An Important Birthday. Reminiscences about and around Hellmuth Stachel

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1 Introduction

This is a journey from 1988 when I became aware of him in Wien. It touches in 1992 on first acquaintance that started in Ebernburg and continued on in Melbourne and Sydney. Therefrom there was a sequence of happy and mind altering events usually centred around various ICGG conferences — their names have changed throughout the years — that began at the fateful one in 1988, Wien, and progressed to the two in 1992 in Germany and Australia respectively. These meetings began my lifelong friendship with Manfred Husty and many, many other Austrian geometers. The ICGG conferences and other benchmarks are Melbourne 1992, yearly trips to Austria and my 1997–98 sabbatical at TU-Graz in which Hellmuth played little part but his influence was ever present in my entertaining geometric life adventures. The ICGG meetings with Hellmuth continued: 2000 Johannesburg, 2004 Guangzhou, 2006 Salvador da Bahia, 2008 Dresden, 2010 Kyoto, 2012 Montreal, 2014 Innsbruck, 2018 Milano. Then covid threw the proverbial monkey-wrench into the works. The time line above is more to keep me on track and quasi-coherent rather than for the reader's benefit. In apology, as my friend David Hyder, the philosopher, sums up in his treatise on Kant and Einstein, "Time is a bad parametrization of reality." That's why I may skip around a bit in what follows.

2 Skipping Around

1988: Hellmuth had just attained the O. Prof. position in geometry at TU-Wien when he organized the conference. It was pure serendipity that I'd submitted a paper which was accepted. The large and wide swath of attendees was remarkable. Sessions were punctuated by short intermission where a brass quintet performed. Hellmuth said, "It's hard to find a music ensemble in August but I managed." which indeed he had along with dragooning grad students and young profs from Graz, Leoben and Innsbruck into service. This made things run smoothly. High points included an upriver excursion to Dürnstein and Melk but the pièce de resistance was a fancy ball in Hofburg Palace with the Lord Mayor in attendance. During the conference Hellmuth mentioned that Johann Strauss studied descriptive geometry

Figure 1: 2008 Tschupik's 80th Birthday

and composed "Waltz of the Cyclides". This seemed far fetched but Wolfgang Rath[†] assured me, "Hellmuth never jokes about geometry." My trip allowed me to connect with my roots in Sopron and Budapest ending with a pre-departure supper with wife Wendy[†] in a restaurant in the district containing many embassies guarded by armed Austrian military. A very attractive middle-aged matron arrived soon joined by a pair of beautiful young ladies. Then one of the soldiers entered with his sub-machine-gun which he placed on the bar and joined the trio whereupon the bartender served them schnapps. After warm greetings and a chat, downing his drink he bid them adieu, picked up his weapon and returned to his post having provided us with an unique *Ausblick* of life in Wien. 1992: The Austrian contingent came to Ebernburg in force; Stachel, Vogler, Pottmann and Husty. The latter asked we might meet again in Melbourne where Stachel invited me to join the "Austrian Geometry Mafia" and Husty planned his post-doctoral sabbatical and his project to effect the ambitious solution to the direct kinematics, *determining end effector pose given six leg lengths*, of general Stewart-Gough platforms.

We joked about how this would lead to his professorship in Innsbruck — We both figured Josef Tschupik would be the next chair-holder in geometry to retire — and my intention to frequently visit and collaborate on problems in kinematic geometry. At the time we did not realize how prophetic these daydreams would turn out less than a decade thereafter. In Fig. 1 we see Hellmuth and me with Josef Tschupik, the former professor of geometry at Uni-Innsbruck on the occasion of *his* eightieth birthday, and Hans-Peter Schröcker, not yet in that position he now holds. In order to complete the three generation pantheon we see, In Fig. 2 from the right, Manfred Husty, then professor of geometry at Uni-Innsbruck, Hans-Peter and wife Emiko. Too numerous to mention, ICGG conferences produce unexpected joyful surprises. This one however cannot be overlooked without comment. With Emiko's help, after Manfred got a haircut in Higashimatsuyama, we paused for ice-cream. This parlor had the added attraction of warm flowing water in a stone trough to sooth the feet of weary pedestrians!

Figure 2: Ice-Cream and Foot-Bath, ICGG 2010, Kyoto

3 Applications -vs- Theory in Geometry

Hellmuth, like most of us, was seduced by the North American mantra "Computers are fast and memory is cheap." CAD and elegant rendering packages like [1] were emerging even in 1988. When Georg Glaeser was completing his PhD at TU-Wien Hellmuth proudly showed impressive images, including flowers, produced with Open GL. Today the package is marketed by a US firm and uses the latest Nvidia graphics processors. There is no mention of Austrian pioneering involvement and who needs geometry or drawing especially DG when, in complete ignorance, one can play expensive video games all day? Even in 1992 Hellmuth still had the applications bug. In Ebernburg he presented an elegant mechanism [7], which he designed, for loading refuse into a garbage truck. What could be more hard-core application of geometry? Around 1996 the pendulum begins to swing the other way. In the US DG had been largely eliminated from engineering curricula. My own experience during sabbatical in 1980–81 at Texas A&M in the department of Engineering Design Graphics was that of a healthy — by US standards — service department that offered a range of courses to engineers, architects and other designers. These contained more than token DG, even. By the next year EDG was merged into Civil Engineering. The long-time chair Jim Earle† described it colourfully as "wading in among the crocodiles". By 1997–98, my next sabbatical at TU-Graz, the DG elimination disease was rampant in Europe. DG had withered in TU-München and Hans Seybold, the proverbial last man standing was retiring and sort of closing the door behind him. In Graz the mechanical engineers were cutting back and graphical kinematics got the axe. Roughly a year prior Hellmuth began to sound the alarm with conference presentations and proceedings contributions summarized in [5]. Another article [6] in the same vein appeared some years later. Both of these identified the importance of geometry education, pled for its retention and enhancement offering powerful arguments in support of the cause. Alas, these inevitably failed to reach the right ears and were largely sermons to the choir. Finally in 2016 [2] and later [4] we come to two books which hit the right note and, if widely read,

would be convincing, to mathematical educators in general, to begin training in geometry at the Gymnasium level and continue on through technical and scientific university curricula even into postgraduate study. If this all seems far fetched it is nevertheless my opinion about elementary geometry and its possible future. The books contain enough meat around which to construct courses at progressively advanced level including many challenging and relevant exercise problems.

4 Why Shall We also Teach the Theory behind Engineering Graphics

This report [5] should be compulsory reading for all detractors of geometry education but they'll never see it. Hellmuth argues for the artistic beauty of geometry and how this is invaluable notwithstanding lack of immediate practical application. After all, no one decries piano or art lessons. His second argument is about important unanticipated application that has occurred countless times in the past and points to location finding with GPS which relies on (at least) four intersecting spheres centred on earth orbiting satellites. The principle harks back to Apollonius' circles tangent to three given ones in a plane. The problem can be solved by construction with ruler and compasses. More important, a modern but still old solution is based on Zyklographie [3] which maps planar circles to points in space thereby mapping circles tangent to a given one to vertical axis right cones with apex angle $\pi/2$ and apex above or below the horizontal plane by the radius of the given circle. In Stachel's GPS example the mapping is upped from two-to-three dimensional to spaces of three dimensions to four. Moreover geometry of satellite juxtaposition shows up quantitatively good, bad and even singular positionings. In 1844 Hermann Grassmann published the principle in his "Ausdehnungslehre" (Extension Theory). It's doubtful he was thinking of GPS. When Einstein said, "I could see so far because I was standing on the shoulders of giants." Newton, Leibnitz and calculus come to mind. I believe he was referring to the geometry of Minkowski, Plücker and Klein. Hellmuth says, "We have to educate not produce."

5 What is Descriptive Geometry For?

Hellmuth wrote [6] that DG uses conjugate pairs of 2D projections of 3D objects to obtain metrical properties. This was presented at TU-Dresden on the posthumous 100th birthday of Rudolf Bereis who was professor of geometry there until his life was cut short. Coincidentally Hellmuth received his honorary doctorate at TU-Dresden some seven years later. DG projection — *viz.*, reducing dimension to reveal information not available in an arbitrary projection falsely often called a "3D view" — may be regarded as the inverse of extension. He goes on to show the importance of so-called principal views even in anatomy where symmetry is not evident. Furthermore the student of linear algebra may not realize eigenvalues and eigenvectors used to reduce coefficient matrices of quadrics or mass moment distributions to diagonal form effectively produce principal views of the quadric, inertial axes of massive object or those of the stress/strain ellipsoid.

6 Conclusion with Apologies and a Bit More

I'm still in the process of rereading [2] and [4] and, at risk of being accused of providing commercial advertising, urge colleagues who have not as yet had opportunity to examine these books to do so and to look for instances, obvious and obscure, of the application of projection and extension therein, tools that Stachel regularly propounds. My second apology is for the many inaccuracies and distortions of memory and interpretation that I'm sure many will notice in what I've written. If you have found nothing of interest or value in this I hope, at least, that my deep admiration and respect for Hellmuth Stachel as inspiring geometrician and teacher show through. I cannot finish with this obsequious whimper. In [8] Hellmuth asks, "What is geometry? Where are the boundaries of geometry within mathematics? Is Geometry a branch of mathematics?" He goes on to present impressive pictures in answer that makes one, not versed in geometric thinking, feel like the seven wise blind men of Hindustan who went to see the elephant. To these so deficient I offer Steve Slaby's[†] short answer when a condescending colleague said to him, "Oh yes, geometry is a branch of mathematics." to which he replied with a bang, "No, mathematics is a branch of geometry."

References

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