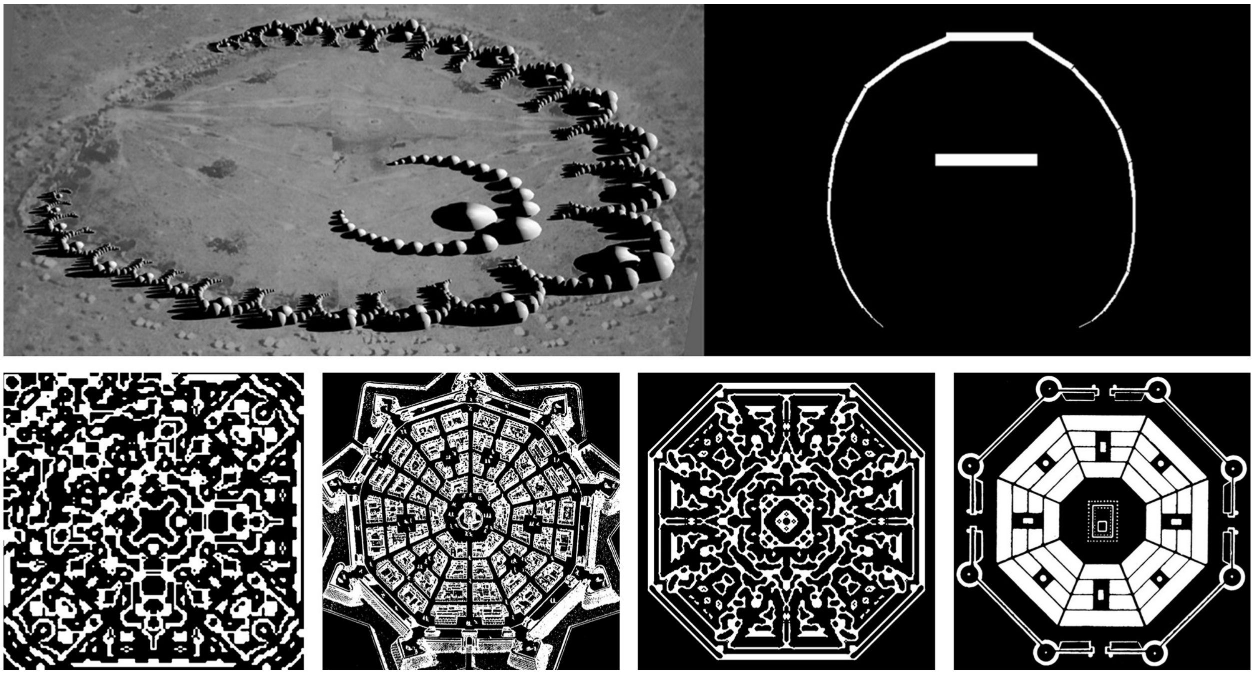

Figure 11: Sample computer simulations prepared with application of the IFS method and the cellular automata (CA) method. Above: African village *Ba-ila*, below: plans of Renaissance ideal cities compared with simulations of cellular automata (CA)

method while analyzing a part of Berlin, Germany. Consecutive stages of the simulation use simple transformations of the city structure (shifting, rotating). Slight *movement* of buildings in Berlin changes completely its urban tissue. Differences in urban systems blur at various stages of the simulation, which can define the degree of the primary organization. In the exhibition of *Images of Complexity* [15] the author made an attempt to interpret complexity. The artistic aim was to look for relations between the structure of a city and dispersion and new complexity (Figure 12d,e).

The visual complexity of a city provides a separate plane for interpretation. Urban structure, reflected in a projection, is a mere simplification. In fact, a man perceives a city as a set of thousands of various views, combined in a certain whole in the minds of inhabitants [10]. Tall buildings have a particular significance for developing the *image of a city;* their range of visual impact is in principle larger [4, 21]. The development of simulation techniques creates the possibility of a precise analysis, which is subject of a research under the *2TaLL project* [1] — focused on developing new methodologies for analyzing the influence of tall buildings on the cityscape. Further research should focus among others on defining comparative relations between the visual complexity and the complexity of the city's geometric structure.

5. Conclusions

In contemporary architecture, a tendency of shift from classic understanding of systematics towards the creation of complex forms can clearly be seen. It is influenced by new techniques of computer modeling and digital CNC prefabrication. In recent years we can also observe an disputable progress in the sphere of modeling and visualization of cities. The quantity and accuracy of accessible 3D virtual city models increases rapidly. While in architectural design

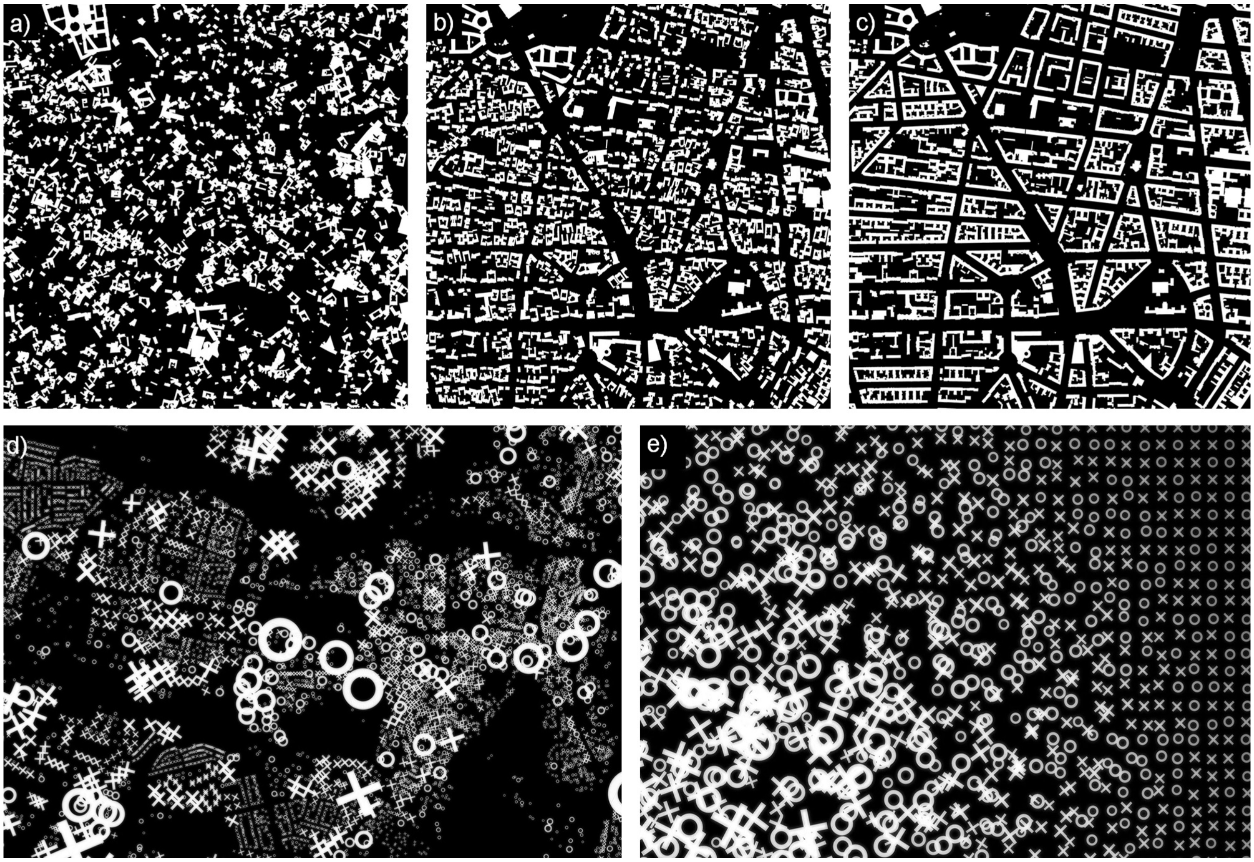

Figure 12: Exploring city complexity: a–c) dispersion method used in analyzing a part of Berlin, Germany; comparison of the real city model Lublin, Poland (d), with a stochastic composition (e). Works form the exhibition *Images of Complexity* in Toruń 2014

the purpose is the creation of new forms (buildings), in urban planning the key importance is the process of analysis of a city. 3D city models allow the application of advanced computer simulations to the exploration of complex urban structures. In both architecture and urban planning, an important subject of creation and subject of analysis is the complexity. Geometry becomes an universal language necessary for the creation of bases for new digital techniques and also for a better interpretation of the structure of complex forms.

The article in hand presents a proposal for a systematics of complex forms within the four methods decomposition, deformation, dispersion, and new complexity. General examples of application of each method to architecture and urban planning are presented as well. The proposed methods allow a better understanding of the construction of complex forms. They can be applied as interpretation tool to both architectural developments and urban structures. Each of the methods can be a basis for the development of specific computer simulations possible to encode geometry. Therefore, the presented methods can be a design tool as well.

Acknowledgments

This research was funded by the Norwegian Financing Mechanism. The digital model of Berlin was provided by Berlin Partner GmbH. All other 3D city models used for simulation were made by Cyber Urban Center at WPUT Szczecin. I gratefully acknowledge this support.

References

[1] 2TaLL: *Application of 3D virtual city models to urban analyses of tall buildings.* Research project, founded by the Norwegian Financing Mechanism. Key staff: K. Czyńska, P. Rubinowicz, A. Zwoliński, 2013–2016.

[2] M. Batty: *Cities and Complexity.* The MIT Press, Cambridge, Massachusetts, 2005.

[3] M. Batty, P. Longley: *Fractal Cities.* Academic Press, San Diego 1994.

[4] K. Czyńska: *Tall Buildings and Harmonious City Landscape.* Space & Form **13**, 267–280 (2010).

[5] R. Eglash: *African Fractals: Modern Computing and Indigenous Design.* Rutgers University Press 1999.

[6] Ch. Jencks: *The New Paradigm in Architecture. The Language of Post-Modernism.* Yale University Press 2002.

[7] R. Krawczyk: *The Codewriting Workbook.* Princeton Architectural Press 2008.

[8] G. Lynn: *Animate Form.* Princeton Architectural Press 1999.

[9] G. Lynn: *Architectural Curvilinearity: The Folded, the Pliant and the Supple.* Folding in Architecture, 24–31 (2004) – first published in Archit. Design **8** (1993).

[10] E. Morello, C. Ratti: *A digital image of the city: 3D isovists in Lynchs urban analysis.* Environ. Plann. B **36**(5), 837–853 (2009).

[11] Z. Paszkowski, P. Rubinowicz: *Toward the Parametric Modeling in Architecture and Design.* Proceedings of 7th ICECGDG, 33–36 (1996).

[12] H. Peitgen, H. Juergens, D. Saupe: *Fractals for the Classroom. Part 1: Introduction to Fractals and Chaos.* Springer, NY., 1992.

[13] J. Portugali: *Self-Organization and the City.* Springer, Berlin 2000.

[14] Ch. Radin: *Miles of Tiles.* American Mathematical Society 1999.

[15] P. Rubinowicz: *Images of Complexity.* exhibition in Wozowania Art Galery, Toruń, 2014.

[16] P. Rubinowicz: *Chaos as the Higher Order in Selected Trends of Contemporary Architecture.* Doctoral thesis, Cracow University of Technology 2011.

[17] P. Rubinowicz: *Chaos and Geometric Order in Architecture and Design.* J. Geometry Graphics **4**/2, 197–207 (2000).

[18] P. Rubinowicz: *Parametric Modeling – Random Factors in Architecture.* Proceedings of 8th ICECGDG, 81–85 (1998).

[19] N. Steinø, E. Obeling: *Developing a Parametric Urban Design Tool: Some Structural Challenges and Possible Ways to Overcome Them.* Architecturae et Artibus **1**, 51–57 (2014).

[20] S. Wolfram: *A New Kind of Science.* Wofram Media Inc. 2002.

[21] A. Zwoliński: *A day in a shadow of high-rise.* Architecturae et Artibus **1**, 67–71 (2014).

Received August 5, 2014; final form November 24, 2015