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Graphic Achievements and Scientific Value of the Works of Mozi

Liu Keming

School of Humanities, Huazhong University of Science and Technology
Wuhan Hubei 430074, China
email: liukeming@mail.hust.edu.cn

Abstract. The *Works of Mozi* is the first history of science and technology that discusses theory and knowledge of graphics and thinking of graphics in ancient China. The records of the *Works of Mozi* show: maps and drawings are the same as text and numbers; they play an irreplaceable role in the advancement and development of human society. However, the study of Mozi, especially concerning the definition of maps and graphics and its status in the history, needs to be further and systematically clarified. This article uses this as the topic. We see the traditional graphics of Chinese culture, practice of graphics and theory and knowledge of graphics, and the spirit of a scientific inquiry reflected by studying the graphical content in the works of Mozi.

Key Words: maps, drawings, graphics, Mozi

MSC 2010: 01A25, 51N05

1. Introduction

The *"Works of Mozi"* are authored by Mozi (ca. 470 – ca. 391 B.C. or, maybe, ca. 480 – ca. 420 B.C.) and his disciples at the most famous *Mozi school*. In the *Works of Mozi* the word 'drawing' appears only twice; the word has been used as a verb and a noun such as 'green drawing'. The word 'drawing' has no meaning of engineering graphics, but the description of graphics and geometry knowledge in the *Works of Mozi* is indeed most abundant when compared with the Pre-Qin literature.

The *Works of Mozi* include basic graphic concepts, definitions of point, line, face, body, and logical relationships between them as well as methods to study the graphic space, finite and infinite issues, the properties of squares, circles, or lines. Mozi and his disciples, most of them engineers, focused on the knowledge of geometry for which there is an urgent need in engineering. These works greatly enriched ancient Chinese practice and the understanding of graphics, in view of graphics knowledge and theory.

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According to the *"Records of Mencius, Xun Qing Biography"* by Sima Qian (145–90 B.C:), who sometimes said that Mozi had the same age as Confucius but was sometimes younger than Confucius (ca. 468 – ca. 376 B.C.), Mozi was a famous thinker and scientist during the Warring States period. Information on Mozi and his school of science and technology and graphics is mainly found in the *"Canons and their Explanations"* (volumes I and II), in the *"Challenges"* of Gong Shu, *Preparation for City Defense*, all of which were collected in the *Works of Mozi*. The basic concepts and the definition of graphics and geometry, as well as the understanding of projection in the *Works of Mozi* were the most systematic studies on graphics during the Pre-Qin period [1, 2, 3].

2. Geometric construction at Mozi's times

Among the disciplines of science and technology, graphics is an interdisciplinary subject, which uses graphical systems to study and understand the objective world. From the view of configuration elements, graphics or drawings consist of points, lines, areas, volumes, etc. The different variations and combinations of these geometric elements form different configurations. A geometric drawing represents best the people's understanding of geometry. This is because in engineering technology we can often encounter graphics which are composed of a large number of points, lines, faces and bodies. Therefore, the basic geometric construction methods and technologies can not only improve the speed and accuracy of engineering drawing, but also reflect the level of development of each era in engineering drawing techniques.

Ancient Chinese cartographers summarized a lot of easy drawing methods. Geometric constructions originating from Mozi's era can be seen at the relics found in summer 1978 in the *Tomb of Ceng Houyi* of Suizhou Warring States and 1986 in the *Baoshan Chu Tomb*. Ceng Houyi and Mozi lived at the same era; Baoshan Chu was later. All these relics are treasures from the Pre-Qin dynasty, which reflect the achievements of ancient Chinese in graphics and graphics technology and the people's understanding of geometry. Relics from the *Tomb of Ceng Houyi* and the *Baoshan Chu Tomb*, either made from bronze or lacquer, show painted

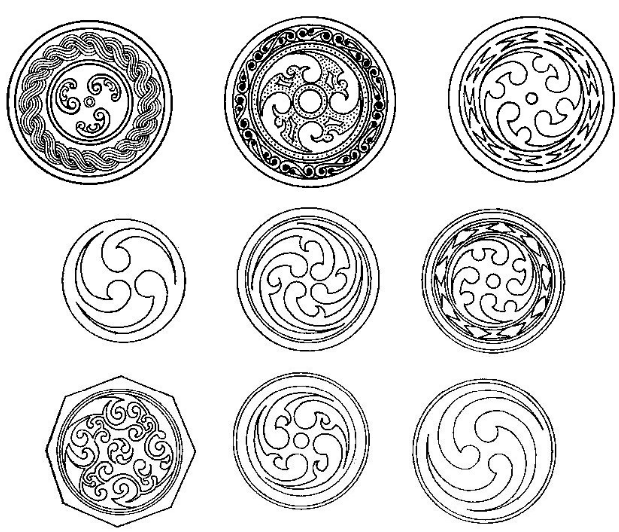


Figure 1: Geometric construction of the cultural relics unearthed from the *Tomb of Ceng Houyi*

Figure 2: Geometric construction of the cultural relics unearthed from the *Tomb of Ceng Houyi*

Figure 3: Geometric designs unearthed from the *Baoshan Chu tomb*

variations of geometric patterns, including a rich content of engineering geometric drawing, such as parallel lines, tangents, rectangles, circles, concentric circles, ellipses, connections of circular arcs, and regular n-gons for $n = 4, 5, 6, 7, 8, 12, 16$, and 20. The plots with ruler, the formal drawings and the approximate geometry drawings show an extremely skilled and accurate geometric construction capability (note Figures 1–4).

At the same time, these geometric patterns and lines and the other decorative patterns with mechanical repetition give the shapes a flavour of an unique artistic painting. Geometric lacquerware patterns consist of points, lines, faces and include the dot pattern, diamond pattern, triangular pattern, web texture, circular pattern, and whirling pattern. It is really amazing that these geometric constructions appeared in the 5th century B.C. in the Warring States.

During Pre-Qin, the ability in geometric drawing was reflected in their geometric drawing practice, and also a sign of advanced drawing technology. As an example, the relics from the *Tomb of Ceng Houyi* and *Baoshan Chu Tomb* cover almost all aspects of geometric drawing, such as ruler plotting, formal drawing and approximate geometric construction. Ruler plottings use tools such as a compass and a rectilinear scale, and they are error-free and accurate. The formal drawings belong to the scope of ruler plottings. Approximate geometric constructions refer to the actual application of most graphics: people used an approximate geometric construction, e.g., for the determination of a regular pentagon or heptagon. So, these paintings can't be proved by axioms and theorems, and these graphics are an important part of modeling (Figures 1–4).

3. Discussion of basic geometry concepts in the *Works of Mozi*

From the viewpoint of the history of graphics development, graphics and drawings have a constantly changing basic concept, which improved along with the progress of human civilization. Today, graphics and drawings in science and technology mostly refers to graphics as being mathematically described, which constitutes the basis for the development of science and engineering. The core of graphics is geometry; graphics is a science which simultaneously

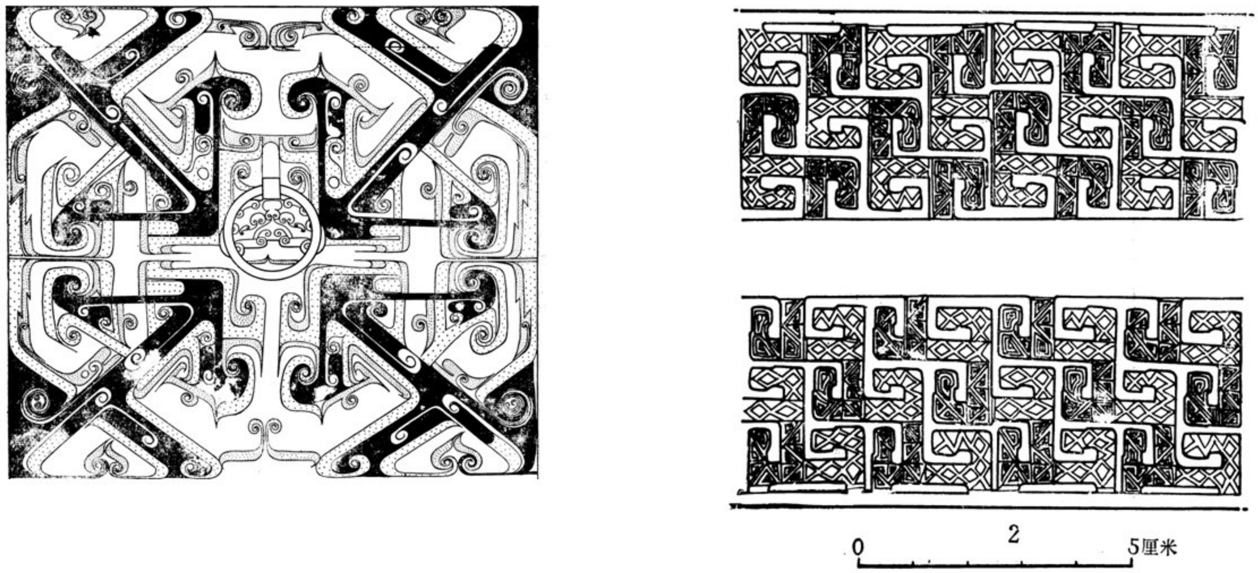


Figure 4: Another cultural relics unearthed from the *Baoshan Chu tomb*

developed with geometry. The history of graphics is also the history of geometry, and the early descriptive geometry is also a part of geometry. Therefore, the graphics basic concepts such as the definitions of point, line, face, and body often reflect people's basic awareness of graphics and their ability.

Meanwhile, according to known conditions, geometry drawing is an essential part of graphics and an important way to show the geometric skills and perspective performed by its maker; but it also reflects the logical reasoning ability. In the *Works of Mozi*, especially the geometry concepts and definitions, graphics can be found in the *Canons and their Explanations* (Chapters I and II) and in the *Exposition of Canon* (Chapters I and II), which contain 29 articles. *Canon I* is about definitions, *Exposition of Canon I* provides the explanations and illustrations for *Canon I*.

3.1. On the definition of points

In the *Works of Mozi*, a point is named a *"Duan"*, which in mathematics denotes the endpoint. As contained in the *Works of Mozi*, points form a line, lines form a plane and planes form a solid. It is said in *Canon I*, Chapter 61, that an "endpoint is the forefront point that divides a line segment at the farthest end". In the *Canons and their Explanations I* endpoint means that there is no other point in front of it.

The word *Duan* appears 15 times in the *Works of Mozi*, in the *Canons* and the *Canons and their Explanations*, which mainly means that in geometry the point is one of the fundamental concepts. Firstly, *Duan* corresponds to the point in geometry. Secondly, *Duan* means the forefront of the object or one of the two terminal points of a line segment. The two meanings are different, but they are also connected. If *Duan* is the forefront point of an object, there is no other points in front of it; so *Duan* means the forefront and cannot be behind others. That is, why the *Canons* say that the "endpoint is the point which devides a line segment at the end". A *Duan* is an essential point. According to the definition in geometry, a single point cannot be measured in size or thickness, which means, two points cannot be in the same position. The analysis and arguments about *Duan* in the *Canons and their Explanations* are arranged from the simple to the complex in a rigorous mathematical concept.

3.2. On the definition of lines

The essence of a line and the relationship between lines and other geometric objects are discussed many times in the *Works of Mozi*. In Chapter 4 "On the Normal Standards" we can read: "The craftsmen draw squares with a square, and draw circles with a pair of compasses, they use the capenter's line to draw straight lines and find the perpendicular by a pendulum".

Using the capenter's line to draw a straight line is one of the important straightening methods in engineering technology. Tightening the carpenter's line, when drawing lines, should be taken as a standard to meet in order to shape the wood more straight, which is obtained from the process of engineering and manufacturing by ancient craftsmen. Thus, the perceptual experiences about lines has gradually been raised up to a form of theory.

Mozi's treatise about "using carpenter's line to draw straight lines" is similar to what is expounded about the essence of straight lines in geometry — for any two points there is a connecting straight line. Based on the former treatise, another law concerning straight lines is put forward in *Works of Mozi*, that is: a line segment has an infinite number of points, which is consistent with the definition of a straight line in geometry.

3.3. On parallel lines

Chapter 52 of *Canon I* says that "equidistance means the same length shared by straight lines". The *Canons and their Explanations* say: "necessity as something inevitable and everlasting may be exemplified by the levelness of a platform or the equality of two brothers. One possibility is that they are brothers, the other is that they are not. It cannot be decided by one's own mind, what is correct."

The definition of equidistance here refers to parallel lines in geometry. 'The same length' means that the two straight lines are parallel, and then the shortest distance between them (called vertical or height) is equal, as shown in Figure 5.

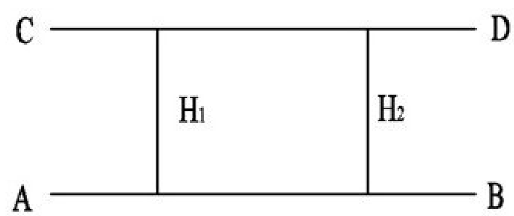


Figure 5: Mozi about the definition of parallel lines

3.4. On line segments of the same lengths

Chapter 53 of the *Canon I* says "*Tung Chang:* to use the straight square for measurement". The *Canons and their Explanations* says "*Tung Chang:* the same lengths of door bar and doorframe are all straight". The statements above are definitions of line segments of the same lengths, and 'the same lengths' means that the two line segments can just coincide.

3.5. On the definition of planes

Chapter 62 of the *Canon I* says: “a gap is the space to be flanked on both sides”, and the *Canons and their Explanations* say: “to have a gap is to be flanked on both sides”. Chapter 63 says: “a gap is an unfilled space”, and the *Explanations* say: “a gap is the space to be flanked on both sides, a line is not to be flanked by a surface or a point, although it is formed prior to the formation of a surface and after the formation of a point”. The word ‘unfilled’ in ‘unfilled space’ does not mean ‘reaching somewhere’.

3.6. On the definition of circularity

The *Canon I* says: “*Yuan* (circularity): one center has the same length”. The *Canons and their Explanations* say: “*Yuan:* the compass can draw a circle and the line will meet”. Chapter 54 in *Canon I* says: “*Chung* (center): from which one shares the same length”. The *Canons and their Explanations* say: “*Chung:* distances outward from this are all the same”. The statements above are Mozi’s study about the definition and properties of circularity and he highlights a closed plane curve such that all its points are equidistant from a given fixed point, the center.

According to the definition of a circle in geometry, a circle is a set of points at given distance from a given point in the plane. The given point is the center and the given distance is the radius. Suppose that $ABCD$ are four points on the same circle and O is the center, then OA, OB, OC, and OD are equal to each other. So, the *Works of Mozi* use *Zhongxin* (center) to show that each point of the circle from the center is equidistant. “A circle is a plane curve every point of which is equidistant to the center”. Points on the circle are at equal distance to the center, while conversely, each point whose distance to the centers equals the radius must be on the circle.

Suppose $ABCD$ are four points on the circle with center O, such that O is located on the chords AB and CD (Figure 6). Then AB and CD must be of the same length. Guo Ye Wang’s Yupian says: “a compass is an instrument for standardizing circles”. When the wheelwright uses the compass to test the roundness of objects in the world, he is often heard to say: “if they match the line of my compasses, they are round; otherwise they are not”.

The *Canons and their Explanations* say: “a circle is drawn by turning a pair of compasses through 360°”, which is called a construction in modern geometry.

Mozi devoted his life to the work of machine building and he was a skilled handcraftsman who can match the famous skillful workman Gongshu Ban. He acquired experience and knowledge from practice and stories that circles should be drawn with a pair of compass. “He knows the meaning of what others wanted to express from listening and seeing”. And then he

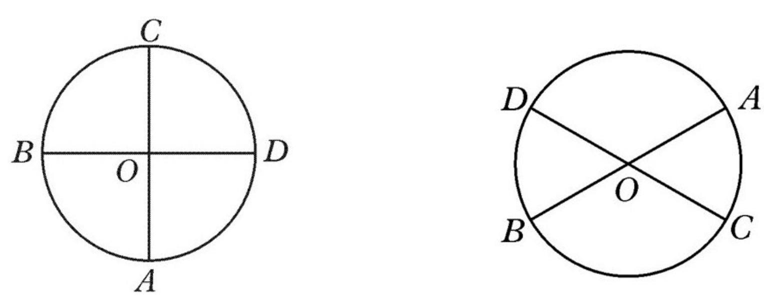


Figure 6: Mozi about the definition of a circle

formulates the recognition that "a circle has a center that is equidistant to all points on its circumference, on the basis of which the roundness of objects in the world can be tested with a compass. If they match the line of my compasses, they are round; otherwise they are not." In this way, Mozi made huge progress in the understanding of circles, in comparison with predecessors.

3.7. On the definition of a square

Canon I says "*Fang* (square): the four corners are at right angles". The *Canons and their Explanations* say "*Fang:* the carpenter's square can draw a square and the lines will be straight".

The definition of a square here is "a shape with equal sides and four right angles". As shown in Figure 7, four sides, namely AB, BC, CD, and DA, are equal in lengths and four angles, namely $\angle DAB$, $\angle ABC$, $\angle BCD$, and $\angle CDA$, are all right angles. Meanwhile the *Works of Mozi*, Chapter 26 says in "The Will of Heaven": "if they match the line of my rules, they are squares, otherwise they are not".

The works of Hsun Tzu (Xunzi) and their explanations, written by Yang Liang (or Yang Jing), say: "*Ju* (spline or carpenter's square) is an instrument to draw squares". Draw a square with carpenter's square, and the four sides will form a closed shape and adjacent sides will be perpendicular to each other. *Zhou Bi Suan Jing*, a work on astronomy and mathematics in ancient China, says: "A circle can be obtained if the square is cut to a certain extent; squares can be drawn with a spline". As is shown at the square $ABCD$, AB intersects BC at B, and BC intersects CD at C. It can proved true that A and D are also intersection points between two straight lines, so we can say: "Squares can be made up with intersecting lines". The definition of rectangles, drawn by a spline, is shown in Figure 7.

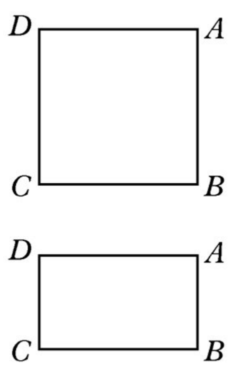


Figure 7: Mozi about the definition of a rectangle

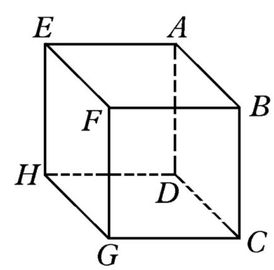


Figure 8: Mozi's definitions about a body

3.8. On the definition of solid

Chapter 55 in *Canon I* says: "the size of an object has dimensions". The *Canons and their Explanations* say: "a solid cannot be formed without dimensions".

Suppose $ABCD$ and $EFGH$ are two figures in different planes. Connect them and they form a solid, and any side can be extended to the dimension of the solid. As *Canons I* formulated, "the size of an object has dimensions, such as length, width and height", which means, the size is the volume acumulated by length, width and height. The definition on solids is shown in Figure 8.

There are also some statements concerning the definitions and properties of solids. Chapter 2 in the *Canons I* says "*Ti* (solid) is a portion in the total or the whole". The *Canons and their Explanations* say "*Ti* is the aggregation of parts, just like a line is the aggregation of points".

The analogy of points illustrates the meaning of solids; if the difference between the whole and the part is well illustrated, the definition of solids is made clearer: It is characteristic to say, "parts are taken from the whole" in *Works of Mozi*, for example, one is a part of two, but two is not all; a point is a part of the line, however, the line is all. Time and space is infinite, the so-called 'whole' is perhaps relevant in a relative sense [8, 9, 11].

In geometric statements and definitions in the *Works of Mozi*, a point is called *Duan* (the endpoint), a line is called *Chi*, and a plane is called *Qu*. When Mozi used the concepts of *Duan, Chi* and *Qu* to study the shape of a space object, the idea of geometrizing graphical issues and simplifying complex issues was born. This is in accordance with statements concerning graphics and visualization, vision and image, geometric design and calculation, which are widely applied in methods of engineering graphics and computer graphics. From the viewpoint of *"Xing"* (shape) in geometry , Mozi revealed the essence of *"Tu"* (chart); therefore he provided scientific statements on shape and chart.

4. Discussion of projection theory in the *Works of Mozi*

With regard to the concept of geometry, the *Works of Mozi* do not intend to use any deductive reasoning to prove geometric theorems, but to explore projective geometry and measurement relevant facts. This becomes obvious in Mozi's theory of graphics. This is because during the process of civilization in human history, graphics has been a method to understand nature and to express and exchange ideas. From the generation of Hieroglyphs on to the development of modern science and technology, human activities are always closely linked with graphics. The importance of graphics can never be replaced by any other expression. The graphic method mainly focuses on how to use projections to express spatial geometry, i.e., the geometry of points, lines and planes and their relative positions, on a drawing plane, and conversely, to infer the exact spatial object from the graphic plane prototype. That is, to establish a one-to-one relationship between the two-dimensional graphics and the three-dimensional physical space.

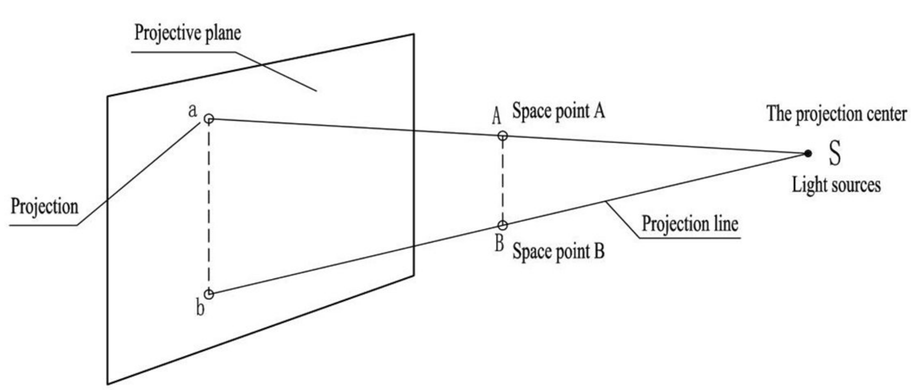


Figure 9: The principle of projection in the modern theory of graphics

As a basic concept of projection, we need a light source, a projected object and a projection plane. This constitutes the three basic conditions for projection. When an object is placed between the light source and the projecting plane, the images of the object will be shown on the projecting plane. If we abstract this natural phenomenon, we obtain the projection method.

Projection principles of modern geometric theory, as shown in Figure 9, need a projection plane P and a fixed point S, not belonging to P, as the center. All projection lines pass through the center of projection. The projecting line through the space point A intersect the projection plane P at the point a which is called the projection of A. The method of representing a space object in a plane according to this geometric law is called the projection method. Projection is the basic theory of graphics [12].

In the *Works of Mozi* projection theory can be found in the *Canons and their Explanations*, which in total has 8 articles and threehundred and forty words. In the *Works of Mozi* there are 20 articles which discuss graphics and eight articles about optics. The first article discusses the definition and generation of a shadow, then the relationship between light and shadow, which implicitly refers to straight lines of light. The third article discusses straight lines of light.

4.1. Understanding the basic projective geometry concepts in *Works of Mozi*

Projective geometry is the most important basic of modern graphics; the projective geometry method is to draw the geometry of the three-dimensional space on a two-dimensional plane. Solving problems of space geometry is the starting point of graphics. In the *Works of Mozi* projection geometry is addressed, as listed below:

Canons and their Explanations II, Article 16: "the image does not move, for what moves is either the object or the light".

Canons and their Explanations: "when the light arrives, the image disappears. If the image is to remain forever, the light and the object should remain still forever".

The *Works of Mozi* discuss the physical properties of a moving object and its projection. The word "shadow " appears 36 times in the *Works of Mozi*, and 32 of them refer to figures or optics. And it often appears in the *Canons and their Explanations*.

The word "shadow" has three meanings in the *Works of Mozi*. Firstly, it is interpreted as light, just as the projection center of projection geometry. Secondly, the shadow formed by an illuminated object. Thirdly, the graphics and images formed by light when reflected in a reflecting surface.

This has clarified the three elements of projection theory, 1. the light , namely the projection center; 2. the projected objects; 3. the projections of the objects. Mozi's idea is consistent with the thinking of modern projective geometry theory. In Article 16 of the *Canons and their Explanations II* and the *Canons and their Explanations I* the term 'shadow' is explained, according to the second meaning, as the shadow that is formed by an object shielded from light.

Mozi clearly stated in the *Canons and their Explanations II* that shadow can't move; if shadow is moving this is caused by a relative movement between the light source and the projected object. Mozi expounded "the shadow formed by an object shielded from light" in the *Canons and their Explanations II*. The projection is formed because the enlighted object is projected onto a screen or ground. When the object is moving, there appears the projection on the ground, because the light is shielded, but after a moment the shadow will disappear, when the ground is enlighted. So MOZI says: "it is unimaginable that the projection is always present at the same place during the movement of objects. Why it seems that the projection moves with the movement of objects? This is because the movement of objects behaves continuously in space and time, and the projection is continuously updated as time changes." MOZI's statement reflects his keen observation and deep insights.

After discussing the object between the light source and the projection, the *Works of Mozi* expound the relation between light source, projected object and object's projection. The *Canons and their Explanations II* states in Article 17: "the shadow has ghosting and penumbra kinds because the shadows overlap".

In the *Canons and their Explanations I* one can read: "ghosting can only be formed when an object is sandwiched between two light sources". MOZI discusses here the relation between light and shadow even deeper than in the previous article.

4.2. The *Works of Mozi* discuss the property of projective geometry

There is no doubt that the basic conception discussed in *Works of Mozi* and the thinking of modern projective theory are consistent. The projection formed by parallel light is called parallel projection, the one formed by a point light is called central projection, the one formed by light that is perpendicular to the plane of projection is called orthographic projection. The projection formed by oblique light is called oblique projection. Shape and size of any object's projection is relative to the projection plane's location and angle.

In shadow theory it is assumed that during light irradiation some part of a plane is deprived of light. In the language of graphics 'a shadow falls on the plane'. The contour of the shadow is the shadow line, and the shadow appears since light is blocked by an object, and the line on this object, which separates the side facing light and the side back to light is called the counter line of shadow. Whether it is a shadow or a projection, they all must have a light source, a projected object and plane where the projection is located. In the *Works of Mozi*, the discussion about the projected object and projection light source, that is, the relation between projection, location, size and the experimental results on the projection space can be seen in the *Canons and their Explanations I* and *II.*

The *Canons and their Explanations II* say: "an object's shadow is sometimes large and sometimes small, because the object is sometimes inclined and sometimes upright, the light source is sometimes near and sometimes far".

The *Canons and their Explanations II* say: "concerning the length of an object's shadow, take a wood stud for example. If the stud is askew the shadow will be short and large. If the stud is upright the shadow will be long and short. If the light source is smaller than the stub the shadow will be larger than the stud. Not only the size of the light source influences the size of the shadow, but also the distance".

In an experiment, the stud is a standard wooden pole used to be projected. This article expounds the principle and laws about how the size of its projection depends on the relative location of light source, object and projection plane. MOZI's discussion about this principle can be proved mathematically: assume that a light source projects from point S onto the ground EF, and the angle with the horizon is θ. A wooden pole AP rotates about the center A in the plane SAE, such that P traces a semicircle B_1PB_2. The length of the projection AB of wooden pole varies on EF. Assume that the angle between the wooden pole AP and horizon is α (as shown in Figure 10).

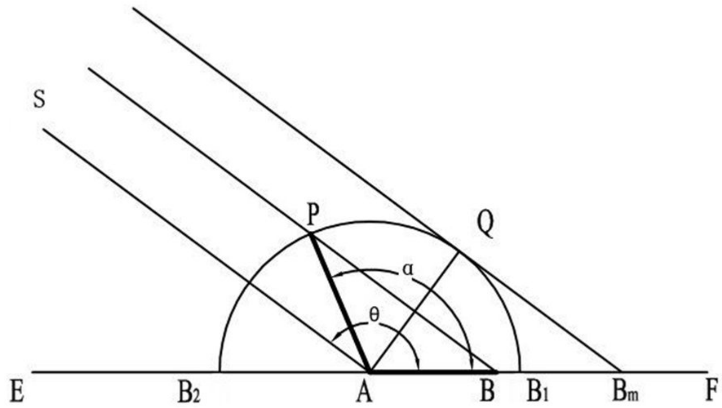


Figure 10: The shadow AB of the rotating pole AP varies

According to the Sine-Theorem we have

$$\frac{AB}{\sin\angle APB} = \frac{AP}{\sin\angle BPA} \tag{1}$$

From the description above we know $\angle BAP = \alpha$, $\angle SAB = \theta$, $\angle SAP = \angle SAB - \angle PAB = \theta - \alpha$, and $\angle APB = \angle SAP = \theta - \alpha$. That means, interior angles between parallel lines are equal. For the angles of the triangle ABP we have

$$\angle PBA = \pi - \angle APB - \angle PAB = \pi - (\theta - \alpha) - \alpha = \pi - \theta.$$

We plug this into the Eq. (1) and obtain

$$\frac{AB}{\sin(\theta - \alpha)} = \frac{AP}{\sin(\pi - \theta)}, \quad AB = \frac{AP}{\sin(\pi - \theta)}\sin(\theta - \alpha) = \frac{AP}{\sin\theta}\sin(\theta - \alpha).$$

When $\alpha = 0$ then $AB = AP$, i.e., the length of shadow is AB_1. When $\alpha = \theta - \frac{\pi}{2}$ then

$$AB = \frac{AP}{\sin\theta}\sin\left(\theta - \theta + \frac{\pi}{2}\right) = \frac{AP}{\sin\theta} = AB_m.$$

If α continues to increase the projection will shorten. When $\alpha = \theta$ then

$$AB = \frac{AP}{\sin\theta}\sin(\theta - \theta) = 0,$$

which means that there is no projection. If α continues to increase the projection will elongate to the left. When $\alpha = \pi$, that is, the pole is parallel to the ground, the length of the shadow is $AB_2 = AP < AB_m$.

The conclusion can be drawn according to above-mentioned mathematical derivation: if $\alpha = 0$, that is, the wooden pole AP rests on the ground, the shadow is AB_1, the length is AP. If α continues to increase the end of shadow will approach B_1. If $\alpha = \theta - \frac{\pi}{2}$ then $\sin(\theta - \alpha) = 1$; the end of shadow reaches the farest point B_m, and $AB_m = AP/\sin\theta$. At this time, the pole is orthogonal to the light AS in AQ, that is called "wood is vertical". After then, when α increases, the length of the shadow will shorten instead. When $\alpha = \theta$ it shrunks to A, that is, the length of the shadow becomes a minimum. If α continues to increase, the shadow moves left from A. When $\alpha = 2\pi$, that is, the pole rests again on the ground, the length of the shadow is $AB_2 = AP$, shorter than the length of AB_m.

The experiment mentioned above is consistent to the statement in the *Works of Mozi*: "if the pole is slant the shadow will be short and large. If the pole is vertical, the shadow will be long and small".

There is a light source AB of size smaller than the pole CD as shown in Figure 11. Place a screen behind it, and we can get the shadow $E'EFF'$, which is always larger than the object CD. So, we can say: "if light is smaller than wood then the shadow is larger than wood". If, contrary, the light source AB is larger than the pole CD, as shown in Figure 12, the shadow $E'EFF'$ on the screen is also larger than the pole CD, although the portion EF of the shadow is smaller than CD. Even, if the screen moves backward the shadow remains larger than CD. So we say: "if the light is larger than the wood then the shadow is also larger than the wood".

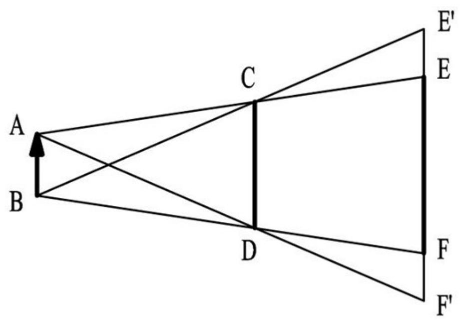


Figure 11: light is smaller than wood ⇒ the shadow is larger than wood

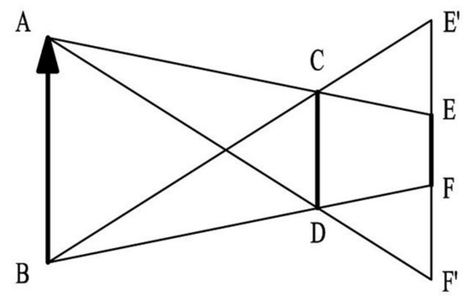


Figure 12: light is larger than wood ⇒ the shadow is larger than wood

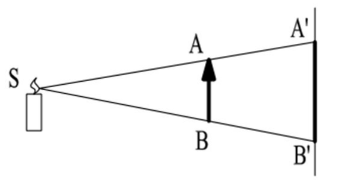
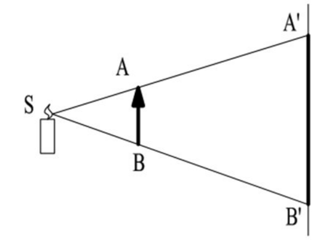
No matter whether AB is smaller or larger than CD in the above discussion, if the distance between the screen and light source is fixed then, the closer CD to the screen, the shorter $E'EFF'$, and, the shorter the penumbra $E'EFF'$, the darker the shadow. On the contrary, the closer CD to AB, the longer the shadow $E'EFF'$, and, the larger the penumbra $E'EFF'$, the lighter the shadow color. So we say: "if the pole is slant, the shadow will be short and large. If the pole is vertical, the shadow will be long and small".

Mozi says in the *Canons and their Explanations II*: "the longer the distance between light source and pole, the smaller is the shadow, and the smaller the distance, the larger is the shadow." From this we know that Mozi's exposition is based on an analysis of projection object and the corresponding proportion of the projection. Mozi discussed the central projection and its properties as shown in Table 1.

Therefore, in Mozi's exposition of projective geometry the principle is involved that the shape of any object's projection is determined by the position of the viewpoint and the object. The front part of the *Canons and their Explanations II* deals with the relation between the attitude of standard wood and the size of image. In the middle part the relationship between the size of the light source and the size of the image is treated, while the last part discusses the relationship between the distance of standard wood and the size of the image (i.e., light source and object). This is done in a unified way. Parallel light beams like that originating from solar light are used to explain properties of parallel projection.

The following statements are related to central projection: "if standard wood is skew, the shadow will be short and big; if standard wood is vertical, the shadow will be long and small." The principle of central projection is that there is a light source and an object, and the light source illuminates the object to generate a projection in a divergent way. So, this passage of

Table 1: Mozi discusses central projection and its properties

from the *Canons and their Explanations II*	the shadow is longer and smaller. It is not only because its size is smaller but also because its distance is farther	
about the explanation	if the light source is far away from the standard wood AB, the projection will be small	if the light source is close to the standard wood AB, the projection will be big
illustration of the explanation		

the *Canons and their Explanations* means: if the standard wood is inclined its projection is short, thick and fat, while, if standard wood stands, the projection is long, narrow and light. If the light source is smaller than the standard wood, then the projection will be bigger than the standard wood itself; if not, the projection will be smaller than the standard wood. If the distance between the light source and standard wood is far, the projection will be small; if not, then the projection will be big.

Mozi's words about the generation of the projection in the *Canons and their Explanations* are on the top of the list on theories on optics and come straight to the point. The definition of a projection and the relationship between light source and projection is easy to understand, meanwhile it is consistent with modern projection theory. Although in the *Works of Mozi* and the *Canons and their Explanations* no graphic method is used to solve the problem of drawing space geometry, it covers projective geometry and the basic concept of drawing, which reflects Mozi's achievements on the understanding of projections [6, 10].

5. The records about drawing instruments in *Works of Mozi*

Beside language, graphs and drawings are the most important ways of transmitting messages. Graphics, as the basic language of engineering technology, can project people's imagination into the real world, which means, graphs and drawings can realize engineer's imagination. The key to realizing this process is drawing. In the process of drawing, common drawing tools are the basis for the accuracy of drawings and an important aspect of measuring the level of drawings. A correct use of drawing tools is not only a basic skill of engineering and technical personnel, but also an important method of learning and consolidating the knowledge of cartography. This holds for East and West.

The ancient drawing tools in the *Works of Mozi*, named *"Gui"* (compasses) and *"Ju"* (squares), have detailed records and higher frequency, which is rare in the Pre-Qin literature. Though Mozi's work is said to be hard to read, it has a strict, strong logic when it refers to *Gui* (compasses) and *Ju* (squares); The word *Gui* (compasses) appears eleven times, while *Ju* (squares) appears ten times. They all appear in pieces of "On the Normal Standards", "The Will of Heaven" and the *Canons and their Explanations*, as given below.

In the Chapter 4 "On the Normal Standards", Master Mozi said: "No matter what kind of business one is engaged in, one must have standards. No one can do his job well without sticking to standards. Even those gentlemen who have been promoted to the rank of generals and ministers have to observe standards. The same is true with craftsmen. They draw squares with a square ruler and draw circles with a pair of compasses. They use the carpenter's line to draw straight lines and find the perpendicular by a pendulum. All craftsmen, whether skilled or unskilled, follow these five methods as standards in their work. The skilled workers can meet the standards. Though the unskilled workers cannot fully meet the standards, yet by abiding the above-mentioned standards they can surely do a better job. Thus, all craftsmen observe standards in their work. Now in administering the land under heaven or managing the state affairs of large countries, we seem to lack standards. This shows that we are less discerning than craftsmen."

In Chapter 26, "The Will of Heaven I", Master Mozi said: "The will of Heaven is to me what a pair of compasses is to a wheelwright or what a square is to a carpenter. The wheel-wrights and the carpenters use their compasses and squares to measure the roundness or squared objects in the world and accept those, that fit these measurements, as right and those, that do not fit them, as wrong. Now the books of the elite gentlemen in the world are too many to be listed, and their sayings are too many to be recorded in full. They endeavor to convince the feudal lords on the one hand and men of honor on the other. But they move far away from the principle of benevolence and righteousness. How do I know that? Because I have the clear guideline, the will of Heaven, to measure their speech and conduct."

In the Chapter 27, "The Will of Heaven II", Mozi said: "the will of heaven to us is the same as a compass to wheel craftsman and a square ruler to carpenter. A wheel craftsman uses his compass to measure whether everything is circular or not, and say: 'if things are in line with the my compass, they are circular; if not, they aren't circular.' Therefore, it is known that whether things are circular or not. Why is this? Because the rules of determining the circle are very clear. A carpenter uses his square ruler to measure whether everything is square or not, and say: 'If things are in line with the my ruler, they are square; if not, they aren't a square.' Therefore, it is known whether things are a square or not. Why is this? Because the rules of determining a square are very clear."

In Chapter 28, "The Will of Heaven III", Mozi said: "taking the will of heaven as a law, like wheel craftsman have a compass and carpenter has a square ruler. Now wheel craftsman uses the compass and no carpenter uses the square ruler to know the difference between square and circle. Thus, Mozi takes the will of heaven as a law."

In Chapters 40–41 of the *Canons and their Explanations I*, Mozi said: "circle: use a compass for drawing a circle, and the curve becomes a circle."

In Chapter 42 Mozi said: "an idea in combination with a compass and circle can be made into a real circle model."

In Chapters 40–41 of the *Canons and their Explanations I*, Mozi said: "square: use a square ruler for drawing straight lines, then the lines intersect at a square."

From this we can know, the terms 'compass' *(Gui)* and 'square' *(Ju)* in the *Works of Mozi* has two meanings: The first is a drawing tool; the second means the rules as standards.

5.1. The drawing tools *Gui* (compass) and *Ju* (square)

The drawing tools *Gui* (compasses) and *Ju* (squares) were already known in the Pre-Qin period. Mozi said, "if things are in line with the my compass, they are circular; if not, they aren't circular". Moreover, the statement "use a square ruler to draw a square, use a compass to draw a circle, use a line marker to draw straight lines" reflects the application of drawing tools at that time.

In geometry, when the drawings tools are limited to ruler and compass, one speaks of a construction with ruler and compass. Drawings produced with ruler and compass are named basic drawings. Since a large class of graphics can be completed by several basic drawing methods, they synthetically and skillfully integrate geometry and mathematics into the problems of plane geometry. Therefore, as a combination of the strengths of various disciplines, drawing with ruler and compass is a kind of 'shape computing' in nature. Drawing with ruler and compass can reflect the level of the geometry development of a civilization.

The geometric drawings from the unearthed cultural relics in the *Tomb of Ceng Houyi*, which is contemporary with Mozi, is enough to represent the high achievements of drawings with ruler and compass in that period. These unearthed cultural relics show not only strict standard geometric drawings, but also a large number of others, such as an approximate partition of a circle into five equal parts, which is so accurate and strict that even from our present point of view it is amazing! This is far beyond ancient Greek ruler drawings, though they appeared more than one hundred years later. Comparing ancient and modern, Chinese and Western, it is hard to find non-standard geometric drawings like those in the *Tomb of Ceng Houyi* in the ancient Greek period.

5.2. *Gui* (compass) and *Ju* (square) as norm and standard

Mozi not only discusses in detail *Gui* (compass) and *Ju* (square) as ancient drawing tools as well as their application for measuring squares and circles, but also makes an analogy between the application of drawing tools and the application of standard laws, such as "the theory to govern", "the principle of working", "the rule of behaving". That is rare in the Western literature.

Even today, when science and technology become more and more developed, any research project or construction project cannot be accomplished without norms or standards. Mozi" emphasized the rules: "doing anything, people cannot have no standard. It's impossible to make careers without any rules." "Everybody, who wants to accomplish anything, must have his normal standards. No one can accomplish anything without them." This is just like the importance of strengthening drawing standardization construction. As a communication language of science, engineering, art, and a tool to transfer ideas and knowledge, graphs and drawings need to follow certain norms and standards. Graphical standardization supports the exchange of human ideas. It comes from the discipline innovation, and constantly promotes the innovation of the subject, also as a result of graphics, thus, "they are still far away from benevolence and righteousness." That's why the *Works of Mozi* can become one of the important ancient drawing technical literatures [7].

6. Conclusion

1. The *Works of Mozi* show the Chinese tradition in emphasizing geometry and graphics. They summarize practice and theory of graphics and reflect the spirit of a scientific inquiry. In addition, Liang Qichao claimed in [4]: "In all Chinese classics, only *The Works of Mozi* match modern Western scientific spirit". This is a very insightful comment.

2. Mozi's theory of graphics, thought of graphics and concept of geometry are not intended to use deductive reasoning for proving geometric theorems. He attempts to investigate the facts related to measurement of projective geometry, and this way of thinking has influenced the development of Chinese culture and technology. Chinese ancient graphics technology couldn't achieve these graphics outcomes without the understanding of geometry by ancient mathematicians represented by Mozi [5].
3. The investigation of graphics in the *Works of Mozi* can help us to rediscover and locate the historical status and function of graphics. In the *Works of Mozi*, as a collection in Qin Dynastic of the Qin Dynasty, a considerable part deals with natural sciences and production technologies, such as mechanics, geometry, mathematics, optics, metrology and so on. Concerning the included content on graphics and basic theory, although it's text is simple and complete and it's words are esoteric, this is a summary of practical theory in Chinese graphics, and is not found in any other Pre-Qin literature. Therefore the *Works of Mozi* is the first history of a scientist, the first history of technical ideas and even the first history on graphics in Chinese classics.

References

[1] Hubei Provincial Museum and the Beijing Institute of Arts and Crafts Co written: *Patterns of cultural relics unearthed from the tomb of Ceng Houyi in the Warring States Period*, Changjiang Literature and Art Publishing House, Beijing 1984, pp. 15–59 and 87–89.

[2] Hubei Provincial Museum: *The Tomb of Ceng Houyi, Beijing*, Cultural Relics Publishing House, 1989, pp. 167–175 and 329–330.

[3] Hubei province Jiogsha Archaeological team: *Baoshan Chu tomb*, first ed., Cultural Relics Publishing House, Beijing 1991, 64 p., 112 p., 194 pages.

[4] Liang Qichao: *The Mo Jing School Releases*. Commercial Press, Beijing 1922, p. 1.

[5] J. Needham: *Science and Civilization in China, Volume 3, Mathematics*. Cambridge University Press, London 1959, pp. 91–95.

[6] Qian Linzhao: *Explains "Mozi" the related optics, mechanics elaboration*. In Fang Lizhi (ed.): *Scientific Historical Treatise Collection*, University of Science and Technology of China Publishing House Publication, Hefei 1987, pp. 1–36.

[7] Sun Yiranh: *Mozi Xiangu, "Sub-Integration", the fourth set*. Zhonghua Book Company, Beijing 1954, pp. 11–12, 122, 128–129, 133, 208–209.

[8] Sun Yiranh: *Mozi Xiangu, "Sub-Integration", the fourth set*. Zhonghua Book Company, Beijing 1954, pp. 190–192, 202–203, 207–208.

[9] Tan Jiefu: *The Research of Mozi's theory, the new scholars integrated the first series*. Zhonghua Book Company, Beijing 1976, pp. 77–78 and 136–144.

[10] Xu Xiyan: *Mozi Research*. Commercial Press, Beijing 2001, pp. 187–189.

[11] Yang Junguang: *Mo Jing study*. Nanjing University Press, Nanjing 2002, pp. 56–61, 386–395, 396–407, 409–427, 431–459.

[12] Zhang Shijun (et compiler): *The Projective Geometry*. Northeast China Education Press, Shenyang 1951, pp. 6–36.

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