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Geometric Aspects of Scalalogy

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Abstract. Stairs are important elements in architecture and can be studied under various aspects. Fundamental rules for stairs had been fixed already by Vitruv, Alberti, Palladio, and Blondel. The German stair researcher Friedrich Mielke played a significant role in the development of scalalogy, the science of stairs, on this basis in the late twentieth century. After an introduction to the function, history and design rules of stairs, the studies of geometric aspects of scalalogy are in the focus of this research and educational project in architecture. The geometric aspects of stairs can be related to their typologies and the resulted movements in space. Moving directions can be derived from the geometries of stairs. Spiral staircases in different variants, single and multi-run or single and polycentric, are of particular geometric interest. Their respective geometric concept of screwing produces different movements. Staircases as fascinating dynamic geometric objects are the results. Finally, the kind of movements on stairs related to the inclination is the theme of the sinus stairs. The sinus stairs offer, with continuously changing step height and step depth according to the sine curve, smooth movements, as if walking onto a natural hill.
The students built a sinus stair as a walk-in project in order to test the thesis, which had been confirmed by the built project. The stairs with their geometries were analysed and presented by 3D-models, drawings and physical models.

Key Words: Staircase, Architectural Geometry, Scalalogy, movement in space.

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1. Function, history, and design rules of stairs

Stairs are important and interesting elements in architecture, especially from the geometric point of view. Besides the function of overcoming height differences, they had been developed in architecture as independent spatial elements with stay qualities.

They make it possible to achieve views and overviews. In stair towers, they get even independent building structures. The staircases play an essential role inside the houses, but also outside in the city and landscape (Figure 1). They are always related to the human movements. Staircases facilitate relationships between movements in space and spatial concepts in architecture.

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Figure 1: Landscape stairs in Croatia and spiral stair in “Neues Museum” Nürnberg, Germany.

Rules for the construction of staircases in relation to their inclination can be found in VITRUVIUS’ *“Ten books of architecture”*, 15 B.C., [13]. He suggested for the rise of steps for a temple between 22.8 and 25.4 cm, for the treads between 45.7 and 61.0 cm, in order to achieve a comfortable going up. These relationships between riser and tread of a staircase had been resumed by ALBERTI in *“De re aedificatoria”* in 1452 [1] and PALLADIO in *“I quattro libri dell’architettura”* in 1570 [10]. Figure 2 shows the main stair notions: rise or riser and run or tread in their relation to the inclination angle of the stair.

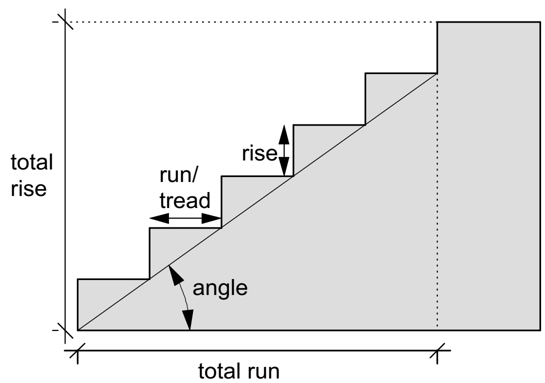

Figure 2: Stair notions: rise or riser and run or tread, inclination angle.

ALBERTI explained, that the number of steps in one *flight* should be always an odd number, starting with the right foot and step out with the same foot. Good architecture should take not more than seven or nine steps in one flight, *landings*, or also called resting places, should follow. According to ALBERTI and then PALLADIO, the recommended risers and tread widths have a wider range.

PALLADIO admitted eleven to thirteen steps in one flight. PALLADIO explained also various kinds of spiral staircases, illustrated by drawings. He described and presented in drawings an extraordinary quadruple spiral staircase (Figure 3, left).

The French architect and engineer François BLONDEL referred in his book *“Cours d’architecture”* in 1675 [3], to the historical examples of stair rules and especially the drawings of PALLADIO. In Figure 3, right, we see some stairs drawings by BLONDEL with similarities

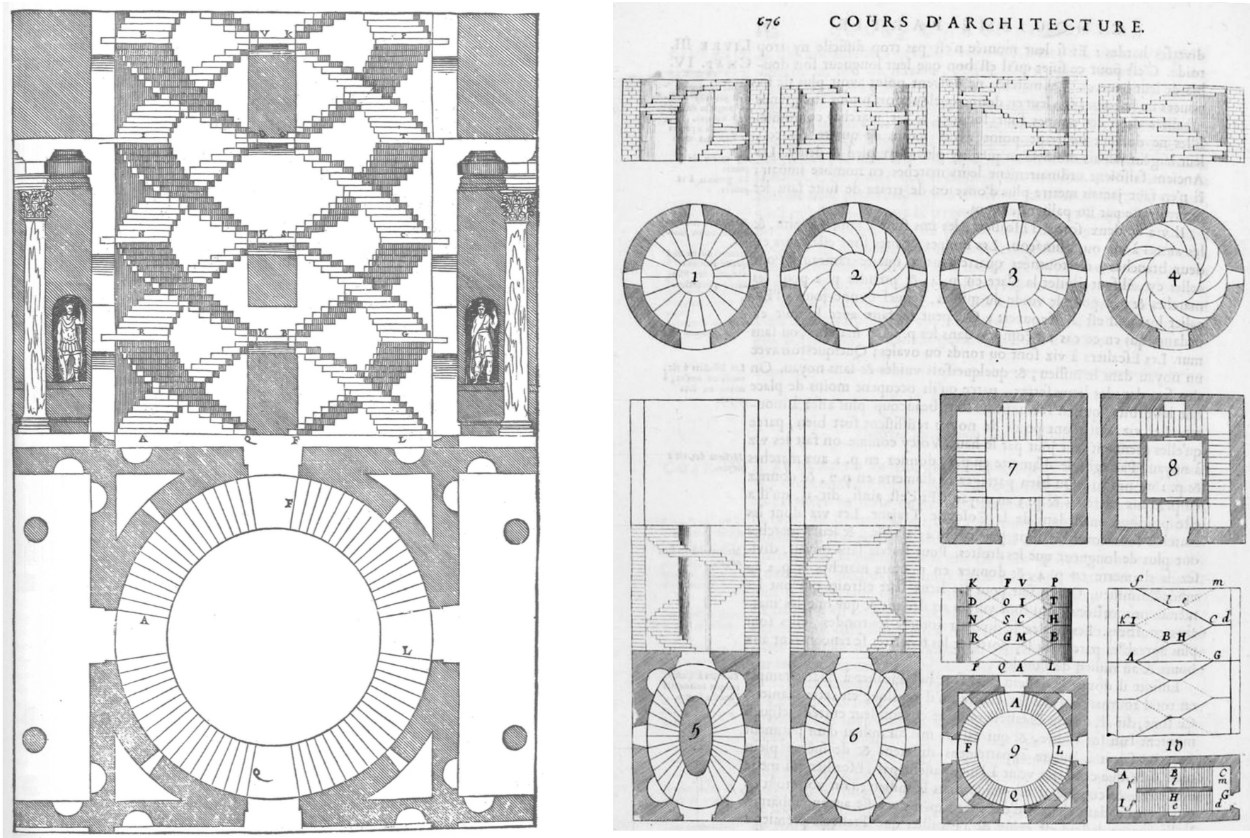

Figure 3: Left: Quadruple spiral staircase drawing by PALLADIO [10],
Right: stair drawings by BLONDEL [3].

to those of PALLADIO. In the book, BLONDEL introduced a formula for the measurements of stairs, which he based on measures of the human step. The formula is still used today. He postulated that two riser heights and one tread/run depth should together make the length of 65 cm. Today we take usually the rule that the step length for the calculation of stairs with normal inclination of 30° should be around 63 cm.

2. Typologies of stairs

These early documents show the long way of efforts to develop rules for the design of staircases as an important and interesting element of architecture. For the development of a staircase science, called *scalalogy*, the German building conservator and stair researcher Friedrich MIELKE (1921–2018), played a significant role. Friedrich MIELKE collected and explored stairs over six decades. In his *"Handbuch der Treppenkunde"*, published in 1993 [7], he developed a nomenclature and typology with over 60 variants of staircases.

He developed the following nomenclature: A sequence of steps is called flight of stairs. In the drawings, the ascent of the staircase is characterized by a running line, which must always be drawn in the middle of the running width. If the sequence of steps is interrupted, the running figure decides, whether it is a longitudinal landing, a corner landing, an intermittent landing or a platform landing. Various flights are distributed from a central platform landing. The spiral staircases are differentiated in single or multi-centered spiral staircases. In Figure 4 the stair typologies are illustrated according MIELKE's stair nomenclature. These results in

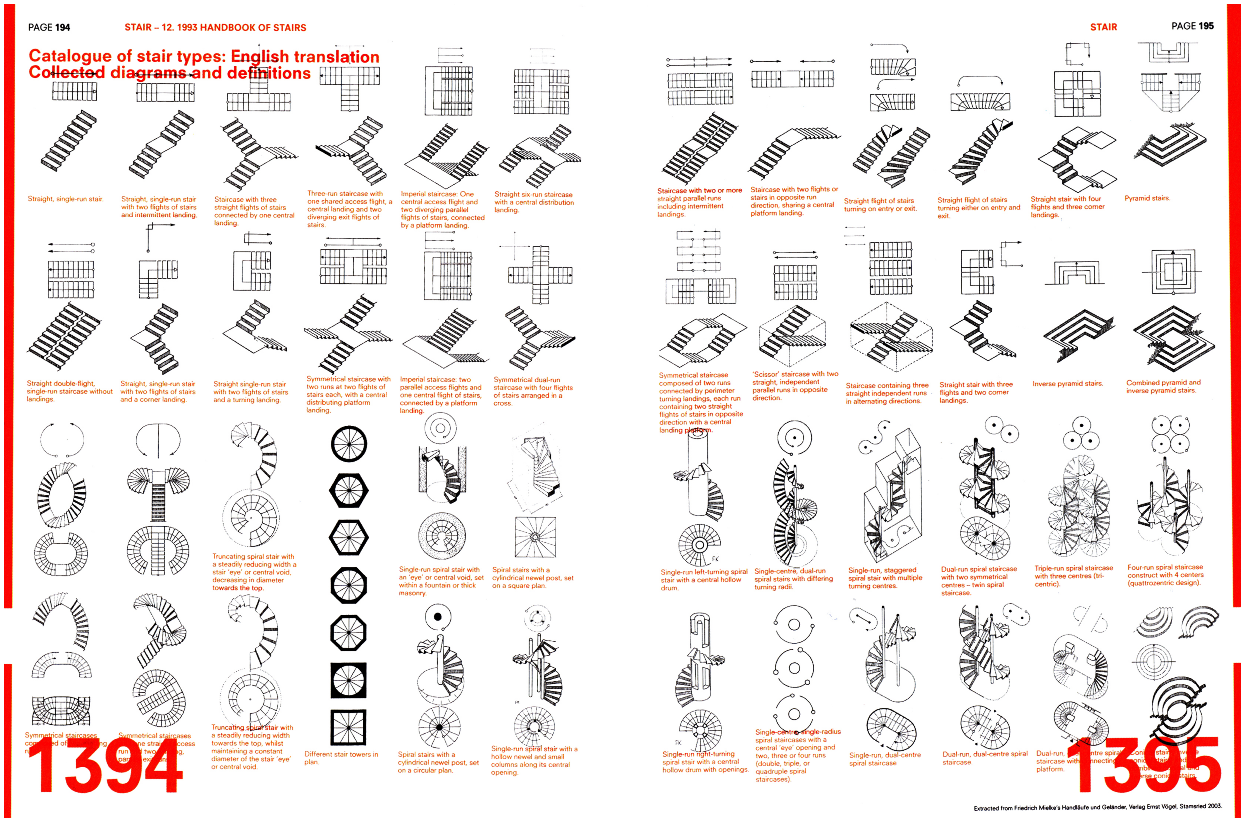

Figure 4: Catalogue of stair types according F. Mielke in *"Elements of Architecture. Stair"* [12].

Figure 5: Examples of stair models in the Friedrich-Mielke-Institute for Scalalogy [4].

the staircase typologies form the basis for scalalogy, the staircase science, which had been worked out by Mielke in many studies and example analyses. But until now his extensive work and collection is not fully worked up yet. We visited with our students the Friedrich-Mielke-Institute for Scalalogy of OTH Regensburg, directed by Joachim Wienbreyer [4] in November 2017. He, and now his successor Ulrike Fauerbach, are still reappraising the comprehensive work of Friedrich Mielke on stairs.

Parts of the work of Friedrich Mielke had been shown at the Architecture Biennale in Venice 2014 in the central pavilion as part of the exhibition *"Elements of Architecture"*, curated by Rem Koolhaas, also some of the stair models as shown in Figure 5.

We studied the geometry of stairs in an optional course in the established subject *"Design Geometry"* (*"Gestaltungsgeometrie"* in German) at the faculty of architecture. The students started with an empirical approach. Each student chose an example of a fascinating staircase, analysed it and related the example to the studied typologies of stairs. The research formed the basis for the development of their respective staircase project and the common project of a sinus stair as a walk-in example, realized out of wood in the faculty building [6]. In the following, some examples of the students' projects will be presented.

3. Relationships between stair geometries and directions

The geometries of stairs influence the movements and directions. The relationships between the directions and the geometry of stairs had been studied with the centered stairs by the

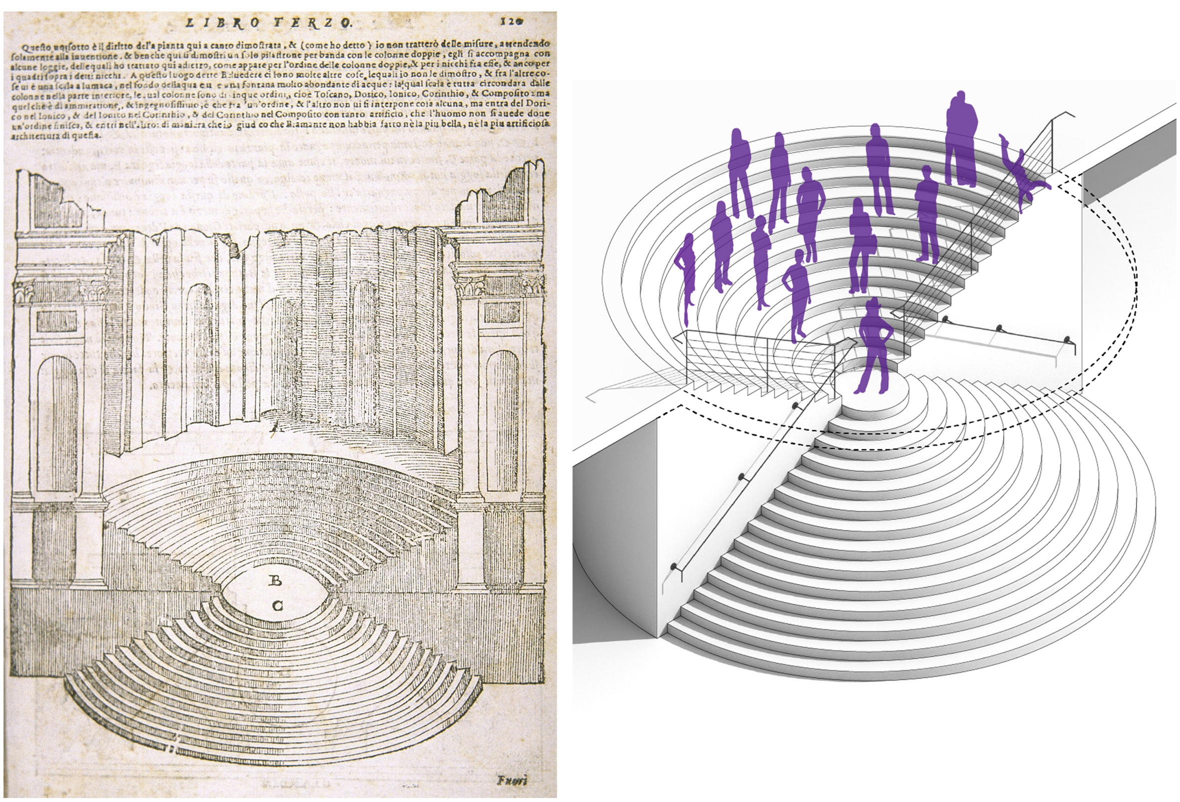

Figure 6: Conical stairs, Bramante's project for the stair of the Belvedere Court in Vatican [11] and visualization of a 3D-model of the stair in Kunstmuseum Bonn by student Marian Buchheiser.

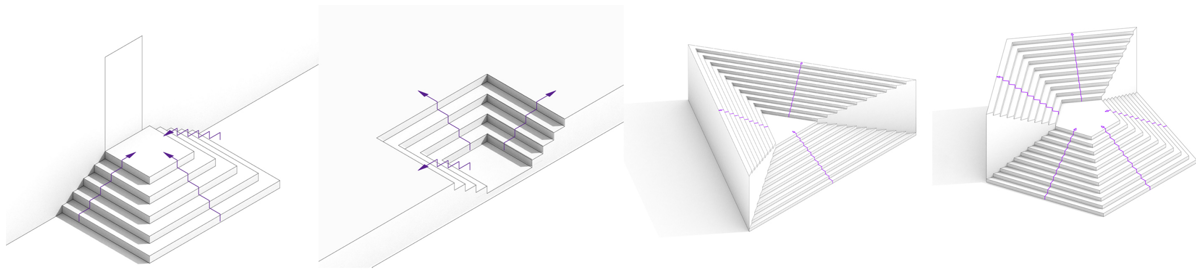

Figure 7: Centered stairs and directions of the walking line related to their geometries, (3D-models by student Marian Buchheiser).

student Marian Buchheiser. Starting with the historical example of Bramante's project for the stair and facade of the Belvedere Court in Vatican, illustrated in the book of Serlio [11], he analysed the contemporary similar example in the Kunstmuseum Bonn, built by the architect Axel Schultes in 1992. Figure 6 shows the drawing in Serlio's book and the visualization of a 3D-model of the conical stair according the Kunstmuseum Bonn example.

A concentration arises on the centered platform of the cone like on a small theatre stage, coming from and going in all various directions. Therefore, the conical staircase leads to this center without preferring a direction before or after. The student studied how the situation is changing, if we have a pyramid or inverse pyramid staircase with a rectangle, triangle or even pentagon as the centered platform. Preferred directions are created by these platforms. With an odd-numbered polygonal center, there arises an imbalance in the entrances and exits, shown in Figure 7.

The studies of the student Jonas Heuser started with the analyses of the famous Spanish Steps in Rome, or called in Italian *"Scalinata di Trinità dei Monti"*. It is divided into eleven stair segments which are used through concave and convex step geometries as podiums, squares and terraces. Each segment consists of twelve steps. The stairs receive multifunctional importance. They are passageway, resting place, stage, studio, playground, concert hall, gallery, or lawn at the same time. The relationship between multifunctionality of the stairs and the geometries of the path routing had been reflected with the help of drawings and models by student Jonas Heuser, as for example shown in Figure 8.

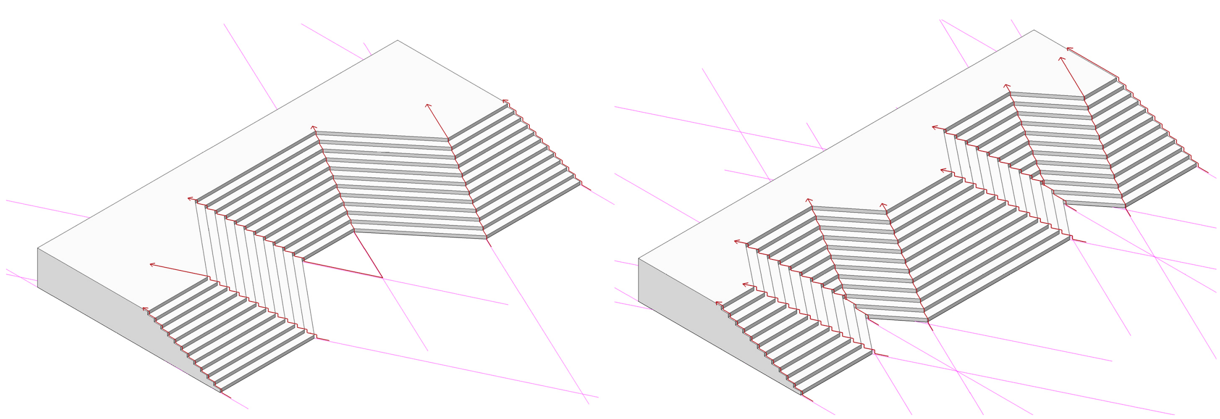

Figure 8: Path routings with the walking lines and resting places related to the stair geometries (drawings by student Jonas Heuser).

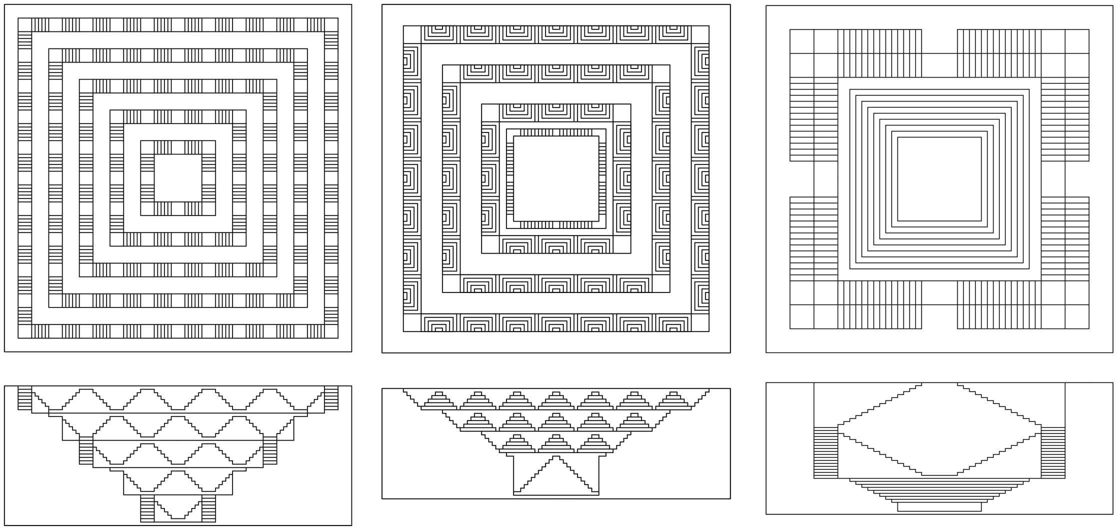

Figure 9: Top views and sections of Indian stepwells: Chand Baori, Hampi, and Rajasthan (drawings by student Sarah Lutgen).

Geometric patterns are developed by those path routings with their consequences for the use of the stairs. The cultural heritage of the Indian stepwells demonstrates the interrelation between the geometric patterns and the routings in an extraordinary way. The student Sarah Lutgen encountered the stepwells [5] in her research on the geometries of stairs. Figure 9 shows three different examples in geometrically abstracted 3D-models in top views and section drawings by student Sarah Lutgen: the Chand Baori stepwell, the stepwell in Hampi, and another in Rajasthan.

4. Characteristics and types of spiral stairs

The typologies of stairs in Figure 4 list also various kind of spiral stairs. Spiral stairs may be single-, dual-, triple-, or even four-run, with an ‘eye’ or central void or with a cylindrical newel post, left- or right-turning, one or two or more centers, truncating with steadily reducing width. Additionally, they can be combined with landings and fit in various shapes of plans. We studied also different kind of spiral stairs. Some examples, analysed and built in 3D-models, by the student Benedikt Blumenröder are presented in the following figures.

The dual-run single-center spiral staircase (Figure 10, left) can be found in various buildings, for example in the Neupfarrkirche in Regensburg or in Château de Chambord in Blois. The way up and down can be separated by that type. In a single-run, dual-center spiral staircase (Figure 10, middle) the stair treads wind around two centers, whereas in a twin spiral staircase, two axisymmetric runs wind in opposite directions around two centers, which can be formulated as spindles, hollow spindles or eyes (Figure 10, right). In the castle of Graz exists an example of such a twin spiral staircase, constructed in 1499. Figure 11 shows the drawings and model of a twin spiral staircase. Many other variants can be created with more centers, one or more runs.

An unusual single-run, triple-centered spiral staircase, probably never realized, is shown in a narrower or wider variant in Figure 12 in drawings. The model photo in Figure 12, right, presents an extraordinary creation by the student as combination of the studied stair types. It is a single-run tricentric staggered spiral staircase, where the flight of stairs follows the

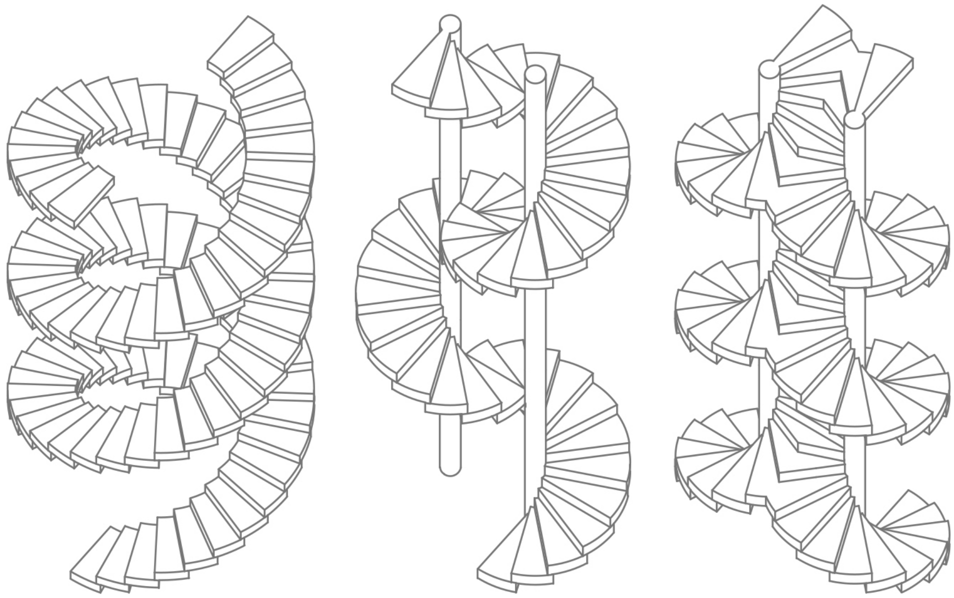

Figure 10: Dual-run single-center (left), single-run dual-center (middle) and dual-run with two symmetrical centers – twin spiral staircase (right) (drawings by student Benedikt Blumenröder).

shape of a triangle spanning the spindles. It turns alternately clockwise and counterclockwise around the three centers, repeating only after six floors itself.

Triple spiral staircases as tri-run monocentric spiral stairs had been realized in minarets, for example in the mosque Selimiye in Edirne. A quadrupel spiral staircase, also described as double double spiral stairs, is often referenced in the literature, but never realized. Palladio [10] draw such a staircase (see Figure 3). He referred there to the double spiral staircase in

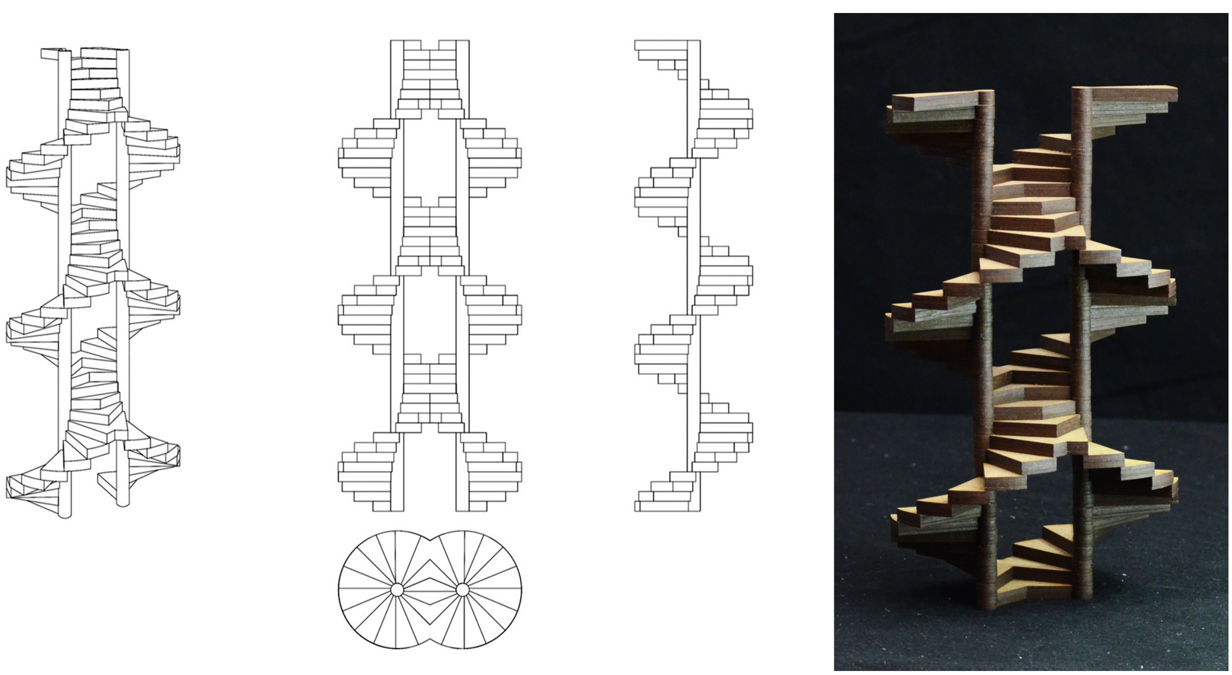

Figure 11: Drawings and model of a twin spiral staircase by student Benedikt Blumenröder.

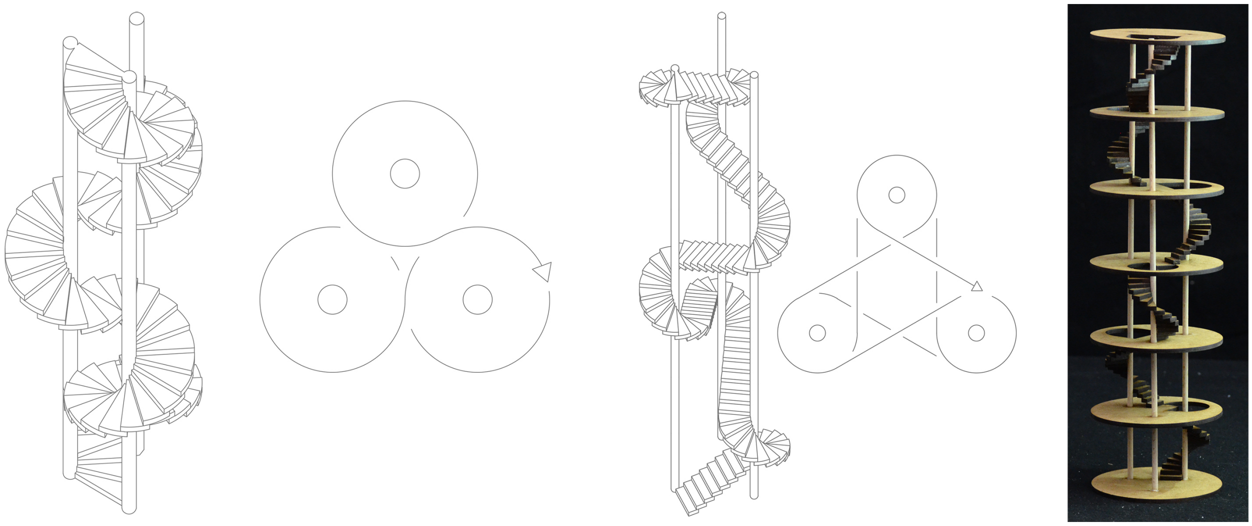

Figure 12: Single-run triple-centered spiral staircase, variants drawings and model of a single-run tricentric staggered spiral staircase by student Benedikt BLUMENRÖDER.

Chambord, but described it in detail as a quadrupel staircase.

Friedrich MIELKE analysed many of these special stair concepts and realized them in models [8]. There, he depicted the exceptional staggered spiral stair above the south portal of Prague's St. Vitus Cathedral, the so-called Golden Gate, built by Peter PARLER in 1372. The semicircular branches change their direction three times and wind around three stony spindles, forming a serpentine line.

The spiral stairs demonstrate the wide range of geometries influencing the moving and application possibilities.

5. The idea of a smooth stair – sinus stair

Werner BÄUMLER-LAURIN's idea for sinus stairs had been to climb up a staircase like a smooth hill [2, 14]. Starting from the plain with a slight rise continuously to the steeper part and again the transition to the plain. This results in line form in a sinus curve. According to

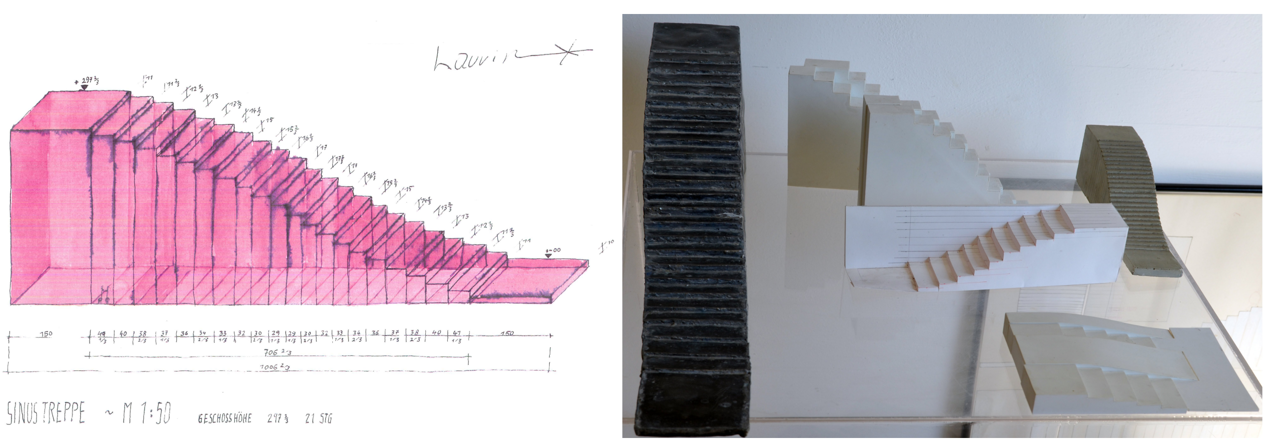

Figure 13: Drawing and models of variants of sinus stairs by Werner BÄUMLER-LAURIN [2] © Werner Bäumler-Laurin.

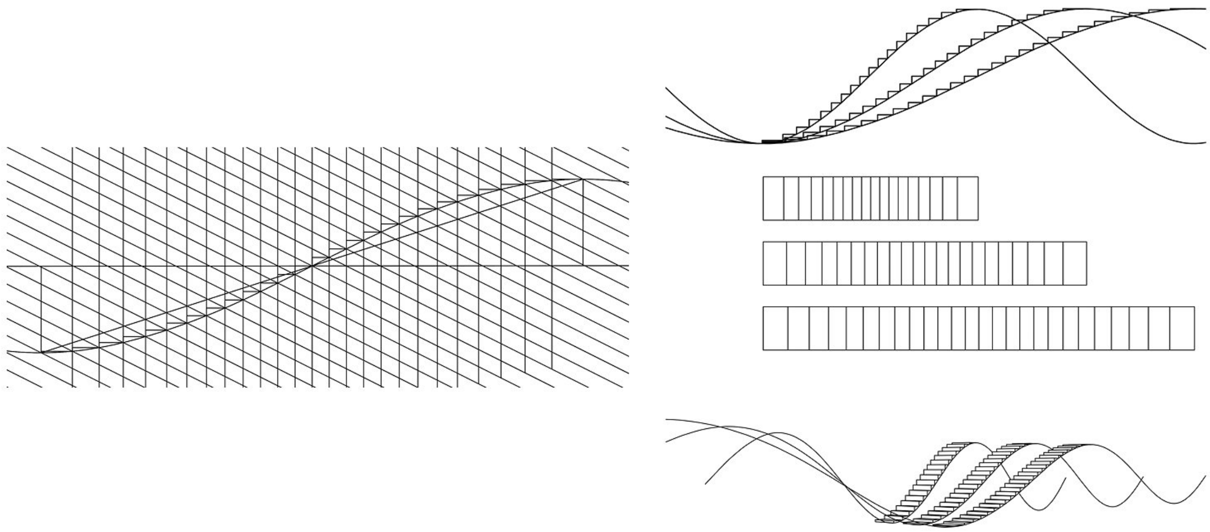

Figure 14: Graphical method with results of variants of sinus stairs
by student Moritz Brucker.

the fundamental concept of Blondel, the slope ratio of the steps is always related to the step length. In our example we took the step length of 63 cm. If we want to create a stair which follows the sinus curve, but at the same time maintains a constant step length, changing riser heights and tread depths are the result. As a consequence, each step has another rise and tread.

The artist Bäumler-Laurin, living and working in Venice, explored his idea of the sinus stair in many drawings and models (Figure 13). He created a sculptural sinus stair, exhibited at Bauhaus Dessau in 1991 and later in Bonn in front of the ministry of building. He realized another sinus staircase in steel 1994 for the administration building of a company in Landshut.

These changing heights and depths can be calculated, but also determined graphically. The students Benedikt Blumenröder and Moritz Bucker developed the graphical method. From Blondel's formula 2 risers + 1 tread = 63, the mathematical function can be derived by transformation:

$$f(x) = -0.5\,x + 31.5$$

This linear function is based on the slope triangle. To create the family of lines in Figure 14, the graph of the linear function $f(x)$ must be moved at its slope triangle. This group of parallel lines is then superimposed with the respective sine function. The steps are located at the intersections of the sine function

$$g(x) = \sin x$$

with the family of lines. The normal sine function gives rise to a relatively steep staircase. Variants with stretched sine functions lead to more comfortable staircases. The sinus staircase in the middle is probably the most comfortable staircase, as it is not too steep and not too low. The selected sinus staircase variant had been built in a concrete model by our students (Figure 15).

Only by walking on the sinus staircase, it is possible to experience the claimed pleasant increasing and decreasing stair slope. Therefore, our students decided to realize a walk-in project in the main entrance hall on the existing staircase in our faculty building. Figure 16 shows the ground plan and a rendering of the concept of the sinus stair.

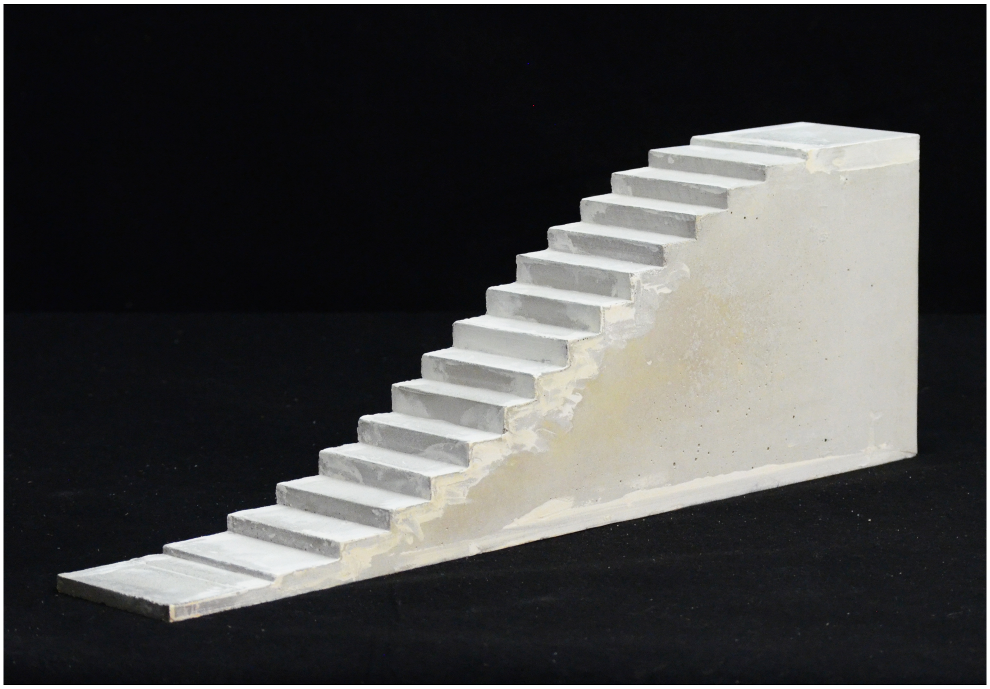

Figure 15: Concrete model by our students according the later built sinus stair.

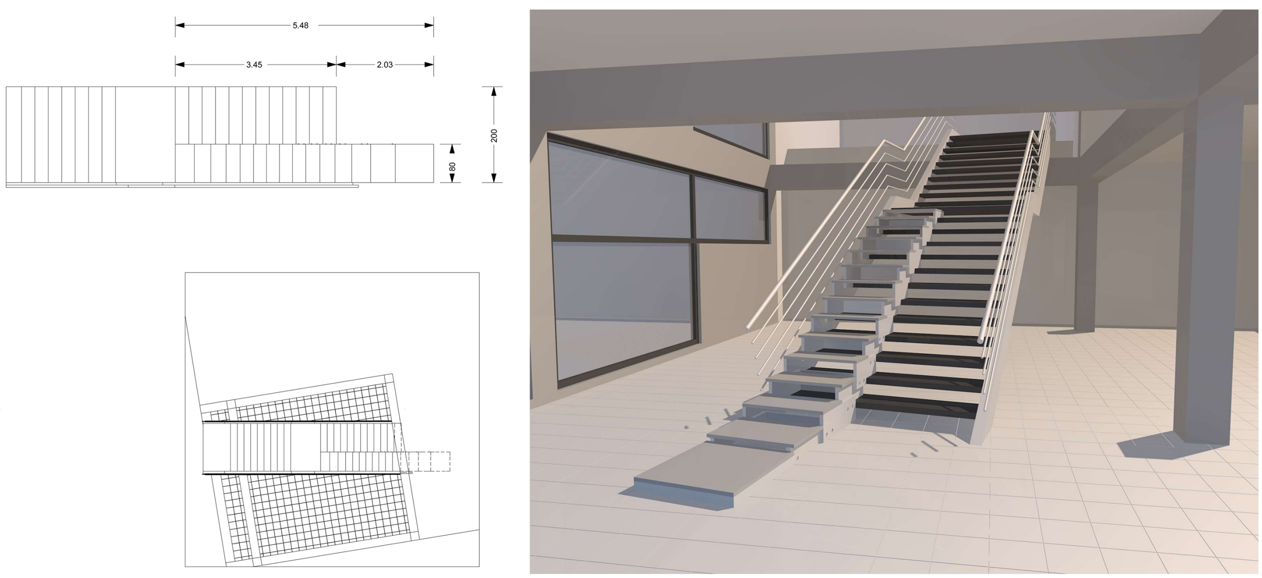

Figure 16: Concept with ground plan and rendering of the walkable sinus stair.

In the planning process, the students tried to find out a way to construct the sinus stair by themselves out of wood. They could use the faculty's lab and the material of an old staircase. The old wood had been carefully planned, nails removed, and glued together for the various treads of the sinus stair (Figure 17). The stair stringers had been cut out of blockboard panels. Finally, we realized the walkable sinus stair on the entrance staircase.

The difference between the normal and the sinus stair could be directly experienced. Each of us had been surprised, how smooth, supple, and fluid it appears, while walking on the sinus stair. In a public exhibition, the visitors confirmed the surprising comfortable experience with the sinus stair (Figure 18).

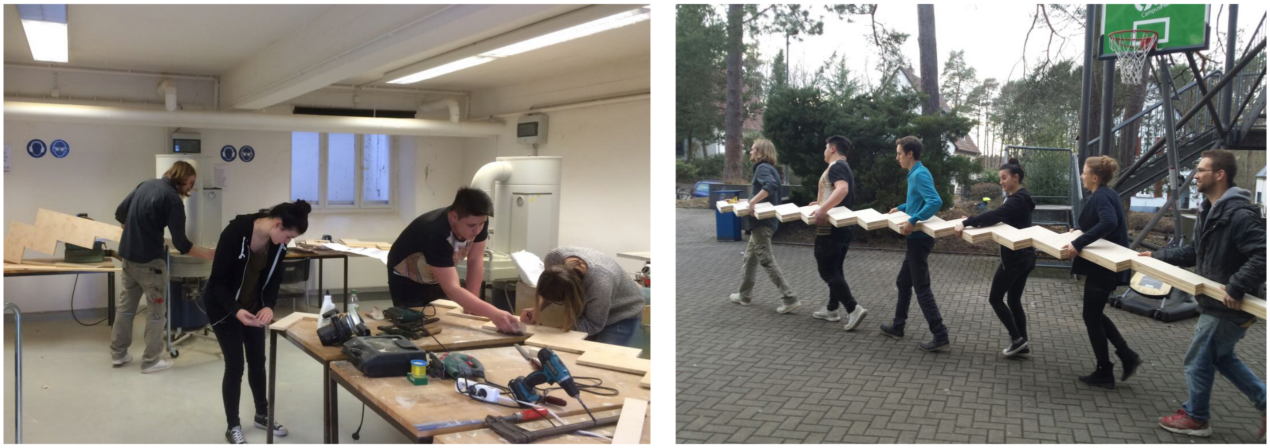

Figure 17: Construction work for the sinus staircase by our students.

Figure 18: Installation of the walk-in sinus staircase in the faculty building: view from below, from the side, and from above.

6. Conclusions

Stairs are varied elements of architecture and especially interesting from a geometric point of view. They are part of the design studios in architecture, inside the building, but also in urban and landscape context and there treated in their function, form, material, and conditions. During our design geometry course on the geometric aspects of scalalogy, it turned out that the topic offers many possibilities for research on the relationships between geometry and spatial concepts in architecture. The fascinating element gives the chance to thematise the relation between path routings, directions, and geometries. The human movement gets interrelated with geometric designs. Besides these research topics on stairs, the students could consider different representations like drawings, 3D-modelling, renderings, animations, digital and analog fabrication tools as well as present their research in an exhibition and explain their work to a public audience. The course was a good experience in linking geometry with perceptible, tangible, and experiential architecture. Especially the sinus stair gave the chance to follow a creation process from the geometric concept to the building and user experience.

Acknowledgments

We would like to thank Prof. Joachim Wienbreyer, University of Applied Sciences in Regensburg, for guiding us to special staircases in Regensburg and giving us an insight into the work, research materials and models, collected and created by Friedrich Mielke. The inventor of the sinus stairs, Werner Bäumler-Laurin, provided us with his documents and material to the development of his ideas and gave me an inside view by visiting him in his studio in Venice. Finally, I thank our students Benedikt Blumenröder, Yuliana Brehmer, Moritz Brucker, Marian Buchheiser, Sabrina Funk, Jana Gretz, Jonas Heuser, Sarah Lutgen, Emmanuel Niyodusenga, Ernst-Markus Rauska, Anna Specchio, Angelika Walz, and Philipp Weber for their committed work.

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