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Theoretical Approach to Practical Perspective in the Early 17^{th} Century

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Abstract. During the years in which perspective laid the theoretical foundations of its scientific definition, it appeared in various forms in the applied arts. The perspective construction sites of architectural perspectives, theatrical scenography, and large anamorphoses spread all over Europe, attracting the attention of mathematicians and artists, who enriched written treatises with contributions of a theoretical and experimental nature. These perspective laboratories, where projective principles acquire physical form reproduced by ropes, shafts, and lamps, become privileged venues for experimenting theories and validating results. Speculative interest in operational practice is thus found in the works of various mathematicians, intent on legitimizing projective procedures used in current practice and optimizing constructions. "Like a stream from its source" the practice thus originate from an evolving projective theory that saw, in the operativity of constructions, the expression of its demonstrative thought.

Key Words: perspective, punctum concursus, Guidobaldo Del Monte, Girard Desargues, theatrical scenography

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1 Introduction

Between the 16^{th} and 17^{th} centuries, the problem of practical construction in perspective emerges as a central issue. This science, used for deluding and amazing, appears in different forms in the applied arts: architectural perspectives enrich the walls and the vaults of palaces and religious buildings, theatrical and liturgical scenography accompany the evolution of the scenic space, just as optical-perspective *divertissements* find expression in the large anamorphoses. Thus, perspective construction sites spread in courts and religious complexes all over Europe, attracting the attention of mathematicians and artists, interested in the operational solution of different problems posed by the practical execution of perspective. These construction sites became areas of privileged experimentation. In fact, they were considered full-scale perspective laboratories where theoretical projective principles acquired physical

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form reproduced by ropes, shafts, lamps, and on-sight alignment operations. In those years, various solutions enrich the pages of theoretical and practical perspective treatises, oscillating between theoretical speculations and experimental approaches. In the historical evolution of operational perspective processes, a privileged role is played by mathematical contributions, capable of developing, through the force of theory, practical solutions founded on the principles of projection science. These contributions are images of an evolving perspective theory that focuses its projective foundations on the definition of the concurrence points theory and in the consequent use of the projecting planes, principal tools for the operational construction of applied perspective.

2 Projecting planes in practice procedures

The contributions given by mathematical perspective to the evolution of scientific thought distanced the science of projections from the needs of the artists [7, pp. 106]; however, it was from the theoretical foundations of this science that simple and effective practical solutions resulted. When scientific culture was broadening its sphere of interest by including operational aspects that could be made scientific [13, pp. 303], the awareness of the natural derivation of practice from theory animated the work of two great geometers, Guidobaldo del Monte and Girard Desargues who, at different times and in different forms, found, in geometry applied to the arts - particularly in applied perspective – the natural consequence of the force of theory. Thus, Guidobaldo introduces his *Perspectivae libri sex*:

> "[. . .] tentare tamen sum au sus aliqua in medium afferre fortè non iniucunda, & ea solidis adeò rationibus (quod ab alijs omissum videtur) comprobare, ut praxes, veluti è fonte riuuli, scturire e manare videantur"[1] [4, p. 3].

Guidobaldo's work and the arguesian *Brouillons* on perspective, although intentionally addressed to the artists, produced the opposite effect, contributing to feeding the gap between art and science that the mathematical interest in perspective had triggered a few years earlier, with the theoretical contributions given by Federico Commandino and Giovanni Battista Benedetti[2]. The strictly Euclidean demonstrative approach with which Guidobaldo Del Monte introduced the theory of concurrence points, inaugurating a new season for the science of projections, was difficult to understand by those without a mathematical background, and similarly difficult for the artists Desargues' writings must have appeared, which were concise, devoid of demonstrations and rich in neologisms. Nevertheless, the seeming complexity of these treatises revealed its simplicity in the effectiveness of the constructions, through an operational practice that in demonstrative terms legitimized the projective operations executed with ropes, lamps and on-sight alignments.

The practical construction of perspective posed difficult problems to resolve. The surfaces to be painted, generally large in size, such as the walls or vaults of halls and churches or the

[1] "[. . .] I dared to provide a contribution, hopefully in a comprehensible and agreeable manner; and to prove the arguments treated with such logical reasoning (which does not seem to have been done by others) that the applications seem to flow and develop like a stream from the source" (author's translation from *I sei libri della prospettiva di Guidobaldo dei Marchesi Del Monte dal latino tradotti interpretati e commentati da Rocco Sinisgalli*) [11, p. 40].

[2] According to Vagnetti, Benedetti's contribution aims to amend the errors made in current perspective practice, effectively appropriating a domain that until then had been the prerogative of the artists [13, pp. 340–341].

wings of theatrical scenography, were confined within assigned spaces. The former bounded by the other adjacent walls, the latter constrained within the building's courtyards used for hosting theatrical installations. These contextual conditions severely limited the on-site construction of perspective because they prevented the use of distance and concurrence points.

At the end of the 16th century, we have written evidence of the various practices in use for the construction of perspective tracings. In the years when Giovanni Paolo Lomazzo, now blind, was dictating the pages of his *Trattato dell'arte della pittura [...]*, describing a transport procedure for the construction of large anamorphoses by the graticulation technique[3], set designers questioned the effectiveness of the projective procedures proposed, for scenography, by Daniele Barbaro and Egnazio Danti[4]. Shafts, ropes, lamps and on-sight alignment operations appear in the pages of perspective treatises devoted to the applications. These instruments, useful for the construction of perspectives from below, theatrical scenographies and anamorphoses, were indispensable tools for the application of projective procedures that demonstrate the ability of prospective artists of the time to operate in space with projecting planes, showing a capability that anticipates the theory through the practice.

In perspective, the center of projection, the line to be represented and its image belong to the same projecting plane. The perspective image of the line in question is simultaneously the image of all lines belonging to this plane. Considering the real line infinitely extended, we observe that its image remains unchanged if the observer moves his eyes to any point on the projective plane. This idea, which was far from the late 16th-century perspective from a theoretical point of view, became evident in operational practice, where the perspective image of a line was obtained by the projection of another line belonging to the same projecting plane. The projective operation could be performed using another rope, by placing a lamp in the center of the projection or by on-sight alignment operations (Figure 1). Guidobaldo Del Monte testifies to these practices in *De Scenis*, book VI of his *Perspectivae libri sex*, describing one of the procedures in use at the time for the construction of a theatrical scenography [4, pp. 287–289]. This procedure was practiced by means of taut ropes for constructing, on the oblique wings, the perspective of horizontal lines that must appear to the observer orthogonal to the backdrop plane. The method, which is flawless, recalled the one described by Egnazio Danti in his *Commentari* to *Le due regole della prospettiva pratica di M. Iacomo Barozzi da Vignola*[5] [14, pp. 90–94]. To construct a horizontal line **EB**, exemplifying of his *modus operandi*, Danti establishes in the theatre the position of point **A** (centre of projection) located at the end of a wooden shaft embedded in the ground (Figure 2). Then, he conducts from this point a taut rope perpendicular to the backdrop plane determining the point **C**, principal point of perspective. From this point, he stretches a second rope to a point **E**, chosen on the wing in correspondence to the front of the scene, where the searched perspective must pass. Finally, he stretches a third rope from the point **A** to the wing, touching **CE** rope at point

[3]Lomazzo's procedure, described in the XIX Chapter of the sixth book of his treatise, entitled *Modo di fare la prospettiva inversa che paia vera, essendo veduta per un solo forame*, which reinterprets Albrecht Dürer's window on a grand scale, is the same used, years later, by Maignan for the construction of the large anamorphosis in the convent of *Trinità dei Monti* in Rome, published in 1646 by Niceron in his *Thaumaturgus Opticus* [8, pp. 335–336].

[4]The dissertation is developed in certain passages of the dialogue by Ercole Bottrigari *La Mascara, overo, della fabrica de' teatri, et dello apparato delle scene tragisatiricomiche*, a manuscript dated 1596, where the four interlocutors question the efficacy of the projective methods proposed by Barbaro and Danti for the construction of theatrical scenography.

[5]Danti's contributions to practical perspective concern the construction of scenography, anamorphoses and perspectives from below; as Danti himself reports, the latter his addressed for the first time in a treatise on perspective [14, p. 89].

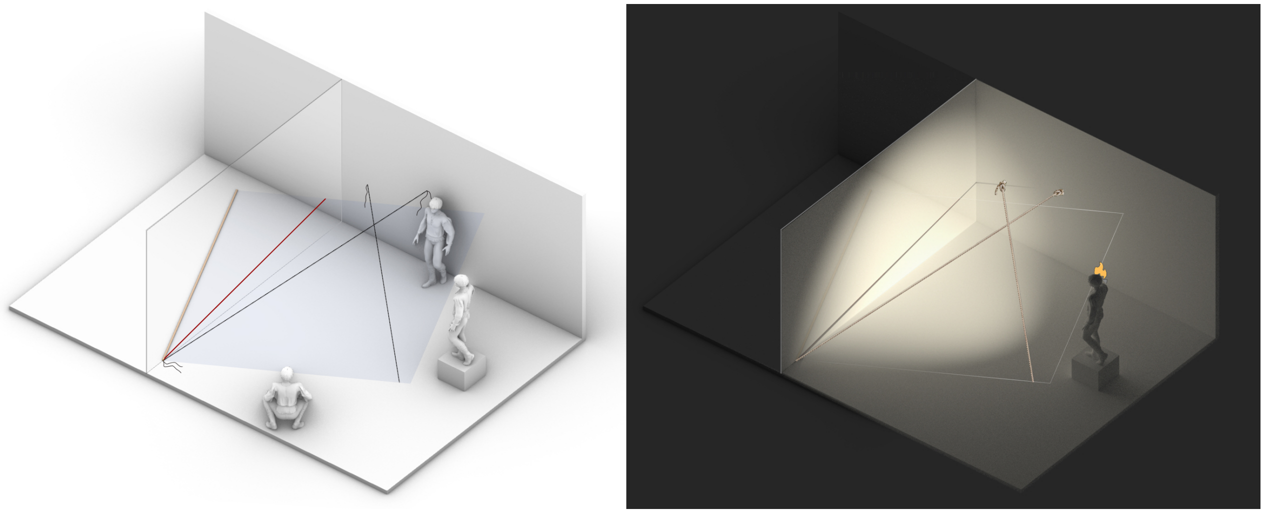

Figure 1: Perspective image of straight lines belonging to same projecting plane obtained by on-sight alignment operations and by the lamp.

D. This rope intersects the wing in point **B**, which belongs to the searched perspective of the horizontal line, that results thus determined. The taut ropes used by Danti in the construction evidently all belong to the same projecting plane, whose use recurs in all practical perspective constructions described in those years. However, there are no demonstrative references in the *Commentari* as well as in the contributions proposed by other authors of the time dealing with similar procedures. Guidobaldo himself believes that the people operating with perspective were not aware of the demonstration, presumably limiting themselves to the experimental verification of the effectiveness of the procedure [4, p. 287].

Guidobaldo's intent in writing *De Scenis* is, instead, deliberately demonstrative. This mathematician intends to show how the practice in use responds in an unexceptionable way to the theory of concurrence points and how the effects of this theory contribute to the simplification and the optimization of perspective construction procedures.

3 Demonstrative approaches of the science of the art

In the first book of *Perspectivae libri sex* Guidobaldo Del Monte demonstrates that the perspective of a system of parallel lines is a set of concurrent lines, defining for the first time its center as *punctum concursus* [9, pp. 15–19]. The idea that classes of generically oriented lines in the horizontal plane converged in *punti paricolari* (special points) had been introduced by Vignola in Chapter three of his perspective, *Delle linee parallele diagonali, & poste a caso* and commented on by Danti in related annotation III. This concept was revised in the ninth Chapter *Digradazione del quadro fuor di linea*, also commented on by Danti in the four annotations that follow it [14, pp. 101–103; 114–116], but it was with Del Monte's contribution that these anticipations found their geometric reason. In fact, the *punti particolari* or *coadiutori* (vanishing points of the lines that are generally oblique to the picture plane, as **G** and **Q** in Figure 3) introduced by Vignola were obtained *ex post*, by intersection of the perspective images of a pair of *linee parallele secondarie* (secondary parallel lines), such as the edges of a square rotated with respect to the picture plane. The perspective of this pair of parallel lines was obtained by degrading the vertices of the square through the use of the distance point and the principal point. Thus, these special points, obtained by intersecting the perspectives

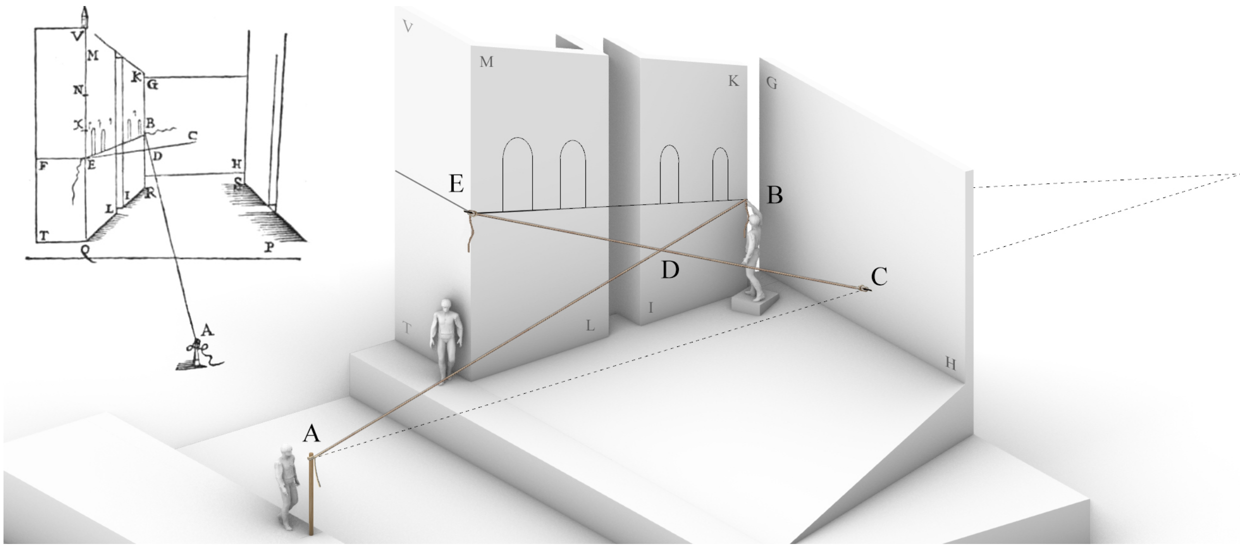

Figure 2: Method proposed by Egnazio Danti in the *Commentari* to *Le due regole della prospettiva pratica di M. Iacomo Barozzi da Vignola* for the construction of a theatrical scenography.

images of a parallel couple of given lines, were only used to facilitate the operability of the constructions (Figure 3) [1, pp. 136–138].

Guidobaldo Del Monte introduces the theory of concurrence points in Proposition XXVIII of the first book. Through the similarity of triangles, he demonstrates the convergence at the same point of the images of a set of parallel lines belonging to the ground plane and oblique with respect to the *sectio* (picture plane). The dissertation is developed according to increasing levels of complexity up to Proposition XXXII, which synthesizes the previous ones by considering a condition of maximum generality, related to a set of parallel lines belonging to a generically inclined plane (Figure 4).

Guidobaldo conducts to the eyes of the observer a line that is parallel to the given straight lines and therefore to the plane passing from them. This line meets the picture plane in a point named *punctum concursus*, where the perspective images of the given straight lines converge; this point is as high as the observer in relation to the given inclined plane. In this proposition, Guidobaldo observes how a line and its parallel passing through the observer

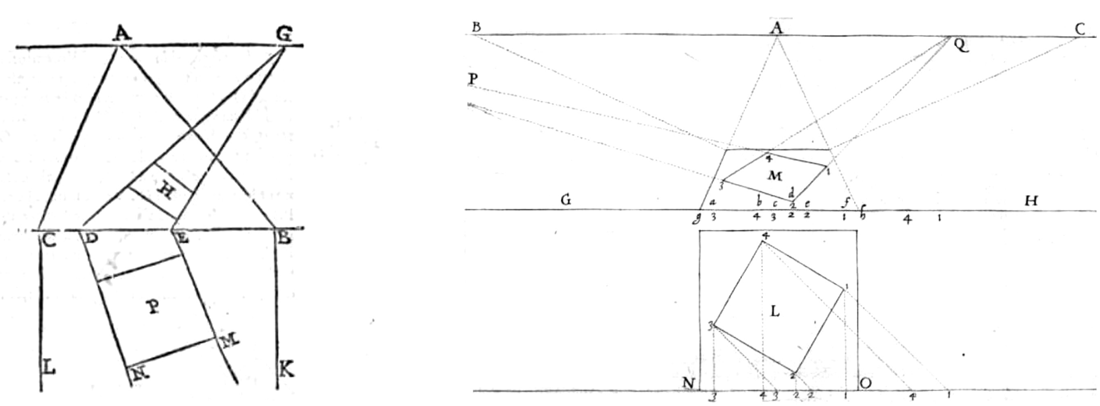

Figure 3: llustration by Danti to the third annotation to Chapter Three *Delle linee parallele diagonali, & poste a caso* (left) and illustration by Vignola to Chapter nine *Digradazione del quadro fuor di linea* (right) from *Le due regole della prospettiva pratica [. . .]* (1583).

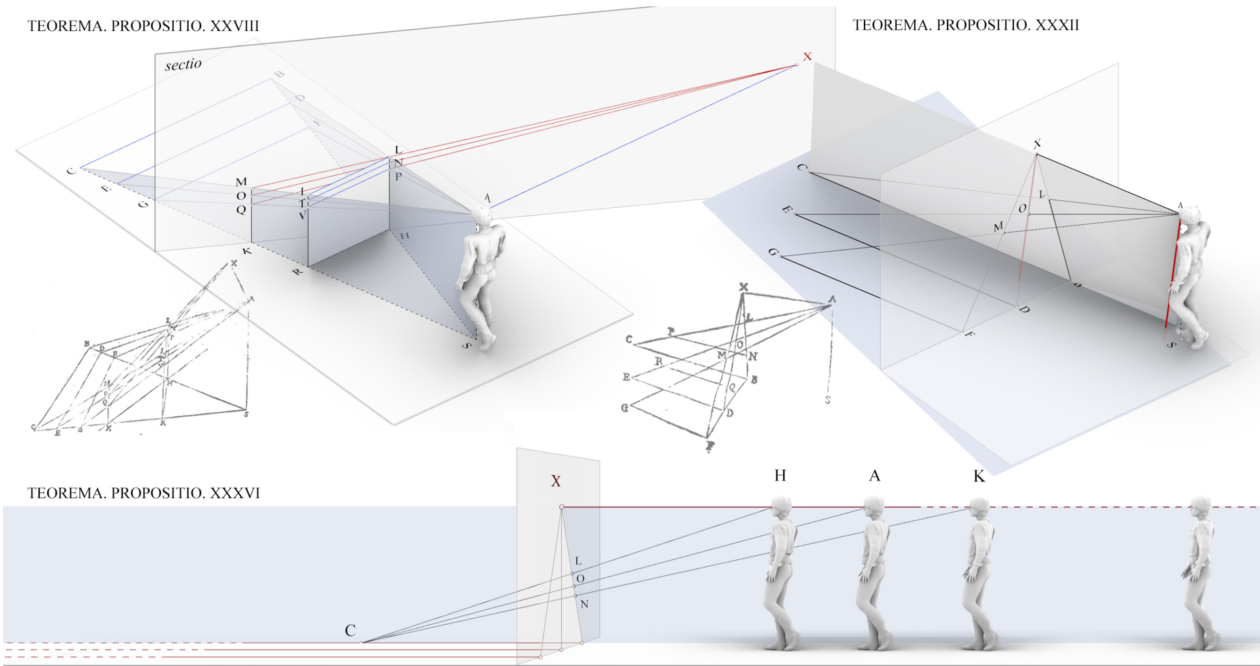

Figure 4: Concurrence points theory in propositions XXVIII and XXXII (above) and movement of the observer along an infinite projecting line in propositions XXXVI (below), from Guidobaldo Del Monte's *Perspectivae libri sex*, first book (1600).

belong to the same plane. The visual lines that project the points of the given parallel lines also lie on this plane, as do their perspective images [4, pp. 35–44]. The idea of this plane, today called "projecting plane", passing through the objective line, the observer, the visual lines and the perspective image, recurs in Proposition XXXVI, which concludes the first book. Here, Guidobaldo works with straight lines of infinite length, imagining that the objective line and its parallel passing through the observer are infinitely extended in one direction and the opposite with respect to the picture plane. He finally imagines sliding the observer's eyes along the straight line – which today we call "projecting line" – letting the center of projection to move closer and further away from the picture plane, demonstrating how, during this movement, the perspective image of the line remains unchanged [4, pp. 48–50]. The theory of concurrence points and the idea of a plane where performing projection operations found fruitful application in scenography, which Guidobaldo treats in *De Scenis.*

During the Renaissance period, theater building had not yet found a stable conformation and was generally located inside palace courtyards where, due to evident problems of space, a full-scale setting could not be reproduced on stage. Thus, the scenic box was contracted, entrusting the perspective illusion to a set of linear perspectives, today anamorphic perspective, painted on differently oriented picture planes, the wings, all viewed from the same center of projection [2]. Since the image of the scenic box had to show a unified parallelepiped space, the conformation of the wings needed to be designed in order to make appear its edges as orthogonal lines to the front of the scene.

The first picture plane that Guidobaldo considers is the stage plane, which was inclined to widen the audience's view of the actors on stage. Since a portion of the stage plane was to appear rectangular to the eyes of the observer, Guidobaldo considers the point of concurrence of the straight lines perpendicular to the front of the scene **X**, where all the edges of the

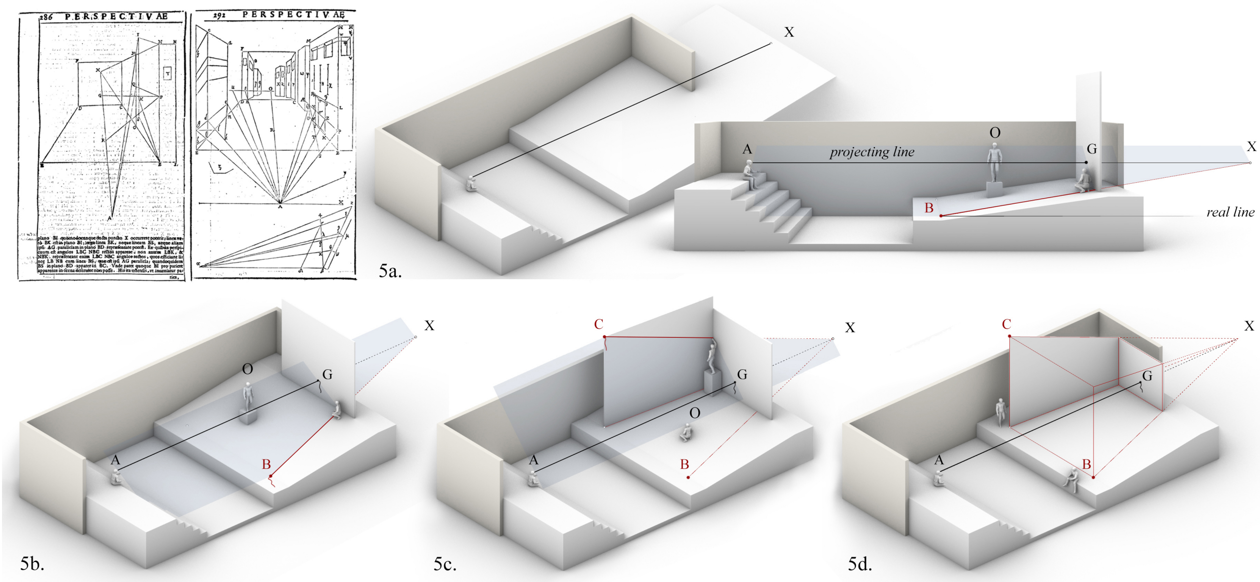

Figure 5: Projective method proposed by Guidobaldo del Monte for the construction of a theatrical scenography applied to the construction of the contracted scenic box.

contracted scenic box – and some of the houses represented therein – would have concurred (Figure 5). That point, intersection of the stage plane with the perpendicular line to the front of the scene conducted through the observer, was evidently beyond the backdrop plane, thus resulting inaccessible for the constructions.

Consequently, a method for operating without it had to be devised, and the solution is precisely given by the projecting plane. Because the line parallel to the one to be represented and its perspective image are in the same plane, it is possible to construct its perspective image through an operation of on-sight alignment, by placing the observer's eyes in any point on the projecting plane. Having defined the direction of the line to be represented, materialized by a taut rope passing through the observer (as **AG** in figure), and having then chosen a point (as **B** in Figure 5b or **C** in Figure 5c) through which the perspective should pass, an auxiliary observer **O** would move around the stage plane observing the taut rope **AG** until its image overlapped with that of the chosen point. In this position, the projecting line and the sought perspective would appear into a single image, easily reproducible by means of another taut rope fixed at the given point and tensioned from this point to the backdrop plane[6] (Figure 5b).

Today we would say that the projecting line is the axis of a set of planes that belong to the infinite straight lines having the same direction. This idea will be developed a few years later by Simon Stevin and used by Girard Desargues as theoretical foundation for construction of perspective scales in his practical perspective.

[6]Guidobaldo explains how, in place of the observer's eyes, it was possible to use a lamp that would project the shadow of the taut rope. The idea of sighting from any point on the projecting plane allowed a reduction of distances between the point of view and the projected line, increasing the sharpness of shadows if lamps were used and reducing the bending of the ropes in case of considerably long ones were used.

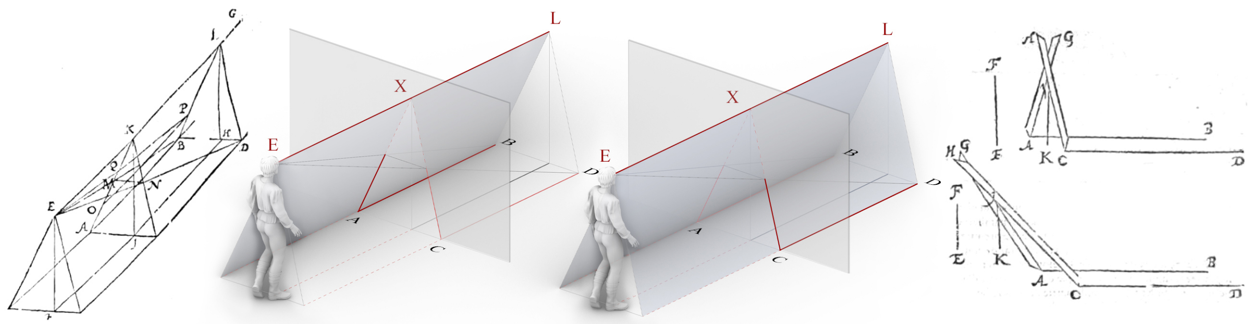

Figure 6: Common section of unlimited planes belonging to parallel lines in Simon Stevin's work.

4 Consequences of *punctum concursus* theory

In the conclusive pages addressed to the *contemplatifs* (theorists) of the *Exemple de l'une des manières universelle du S.G.D.L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre*, dated 1636, Desargues resumes the concurrence points theory, that enunciates in all its generality. Recognizing the value of this theory, he revisited some formulations developed a few years earlier by Simon Stevin and published in 1605 in *La Scenographie* [10, p. 267]. Stevin had developed the theory of concurrence points enunciated by Guidobaldo del Monte starting from Theoreme 3. Proposition 3 of his treatise:

> "Les lignes paralleles ombrageables estant vuës par le vitre qui n'est parallele avec les paralleles, et leurs ombres éstant prolongées en iceluy ells conjoignent en un mesme poinct du rayon qui est parallele avec l'ombrageable parallele, & icelles ombrageable estant pareillement paralleles avec le pavé, leur poinct de conjonction vien aussi haut dessus le pavé comme l'œil"[7] [12, p. 526].

Stevin's demonstration in fact uses the 'projecting plane', considered unlimited, that belongs to the given line (**AB** in Figure 6) and that intersects the *vitre* (picture plane) in its perspective (line **AX**). Reiterating the procedure for a second straight line parallel to the first one (as **CD**), he demonstrates how the images of two parallel lines intersect each other in the *vitre* in the same point **X** where it is crossed by the parallel straight line passing through the observer **E**. In fact, the line **EX** is the common section of the unlimited planes that belong to the given parallel lines[8].

Thus, in the pages devoted to theorists Desargues elaborates these theoretical assumptions. He imagines an indeterminate line passing through the observer's eye, called *ligne de l'oeil* (line of the eye) that, capable of moving in all directions, can be conducted parallel to any given line. This line is the common axis of a set of planes passing through sets of parallel or intersecting given lines (line ***lo*** in Figure 7). Desargues considers two sets of straight lines

[7] "If the lines to be drawn are sighted through a *vitre* that is not parallel to them, their prolonged images will concur at the same point of the ray parallel to the line to be drawn, and if the lines to be drawn are parallel to the ground plane, the point of concurrence rises above the ground plane to the same height of the eye" (author's translation).

[8] Stevin also proposes an experimental verification of this statement, practised by the *Most Illustrious Prince*, which consisted in drawing two parallel lines with chalk on a horizontal surface and placing at the ends of these lines, on the side of the observer, two wooden rods crossed at x in a point at the same height as the observer's eyes. By observing this system of lines, the wooden rods would cover the drawn chalk lines [12, p. 527].

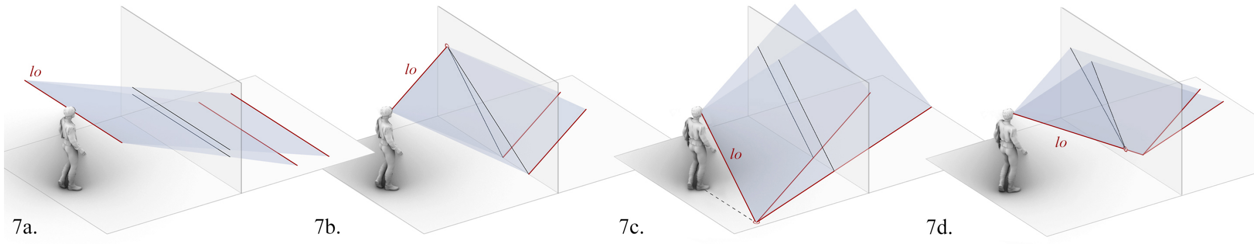

Figure 7: Perspective images of parallel and intersecting straight lines discussed in the pages devoted to the theorists of Girard Desargues perspetive work.

parallel to each other, respectively parallel and oblique with respect to the picture plane (Figure 7a and 7b), and then he considers two sets of straight intersecting lines (Figure 7c and 7d). In the case of parallel lines, the *ligne de l'oeil* is conducted to the *tableau* (picture plane) parallel to the given lines forming, with these, a set of planes having their common axis in this line. Desargues first considers the classes of parallel lines to the picture plane whose perspective images are parallel to each other (Figure 7a), then he considers the parallel lines rotated with respect to the picture plane (Figure 7b), whose perspective images converge in a point (today vanishing point). Then he considers the case of intersecting lines. In this case the *ligne de l'oeil* is conducted at the intersection point between the given lines, because it is the common axis of the set of planes that belongs to them. If the *ligne de l'oeil* is parallel to the picture plane (Figure 7c), the perspective images of the given lines will be parallel to each other. This case is particularly interesting because it anticipates the properties of a special plane, the front plane passing through the observer, whose points, when projected, meet the picture plane in a point at infinity. Finally, if the *ligne de l'oeil* intersects the picture plane (Figure 7d), the perspective images of the given lines will converge at the point where this line meets the picture plane [6, pp. 11–12].

These theoretical assumptions are the core of Desargues' demonstrative thinking, that legitimizes the construction of the scales at the basis of his practical perspective. Like in the previous cases, the problem posed by Desargues consists in constructing perspective without the use of *tiers points* or *points de distance*. The idea at the basis of the arguesian perspective consists in constructing a two-dimensional system of Cartesian coordinates arranged on the ground plane, to which he refers the vertices position of any subject to be represent. This reference system, which we read in true form in a planimetric representation, must be degraded in perspective. For this purpose, Desargues elaborates two perspective scales: the *échelle d'éloignemens* for the degradation of depth and the *échelle de mesure* for the degradation of width. The subject, chosen for illustrative purposes, is a cage rotated with respect to the picture plane and embedded in the ground plane (Figure 8a). This kind of subject should not have been a novelty for Desargues, because it is similar to the tower used in Problem 5. Proposition 11 by Simon Stevin in his treatise, for introducing the inclined picture plane in perspective (Figure 8b).

Desargues illustrates the text employing a single image composed by three figures: the first devoted to the planimetric representation of the problem, showing the plan of the cage, the reference system with its coordinates and unit of measurement (feet), the position of the observer and his height; the second describing the theoretical setup of the perspective scales; the third showing the operative application of these scales to the construction of perspective of the cage. Once the first of these three figures is illustrated, Desargues describes the other two together [6, pp. 1–11] (Figure 9).

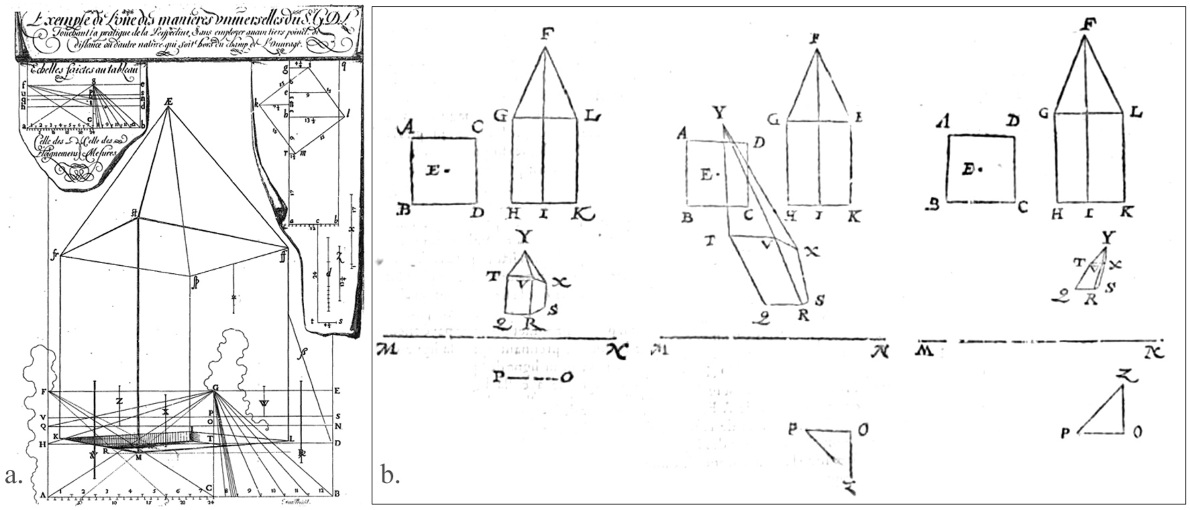

Figure 8: Perspective of the cage in Desargues' *Exemple [. . .]* of 1636 (8a) in comparison with the tower chosen by Simon Stevin in 1605 to introduce inclined picture plane (8b).

He imagine to work on a strictly delimited portion of the picture plane and divide the straight-line **AB**, intersection between the picture plane and the ground plane, in equal parts of one foot of length (12 in figure). Then, he draws the height of the observer's eyes in this perspective by constructing a horizontal line **FE** parallel to **AB**, passing through **G**, which is the foot of the perpendicular line conducted from the observer to the picture plane. Then, from this point he vertically divides the perspective space into two parts, in which he will construct respectively the *échelle d'éloignemens* (left half) and the *échelle de mesure* (right half). The first of these perspective scales is the *échelle d'éloignemens* or *échelle d'optique*. The construction of this scale is geometrically demonstrated by the theory of concurrence points, in particular by the above-mentioned theoretical considerations regarding perspective images of parallel lines. However, being an operational example, *de pure pratique*, the procedure, lacking in demonstration, is didactically described in algorithmic form. Desargues conducts the **GA** line from point **G** and the **FC** line from point **F** and, through their point of intersection he constructs the horizontal line **HD**. Referring to the third figure, he conducts the **GH** line from point **G** and, at the intersection of this with **FC**, draws a second horizontal line **QN**. The same construction is proposed in relation to the second figure (Figure 9). In this case, it conducts the line **FT** from point **F** and, at the intersection of this line with **GA** draws a second line **QN**. The horizontal lines indifferently obtained in this manners are progressively distant from the picture plane and from each other by 24 feet, just as far as the observer is from the picture plane. Reiterating the operation as necessary, it is possible to degrade an infinite series of intervals. If we consider the construction referable to the second figure, we observe how the parallel straight lines having their concurrence point in **F** are capable of detaching equal intervals on the perpendicular line to the picture plane passing form point **A**, and therefore they are skilfully employed by Desargues as *ante litteram* measuring straight lines. Then, Desargues proceeds with the *échelle de mesure*, obtained through the perspective construction of straight lines perpendicular to the picture plane passing through the one-foot intervals marked on the **AB** line. Once the perspective scales are defined, Desargues demonstrates the operational effectiveness of its method through the perspective construction of the cage. First of all, he constructs the point **M** (Figure 10). To determine the degraded position of this point, which is 17 feet from the picture plane,

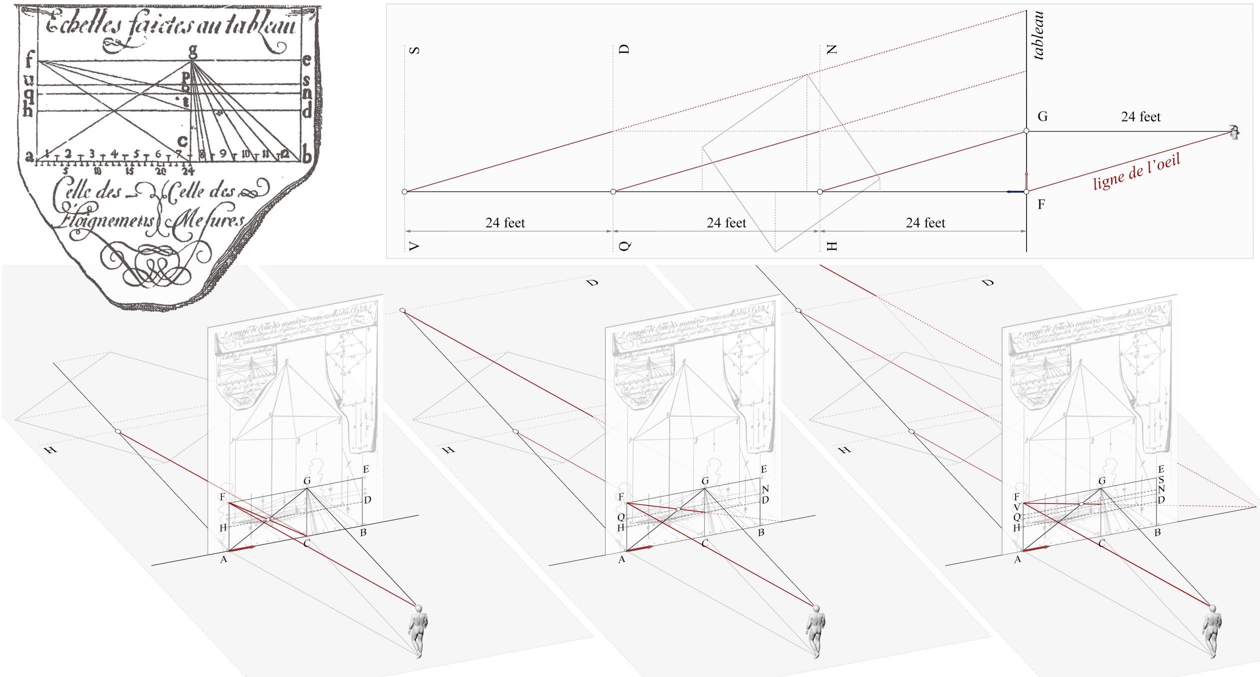

Figure 9: Theoretical foundations of *échelle d'éloignemens* construction in Desargues' perspective.

Desargues constructs the point **R**, located at the same distance, on the **AG** line (ordinate). To determine this distance, he divides the **AC** line into 24 feet, the same as the length of the first interval. Then, he draws a horizontal line to point **R** entirely passing through the useful portion of the picture plane (abscissa). Because the point **M** is located 1.5 feet from **R**, he takes this interval with a compass on the *échelle de mesure* and determines, in perspective, its correct position.

Desargues proceeds in the same way for the other points of the cage, constructing the relative heights through their overturning on the ground plane, according to a method already described by Danti in his *Commentari* [14, p. 93]. The arguesian procedure led to the construction of a proper perspective machine where, in order to avoid the multiplication of occult lines, the *échelle d'éloignamens* operated by tense wires having one end fixed at the concurrence point and the other movable, while the *échelle de mesure* operated through the

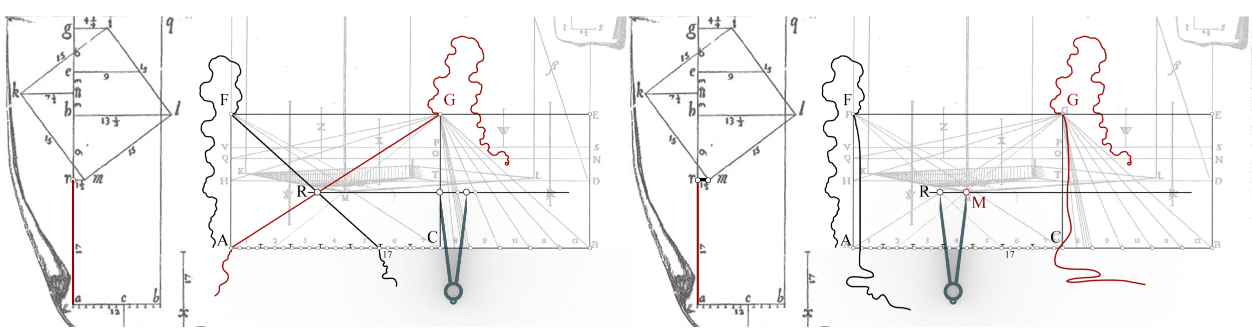

Figure 10: Use of the *échelle d'éloignamens* and the *échelle de mesure* for the construction of a point (M) of Desargues' cage.

use of a compass[9].

Desargue's perspective could be directly realized on the wall or transported onto generic shaped surfaces, as described year later by Abraham Bosse in his *Moyen universel de pratiquer la perspective sur les tableaux ou surfaces irrégulières* published in 1653. The transport operations were performed through taut ropes capable of reproducing a grid in space, through the lamps free to move along a rope perpendicular to the picture plane for obtaining sharp shadows, or by sight, positioning the eyes on the projecting plane, according to the Guidobaldo Del Monte's manner [3, pp. 50–59].

5 Conclusions

This recognition regarding the theoretical reasons of applied perspective in the early 17th century is intended to trigger a reflection about the relationships existing, in those years, between the practical construction of perspective and the development of demonstrative perspective thinking, to which it had to respond. The possibility of operating full-scale in projective space with ropes, lamps, and on-sight alignment operations, able to reproduce projecting lines and projecting planes, made the perspective building sites of theatrical scenographies, architectural perspectives and anamorphoses, ideal spaces for experimenting theories and validating results. Hence, practice looked to theory, but at the same time the theory recognized in the practice the effective operational strategies which were consolidated through experience and experimental verification. The prominent role that perspective applied to the arts played in the customs of Baroque society is the engine that fuelled the cultural ferment surrounding this research. While the contribution of mathematicians introduces a clear separation between theoretical perspective and that of the artists, it is useful to reflect on the fortunate union between art and science that nourishes and effectively structures scientific perspective thinking at the turn of the century. In its early scientific maturity, theoretical perspective participates in the on-going scientific revolution, defining its own projective foundations and laying the basis for future projective geometry.

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