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A Suggestion for a Differentiated Presentation and Feedback Method (DIAM) for the Spatial Orientation Test (SOT)

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Abstract. The test battery of the spatial ability research project *GeodiKon* consisted mainly of four spatial ability tests: Three Dimensional Cube Test (3DW), Differential Aptitude Test (DAT), Mental Rotation Test (MRT), and Spatial Orientation Test (SOT). The results of the analyses of the SOT will be compactly summarized and visualized (average deviation, pointing accuracy and pointing direction and gender effects). The analyses of the data of the SOT brings up three challenging aspects:
1) We had the goal to provide a sufficiently helpful and meaningful feedback to test persons.
2) The very large deviations from the correct solution angle compared to other research projects motivated for deeper analyses.
3) The analyses of the data show the fact, that students solve the tasks of the SOT within two different steps. The first step is to locate the solution angle in the correct quadrant/semicircle and the second step is to place the best possible solution angle.
These three aspects motivated to develop the differentiated presentation and feedback method DIAM, which is introduced in this paper. The two steps of the differentiated presentation and feedback method DIAM are ordered in accordance to the chronological considerations of test persons.

Key Words: spatial ability, spatial orientation.

MSC 2010: 97G80, 51N05

1. Motivation for the project

Starting from the beginning of the twentieth century, researchers have increasingly been convinced that intelligence is not just one-dimensional. They identified differing aspects of intelligence and defined it as a multi-dimensional term. Ever since there has been clear evidence

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that Spatial Ability is one of the fundamental parts of human intelligence [6, 30, 33]. In a further step investigations were made to find the constituing parts of Spatial Ability. The research was centred on the definition of structured, psychometric, and factor based models of Spatial Ability [4, 6, 7, 11, 18, 34, 35]. Continuing research made it evident that Spatial Ability is not defined only by genetical inheritance and is therefore not limited. Individuals can improve their Spatial Ability actively through specific support and well balanced training [9, 10].

This awareness is one of the reasons why it is a central aim of geometry lessons to support and train Spatial Ability of the students. The teachers' preparation for each lesson is based on the curriculum for the subject. In addition to this, the competence model for "Geometrisches Zeichnen (GZ)" (this subject can be described as the subject "descriptive geometry" for lower secondary schools) [27], the competence model of "Darstellende Geometrie (DG)" (descriptive geometry for upper secondary schools) [16], the competence model for mathematics (4th and 8th school level: [3], 12th school level: [17]), and often school books are relevant for the preparation of lessons. Does also the psychological knowledge about spatial intelligence and the factors of Spatial Ability — especially because of the fact that we can actively support and train and improve Spatial Ability of students — play an important role for the didactical design of teaching? The analysis of geometric tasks from mathematics lessons of the first up to the tenth school level shows that mainly the factor Visualization is addressed. The other factors are not affected or play just a subordinate role [18, p. 237]. This situation raises the question whether it is possible to train and improve Spatial Ability of individuals if the tasks of geometry lessons are selected with the focus on addressing all the factors of Spatial Ability in a well-balanced proportion? There is a clear focus on this question in the research project *GeodiKon* (development of a didactical concept for geometry education).

The first of two mainly addressed fields of geometry research at *GeodiKon* is the topic *"factors"*. The second issue is *"strategies"*. The measurement of spatial intelligence is usually performed with the help of tests. These tests are generally compiled as tests with different groups of tasks and with different types of tasks which address special factors of spatial intelligence. As a result, the different developments of each of the factors of spatial intelligence of an individual can be identified. Those special tasks which address (almost) exactly one factor of Spatial Ability are called *'marker'* of the factor [13, pp. 121–169]. But only if all the probands solve the special marker-tasks with the same intended solving strategies, and the correct factor can be ascertained, valid results of the test can be expected.

The recent literature on spatial ability research points out the problem that individuals solve spatial ability test tasks with highly differing strategies [9, 14, 29]. This fact opens a wide field of questions, such as: How far can spatial ability tests really address and finally measure the competence of probands at the different factors of Spatial Ability? and: Which and how many strategies for assessing spatial ability tasks do individuals use? The focus, therefore, was shifted to the identification and analysis of diverse strategies used for solving spatial tasks. The statements below underpin the intention to put more emphasis on strategies:

> *"The flexible use of strategies or the adequate use of a strategy depending on the given task is a key factor for the optimal solution of spatial tests"*

(citation translated from [10, 14]),

> *"The amount of strategies and the flexibility in adapting them to the respective task is more relevant for achievement than simple basic cognitive processes"*

(citation translated from [9, pp. 325–326]). Not least, Maier pointed out that

> *"Common alternative solving strategies using other cognitive abilities or different*

spatial and visual references should thus find recognition"
(citation translated from [18, p. 55]).

In the research project *GeodiKon* the topic "strategies" was consciously chosen to answer questions, such as "Which strategies for solving spatial tasks do students use?", and "Does the training of many different strategies for assessing spatial tasks and thereby the expansion of the individual repertoire of strategies lead to an improvement of Spatial Ability ?"

The two areas — factors and solving strategies for spatial ability tasks — represent the main focus of investigation within the project *GeodiKon*. The huge amount of collected data for the project makes it possible to discuss not only the two research hypotheses, but also a big variety of questions regarding leisure activities and spatial abilities, computer usage and spatial abilities, effects of geometry lessons, the accuracy of the ability of pointing to objects, and gender specific effects [22, 25, 26]. The following two models form the scientific basis of the project:

1. Model of the four factors of Spatial Ability: *Visualization*, *Spatial Relation*, *Mental Rotation*, and *Spatial Orientation*
2. Model of the four pairs of strategies for the solution of spatial ability tasks: Holistic Strategy — Analytic Strategy, Spatial Thinking — Planar Thinking, Move Object — Move Self, and Verifying Strategy — Falsifying Strategy.

A detailed argumentation of both models is provided in [19, 20, 21].

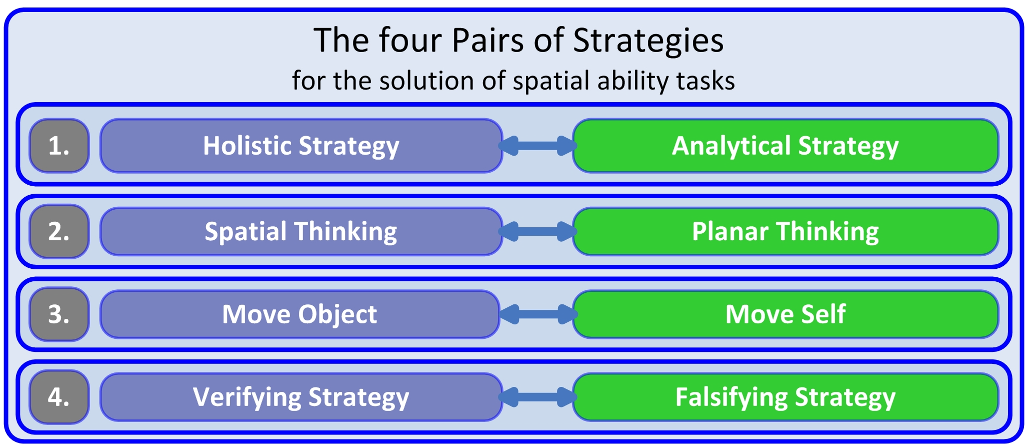

Figure 1: The model of the "four pairs of strategies for the solution of spatial ability tasks"

2. Description of the study – embedding of the SOT in the research project

The research project *GeodiKon*, funded by the Federal Ministry for Education, the Arts and Culture (BMUKK-20.040/0012-I/7/2012) and the University of Education of Salzburg, was carried out from 2013 to 2014 in the three Austrian provinces of Salzburg, Styria, and Lower Austria. 46 classes with 903 students at the age of 12 to 14 years from the school types Hauptschule (HS), Neue Mittelschule (NMS), and Allgemeinbildende Höhere Schule (AHS) took part in the project. The partners of the project were the private University of Education of Krems/Vienna, the University of Education of Styria, the University of Innsbruck, the University of Salzburg, the University of Vienna, the Technical University of Vienna, and the Workgroup for Didactical Innovations (ADI).

The project was carried out in a pretest-posttest-design. During the first phase of the project (January until September 2013) the project team compiled learning material for 12 weeks of lessons in Geometry and Mathematics. The learning material contains more than 100 specific spatial ability tasks, training the factors of Spatial Ability and the different strategies for solving spatial tasks. All the tasks can be edited only by the use of a pencil. For some tasks you need coloured pencils. No ruler or divider is needed. The structured model of the four pairs of strategies for the solution of spatial tasks was developed and each of the test-classes got age-based posters with all the strategies and explanations. In each geometry lesson, the teachers had the order to speak in the classes about different strategies to widen the repertoire of strategies of their students. In September and October 2013 the pretests were carried out. Directly after the pretests the twelve-week long learning phase began for the treatment groups. In January and February 2014 the posttests were carried out in all project classes. March until November 2014 was the time for digitalization, preparation and analyses of the collected data, compilation of the user-friendly book with all the special learning material [23], training for teachers and lecturers how to use the material in classes, and dissemination of the results of the project in conference presentations and papers.

The test battery of the pretests and the posttests consisted mainly of the four spatial ability tests: *Three Dimensional Cube Test* (3DW, [8]), *Differential Aptitude Test* (DAT, [2]), *Mental Rotation Test* (MRT, [28]), and *Spatial Orientation Test* (SOT, [12]) together with additional questions regarding to which strategies students used to solve spatial tasks and questions about age, gender, computer usage, leisure activities, school marks in Mathematics, German and English and type of learner. We mainly analysed the difference in performance of the students between pretest and posttest. The pretest lasted for 85 minutes and the posttest for 77 minutes.

This paper focusses on the SOT. So in the following this test will be briefly introduced. The SOT is a test of one's ability to imagine different perspectives or orientations. In each task one can see a picture of an array of objects and an "arrow circle" with a question about the direction between some of the objects. For each question one should imagine to be standing next to one object in the array (which is named in the center of the circle) and facing another object, named at the top of the circle. The task is to draw an arrow from the center object showing the direction to a third object from this facing orientation [12]. In this test you can get no points. At each task the deviation angle from the right answer is measured. The angle is measured not orientated, so therefore all the deviation angles are in the range between 0° and 180°. The SOT consists of 12 tasks and lasts for 8 minutes.

3. The results of the SOT at GeodiKon

903 students from the 46 project classes participated in the tests. 786 students out of the 903 students were present at both tests. There were no systematic errors, so we accepted missing completely at random (MCAR). Of the 786 students who took part in both tests 771 students delivered evaluable tests. Those 771 students (413 male and 358 female) formed the basis of the data analyses. In this chapter the results of the analyses of the SOT at the project *GeodiKon* will be compactly summarized and visualized (average deviation, pointing accuracy and pointing direction and gender effects). In the following chapter the limitation of the significance of the results of the SOT will be shown and therefore a suggestion of the differentiated presentation method DIAM of the results of the SOT will be developed.

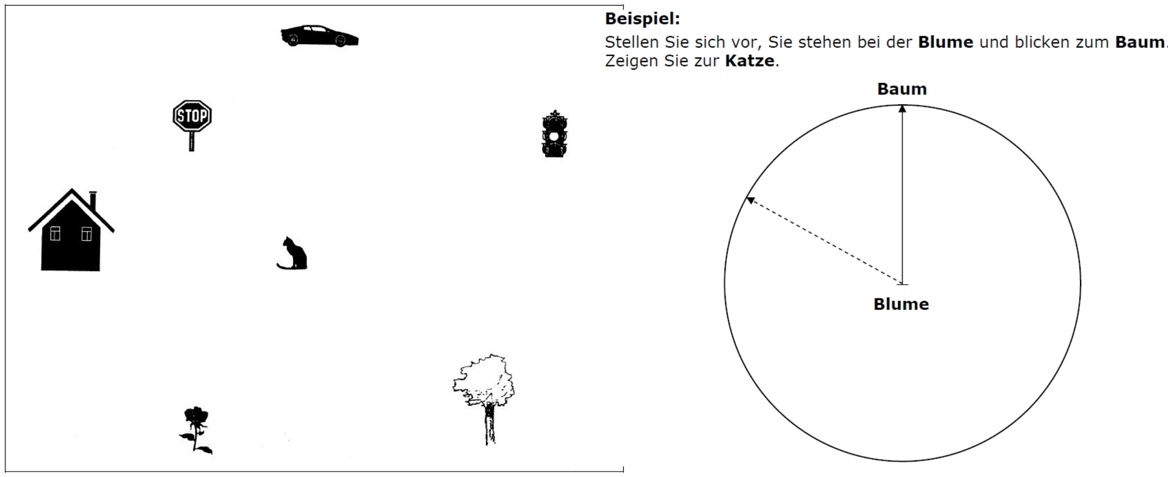

Figure 2: An example of the Spatial Orientation Test (SOT)

3.1. Average deviation

In the SOT we can see that the average deviation of the 12–14 year old students in the pretest is 59.04° [31] and in the posttest 50.64° [32]. These values are nearly twice as large as the performance of 17–18 year old students who have an average deviation of 30° at the pretest and 27° in the posttest [5]. There are no comparable big differences in the other three tests (3DW-Test, DAT and MRT).

3.2. Pointing accuracy and pointing direction

We analyzed the assumption whether the absolute angular error increases with the angular deviation of one's imagined heading from the orientation of the array [15]. Figure 3 shows the angle between the orientation of the array and the perspective to be imagined at tasks of the SOT. The task would be: "Imagine you are standing at the stop sign and facing the cat". To solve this task you mentally have to turn right through 145° to find yourself in the starting position. The angle between the orientation of the array and the perspective to be imagined is therefore 145°.

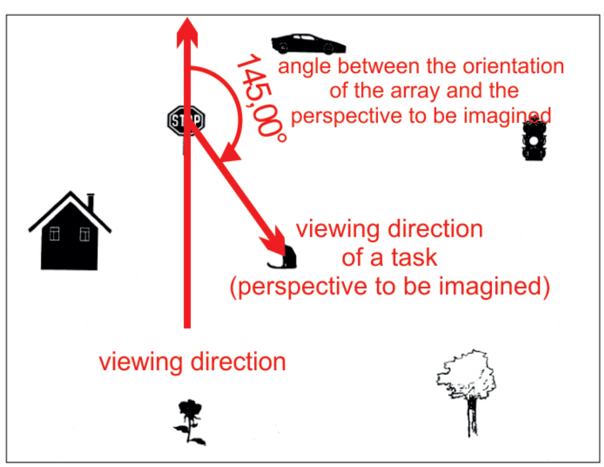

Figure 3: Angle between the orientation of the array and the perspective to be imagined at tasks of the SOT

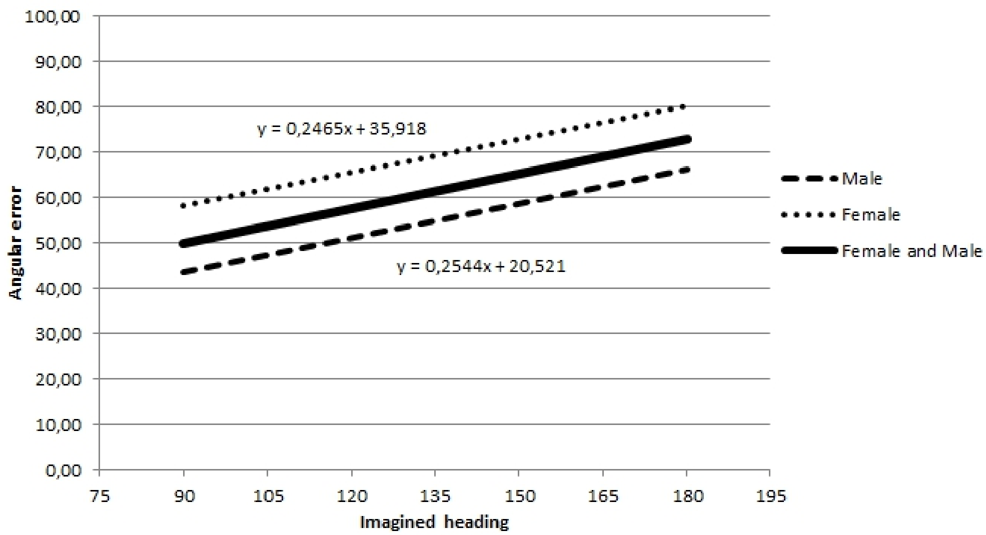

Figure 4: Pointing accuracy as a function of imagined heading

At Figure 4 we can see clearly that the absolute angular error does increase with the angular deviation of one's imagined heading from the orientation of the array in general. We therefore can confirm the results of Kozhevnikov and Hegarty [15]. In the SOT there are only tasks which have an angle of 90°–180° between the orientation of the array and the perspective to be imagined (see Figure 4). Kozhevnikov and Hegarty [15] point out that only for items that require a perspective change of more than 90° probands use the *"move self" strategy. "In contrast, most items that require a perspective change of less than* 90°*, are solved by object manipulation strategies"* [15].

The *"perspective change" strategy* is called "move self" strategy by Barratt [1] and is an indicator for a test which measures Spatial Orientation while the *"move object" strategy* [1] which is called *"object manipulation" strategy* by Kozhevnikov and Hegarty [15] is mainly used for tasks which have an angle between the orientation of the array and the perspective to be imagined less than 90°. The "move object" strategy or "object manipulation" strategy is typical for tasks where probands do not use the factor Spatial Orientation.

3.3. Gender effects

In the SOT the average of the deviation angle of the correct solution was 59.04° in the pretest and 50.64° in the posttest. So the average reduction of the deviation angle from the correct answer was 8.40°, which means a highly significant value for all classes and groups ($F_{1;44.99} = 80.56$; $p < 0.001$). The deviation angle of male students is smaller than the deviation angle of female students in the pretest (14.4°) and in the posttest (13.5°). This shows that the gender difference stays nearly the same in the pretest and posttest and no significant gender specific treatment effect in the SOT ($F_{1;757} = 0.28$; $p = 0.597$) can be seen.

The results of the four spatial ability tests from all students who worked with the learning material during the project show clearly some differences between the generally strength of female and male students regarding to their Spatial Ability. Male students have more generally strength in the three domains Visualization, Mental Rotation, and Spatial Orientation. The domain Spatial Relations is gender neutral.

The results of the analyses of the four spatial ability tests brought up that female and male students do have different potential regarding to the improvement of their performance. Female students do have more increase of performance at the domains Visualization, Spatial Relations and Mental Rotation. The domain Spatial Orientation seems to be a male domain. Here the male students do have a bigger increase of improvement than the female students [24].

The detailed results of the project *GeodiKon* (statistics, tables, scores, details about the gender specific differences and much more) can be reached at Maresch [25].

4. The problems of the analyses of the SOT

In the SOT we can see that the average deviation from the correct angle of the 12–14 year old students in the pretest (59.04°) and in the posttest (50.64°) is nearly twice as large as of 17–18 year old students who have an average deviation of 30° at the pretest and 27° in the posttest [5]. There are no comparable big differences in the other three tests (3DW-Test, DAT and MRT).

This is the reason why we wanted to make further and detailed investigations concerning the SOT-data. We see obviously that there is no possibility of providing any detailed and differentiated analyses of the results of the SOT when we use the "classical" method for providing the results — summarize all the deviations from the correct angels of the twelve tasks from the SOT and determine the arithmetic mean of the sum. On the one hand there is no possibility to see if a proband nearly sets the correct answer at some tasks or she/he had quite a large deviation (e.g., more than 90°) at some of the tasks. On the other hand if a student has a deviation at just one task of nearly the maximum deviation (180°) this could have considerable effects on the overall results of this person. If a student has an average deviation of, e.g., 15° one could not determine if the proband has a deviation of 180° at just one of the twelve tasks and all the other eleven solutions were absolutely right or she/he has for example at six tasks a deviation of 30° and all the other six tasks the correct answer or if the proband has a deviation of 15° at all twelve tasks. Each of these results would imply a completely different quality of the factor Spatial Orientation of the person. This means that when only the average deviation of the correct angle of the tasks is presented as the result of the SOT there is just a very limited chance to provide any differentiated statements regarding to the factor Spatial Orientation of a proband.

In the following chapters the differentiated presentation method DIAM of the results of the SOT will be developed and presented. It should help researchers for research purposes and it should also provide a differentiated and therefore helpful feedback for probands of the test to get many detailed comments and results of their performance at the factor Spatial Orientation of Spatial Ability.

5. Considerations on the way to the differentiated presentation and feedback method DIAM

As a first step, to understand the solution behaviour of the 903 probands at *GeodiKon* especially at the SOT, scatter diagrams of all the twelve tasks were generated where one can visually find all the deviation angles of the students (Figure 5).

Figure 5 clearly shows that four different areas can be recognised where the results of the students accumulate. At task 6 the correct answer is 235°. The following four areas of accumulation can be identified visually:

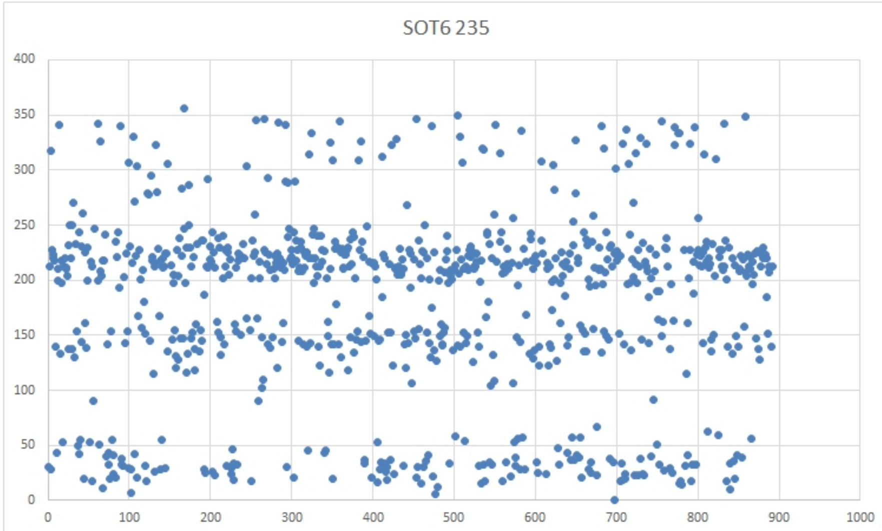

Figure 5: Scatter diagram of the task 6 of the SOT. The horizontal axis shows the probands and the vertical axis presents the deviation angle of the correct solution.

1. The quadrant with the correct angle. At task 6 this is the quadrant between 180° and 270°.
2. The quadrant which is mirrored left or right from the correct quadrant on the vertical diameter of the solution circle. At task 6 this is the quadrant between 90° and 180°.
3. The quadrant which is mirrored front or back from the correct quadrant on the horizontal diameter of the solution circle. At task 6 this is the quadrant between 270° and 360°.
4. The quadrant which is mirrored left/right and front/back (or is rotated through 180°). At task 6 this is the quadrant between 0° and 90°.

Figure 6 presents the four possible quadrants of the task 6 of the SOT. The "cQ"-quadrant is the quadrant of the correct solution angle. The "lrQ"-quadrant is the quadrant which is mirrored left/right in the vertical diameter of the solution circle. The "fbQ"-quadrant is mirrored in the horizontal diameter of the circle and finally the "lrfbQ"-quadrant is the one which can be identified when the correct quadrant is rotated through 180°.

Figure 7 presents a combination of the information of Figures 5 and 6. The background of Figure 7 is exactly the scatter diagram from Figure 5. In the foreground it shows the four quadrants marked with boxes with the same name as in Figure 6. This compilation should make it easy to identify visually the four main areas where the probands edited their solution angle.

The analyses of all twelve scatter diagrams of the tasks of the SOT brings up that there is a comparable visually impression at the tasks 1, 2, 4, 5, 6, 7, 10, 11, and 12 with the task 6 (Figure 5 and 7). Only two instead of four areas where students provide their solutions can be identified at the tasks 3, 8 and 9 (Figure 8).

Table 1 enables us to recognise the reason for this effect: The correct solution angles at tasks 3, 8 and 9 deviate from the quadrant-borders at 0°, 90°, 180° and 270° of a maximum of just 10°. This is the reason why at tasks 3, 8 and 9 it is not possible to differentiate if an error occurs because the student makes a front/back-error or because of her/his inaccuracy of the estimation of the solution. At task 3, for example, if a proband provides the solution angle of 96° it is not possible to find out if the deviation of the correct solution is the result

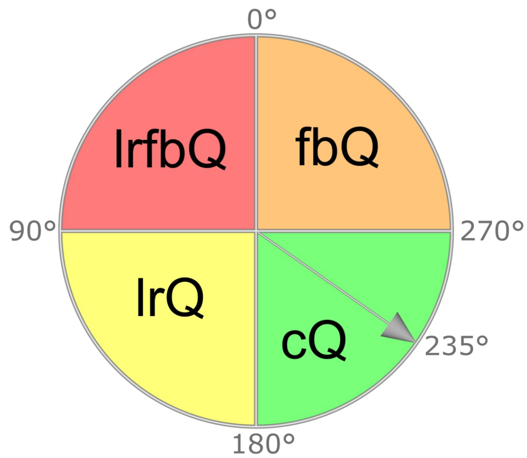

Figure 6: Correct solution arrow of task 6 and the four possible quadrants (one correct solution quadrant and three error quadrants)

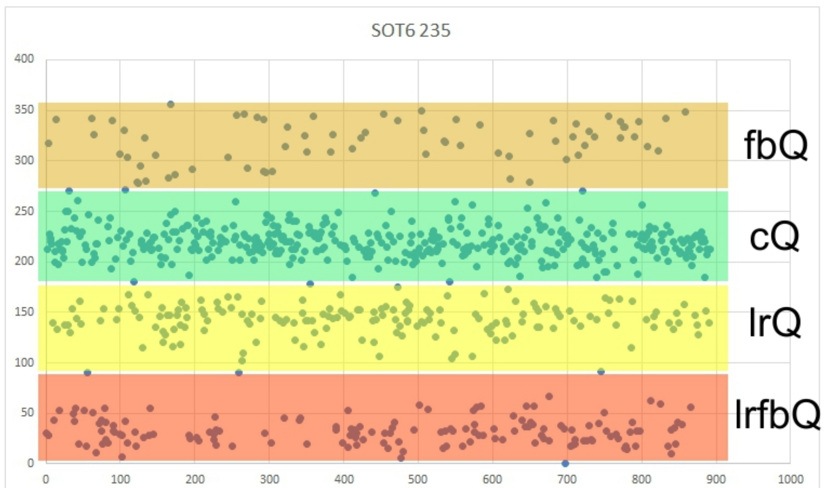

Figure 7: Scatter diagram of task 6 of the SOT and the four possible areas where students provide their solutions. The horizontal axis shows the probands and the vertical axis presents the deviation angle of the correct solution.

of a front/back-error or is the result of an inaccurate estimation. The nine tasks 1, 2, 4, 5, 6, 7, 10, 11, and 12 have a deviation of at least 26° of the quadrant-borders. These tasks were analysed in regards to the aspect in which quadrant the students provided their answers. The tasks 3, 8 and 9 were analysed in regards to the aspect in which semicircle (between 0° and 180° or between 180° and 360°) the probands edited their solution.

At the basic analysis of the SOT at *GeodiKon* (see chapter 3) only those data were used

Table 1: Correct solutions angles of the twelve tasks of the SOT

Task	1	2	3	4	5	6	7	8	9	10	11	12
Solution	123°	237°	83°	156°	319°	235°	333°	260°	280°	48°	26°	151°

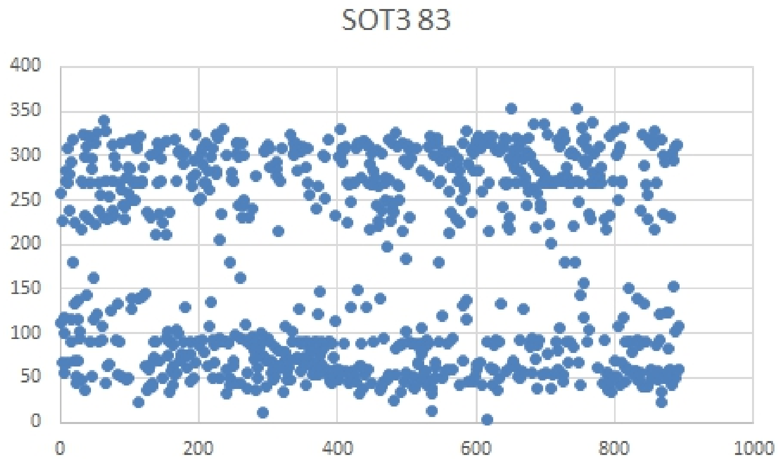

Figure 8: Scatter diagram of task 3 of the SOT. The horizontal axis shows the probands and the vertical axis presents the deviation angle of the correct solution.

by students who were present at both tests and who delivered evaluable tests at the pretest and posttest. At the detailed analyses of the SOT in this chapter the tests of all students were included. 823 students took part in the pretests and 834 took part in the posttest. Table 2 provides the number of answers for each task (pretest and posttest). At the basic and detailed analyses of the SOT only those data were used, which actually were edited by the students. Kozhevnikov and Hegarty (2001) assigned a score of 90 for all missing values, which corresponds to the mean of the possible error angle (which is between 0° and 180°). Both methods for the analysis of the SOT were compared by Svecnik [31], which led to nearly identical results (r = 0.99). Therefore in the following only those solutions were used,

Table 2: How many students provided their answer in which of the four possible quadrants at the pretest and posttest at the nine quadrant-tasks ? Correct Quadrant (cQ), Left/Right mirrored Quadrant (lrQ), Front/Back mirrored Quadrant (fbQ), and Left/Right and Front/Back mirrored Quadrant (lrfbQ)

Pretest: Task	1	2	4	5	6	7	10	11	12
cQ	352	469	437	432	402	507	441	427	427
lrQ	260	163	278	253	173	110	183	213	78
fbQ	64	137	50	73	73	124	56	29	140
lrfbQ	146	51	49	65	150	57	38	34	28
Probands	822	820	814	823	798	798	718	703	673

Posttest: Task	1	2	4	5	6	7	10	11	12
cQ	459	511	474	495	472	525	524	543	536
lrQ	169	140	250	182	160	100	187	196	80
fbQ	61	127	41	51	69	140	48	35	126
lrfbQ	142	45	52	101	120	61	42	40	42
Probands	831	823	817	829	821	826	801	814	784

Table 3: How many students provided their answer in which of the two possible semicircles at the pretest and posttest at the three semicircle-tasks?

	Pretest: Task			*Posttest: Task*		
	3	8	9	3	8	9
Correct Semicircle (cS)	424	551	381	533	599	470
Mirrored Semicircel (mS)	406	214	369	297	217	349
Probands	830	765	750	830	816	819

which were actually edited by the students. 9314 angels were edited by the 823 students at the pretest, which means that an average of 11.32 solutions were edited. 9811 answers were given at the posttest from 834 probands. This means an average of 11.76 answers per student. Table 2 shows that the six tasks 1, 2, 4, 5, 6, and 7 were edited by nearly the same number of students. At the pretest there are considerably less edits at the tasks 10, 11 and 12 as at the posttest.

Table 3 shows the analyses of the tasks 3, 8 and 9. At task 3 there were exactly as many probands at the pretest as at the posttest. At task 8 and 9 more edits can be recognised at the posttest than at the pretest. Tables 2 and 3 present the absolute figures of students who attend the pretest and the posttest. Tables 4 and 5 show the precentages of how many probands edited their solution in which of the four possible quadrants and in which of the two possible semicircles.

An increase of performance (more answers in the correct quadrant/semicircle) can be recognised between pretest and posttest at every task (Figure 9). The increase ranges between 3.55% (at task 7; from 507 up to 525 students) and 30.40% (at task 1; from 352 up to 459 students).

Table 4: How many students provided their answer in which of the four possible quadrants at the pretest and posttest at the nine quadrant-tasks? All the numbers represent percentages. Correct Quadrant (cQ), Left/Right mirrored Quadrant (lrQ), Front/Back mirrored Quadrant (fbQ), and Left/Right and Front/Back mirrored Quadrant (lrfbQ)

Pretest: Task	1	2	4	5	6	7	10	11	12
cQ	42,82	57,20	53,69	52,49	50,38	63,53	61,42	60,47	63,45
lrQ	31,63	19,88	34,15	30,74	21,68	13,78	25,49	30,30	11,59
fbQ	7,79	16,71	6,14	8,87	9,15	15,54	7,80	4,13	20,80
lrfbQ	17,76	6,22	6,02	7,90	18,80	7,14	5,29	4,84	4,16
Posttest: Task	1	2	4	5	6	7	10	11	12
cQ	55,23	62,09	58,02	59,71	57,49	63,56	65,42	66,71	68,37
lrQ	20,34	17,01	30,60	21,95	19,49	12,11	23,35	24,08	10,20
fbQ	7,34	15,43	5,02	6,15	8,40	16,95	5,99	4,30	16,07
lrfbQ	17,09	5,47	6,36	12,18	14,62	7,38	5,24	4,91	5,36

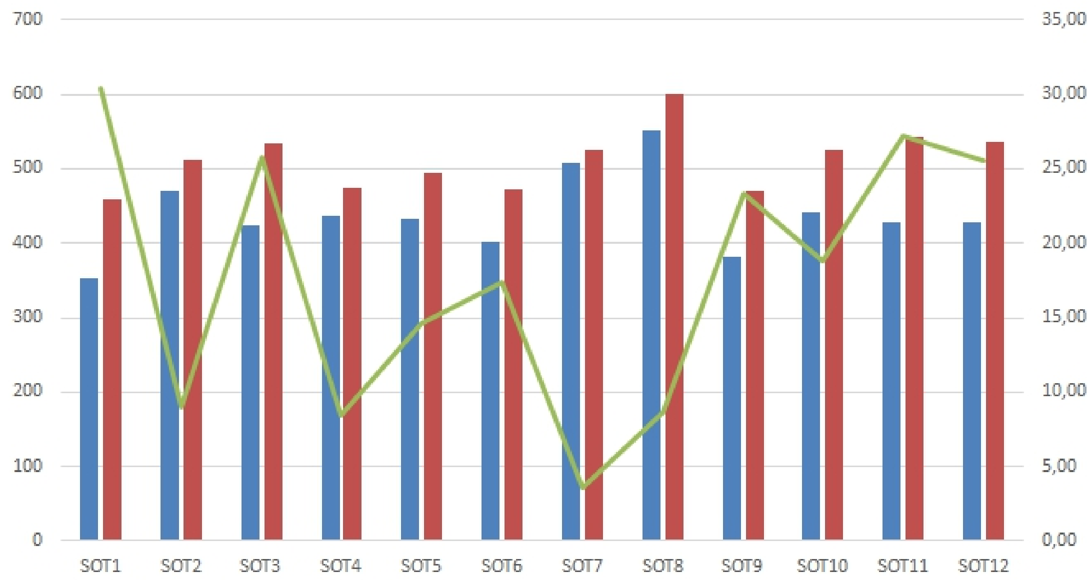

Figure 9: Number of solutions in the correct quadrant or correct semicircle at the pretest (left bar) and the posttest (right bar) and the percentage of the increase of performance (polygon)

The absolute numbers of probands are differing at all tasks. Because of this fact, in the following we will argue with percentages (and not with absolute numbers). Tables 4 and 5 show the increase of performance of the students at each task. Table 6 presents the summative results (precentages) of the probands.

The numbers in Table 6 show that there is an increase of students who edited their solution in the correct quadrant of 5.65%. 4.46% of this increase comes from students who had a left/right error at the pretest and 1.25% comes from students who had a front/back error at the pretest. The amount of probands who edited their solution in the left/right-and-front/back-error-quadrant (lrfbQ) stays nearly constant. A substantial increase of performance from 57.97% to 65.00% can be recognised at the semicircle-tasks. At the pretest more probands (15.94%) edited their answer in the correct semicircle than in the incorrect semicircle. At the posttest this number raises up to 30.00%. As part of the detailed analyses of the SOT-data, the following questions were also studied:

- Is there a correlation between the size of the solution angle and the deviation of the correct angle of each test person?
- Is there a correlation between the angle between the orientation of the array and the perspective to be imagined and the deviation of the correct angle for the probands?

Table 5: How many students provided their answer in which of the two possible semicircles at the pretest and posttest at the three semicircle-tasks ? All the numbers represent percentages.

	Pretest: Task			*Posttest: Task*		
	3	8	9	3	8	9
Correct Semicircle (cS)	51,08	72,03	50,80	64,22	73,41	57,39
Mirrored Semicircel (mS)	48,92	27,97	49,20	35,78	26,59	42,61

Table 6: How many students provided their answer in which of the four possible quadrants/the two possible semicircles at the pretest and posttest at all tasks? All the numbers represent percentages.

9 *Quadrant-Tasks*	*Pretest*	*Posttest*	*Difference*
Correct Quadrant (cQ)	56,19	61,84	5,65
Left/Right mirrored Quadrant (lrQ)	24,36	19,90	-4,46
Front/Back mirrored Quadrant (fbQ)	10,77	9,52	-1,25
Left/Right and Front/Back mirrored Quadrant (lrfbQ)	8,68	8,74	0,06
3 *Semicircletasks*	*Pretest*	*Posttest*	*Difference*
Correct Semicircle (cS)	57,97	65,00	7,03
Mirrored Semicircel (mS)	42,03	35,00	-7,03

- Is there a relationship between the sum of the two angles "angle between the orientation of the array and the perspective to be imagined" plus "correct solution angle" and the deviation of the correct angle for the probands?
- Is there an improvement in the quality of all angles which were recorded in the correct quadrant between pretest and posttest?

For all four investigated contexts above there must be stated that none of the supposed dependencies could be actually observed.

6. The differentiated presentation and feedback method DIAM

The "classical" method for providing the results of the SOT is to summarize all the deviations from the correct angels of the twelve tasks and determine the arithmetic mean of this sum. So one value is the only result-value and feedback-value of one's SOT. In chapters 4 and 5 we discussed that first of all the probands had the aim to locate the solution angle in the correct quadrant/semicircle. And then the second step is to have a best possible pointing accuracy. When processing the tasks, three fundamental errors in the considerations for the localization of the solution at the correct quadrant potentially occur:

1. Left/right error
2. Front/back error
3. Left/right and front/back error

The solution angle is recorded either in the horizontal mirrored quadrant (at error 1), in the vertical mirrored quadrant (at error 2) or in the horizontal and vertical mirrored quadrant (at error 3) [15]. This fact explains the four areas where the recorded solutions can be observed at Figures 5 and 7. Thus test persons used two steps to edit the tasks of the SOT:

1. Locate the solutions angle in the correct quadrant (at the nine quadrant-tasks) or semicircle (at the three semi-circle-tasks), and
2. record the best possible solution angle.

These two steps of considerations form the basis of the differentiated presentation and feedback method DIAM. The first step is to record in which quadrant/semicircle the probands located their solutions at the pretest and the posttest (Table 7). Then, the second step is to provide the deviation-value from the correct solution if the proband's solution was recorded in the correct quadrant/semicircle. If a student set her/his solution in a wrong quadrant/semicircle then the deviation of the corresponding angle is listed in the table. To mark that in some cells the deviation is not calculated from the correct solution but from the corresponding solution, those results are marked with a "*". For example: The correct answer of task 6 is 235°. The proband recorded the value 30°, which means that she/he made a left/right-and-front/back error. The corresponding error angle therefore is $235° - 180° = 55°$. Thus Table 7 records $55° - 30° = 25°$ as the deviation of the proband at the pretest at task 6, and the value 25 is marked with an "*". The last column provides the solution-probabilities of all test persons as a "fair comparison" for all probands.

Table 7: Overview of the differentiated results of the SOT. The indication of all values (except of the last column) is in degree.

Task	*correct solut.*	*Test*	*my value*	*quadrant semicircle*	*my deviation*	*Average solution-probability (cQ, lrQ, fbQ, lrfbQ; cS, mS)*
SOT1	123	Pre	115	cQ	8	42,82%, 31,63%, 7,79%, 17,76%
		Post	130	cQ	7	55,23%, 20,34%, 7,34%, 17,09%
SOT2	237	Pre	270	cQ	33	57,20%, 19,88%, 16,71%, 6,22%
		Post	180	cQ	57	62,09%, 17,01%, 15,43%, 5,47%
SOT3	83	Pre	112	cS	29	51,08%, 48,92%
		Post	93	cS	10	64,22%, 35,78%
SOT4	156	Pre	180	cQ	24	53,69%, 34,15%, 6,14%, 6,02%
		Post	134	cQ	22	58,02%, 30,60%, 5,02%, 6,36%
SOT5	319	Pre	278	cQ	41	52,49%, 30,74%, 8,87%, 7,90%
		Post	271	cQ	48	59,71%, 21,95%, 6,15%, 12,18%
SOT6	235	Pre	30	lrfbQ	25*	50,38%, 21,68%, 9,15%, 18,80%
		Post	335	fbQ	30*	57,49%, 19,49%, 8,40%, 14,62%
SOT7	333	Pre	135	lrfbQ	18*	63,53%, 13,78%, 15,54%, 7,14%
		Post	223	fbQ	16*	63,56%, 12,11%, 16,95%, 7,38%
SOT8	260	Pre	270	cS	10	72,03%, 27,97%
		Post	219	cS	41	73,41%, 26,59%
SOT9	280	Pre	134	mS	34*	50,80%, 49,20%
		Post	135	mS	35*	57,39%, 42,61%
SOT10	48	Pre	40	cQ	8	61,42%, 25,49%, 7,80%, 5,29%
		Post	46	cQ	2	65,42%, 23,35%, 5,99%, 5,24%
SOT11	26	Pre	329	lrQ	5*	60,47%, 30,30%, 4,13%, 4,84%
		Post	39	cQ	13	66,71%, 24,08%, 4,30%, 4,91%
SOT12	151	Pre	52	fbQ	23*	63,45%, 11,59%, 20,80%, 4,16%
		Post	131	cQ	20	68,37%, 10,20%, 16,07%, 5,36%

Table 8: Compact summary of the performance of a test person at the SOT

Where did I locate my solution angle ?	*Pretest*	*Posttest*
Correct Quadrant (cQ), Correct Semicircle (cS)	7	9
Left/Right mirrored Quadrant (lrQ), Mirrored Semicircle (mS)	1	0
Front/Back mirrored Quadrant (fbQ)	2	3
Left/Right and Front/Back mirrored Quadrant (lrfbQ)	2	0
Jumps between Quadrants/Semicircles		
Improvements	4	
Deteriorations	0	

As second part of the differentiated presentation and feedback method DIAM the results from Table 7 are compactly summarized in Table 8. The sum of the correct answers and the answers in the mirrored quadrants/semicircles is provided separately for the pretest and the posttest. The more answers a proband has in the correct quadrant/semicircle, the better she/he is. The more answers the student has in the three rows of the different error quadrants/semicircle, the worse she/he is. For the sake of completeness it is mentioned, that the values which are shown in Tables 7 and 8 are from a real test person. This proband had an average deviation of the correct solution of 64.33° at the pretest and of 53.28° at the posttest, which are the results of the "classical" analyses of the SOT.

It is not appropriate to calculate and compare the values in Table 7 with the results of the "classical" analyses of the SOT, because at the differentiated presentation and feedback method DIAM the maximum deviation angle at the nine quadrant tasks is 64°, and at the three semicircle-tasks it is a maximum of 100°, whereas the deviation angle of the "classical" analyses method is between 0° and 180°.

7. Relativization

During the analyses of the SOT-data from *GeodiKon* examinations were made if there is a correlation between the direction to the target from one's imagined heading and the absolute pointing error. The results (Figure 10) show clearly that test persons have a small angular error if the direction to the target of one's imagined heading is nearly in front (0°) or behind (180°) the probands. If the target is nearly at the left side (90°) or rights side (270°) of students then the angular error is nearly twice as large. Therefore the presumption can be formulated that it is much easier for a person to point to objects which are in front or behind than to objects which are at the left or right sight of the person. This presumption was first published by Kozhevnikov and Hegarty [15] and was confirmed by Maresch [22, pp. 301–309].

In the light of the differentiated presentation and feedback method DIAM, these presumptions must be relativized and can no longer be kept as a thesis. At least two arguments underpin the relativization:

1. Just 56 – 65% of the test persons locate their solution angle in the correct quadrant/semicircle. According to the arguments of the differentiated presentation and feedback method DIAM only these values can be taken for further calculations. Con-

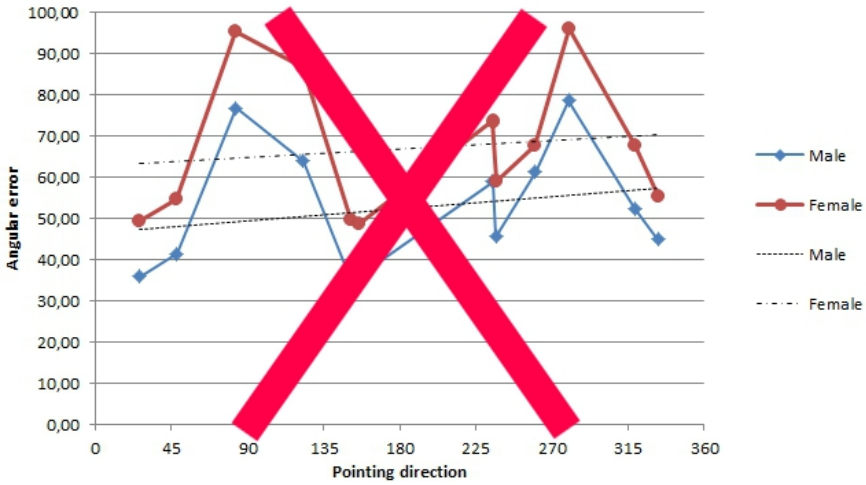

Figure 10: Correlation between the direction to the target from one's imagined heading and the absolute pointing error

versely, 35 – 44% of values are included in the calculation, which stem from probands who located their solution angle in one of the three incorrect quadrants/semicircle.

2. Between 35.00% and 42.03% of the probands locate their solution angle in the mirrored semicircle. Especially at those tasks of the SOT where the correct solution angle is nearly 90° or 270° — this is the fact at the three semicircle tasks 3, 8, and 9 — a mirror error has a very high numerical effect. Thus it is obvious that especially at the tasks 3, 8 and 9 (with the correct solution angles of 83°, 260°, and 280°) very high angular errors can be observed.

Figure 11 presents two examples, which show clearly that a mirror error can have considerably different effects on the size of the error angle. The correct solution of task 11 is 26°. If a proband makes a left/right-mirror error then this would mean that she/he has a deviation of the correct angle of about 52°. The correct solution of task 8 is 260°, which implies, that a left/right-mirror error leads to a deviation of the correct solution of about 160°. Thus a left/right mirror error can lead to error angles with huge differences. The considerations of

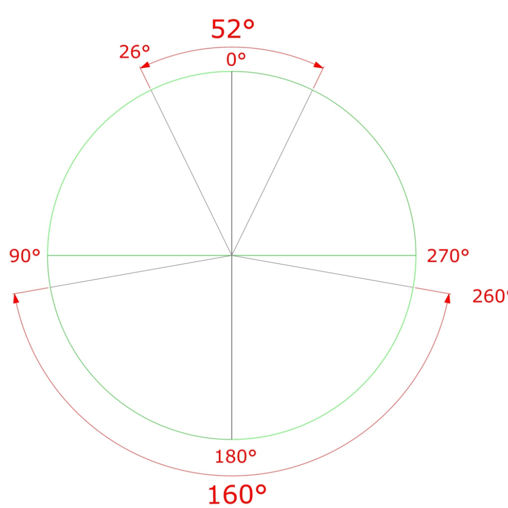

Figure 11: Left/right-mirror errors of task 8 and 11 of the SOT

this chapter once again illustrate the inadequacy of just summing up all the deviations of the correct solution at the SOT. The differentiated presentation and feedback method DIAM, which is introduced in this paper, could defuse problem areas and deliver meaningful results.

8. Conclusion

903 students took part in this project *GeodiKon* in which the major aim was to find out whether the training of each factor of Spatial Ability and of a repertoire of strategies for solving spatial tasks will lead to an improvement of Spatial Ability. The huge amount of collected data for the project makes it possible to discuss not only the two research hypotheses, but also a big variety of questions regarding leisure activities and spatial abilities, computer usage and spatial abilities, the accuracy of the ability of pointing to objects and gender specific effects [22, 25, 26]. The test battery of the pretests and the posttests consisted mainly of four spatial ability tests: Three Dimensional Cube Test (3DW), Differential Aptitude Test (DAT), Mental Rotation Test (MRT), and Spatial Orientation Test (SOT). In addition to the four tests there were different questions concerning which strategies students used to solve spatial tasks, and questions about age, gender, computer usage, leisure activities, school marks in Mathematics, German and English, and the type of learner.

One of the results of the "classical" analyses of the SOT is that the average deviation of the 12–14 year old students in the pretest is 59.04° [31] and in the posttest 50.64° [32]. These values are nearly twice as large as the performance of 17–18 year old students who have an average deviation of 30° at the pretest and 27° in the posttest [5]. There are no comparable big differences in the other three tests (3DW-Test, DAT and MRT). A clear correlation between the absolute angular error and the angular deviation of one's imagined heading from the orientation of the array can be observed. The absolute angular error does increase with the angular deviation of one's imagined heading from the orientation of the array (Figure 4). We therefore can confirm the results of Kozhevnikov and Hegarty [15].

The analysis of the data from the SOT was challenging:

1) The very large deviations from the correct solution angle compared to other research projects (e.g., [5]) motived for deeper analyses.

2) We had the goal to provide a sufficiently helpful and meaningful feedback to test persons.

3) The analyses of the data show the fact, that students solve the tasks of the SOT within two different steps.

These three aspects motivated for the development of the differentiated presentation and feedback method DIAM.

In this paper the two steps of the differentiated presentation and feedback method DIAM are introduced. The first is to locate the solution angle in the correct quadrant/semicircle, and the second step is to place the best possible solution angle. The two steps of the differentiated presentation and feedback method DIAM are ordered in accordance to the chronological considerations of test persons. Table 7 provides a detailed overview of the differentiated results of the SOT. A compact summary of the performance of a test person is shown in Table 8. These two tables form the core of DIAM. They should provide enough information for researchers for a detailed analysis of the results of the SOT, and they should offer a differentiated and therefore helpful feedback for test persons. Further investigations will be made to study the correlations between the new and the classical method and to describe more and more advantages (or even disadvantages) of the differentiated presentation and feedback method DIAM.

References

[1] E.S. Barratt: *An analysis of verbal reports of solving spatial problems as an aid in defining spatial factors*. J. Psychol. **36**, 17–25 (1953).

[2] G.K. Bennett, H.G. Seashore, A.G. Wesman: *Differential aptitude tests, forms S and T*. The Psychological Corporation, New York 1973.

[3] Bifie (Bundesinstitut für Bildungsforschung, Innovation & Entwicklung des österreichischen Schulwesens): *Kompetenzen und Modelle.* [Retrieved from `https://www.bifie.at/node/49`].

[4] R.B. Cattell: *Theory of Fluid and Crystallized Intelligence: A Critical Experiment.* J. Educ. Psychol. **54** (1963).

[5] A. Dünser: *Trainierbarkeit der Raumvorstellung mit Augmented Reality*. PhD Thesis University of Vienna, Department of Psychology, 2005.

[6] A.A.H. El Koussy: *The visual perception of space*. British J. Psychol. **20**, 1–80 (1935).

[7] H. Gardner: *The Unschooled Mind: How Children Think and How Schools Should Teach*. Basic Books, New York 1991.

[8] G. Gittler: *Entwicklung und Erprobung eines neuen Testinstruments zur Messung des räumlichen Vorstellungsvermögens*. Zeitschrift für Differentielle und Diagnostische Psychologie **5**(2) (1984).

[9] J. Glück, O. Vitouch: *Psychologie*. In St. Günzel (ed.): *Raumwissenschaften*, Suhrkamp Verlag, Frankfurt am Main 2009, pp. 324–337.

[10] J. Glück, H. Kaufmann, A. Dünser, K. Steinbügl: *Geometrie und Raumvorstellung – Psychologische Perspektiven*. Informationsblätter der Geometrie (IBDG) **24**(1), 4–11 (2005).

[11] J.P. Guilford: *The Nature of Human Intelligence*. McGraw-Hill, New York 1967.

[12] M. Hegarty, D. Waller: *A dissociation between mental rotation and perspective-taking spatial abilities*. Intelligence **32**, 175–191 (2004).

[13] M. Hegarty, D. Waller: *Individual differences in spatial abilities*. In P. Shah, A. Miyake (eds.): *The Cambridge Handbook of Visuospatial Thinking*, Cambridge University Press, Cambridge 2005.

[14] H. Kaufmann: *Lösung- und Bearbeitungsstrategien bei Raumvorstellungsaufgaben*. Raumgeometrie – intuitiv und konstruktiv, CD 2008.

[15] M. Kozhevnikov, M. Hegarty: *A dissociation between object-manipulation spatial ability and spatial orientation abilities*. Mem. Cognition **29**, 745–756 (2001).

[16] M. Kraker, A. Asperl, M. Forstner, D. Hochhauser, M. Husty, M. Liebscher, G. Maresch, D. Miestinger, O. Röschel, E. Wurm: *Die kompetenzorientierte Reifeprüfung. Darstellende Geometrie. Richtlinien und Beispiele für Themenpool und Prüfungsaufgaben*. bmukk, Wien 2012.

[17] M. Liebscher, G. Breyer, S. Fürst H. Heugl M. Kraker, C. Preis, E. Svecnik, I. Liegl, G. Plattner: *Praxishandbuch Mathematik AHS Oberstufe. Auf dem Weg zur standardisierten kompetenzorientierten Reifeprüfung. Teil 1.* Bifie (Bundesinstitut für Bildungsforschung, Innovation & Entwicklung des österr. Schulwesens), Vienna 2013.

[18] H.P. Maier: *Räumliches Vorstellungsvermögen: Komponenten, geschlechtsspezifische Differenzen, Relevanz, Entwicklung und Realisierung in der Realschule*. Europäische Hochschulschriften: Reihe 6, Psychologie, 493 (1994).

[19] G. Maresch: *Raumintelligenz – Die Phasen der Raumintelligenzforschung*. Informationsblätter der Geometrie (IBDG) **32**(1), 30–35 (2013).

[20] G. Maresch: *Strategies for Assessing Spatial Ability Tasks*. J. Geometry Graphics **18**(1), 125–132 (2014).

[21] G. Maresch: *Spatial Ability – The Phases of Spatial Ability Research*. J. Geometry Graphics **17**(2), 237–250 (2014).

[22] G. Maresch: *The Educational Research Project GeodiKon: Pointing Accuracy, Strategies and Gender-Specific Effects*. In H.-P. Schröcker, M. Husty (eds.): Proceedings 16th International Conference on Geometry and Graphics, Innsbruck 2014.

[23] G. Maresch, T. Müller, K. Scheiber (eds): *GeodiKon. Die Lernmaterialien. Praktische Raumvorstellungsübungen für den Geometrie- und Mathematikunterricht mit Lösungen*. Studienverlag, Innsbruck 2014.

[24] G. Maresch, T. Müller, K. Scheiber: *Das war GeodiKon – Der Versuch einer kompakten Zusammenfassung*. Informationsblätter d. Geometrie (IBDG) **34**(1), 9–11 (2015).

[25] G. Maresch: *How to develop spatial ability? Factors, Strategies, and Gender-specific Findings*. J. Geometry Graphics **19**(1), 133–157 (2015).

[26] G. Maresch: *Wie kann die Raumintelligenz gefördert werden? Faktoren, Strategien und geschlechtsspezifische Befunde*. Mathematik im Unterricht [Retrieved from `http://www.mathematikimunterricht.at`], 2015.

[27] S. Mick, S. Eibl, J. Gabl, D. Hochhauser, S. Ranger, J. Schmied: *Arbeitsunterlagen zu einem kompetenzorientierten Unterricht aus Geometrischem Zeichnen*. [Retrieved from `http://www.geometriekompetenzen.at`].

[28] M. Peters, B. Laeng, K. Latham, M. Jackson, R. Zaiyouna, C. Richardson: *A Redrawn Vandenberg & Kuse Mental Rotations Test: Different Versions and Factors that affect Performance*. Brain Cognition **28**, 39–58 (1995).

[29] E. Souvignier: *Förderung räumlicher Fähigkeiten – Trainingsstudien mit lernbeeinträchtigten Schülern*. Pädagogische Psychologie und Entwicklungspsychologie **22** (2000).

[30] C. Spearman: *General Intelligence, objectively determined and measured*. Am. J. Psychol. **15**, 201–293 (1904).

[31] E. Svecnik: *GeodiKon Pretest – Ergebnisse der Datenanalysen. Statistische Auswertung der Pretests*. [Retrieved from `http://www.geotic.at/docs/Pretest-Analysen-GeodiKon-Svecnik-2013-12.pdf`], 2013.

[32] E. Svecnik: *GeodiKon Posttest – Ergebnisse der Datenanalysen. Statistische Gesamtauswertung des Projekts GeodiKon*. [Retrieved from `http://www.geotic.at/docs/Posttest-Analysen-GeodiKon-Svecnik-140703.pdf`], 2014.

[33] L.L. Thurstone: *Primary Mental Abilities*. The University of Chicago Press, Chicago/IL 1938.

[34] L.L. Thurstone: *Some primary abilities in visual thinking*. Psychometric Laboratory Research Report No. 59, the University of Chicago Press, Chicago 1950.

[35] P.E. Vernon: *The structure of human abilities* (Rev. ed.). Methuen, London 1961.

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