Interactive 3D-Representation of Material Mathematical Models with WebGL

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Abstract. Collections of mathematical models are historical witnesses for "Anschauung" (notion, comprehension by experience); they are a cultural heritage and in recent years appreciated as scientific sources for different disciplines. Since the models are often hidden in single institutes, it is rewarding to make them accessible for online research in a detailed database. This is intended with the Digital Archive of Mathematical Models (DAMM), where we in particular use the current 3D-web-technologies to compensate the lack of physical presence. This 3D approach facilitates the exploration of the material models and adds extra options like virtual cutting and the connection with results from computer algebra systems.

Key Words: Mathematical models, mathematical artefacts, digital archive, 3D-representation

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1. DAMM

The web project *DAMM (Digital Archive of Mathematical Models)* [1] at the TU Dresden aims to provide an infrastructure for multidisciplinary research. We have created a platform not only for mathematical communication but as well for inspiration. In the near future an expansion of the concept to related scientific collections of various subject areas, like crystallography, physics or chemistry, is pursued.

At the beginning, the main goals were to support the preservation of mathematical artefacts as a cultural heritage, to foster the dissemination of previously hidden treasures of scientific collections and last but not least to enable a new approach to the perception of historical models. Therefore we published photographs of the mathematical models online. Within DAMM we take the next step and additionally show interactive 3D-representations. This enhances the user experience during online investigation and broadens the possibilities of interaction with the models.

Various users may be interested in this archive: scientists of mathematics and of the above-mentioned disciplines as well as engineers, didacticians, historians, designers, and artists. The faceted search tool allows to meet the demands of mathematical experts as well as the curiosity of the general public. To list models of interest the user can select characteristics like material or special field. DAMM can be seen as an extensive place for open education, because everyone has access to it and the users can choose their individual way to study the models.

Due to the worldwide connectivity, the future of the archive can be seen as a platform for exchange. Collections from different locations may contribute to individual topics and in this way get connected to one big compilation in DAMM.

2. Mediabox

We have constructed a data framework called *"Mediabox"*, which technically is a special structure of files in a directory on a web server. It can be loaded as a whole by the website.

For every 3D-representation there is a Mediabox containing a standalone presentation with a user interface. Essentially it only needs HTML, JavaScript and WebGL to show the 3D content. Detailed information is given in Chapter 3. Furthermore, the Mediabox is compatible with other techniques to create any new interactive content.

This container "Mediabox" was initially created to compensate the absence of the material models during web research. Of course we cannot imitate the tactility on screen, but at least the models can now be viewed from all directions. Additionally, with the 3D technology there come even more possibilities that radically exceed the ideas of compensation and imitation.

At first we want to report, how we create content for the Mediabox from digitized models. After this we will show, how the user interface supports innovative investigation techniques. Therefore we demonstrate some aspects of user customization. As an outlook we introduce, how the content can be extended with additional virtual data, how several 3D-representations can be linked and finally, how the user can make use of the data for contexts like education and exhibitions.

3. Creation of content

The creation of content for the Mediabox is critically dependent on the material properties of the models. Thus the digitizing process forces a classification of the models in categories like solid plaster, transparent and reflecting plastic, movable and flexible, and string models. Each category demands a different workflow. Our aim is to obtain a very high precision with very limited resources. This is why our processes are mainly built with open source components.

3.1. Solid plaster

The digitizing process for solid plaster combines two techniques. For the 3D model, we use an industrial optical 3D scanner. For the textures, we have created a setup that automatically produces a series of photographs from different angles with a full frame camera. Both datasets are then merged by means of photogrammetry and additional routines, which have been compiled during this project. In [2] the processing of plaster models is described in depth.

As a result we obtain a HTML and a X3D file with an accompanying texture file as PNG. HTML was selected to visualize the content in any of the latest webbrowsers (Figure 1). X3D is a language to represent 3D objects independently from additional plug-ins by using
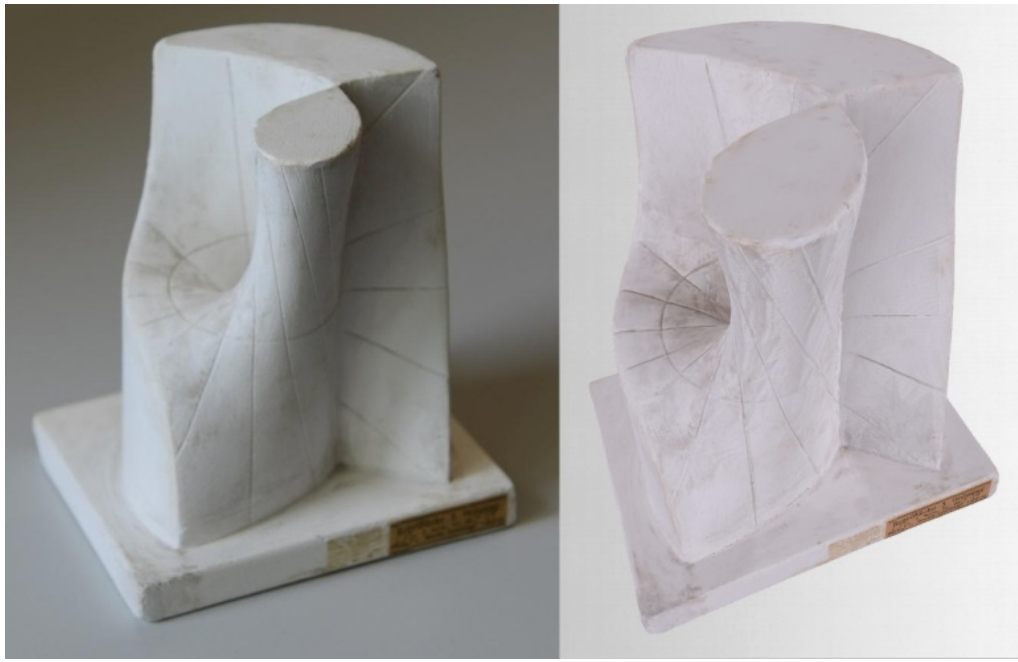
Figure 1: Ruled surface of third order, Schilling catalogue, VII 20 (left: photograph of the historical model, right: rendering of the digitized model with the Mediabox of DAMM)

X3DOM [3]. This framework is open source and uses JavaScript to create 3D scenes. One example for its potential is given in 4.1.

In the following we describe, which alternative techniques can be used to create digital representations with similar accuracy. This is especially relevant for models, which do not fit the optical scanning process.

3.2. String models

We execute the digitization of string models in two different ways. The first practice is an interactive virtual reality tour (like QuickTime VR models), which is simulated by a series of pre-captured photographs (3.2.1). The other method is a kind of a reverse engineering process, which is based on flatbed scans taken from the frameworks of the string models (3.2.2).

3.2.1. Virtual reality

With the same setup as used for the texturing of the solid plaster models, we take a systematic series of photographs of each model from different angles. These photographs are then integrated into the Mediabox with JavaScript. The user can change his viewpoint by clicking on arrows or with mouse movements. This well-known application suggests that the beholder is moving around the model, although he is watching 2D images (Figure 2).

3.2.2. 3D modeling software

At first we scan every side of the metal or wooden framework of the string model with a flatbed scanner. Then we re-model the framework with a graphical 3D software, for example Blender [4]. The scans both serve as source for the measurements and as textures for the constructed
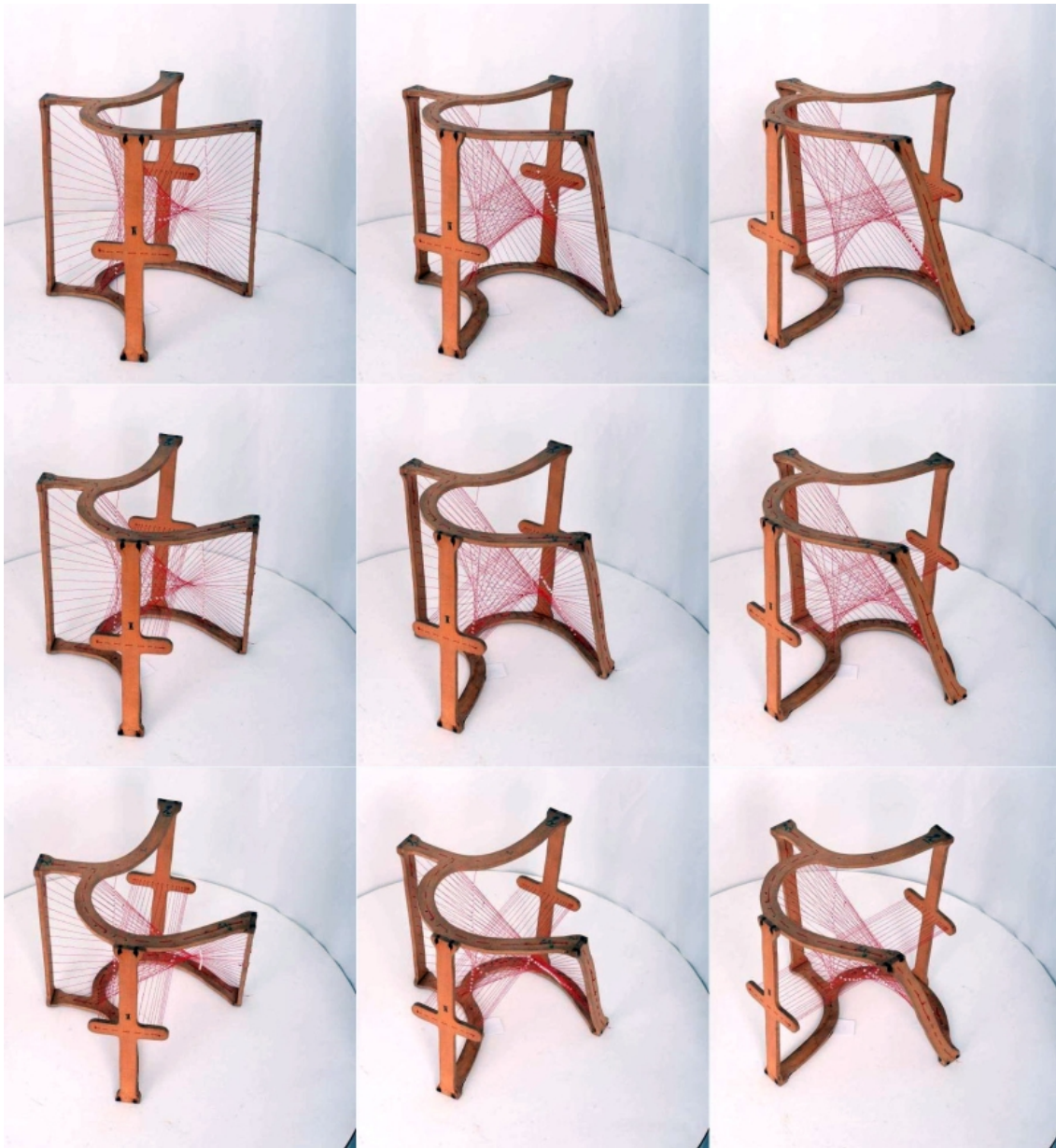
Figure 2: Nine pictures from a set of photographs used to form a VR object,
Schilling catalogue, XVIII 2

3D elements. The threads are represented by cylinders. Each cylinder is constructed within the modelling software and connects two corresponding drill-holes in the framework (Figure 3).

3.3. Transparent and reflecting plastic, movable and flexible models

Of course, a virtual reality model like described in 3.2.1 may always be an alternative for those material models, which do not fit the previously described 3D processes. The advantage of VR models is that they can be produced more or less automatically. But if we want to achieve true 3D models, we have to apply techniques of re-modelling and texturing. One solution is to use a Microscribe Digitizer [8].

Since several methods can be combined, it is essential to document the digitization process
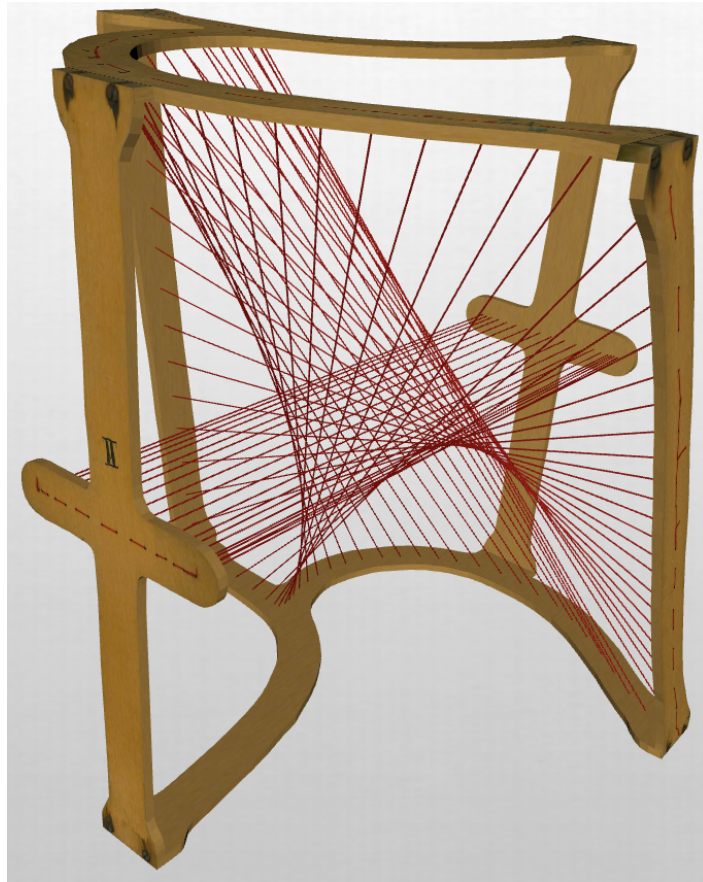
Figure 3: Flatbed scanned and remodelled framework. Cylinders in the place of the threads are also constructed with 3D software, Schilling catalogue, XVIII 2

in the metadata of the model. Thus the user is always informed about the reliability of the virtual model.

For the representation of flexible models an additional parameter can be implemented into the Mediabox to show the movement.

Figure 4: Left: Interface of the Mediabox for optional sections, rotations about special axes, fullscreen view and help bubble; right: a parabolic cyclide rendered in four positions with a clipping plane

4. User interface

The handling of the virtual 3D models is quite intuitive. Three elementary functions are given by the Mediabox: the left mouse button rotates the model, the right mouse button zoomes in or out and the middle mouse button moves the model. In addition a user menu is displayed (Figure 4, left). Implemented functions are a virtual clipping plane (1) (Figure 4, right) as well as an optional limitation to a vertical (2) or horizontal (3) rotation axis. Button 4 restores the view to the preset, 5 starts the full screen mode and 6 gives information about the Mediabox.

4.1. User customization

X3DOM is very powerful. Lighting and shadow effects, especially intensity and size, allow the user to modify the appearance of the Mediabox [5]. Thus the model can be displayed attractively and in different atmospherical settings. Another option is to apply section planes and cross-sections (Figure 5).
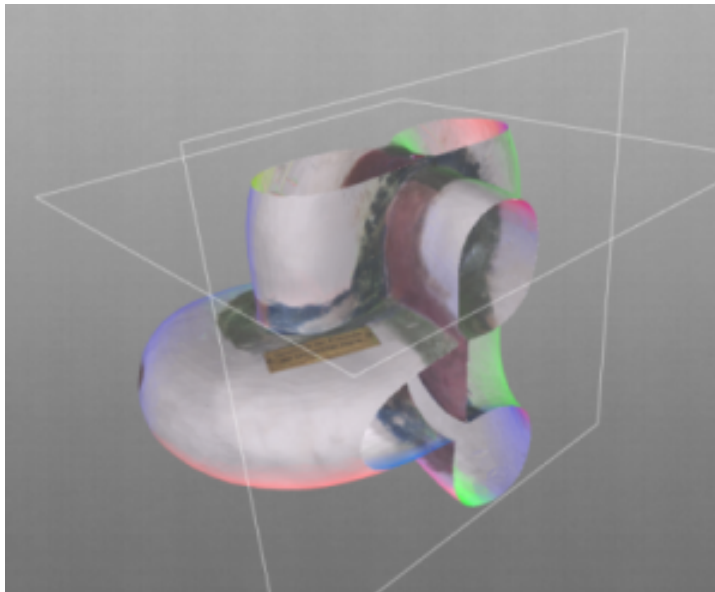
Figure 5: Schilling catalogue, XXX 2, with two virtual section planes and lighting effects

4.2. Additional data

The procedures described in 3.2.2 already may suggest that it is possible to combine the results of the optical scanning process with constructed or calculated data. In particular, such overlays can be used for the visualization of scientific facts and cross-references. Thus the user can easily grasp background information, even beyond the topics, which are directly incorporated in the mere material model. A good technical choice for this purpose within an online environment is to include CindyCS [7], which was recently developed at the TU Munich. Jürgen RICHTER-GEBERT supported DAMM with an example, where the 3D annotations can be selected interactively (Figure 6).

The Mediabox holds even more options for the interaction with the 3D data. For example it is possible to simultaneously load two digitized models. Hence, the user is able to compare two models with regard to their point clouds or study their intersection.
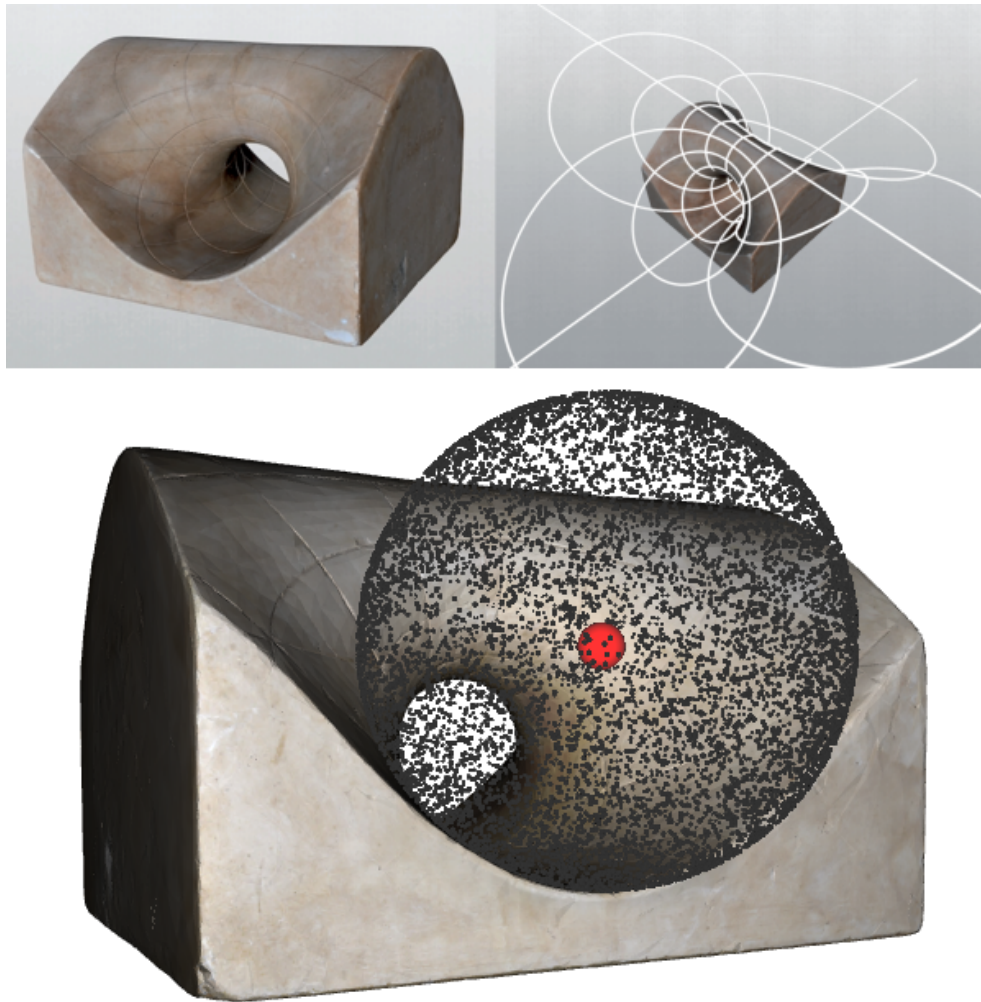
Figure 6: Parabolic ring cyclide from the Schilling catalogue, X 5; top left: rendered in real-time within the Mediabox, top right: the same model with additional circles completing the incised circular arcs and bottom: the same data in CindyJS superimposed with a sphere, which always touches the surface in a circle while the center can be moved interactively

The generation of 3D content is a process, which typically involves some manual corrections because of technical issues during the scanning procedure. Of course, the principal ethics of the digitization is to try to achieve the best correlations possible between the material object and the 3D data. On the other hand, there might be models, which are broken or incomplete. Those can be repaired and amended digitally to achieve a more meaningful representation. It lies in the responsibility of those, who produce the 3D data, to document any manipulation or addition. How this information together with technical notes about the digitization can be represented in the meta-data of the model is an ongoing challenge.

5. Geometry software

The Mediabox framework is open to include third party geometry software like Surfer [6] or CindyJS [7]. Thus the users of DAMM can even experiment with equations to create and
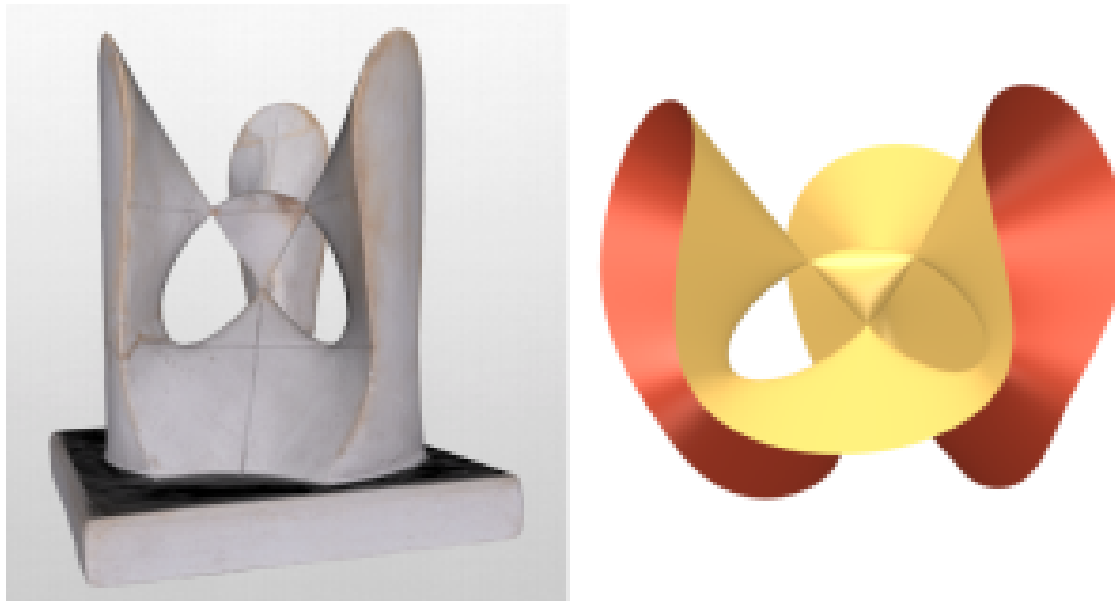
Figure 7: Surface of third order, Schilling catalogue, VII 2 (left: digitized model in the Mediabox, right: rendered with the software Surfer [6])
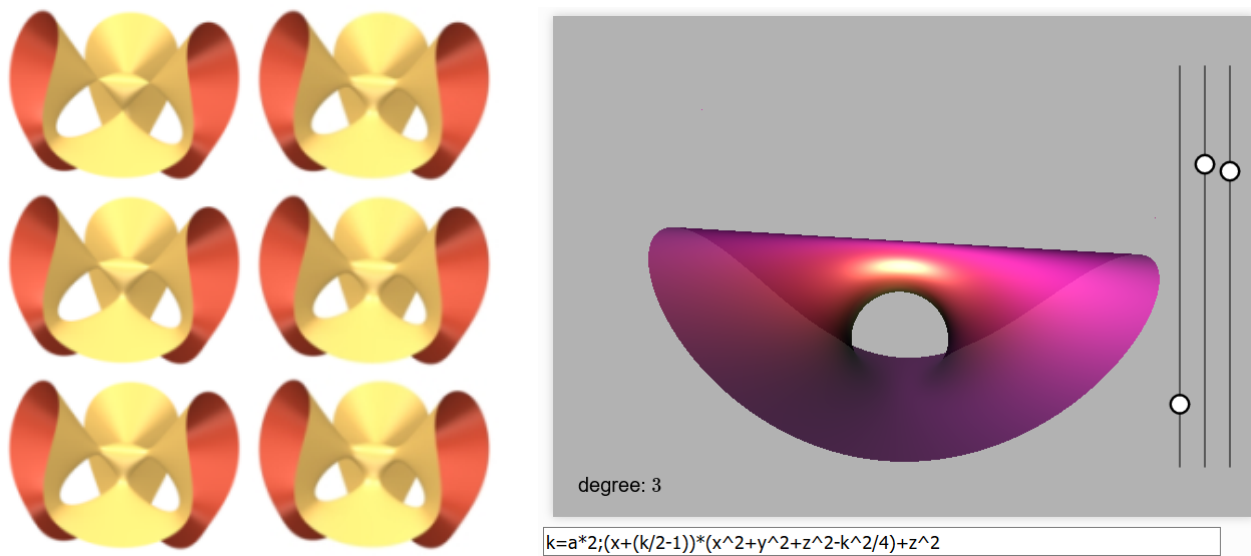

Figure 8: left: A real cubic surface morphs into another real cubic surface, rendered with Surfer [6], right: Parabolic ring cyclide similar to Schilling catalogue, X 5, with its equation, rendered with CindyJS [7]

render algebraic surfaces and directly compare them to material models (Figure 7).

One application is to visualize the linkages between different models. This may for example prove graphically, if two models belong to the same family of surfaces, such as cubics, which differ only slightly in a single parameter value (Figure 8).

Furthermore, the user can check the accuracy of the historical models by superimposing the digitized and the calculated 3D models.

6. Usage of the data and outlook

Every dataset of a material model combines drawings, photographs, digitized 3D models, and metadata with digitized and edited grey literature as well as mathematical and historical background information. It encourages especially mathematical writing and journalism, links current mathematical communicators and offers a broad range of information and aid. Thus the archive can also be seen as a source for open online courses and exchange.

DAMM claims to be a research infrastructure. To develop this aspect we plan to add further functionality for the utilization of the accumulated data (API), insert functionality towards interactivity (user accounts) and make 3D data available for the reproduction of models via 3D printing or as a source for further research projects outside of DAMM.

It is undeniable that the digitization process adds something to the collection, which again produces metadata. It needs awareness to deal with the implications of the new content. A natural effect is the necessity to scale up the database continuously to meet the ever growing list of new research questions.

The previous remarks also imply that the digitization process fosters a connection of the material substance of the presented collections with other disciplines. This is added value to our historical heritage.

Acknowledgments

References

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[3] `http://www.x3dom.org`

[4] `https://www.blender.org`

[5] `http://www.x3dom.org/examples`

[6] `https://imaginary.org/de/program/surfer`

[7] `https://cindyjs.org`

[8] `http://www.3d-microscribe.com`