

A Graphical Analysis of “The Geographer”

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Abstract. This research conducts a graphical analysis of *The Geographer*, a seventeenth-century painting by Dutch artist Johannes Vermeer. Produced in 1669, it creates a contrast between light and shadow by depicting a close-up view in dark tones with a long-distance view on a bright floor. This contrast creates a sense of depth in the painting’s virtual space. A rectangle like the seat of a backless chair at the bottom right of the screen creates a difference in height with respect to the floor in the back, and functions as a motif to give a sense of depth to the picture. This sense of depth would be lost without the bright flooring. Vermeer painted the flooring when *The Milk Maid* was created, which would indicate that the flooring configuration was done in the early stages of his career. However, *The Geographer* belongs to the latter period. Furthermore, checkered tiles were brilliantly placed on the floor of the interior paintings produced before and after 1669. Nevertheless, the flooring in this work does not constitute tiles. Therefore, this study analyzes whether the backless chair depicted in the foreground of this work was a part of the tiled floor that Vermeer intended to depict. As a result, the seat of this backless chair was equivalent to two square tiles. And if we assume that the construction method for those tiles that cover the floor of this painting was the first step, it suggests that a trigonometric ratio may have been used there. From this analysis, a hypothesis is formulated about Vermeer constructing floors of checkered tiles.

Key Words: geometric aspects of technical art, seventeenth-century painting, perspective technique, golden ratio, drawing tiles

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1 Introduction

The Geographer and *The Astronomer* have a typological composition in which a male model is depicted alone in the picture. In both pictures, a carpet is nonchalantly placed on a table whose legs are not visible. This placement of a chair or table whose legs are not visible in

the foreground creates a sense of depth with relation to the front wall. This composition is characteristic of Vermeer’s paintings of interior scenes. It is important to note that the backless chair in the foreground is similar to that used by the painter in *The Art of Painting*. The rectangle depicted in the fore-ground in *The Geographer* may be a chair. Furthermore, considering the age of production, it should have been possible to draw a floor with tiles; however, there is a sense of discomfort in the flooring’s composition. Therefore, the assumption is that the seat of the backless chair was a tile that Vermeer had intended to depict. In this study, the hypothesis stated was based on the analysis of results from determining the relationship between the composition and distance points located outside the canvas, in determining the gradual reduction rate of the tiles.

2 Composition and the golden ratio

Robert Wald, a conservator, analyzed *The Art of Painting* in 2010. He reported that the ratio of the length from the visual center (V_c) to the edge of the picture (Q_2) and from Q_2 to the distance point (D_2) is the golden ratio [5, pp. 312–321]. He also stated that this finding “may be a coincidence.” However, this study supports the probability that this finding may not be a coincidence.

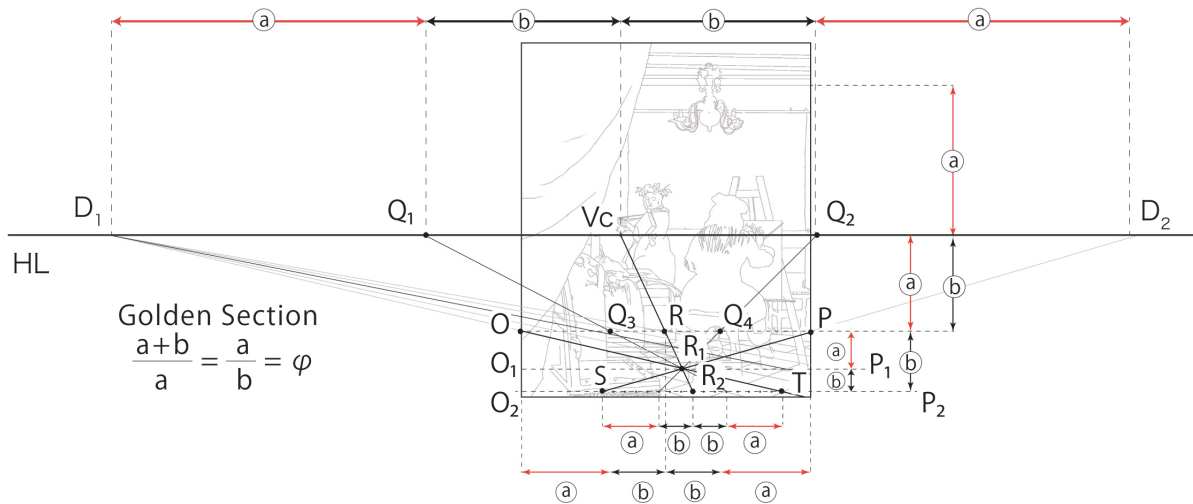


Figure 1: An Analytical Diagram of *The Art of Painting*.
The Art of Painting, 120 cm × 100 cm, Kunsthistorisches Museum, Vienna.

As shown in Figure 1, if we divide the line from the visual center (V_c) to distance point (D_2) according to the golden ratio, the point (Q_2) overlaps the edge of the canvas at the division point. Moreover the division point can be set at the height of the line of sight (HL), by dividing the area between the edge connecting the front wall to the ceiling and the boundary line of the floor (OP) according to the golden ratio. Furthermore, the dotted line (O_1P_1) is created when between the dotted line (OP) and the dotted line (O_2P_2) when it is divided according to the golden ratio. When extended, OT converges at D_1 and overlaps the border of the drawn tile.

In *The Music Lesson* and *The Young Woman Standing at a Virginals*, the relationship between the golden section point from the visual center to the distance points and the com-

position has already been analyzed [4, pp. 176–184]¹. At that time, The Geographer’s composition and golden ratio were not the subject of analysis. However, we investigated the golden section of the width of the screen in *The Art of Painting*, but there were many possibilities to ponder on. Nevertheless, as a result of *The Geographer*’s analysis, the following possibilities can be considered.

3 Analysis of *The Geographer*

The model in *The Geographer* is in the midst of spreading something like a map on a desk and measuring with a divider in one hand. However, his gaze is not focused on the hand holding the tool and seems to be looking out of the closed window and thinking about something. Similarly, the model in *The Astronomer* sits in a chair and stretches his right hand forward to touch a celestial globe on a desk. Our line of sight is naturally guided towards the globe, following the model’s line of sight and the direction of his arm. One theory states that the model in both paintings was Antonie van Leeuwenhoek 1632–1723, who was born in Delft and is considered a pioneer in microscopy [2, p. 104]².

3.1 A Comparison of the Composition in *The Geographer* and *The Astronomer*

When analyzing *The Geographer*’s visual center (V_{C1}), we expected it to overlap around the hand holding the divider, but actually it is located on the front wall behind the model. From the window rail and the depth of the chest of drawers, we derive that V_{C1} is located between the picture in the picture and the chest of drawers, as shown in Figure 2.

In *The Geographer*, the position of V_{C1} is determined from the chest of drawers and the chair on the right side of the picture. Here, the bottom of the chest is indicated by a gray rectangle. There seems to be a space of approximately one window between the chest of drawers and the model. Subsequently, the viewpoint can be obtained by assuming that the motif, like the seat of the backless chair in the close-up view of this image, is rectangular.

On the other hand, in Figure 3, *The Astronomer*’s visual center (V_C) is set around the wrist of the model’s right hand, almost at the center of the canvas. V_C is located at the center of the picture, assuming that the window frame is perpendicular to the screen. The depth line of the chest is toward V_C and is drawn such that the area under the windows cannot be seen. The part between the legs of the chair should ideally reflect the color tones of the floor. However, as the color tone at dusk dominates the entire picture, the front wall and the floor integrate, and their boundary becomes unclear.

Although both paintings have a similar canvas size, *The Geographer* looks wider. In both paintings, the chest of drawers, windows, and the picture within the picture are similar in size and character; thus, these elements cannot be attributed to why the width of the paintings

¹Wald showed that the vanishing point of the chair depicted in the foreground in *The Art of Painting* converges on Q_1 of the golden section in Figure 1. Wald’s analytical diagram is shown in [5] (Fig. 10 on p. 199). In our analysis, the vanishing point of the chair in the foreground in *The Woman Standing at the Virginal* converged to the golden section also.

²According to Nash’s citation, the hypothesis by Arthur Wheelock of the National Gallery of Art in Washington is that there are many similarities to the portrait of Leeuwenhoek created in 1686. In fact, after Vermeer’s passing, Leeuwenhoek was appointed the trustee of his property and it is believed that they were friends.

seem different. Therefore, it is inferred that Vermeer’s control of light in his paintings is a major factor in providing a sense of depth to the room. Both paintings are composed differently with how the light from outside illuminates the room.

In order to clearly depict light and shadow on a painting such as *The Geographer*, the artist wants to create a feeling of contrast. If this is not achieved, that sense of depth will be lost like in *The Astronomer*, and an obstruction will be created on the virtual space in the painting. Therefore, the treatment of light and the composition of the painting play an important role in creating a sense of depth in a limited space.

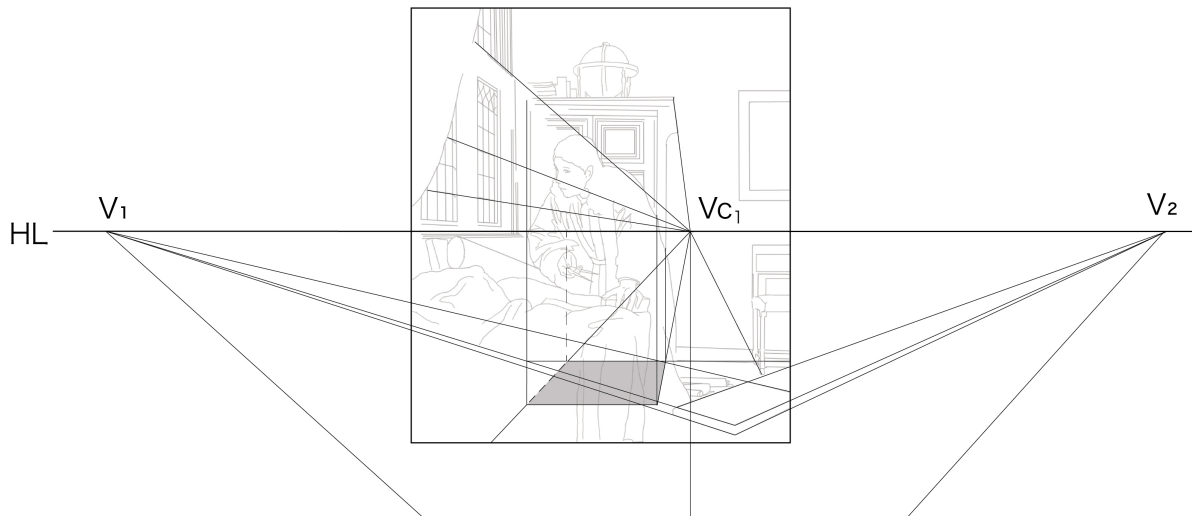


Figure 2: An Analytical Diagram of *The Geographer*’s Visual Center.
The Geographer, 52 cm × 45.5 cm, Städelsches Kunstinstitut, Frankfurt

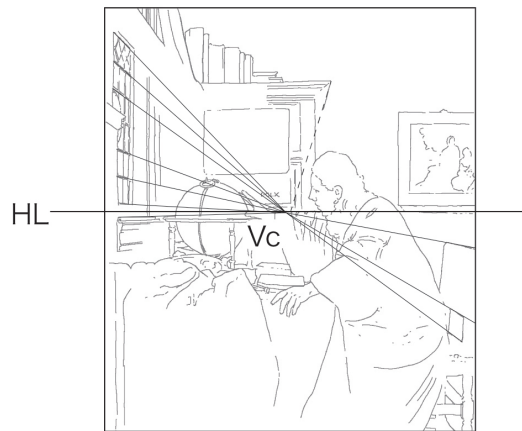


Figure 3: An Analytical Diagram of *The Astronomer*’s Visual Center.
The Astronomer, 51 cm × 45 cm, Louvre, Paris.

3.2 An Analysis of the Seats in *The Geographer*

Sometimes, Vermeer drew a chair with a backrest close to the right or left edge of the picture. Therefore, it is natural to think that the chair depicted in this work was also arranged with the same intention. However, it was observed that the angle of the chair’s outline approximated

that of the flooring tiles depicted in other works of Vermeer.³ Therefore, an analysis of the chair ($ABCD$) was conducted by assuming a square tile with an inclination of 45° to the screen, as shown in Figure 4.

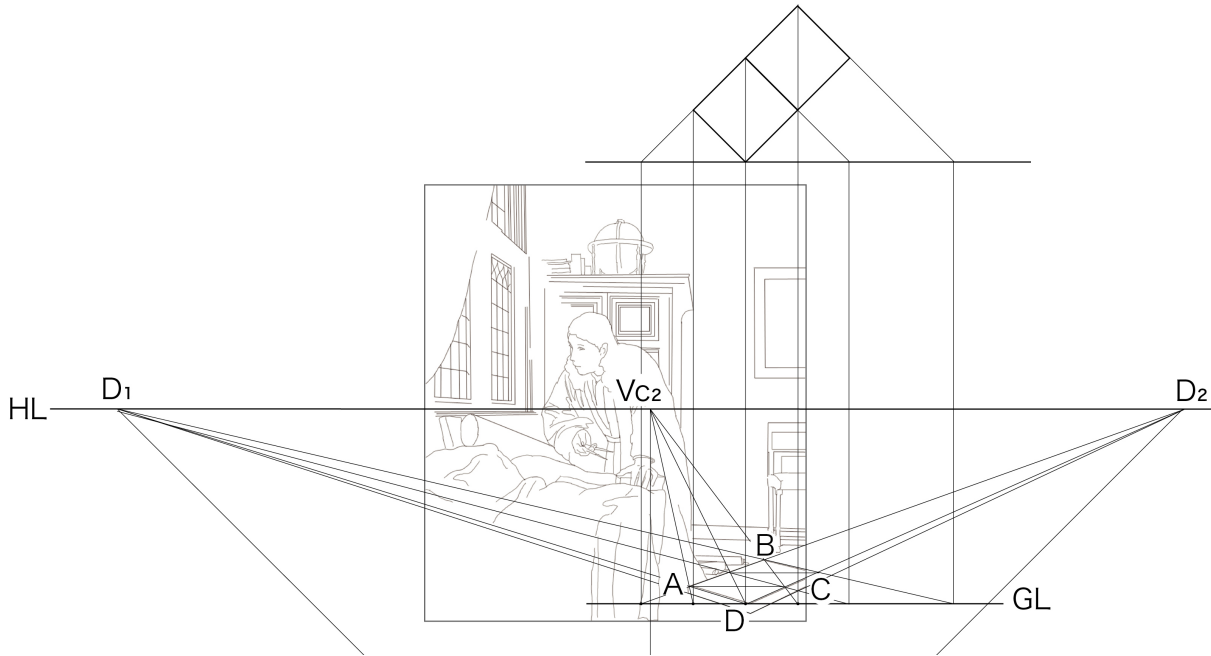


Figure 4: An Analytical Diagram of the Seats in *The Geographer*

The visual center (V_{C2}) was obtained near the left arm of the model. From the results, it can be observed that these lines drawn for analysis converges at V_{C2} and the distance point (D_1 and D_2). The two rectangles shown in Figure 4, like the white and black checkered tiles depicted in other interior paintings, are inclined at 45° to the screen and are assumed to be square. The part drawn as a seat indicates that Vermeer could have been trying to draw tiles halfway through the painting. If so, the position of the visual center was changed from V_{C2} in Figure 4 to V_{C1} in Figure 2 in the middle of the depiction then it would have been necessary to add depth to the chest and change the angle of the crosspiece of the window frames. Alternatively, it is possible that Vermeer tried to construct only the floor separately from the chest of drawers and the window frame. Because the visual center has changed. As the lower part of the chest is hidden behind the gown worn by the model, the consistency of the perspective lines is not a point of concern. However, if the composition is changed in-between, things like the arrangement of motifs and color tones must be harmonized. Nevertheless, if Vermeer changed it, it is probable that he presumed that the new composition would make a better picture.

3.3 The Relationship Between the Composition of *The Geographer* and the Golden Ratio

In *The Geographer*, as shown in Figure 5, the line from the visual center (V_{C2}) to the distance points (D_1 and D_2) is divided into the golden ratio at points Q_1 and Q_2 . When the seat in

³There are several works in which the tiles are organized in the lower right corner of the picture: *The Concert*; *The Art of Painting*; and *The Allegory of Faith*.

the foreground is assumed to be two square tiles, Q_1 and Q_2 are obtained by dividing the area between the obtained visual center V_{C2} and the distance points (D_1 and D_2) according to the golden ratio. Q_1 is under the drawn window frame and above the canvas but Q_2 is outside the picture. In this instance, if the area from the upper edge of the canvas to the boundary line OP of the floor is divided according to the golden ratio, the division point is obtained on the line HL . Further, if the area from the line HL to the lower edge of the canvas is divided according to the golden ratio, the division points will be obtained on the floor boundary OP . After dividing V_{C2} to D_1 into the golden ratio, Q_1 was located as the division point near the left edge of the screen (Figure 5).

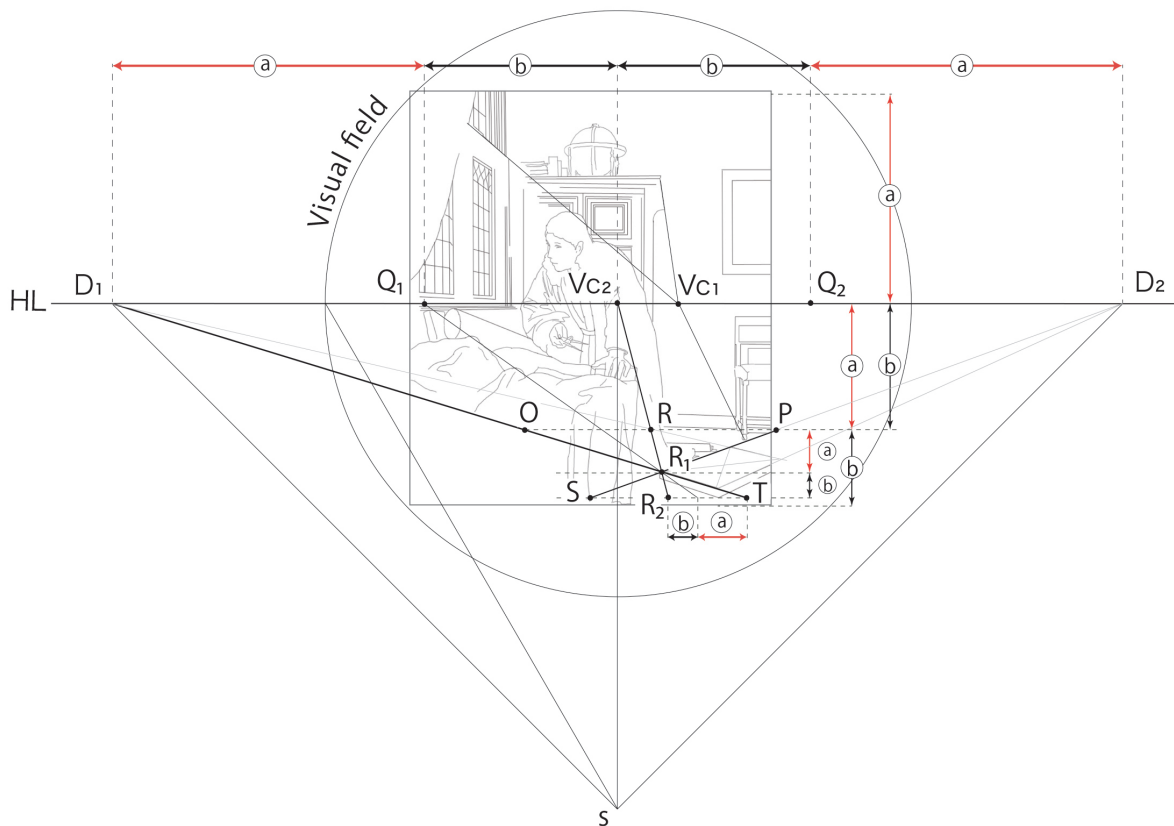


Figure 5: An Analytical Diagram of the Golden Ratio in *The Geographer*

However, the division points of the golden ratio did not overlap the edges of the canvas, as they did in Wald’s analysis. A straight line of any length can be divided according to the golden ratio. However, it is not possible to place the division point unintentionally at the boundary line of the floor and HL which are the predetermined positions for composition. Such occurrences are possible by chance, but it cannot be considered a coincidence if it occurs repeatedly. In Figure 1, it is observed that the division points of the golden ratio overlap at the main positions when the perspective image is drawn. It is observed that the composition of *The Allegory of Faith* is also determined using the golden ratio. The composition of the tiles in *The Allegory of Faith* will be discussed later, along with the results of *The Geographer*’s analysis.

3.4 The Relationship Between the Golden Ratio and Distance Points

Pietro Accolti (1579–1642) uses Euclid’s theorem to demonstrate that the triangles ABE and DCE in Figure 6 are equiangular triangles and their sides are proportional in length [1, p. 12]⁴. If two other lines intersect each other between the two parallel lines, we can draw another straight line on the part created between them, and divide the parallel lines into two. Further, proportional sections can be created between them. He states that this proportional relationship can be used by painters to enlarge a design. This study applies the concept to the triangles $D_1D_2R_1$ and TSR_1 , as shown in Figure 5. It is observed that they shared a similar relationship. Contrary to the analytical process so far, these important distance points outside the canvas can be assumed, if the similarity of the golden ratio triangle is used on the screen.

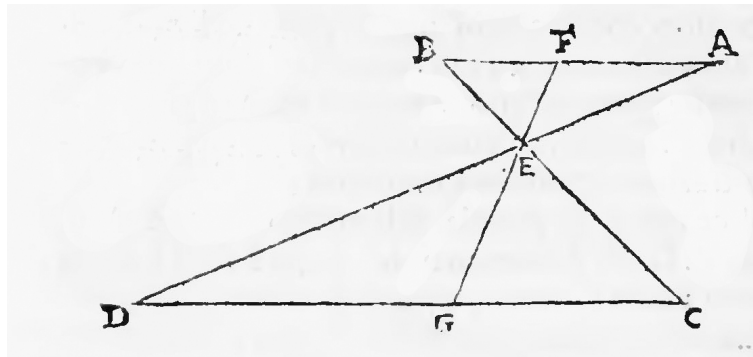


Figure 6: An Image by Pietro Accolti

A painter who used this method would have been able to draw as if he had used that point without having to prepare a distance point outside the canvas. We won’t quite go into how to assume a virtual distance point here, but we’ve already considered the possibility [3]. By setting this point farther from the visual center, the depicted scene will feel as if it were viewed from a distance. The chair, depicted geometrically, pulls the viewer’s point of view away from the screen, and as a result, the depicted scene fits inside a circular visual field giving people a sense of realism. On the contrary, if this chair was not drawn in the near view of the painting’s visual space, the composition would be as if the model was captured from a short distance as in *The Astronomer*.

In *The Art of Painting* shown in Figure 1, the straight lines SP and OT overlap the boundaries of the tiles, and these straight lines intersect at point R_1 on the dotted line O_1P_1 . At this time, the triangle TR_1S is similar to the triangle OR_1P , which is similar to the triangle $D_1R_1D_2$. The point R is located at a position that divides the width of the canvas into two equal parts. The lines OR and PR are divided into golden ratios at Q_3 and Q_4 , respectively. The ratio which divides the floor boundary OP into the golden ratio is proportional to the straight lines D_1V_C and D_2V_C . Analysis shows that Vermeer may have assembled the composition by incorporating the golden ratio vertically and horizontally on the screen.

⁴Quoted from Chapter 13. Same as above. Accolti cites and demonstrates Euclid’s Elements, Volume 1, Proposition 29 and Proposition 15, Volume 6, Proposition 4, and Volume 5, Proposition 11. Accolti is an Italian mathematician, painter, and architect. There is no record of this book ever being translated into Dutch.

4 Discussion on The Gradual Decrease Ratio of Tiles

The canvas size and depiction of the tiled floor in *The Art of Painting* and *The Allegory of Faith* are similar. Figure 7 shows the relationship between the screen and the golden ratio based on the line HL obtained from the analysis of the tiles in *The Allegory of Faith*. Upon analysis it was noted that lines of the tiles in the painting do not completely converge at the distance point (D_1), the height of the line HL which determined by extending the diagonal line of the tile that converges to the visual center.

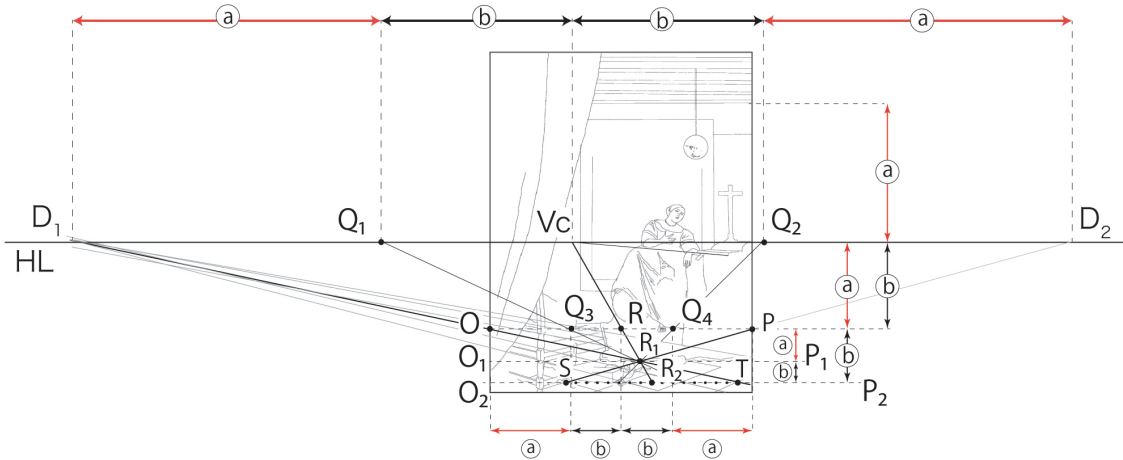


Figure 7: An Analytical Diagram of *The Allegory of Faith*.

The Allegory of Faith, 114.3 cm × 88.9 cm, Metropolitan Museum of Art, New York.

The straight line OT almost coincides with the boundary line of the tile, but only one row of the innermost tile does not follow this straight line. As a result, the area up to the bottom edge of the canvas was not divided according to the golden ratio (below the dotted line OP). However, the gradual decrease ratio of tiles is derived from the intersection of straight lines OT and PS in both *The Art of Painting* (Figure 1) and *The Allegory of Faith* (Figure 7). Also, in both Figures 1 and 7, the floor boundary line OP —on the golden section point in the vertical direction—is divided into two, at point R ($OR = PR$). As mentioned above, the exact location of the border depicted in *The Allegory of Faith* is slightly above the OP line and the row of tiles.

Figure 8 shows the result of drawing by adjusting the golden ratio in the vertical length of the canvas as in *the Geographer* in Figure 5. The HL_1 overlaps the shoulder of the model and divides the vertical length of the canvas into two parts. So, if Vermeer set the visual center without using distance points, it is possible that from VC_1 , from the tiles in the foreground, he had diagonal lines converge. In *the Allegory of Faith*, due to a carpet covering the right section of the tiled floor, the observer can't see from where diagonal lines would extend from the tiles, so establishing the visual center as VC_1 does not make you feel uncomfortable. However, in order to set the golden section point at the edge of the screen, the position of the horizontal line must be moved from HL_1 to HL . This naturally causes the problem in that the vertical length of the screen is not divided into golden sections. To solve this problem as an artist, by adding a ceiling adjoining the front wall, the golden section along the vertical line can be repositioned.

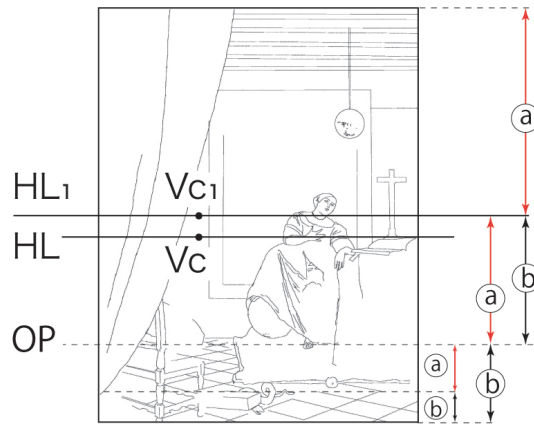


Figure 8: An Analytical Diagram of *The Allegory of Faith* and the Golden Ratio.

5 Conclusions

This study addresses the question of how the sense of depth in Vermeer’s interior paintings was created and examines only one of the probable answers. It does not connect the use of the camera obscura with the needle holes on the picture. Vermeer may have applied the rules for determining composition that was already attempted in the creation of *The Art of Painting* to *The Geographer* to give depth to the room as depicted in *The Astronomer*. Then, considering the order in which the works were painted, it is possible that he may have applied the compositional division of *The Geographer* to *The Allegory of Faith*.

This concept of the golden ratio is nothing special and exists everywhere in nature. However, Vermeer may have considered the golden ratio as a special one in determining the composition. That ratio was inherent in Vermeer’s painting’s even before this analysis was done. And, the golden ratio does not overtly appear visible on the canvas. This fusion of geometry and God is remarkable especially during the Renaissance. And, for Vermeer, the golden ratio may have actually been something Divine.

This analysis shows that distance points can be obtained from similar triangles, even if the camera obscura does not have a tile image. This means that the floor is geometrically created even if there is no physical distance point in the atelier. In brief, it is also assumed that Vermeer painted the tiles proportionally to fit the canvas size.

The Allegory of Faith and *The Art of Painting* were painted on a relatively large canvas. However, checkered tiles were also painted on canvases smaller than these works. It is inferred that Vermeer could not see the scene he painted through the camera lens all at once, even if he used the camera obscura. Moreover, we believe that the performance of the lens Vermeer used was not as effective as some may claim. From the results of analyzing Vermeer’s work, the depictions and compositions left on the screen all show the high level of his unique artistic skill.

Whether this finding can make up for the fact that despite many illustrators having conducted analyses of the tile lines not matching the distance points, some questions remain unanswered. Future research could determine whether ratios are repeated in the composition of other works and are not limited only to the golden ratio, for example.

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