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# Magnification Illusion – Change of Interpretation when Viewing Through a Telescope

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**Abstract.** When looking at a pair of parallel lines placed on a floor receding into the distance with a pair of binoculars, the parallel lines seem diverged instead of converged. This phenomenon has been reported as an illusion. We hypothesized that this perception is caused by the change in the three-dimensional interpretation of lines when a scene is magnified and named this perception the "Magnification illusion." This might be similar to the change of optical slant angle interpretation, so the plane is more slanted to the fronto-parallel plane when magnified. In this work, we investigated the change due to magnification, such as the change in the interpretation of the perspective angle and the optical slant angle, and also tested with stimuli other than the parallel lines. The experiment result showed that the perspective angle is underestimated for all stimuli if magnified only when viewed with binocular vision. Meanwhile, the optical slant is underestimated even when viewed using the naked eye, and it is even more underestimated when magnified, regardless of binocular or monocular vision. In summary, this paper found that the change in the interpretation of perspective angles when magnified does occur in all pair of lines receding into the distance under binocular vision.

*Key Words:* perception, linear perspective, slant perception, illusion

*MSC 2020:* 91E30 (primary), 62P15

## 1 Introduction

The image of the external world on the retina is flat or 2D, but it is still possible to reproduce the 3D information with remarkable precision, even if perceived with a single eye. The visual system relies on depth cues to reconstruct 3D information from the 2D image projected on

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the retina. Depth cues include both physiological and psychological cues. The physiological depth cues are accommodation, convergence, binocular parallax, and monocular movement parallax. Convergence and binocular parallax are the only binocular depth cues, and all others are monocular. The psychological depth cues are retinal image size, linear perspective, texture gradient, overlapping, aerial perspective, and shades and shadows. In this work, we are interested in linear perspective cues that help humans interpret the 3D scene from the 2D image projected on the retina. We will discuss the projected image on the fronto-parallel plane (projected plane as in 3D CG) instead of the retina in this paper because it is easier to discuss the geometrical arrangements of the 3D scene and the 2D image.

As an example of linear perspective cues, the parallel lines, when projected on the fronto-parallel plane, would converge at a vanishing point on the image located on the crossing point of the extended parallel line passing through an eye and the fronto-parallel plane. Figure 1 illustrates the vanishing point $V_0$ of the parallel lines indicated in bold black lines on the projected image on the projected plane (yellow plane). Suppose the two lines on the projected plane would form an isosceles triangle in order to simplify the discussion.

In 1976, Eltenton [4] reported the unexpected perception when looking at the parallel lines placed on a ground through a pair of binoculars that the lines appear diverged instead of converging into the distance. This perception phenomenon is illustrated in Figure 2. Tsuinashi [30] performed the experiment on this perception by having the participants look at the rectangular surface placed on the ground. His experiment result confirmed that some participants reported seeing the rectangular surface diverged when viewed using binoculars. In his later paper [31], he proposed that the retinal image actually diverged when viewed using binoculars. However, the projected image should be merely enlarged when viewed using binoculars. We believe that this unexpected perception reported by Eltenton is caused by the change in the interpretation of the projected image.

The projected image on the yellow plane shown in Figure 1 can be interpreted as any pairs of lines like blue and green lines. The projected image could be produced by the parallel lines placed on the ground (black lines), the converged lines on an upward-slanted plane (blue lines, with $V_1$ as the crossing point), or the diverged lines on a downward slanted plane (green lines, with $V_2$ as the crossing point).

We define the angle between the projected plane and a plane of the interpreted lines as the optical slant angle, $\theta$, while the half-angle of the interpreted lines is the perspective angle, $\beta$. In principle, changes in the interpretation of optical slant angle could result in changes in perspective angle and vice versa. It is known that the visual system would rely on both the angle and bottom width of the isosceles triangle in the projected image to interpret the slant angle of the plane of parallel lines [20, 21].

We think the interpretation of the projected image, especially the perspective angle, may change when using a telescope, not necessarily a pair of binoculars because the projected image is magnified. We define this change of interpretation as "magnification illusion." In this work, we explore magnification illusion by examining the perceived optical slant angle $\theta$ and perspective angle $\beta$ under different viewing conditions, such as naked eye vs. magnified, monocular vs. binocular, and several actual perspective angles.

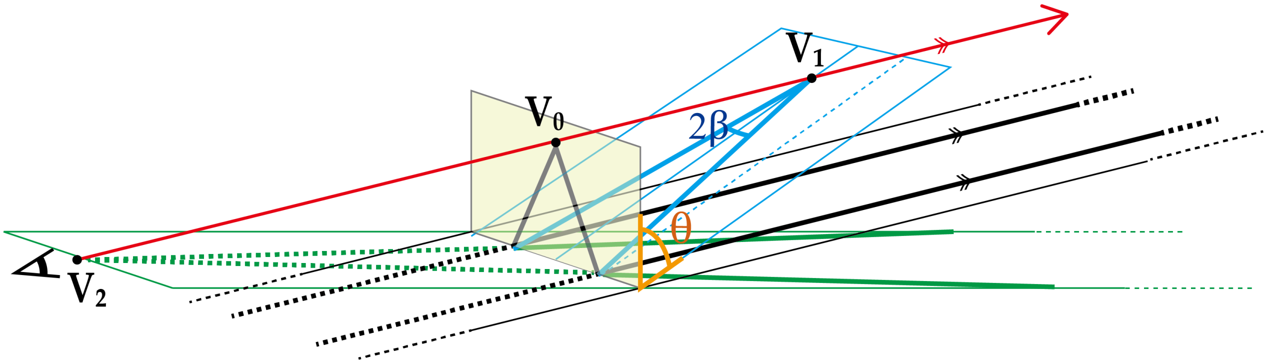

Figure 1: Illustration for multiple interpretations of one projected retinal image (yellow). One can interpret the stimulus as being a parallel line placed on the ground (black lines), or being converged and slanted upward (blue lines), or being diverged and slanted downward (green lines).

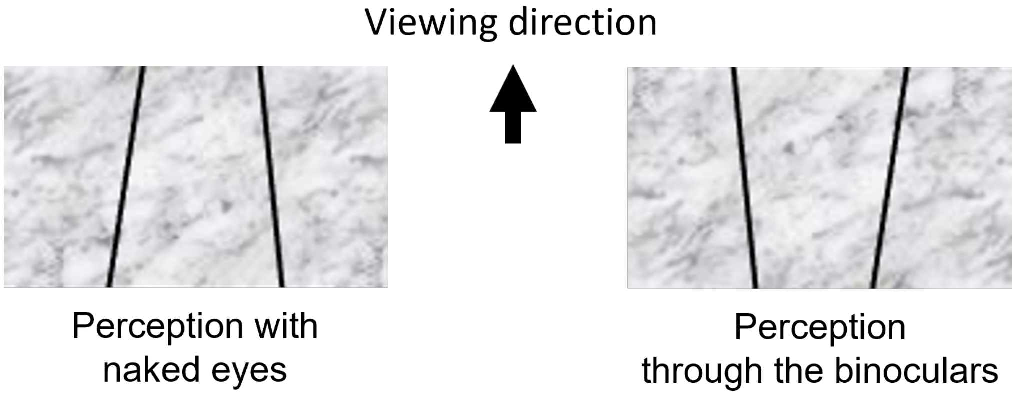

Figure 2: The example of the illusion reported by Eltenton [4]. Left: The perception of parallel lines that are placed on the ground surface with the naked eye(s). Right: The perception of the same stimulus through a telescope.

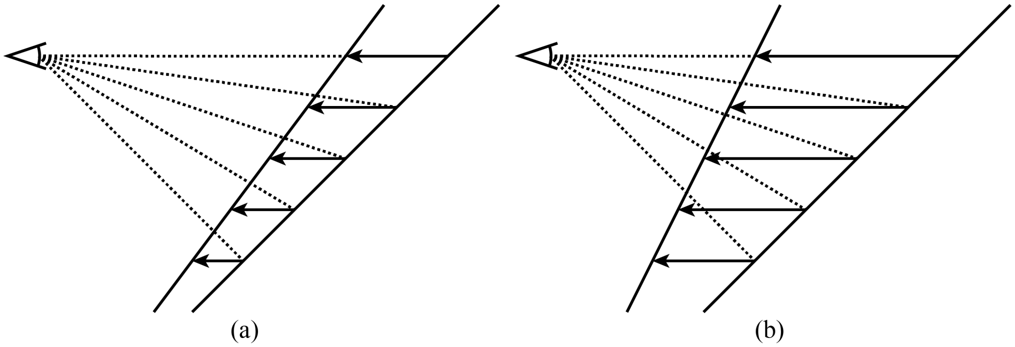

Figure 3: Illustration of slant underestimation. When the perceived distance is reduced, the slant of a plane seems to be closer to the fronto-parallel plane.

## 2 Background

### 2.1 Distance Perception vs. Optical Slant Perception

Over centuries, distance perception in human visual perception has been extensively investigated. Many studies reported that human would compress egocentric distance, i.e., perceived distance from an observer to an object, as much as 0.7 in [11, 16] even seeing with naked eyes.

This depth compression may cause an underestimate in optical slant [15, 22, 32]. Figure 3 illustrates the mechanism of optical slant underestimation. Li and Durgin [2] conducted an experiment to find out perceived optical slant by manipulating a range of physical slants and found that the data fit perfectly to the logarithmic function of viewing distance proposed by Bridgeman and Hoover [1]. They further investigated the cause of slant underestimation by focusing on depth compression theory and intrinsic bias hypothesis theory [18]. Perceived distance is shortened by magnifying the image using a telescope, or in the head-mounted display. Especially when the field of view is restricted [14, 29]. It was found that the distance judgment is significantly more accurate in the larger field of view condition [14, 33].

Monocular depth perception is sensitive to viewing conditions and lacks some direct depth information, such as binocular disparity. It relies on inference and is often more ambiguous than binocular perception. Therefore, monocular depth perception is generally more difficult than binocular depth perception due to the absence of precise binocular cues. However, humans can still judge depth with just one eye, relying on monocular cues such as linear perspective, texture gradient, and motion parallax.

Variations of distance can be clarified better in binocular cues due to mechanisms like fusion and retinal disparity. When one views a scene binocularly, the images are interpreted as three-dimensional. These binocular cues are based on different images that our two separate eyes produce. Then, the brain puts these images together to form a 3-D scene (structure). Some work demonstrated that monocular vision affects the kinematics of skilled visually guided reaching movements in humans [19, 24, 25]. It has been found that some amount of binocular parallax is required to enhance the experience of realistic depth [13]. According to this, monocular vision could affect distance perception differently from binocular vision. Since the availability of multiple binocular cues influences distance perception [10], and distance perception is related to slant estimation [15, 22, 32], it would therefore be reasonable to suggest that binocular cues should affect slant perception. Actually, there have been some studies investigating the effect of monocular observation onto the perceived optical slant [5–7, 9].

## 2.2 Measurement of Slant Perception

To measure perceived optical slant, many methods were employed. A particular method that has been used is haptic measurement. Haptic measurement [1, 22] refers to adjusting an unseen palm board to match the orientation of the observing surface. A palm board is a flat plate that can be rotated by hand about a horizontal axis. The palm board is placed near waist level, and the arm is extended down to meet it. The task of adjusting a palm board is referred to as "visually guided action." Another method used in the early studies is a verbal estimate. However, many studies found that verbal estimates or other types of conscious reports induce a phenomenal underestimation of optical slant (virtual and artificial hills) by 5-25 degrees [2, 12, 17, 22, 27, 28][1]. These verbal slant estimates are accompanied by haptic matching, where the people would hold their unseen hand and/or forearm parallel to the slope. The difference in the slant estimation between the verbal estimate and haptic measurement is said to be because of proprioceptive calibration [3], which causes bias in verbal estimates of visually perceived optical slant and proprioceptively perceived hand orientation. Shaffer [26] also confirmed that proprioceptive calibration exists in a pedal estimate as well as haptic.

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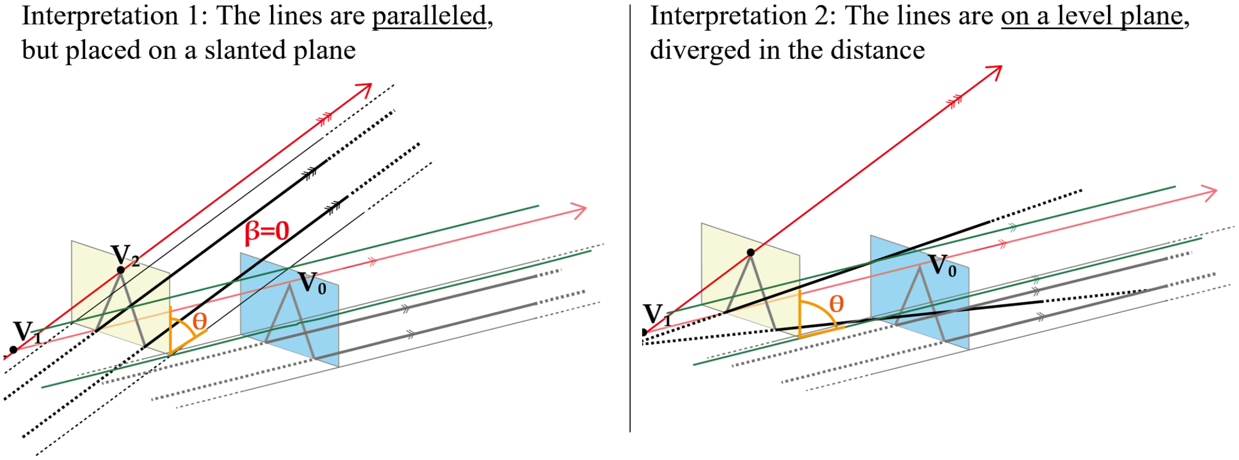
[1] The studies call the error of the perceived angle as over-estimation of the angle from the ground. In this paper, however, we are discussing the slant angle $\theta$ from the fronto-parallel plane. The perceived slant angle should be less than the actual slant angle.


Interpretation 1: The lines are paralleled, but placed on a slanted plane

Interpretation 2: The lines are on a level plane, diverged in the distance

Figure 4: Two possible interpretations when viewing a scene through a telescope.

## 3 Hypothesis

When we look at a scene through a telescope, depth compression occurs due to the magnification characteristic of the telescope. Therefore, the fronto-parallel plane is shifted closer to the eyes. For the case of looking at the parallel lines placed on the level ground through the telescope, a lot of different interpretations can be considered. Figure 4 illustrates two specific cases. Supposed the blue planes are the original fronto-parallel planes when viewed using the naked eye, the yellow planes are the shifted fronto-parallel planes when looked through a telescope. Depth compression caused by looking through the telescope should decrease either $\theta$ or $\beta$, according to the two following specific interpretations;

1. The lines remain parallel, but they are being placed on a slanted plane (the perspective angle is kept, but the optical slant angle decreases)
2. The lines are not parallel, and they are being placed on a level plane (the optical slant angle is kept, but the perspective angle decreases)

These interpretations are examples of looking at parallel lines. Instead, we believe that such a variation of interpretations may occur not only in the parallel lines but in all two lines receding into the distance. In this work, we aim to investigate the change in the interpretation when looking at the stimulus through a telescope. Our hypotheses are as follows

1. The perspective angle $\beta$ and/or the optical slant angle $\theta$ will change when looking at the stimulus through a telescope. (magnification condition)
2. Monocular/binocular vision may affect such changes.
3. The change in the interpretation when magnified may be affected by stimuli having a variety of *actual perspective angles,* or ground truth angles, $\beta_G$.

In order to prove the first hypothesis, we collect optical slant angle $\theta$ as well as perspective angle $\beta$ from the participants in the investigation. Because monocular vision and binocular vision affect distance perception [10], we hypothesize that the interpretation of slant and perspective angles would also be affected. Therefore, we also investigate by collecting the observation data through the use of one eye (monocular vision) or both eyes (binocular vision), which is designed to prove the second hypothesis. To prove the third hypothesis, we tested with different types of stimuli; diverged lines, parallel lines, converged lines, and more converged lines in the experiment.

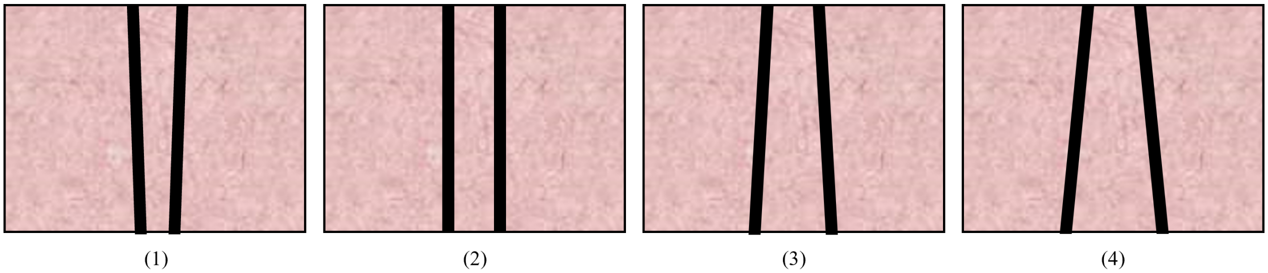


Figure 5: Stimuli used in the experiment. The actual perspective angles, $\beta_G$, are −1°, 0°, 1.4°, and 3.75° for stimulus 1, 2, 3, and 4, respectively.

# 4 Experiment

## 4.1 Participants

Twenty-three students (16 males, and 7 females, mean age = 24.5 years, SD = 4.76 years) participated in the experiment. The research was performed with the approval of and in accordance with Shibaura Institute of Technology Institutional Review Board. Informed consent was obtained from all participants. The participants were unfamiliar with the aims of the experiment and received no feedback about their performance. All participants had normal vision or wore contact lenses. Stereoacuity was tested using four random-dot stereograms and all participants could report all images correctly.

## 4.2 Stimulus

We performed a preliminary experiment to test the perspective angle when changing actual angles between the two lines and decided on additional three angles of stimuli for this study. Figure 5 shows stimuli used in the main experiment. The actual perspective angles, $\beta_G$, for each stimulus are −1°, 0°, 1.4°, and 3.75°. The width of the black lines is 5 cm, and the distance between the two lines is always kept at 12 cm, measuring on the side away from the observer. Lines are drawn using black tape on a 0.5 cm thick white foam board with a textured paper of light pink to make the plane noticeable. We used the 1/f natural-noise texture, which is considered the least helpful in slant discrimination [23]. The mean luminance of the texture is 39.64 cd/m$^2$, and that of the lines is 2.77 cd/m$^2$, measured by Konica Minolta CS-200 chroma meter. The Weber contrast of the texture and the black lines is 13.31, which is considered easy to discriminate the black lines. The lines are diverged lines (1), parallel lines (2), converged lines (3), and more converged lines (4). Every stimulus is horizontally placed on the floor. Therefore, the actual slant of the stimuli is never changed throughout the experiment.

## 4.3 Apparatus

The participant is required to look into the binoculars or a pair of paper cylinders and answer the perspective angle and the optical slant that they observe from the stimulus, by using the answering tools. The binoculars or the paper cylinders are placed on a tripod, which fixes the viewing height and viewing distance. The tripod is placed 3 m away from the center point of the stimulus. The tripod is set to 1 m height from the floor. The size of the stimulus is

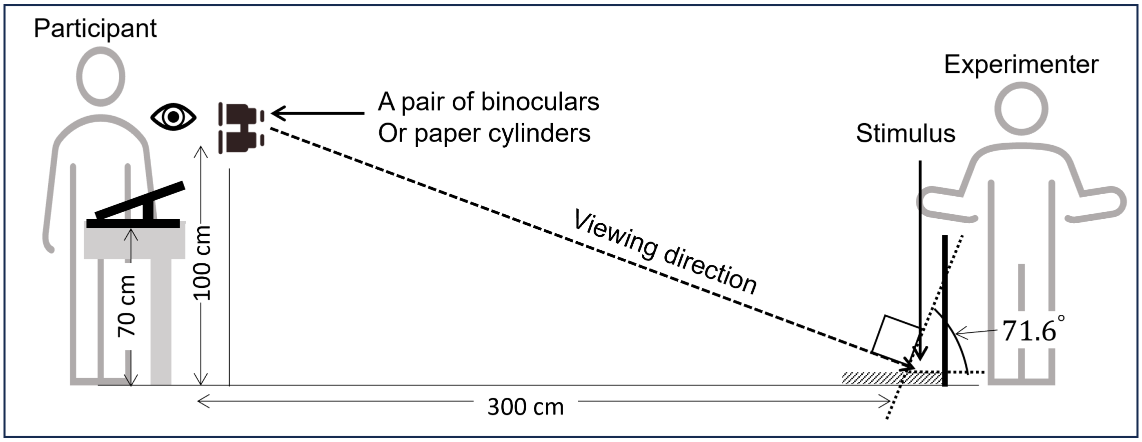


Figure 6: The setup of the experiment.

Figure 7: Photos from the experiment. Left: the participant was using an angle adjuster to report the perspective angle. Right: the participant was using a slant pad to report the optical slant angle.

91 cm×60.5 cm. For the binoculars used in the experiment, we used Olympus 8x21 RCII, with 8x as the strength of magnification, and the size of the objective lens is 21 mm.

The answering tools in our experiment were developed so that the observer could replicate what they perceived. For perspective angle measurement, we create an angle adjuster using two digital angle finders. The angle adjuster can be placed on the participant’s lap while viewing the stimulus. For slant measurement, we create a slant pad by referring to the palm board [22]. A digital angle finder is also attached to the slant pad. The slant pad is fixed to the table, around 70 cm from the floor. Unlike the traditional haptic measurement method, the participant is allowed to look at the stimulus, the angle adjuster, and the slant pad and adjust both the optical slant and perspective angles as many times as they desire. The angle adjuster and slant pad are shown in Figure 7, left and right accordingly. The experiment setup is illustrated in Figure 6. We also show a photo taken from the experiment in Figure 7.

## 4.4 Design

Participants were assigned to four viewing conditions in order. In the first two conditions, participants look at the stimulus through a pair of binoculars, monocularly and binocularly. In the second two conditions, the participants look at the stimulus through paper cylinders, using one eye (monocular vision) or two eyes (binocular vision). The four viewing conditions are described as follows.

- Look through the binoculars (magnified) using both eyes (binocular vision)
- Look through the binoculars (magnified) using only one eye (monocular vision)
- Look through the cylinders (naked eyes) using both eyes (binocular vision)
- Look through the cylinder (naked eye) using only one eye (monocular vision)

The order of viewing conditions was the same as the list above. We showed four stimuli in random order while fixing one of the viewing conditions.

## 4.5 Procedure

The participant viewed the stimulus monocularly or binocularly, with or without binoculars. In the monocular condition, the participant used his/her dominant eye. The participant could only see the experimental setup through the binoculars or a pair of paper cylinders because the experiment area was covered by a curtain with a hole in the middle, only to fit the binoculars or the paper cylinders. For each stimulus, the participant was asked to estimate the stimulus's perspective and optical slant angles. The participant first looked at the stimulus, then adjusted the digital angle finders until both the angle adjuster and slant pad replicated their answer. It is possible to adjust the angles while or after observing the stimulus and to repeat the procedure until he/she is satisfied with the result. The participant recorded their responses by copying the numbers shown on the digital angle finders to an answer sheet. The optical slant angle, $\theta$, and the perspective angle, $\beta$, were calculated from the recorded numbers for analysis. After recording the angles for a stimulus, the participant was instructed to sit on a chair until the experimenter finished changing the stimulus behind the curtain. The participant was not given any feedback regarding their performances.

# 5 Results

The interpretation of $\theta$ and $\beta$ can be changed by the magnification of the scene. For the analysis, we compare how $\theta$ and $\beta$ changed under different viewing conditions (magnification or naked eyes, under monocular vision or binocular vision) using multiple stimuli with different actual perspective angles.

## 5.1 Discrimination Ability for $\beta$

Discrimination ability for $\beta$ was investigated in both monocular and binocular vision conditions, as presented in Figure 8 and 9, respectively. The statistical detail for the pairwise comparison is shown in Table 1. In the monocular vision condition shown in Figure 8, a significant difference was found exclusively between the naked eye condition of stimuli 2 and 3 (representing parallel lines and converged lines), while no significant differences were observed for stimuli 1 and 2 (representing diverged lines and parallel lines), and 3 and 4 (representing converged lines and more converged lines). On the other hand, it can be seen that significant differences were found between all the neighboring stimuli pairs under magnification conditions. In contrast, with

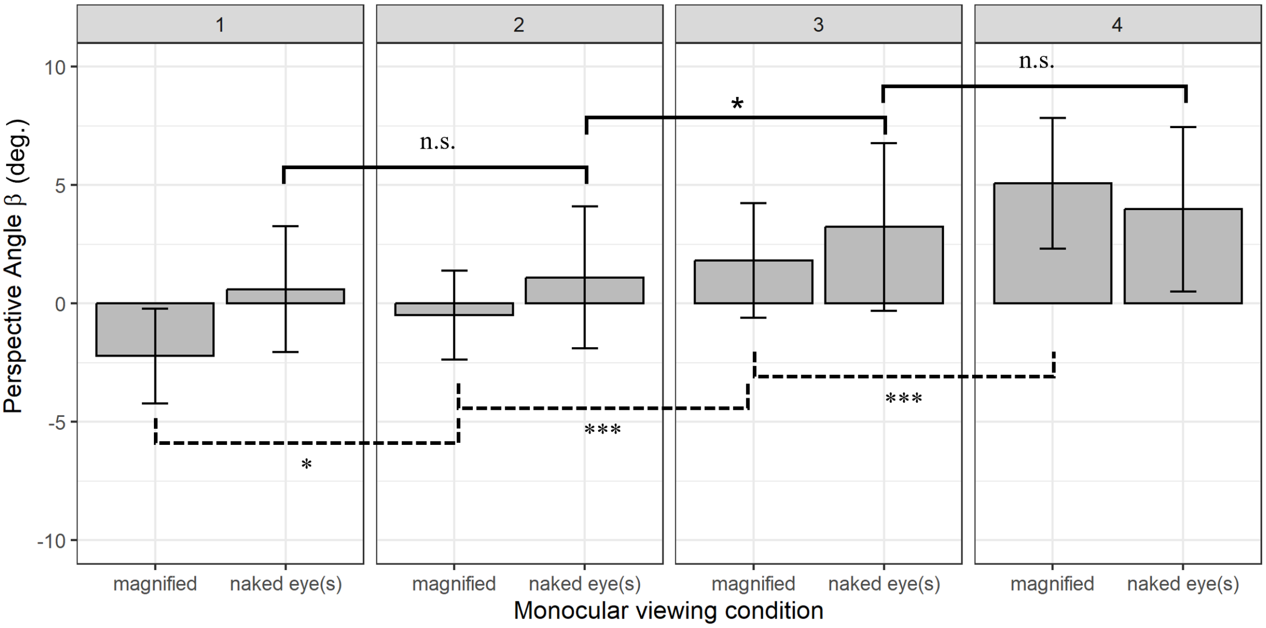


Figure 8: Bar chart comparing perspective angles, $\beta$ for stimuli 1 to 4 under magnified and naked-eyes conditions, viewed with monocular vision. Each graph is denoted with $*$ for significant difference, $* = p < 0.05$, $** = p < 0.01$, $*** = p < 0.001$, or n.s for no significant difference.

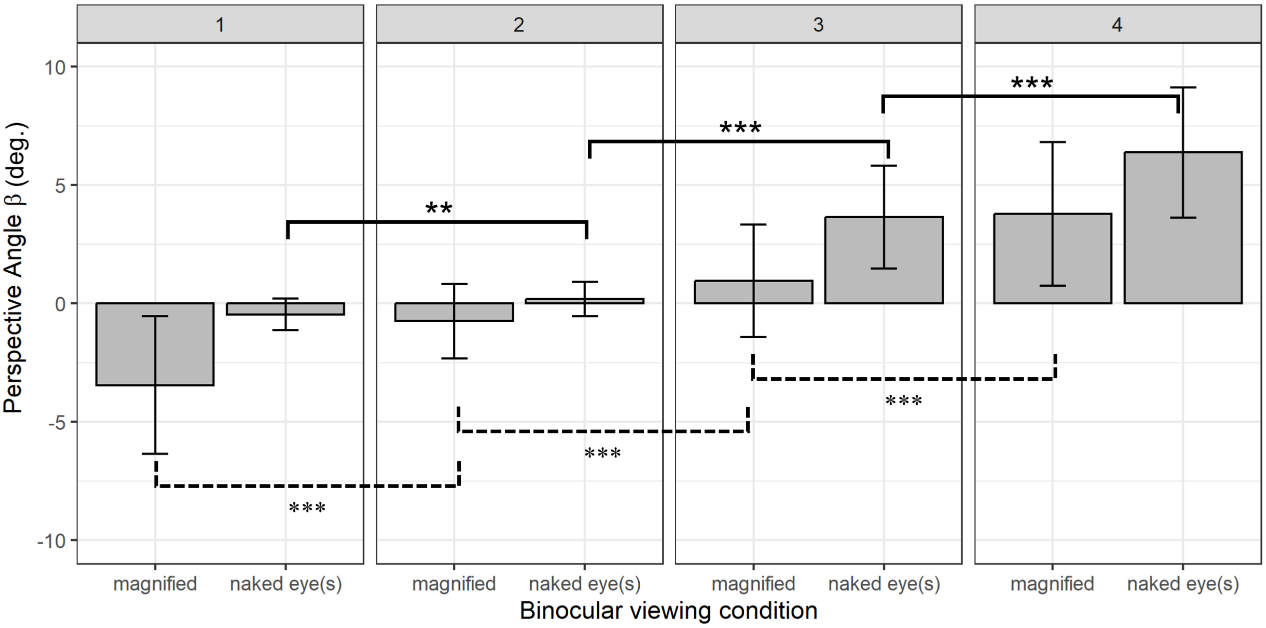


Figure 9: Bar chart comparing perspective angles, $\beta$ for stimuli 1 to 4 under magnified and naked-eyes conditions, viewed with binocular vision. Each graph is denoted with $*$ for significant difference, $* = p < 0.05$, $** = p < 0.01$, $*** = p < 0.001$, or n.s for no significant difference.

binocular vision, significant differences were observed for all neighboring stimuli pairs under both the naked and magnification conditions, as seen from Figure 9. This result emphasizes the remarkable discrimination abilities of participants, even when the stimuli have subtle differences of 1–2 degrees in the actual perspective, especially under binocular vision.

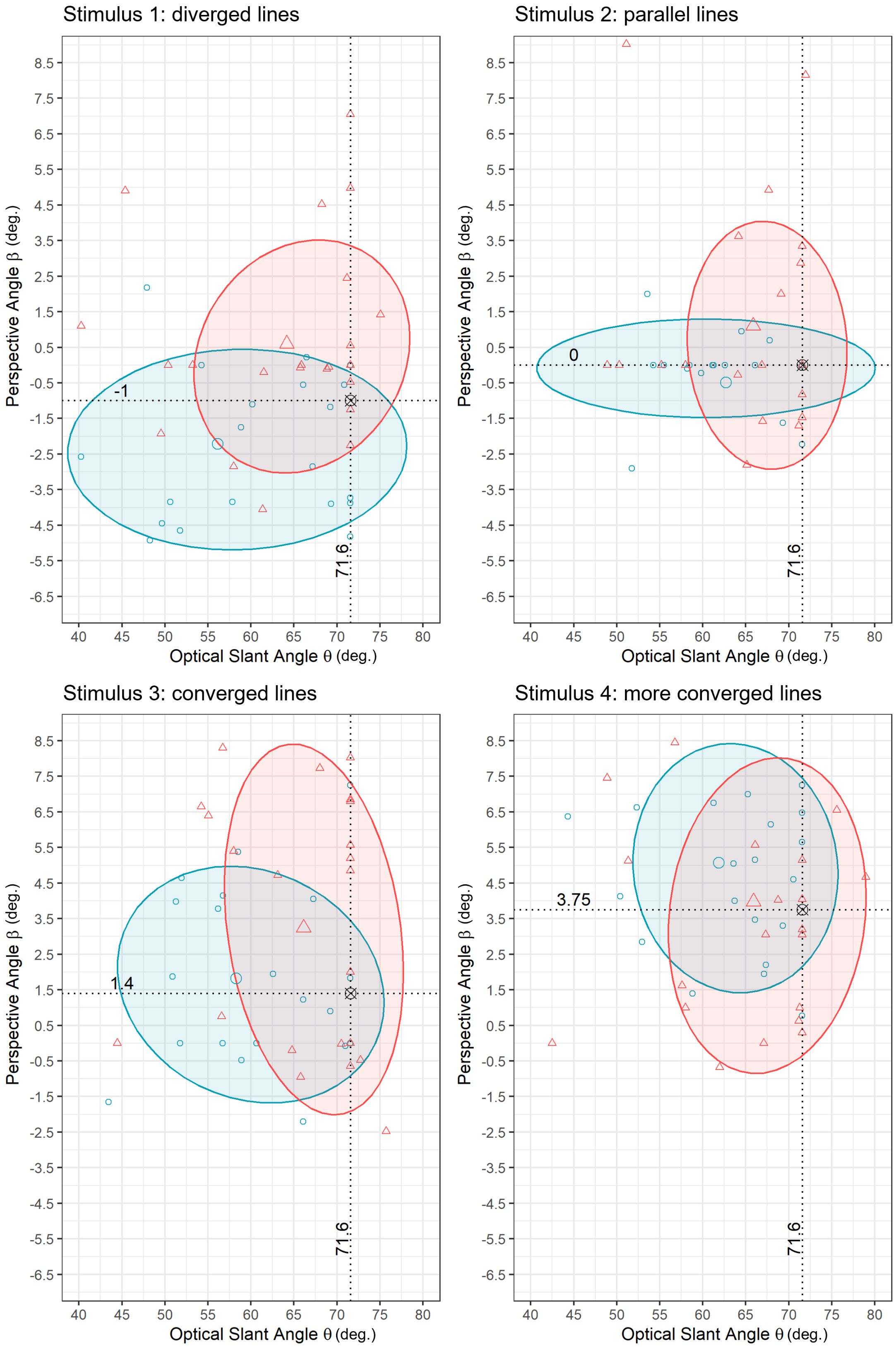


Figure 10: Observation results of four stimuli under monocular vision, shown in each scatterplot. The blue small and large circles show each participant's observation and the average value of $\beta$ and $\theta$ under the magnification condition, while the orange small and large triangles show the observation and average of $\beta$ and $\theta$ under the naked eyes condition. Ground truth observation is denoted using $\otimes$ as the crossing point of the dotted lines.

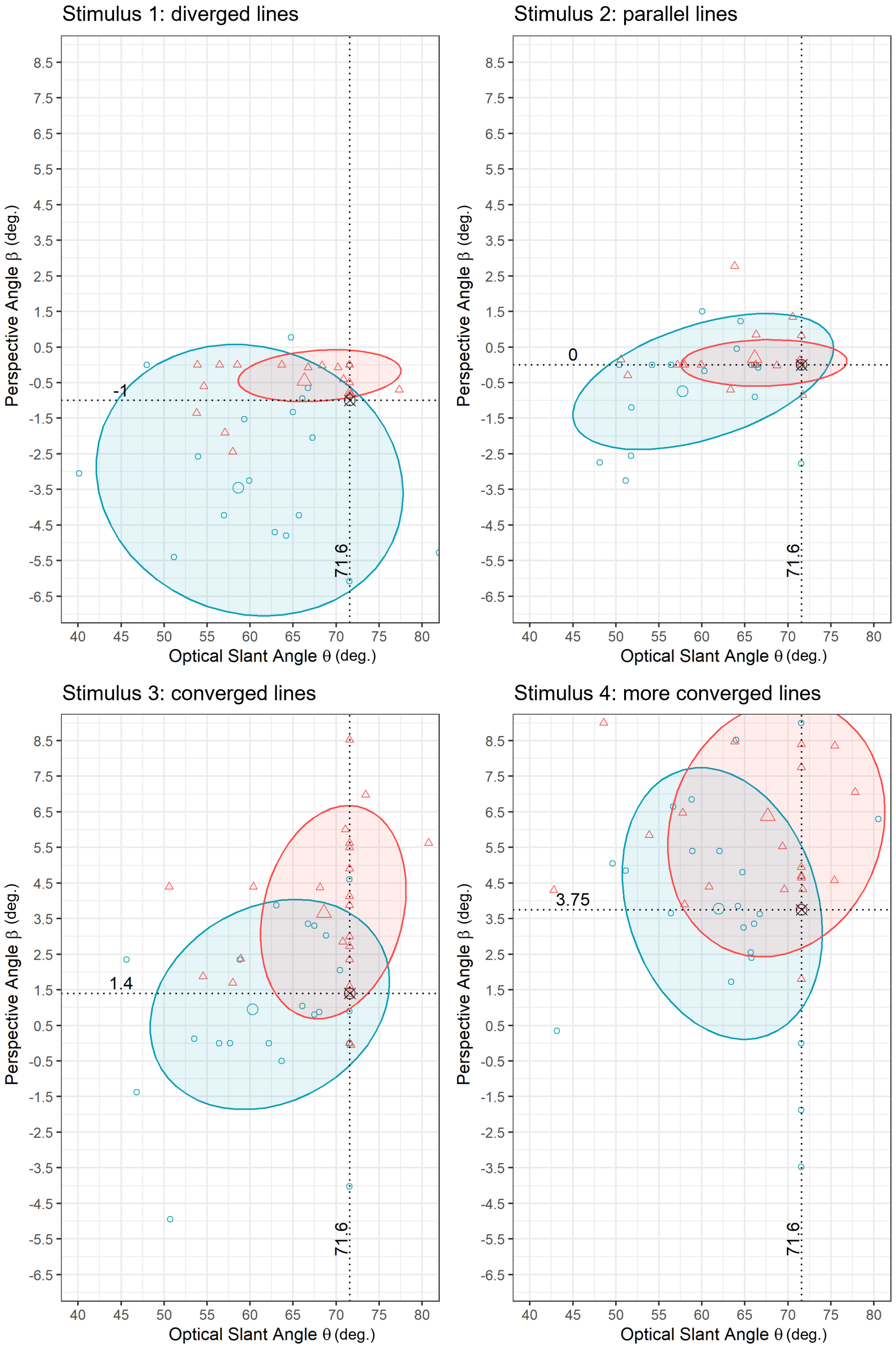


Figure 11: Observation results of four stimuli under binocular vision, shown in each scatterplot. The blue small and large circles show each participant's observation and the average value of $\beta$ and $\theta$ under the magnification condition, while the orange small and large triangles show the observation and average of $\beta$ and $\theta$ under the naked eyes condition. Ground truth observation is denoted using $\otimes$ as the crossing point of the dotted lines.

Table 1: The statistical detail of the paired t-test comparison between the same viewing condition of stimulus $i$ and stimulus $i + 1$.

| Stimulus | | Viewing Condition | Number of eyes | Paired differences | | Statistic | |
|---|---|---|---|---|---|---|---|
| | | | | Mean difference (deg.) | Std. Dev | t(22) | $p$-value |
| 1 | 2 | Naked eyes | Monocular | -0.502 | 4.341 | -0.555 | 0.585 |
| 2 | 3 | | | -2.138 | 4.427 | -2.316 | **0.030** |
| 3 | 4 | | | -0.739 | 5.374 | -0.660 | 0.516 |
| 1 | 2 | Magnified | | -1.726 | 3.067 | -2.699 | **0.013** |
| 2 | 3 | | | -2.305 | 2.425 | -4.559 | **0.000** |
| 3 | 4 | | | -3.260 | 2.129 | -7.342 | **0.000** |
| 1 | 2 | Naked eyes | Binocular | -0.640 | 0.881 | -3.484 | **0.002** |
| 2 | 3 | | | -3.475 | 2.104 | -7.922 | **0.000** |
| 3 | 4 | | | -2.717 | 2.255 | -5.780 | **0.000** |
| 1 | 2 | Magnified | | -2.703 | 2.666 | -4.863 | **0.000** |
| 2 | 3 | | | -1.696 | 1.983 | -4.102 | **0.000** |
| 3 | 4 | | | -2.825 | 3.026 | -4.477 | **0.000** |

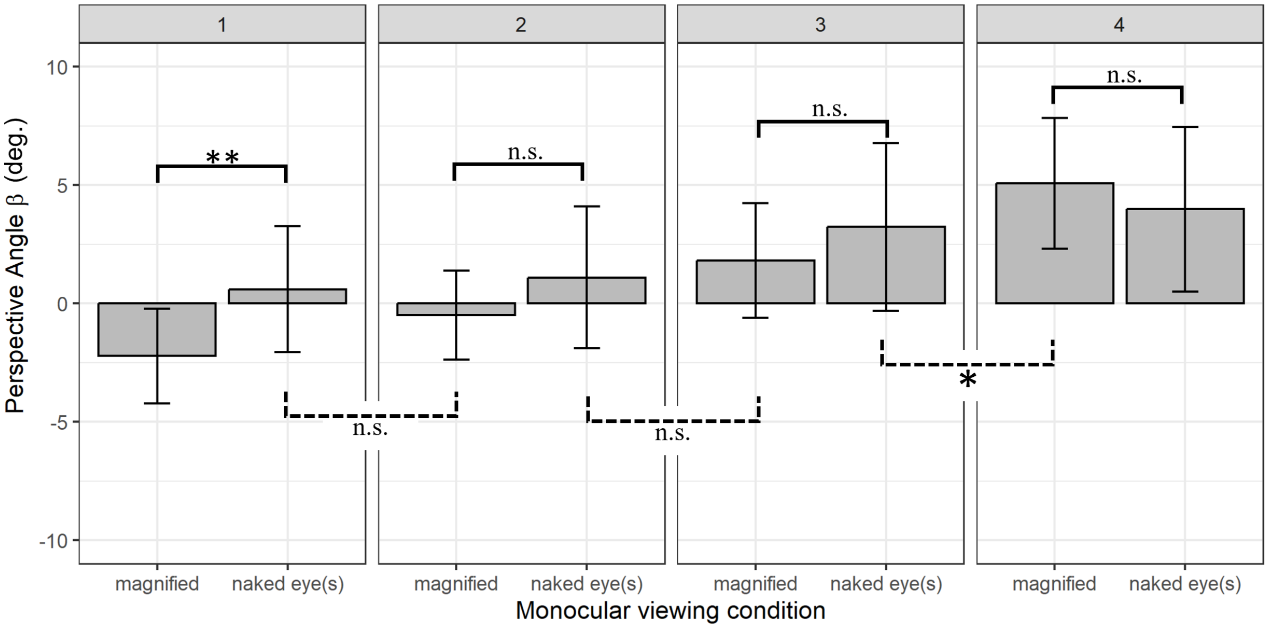


Figure 12: The comparison of $\beta$ between magnification and naked eye conditions, in monocular vision, separated by stimulus (leftmost graph - stimulus 1 to rightmost graph - stimulus 4). Each graph is denoted with * for significant difference, $* = p < 0.05$. $** = p < 0.01, *** = p < 0.001$.

## 5.2 Change of $\beta$ and $\theta$ when Magnified

The change of both values, $\theta$, and $\beta$, for each stimulus, are illustrated using scatter plots in Figure 10, and Figure 11 for monocular vision and binocular vision, respectively. The scatter plots of individual data points are drawn with 68% error confidence ellipses. The average value of each data group is indicated with the large circle (magnification) and large triangle (naked eye) symbol. We also plot the original/individual data using a small circle (magnification) or a small triangle (naked eye) symbol. Additionally, a dotted vertical line shows the actual slant angle, i.e., the ground truth value $\theta_G$, which is 71.6° for all stimuli. We also place a dotted horizontal line for the actual perspective angle of each stimulus, i.e., the ground truth value

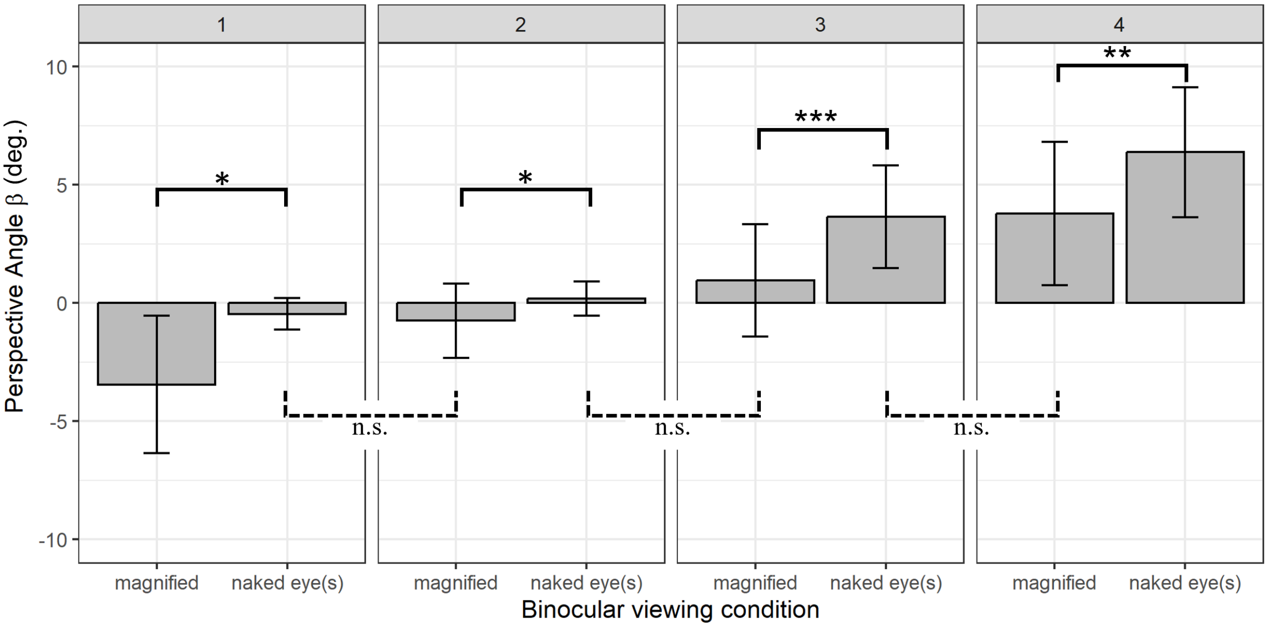


Figure 13: The comparison of $\beta$ between magnification and naked eye conditions, in binocular vision, separated by stimulus (leftmost graph - stimulus 1 to rightmost graph - stimulus 4). Each graph is denoted with * for significant difference, $* = p < 0.05$, $** = p < 0.01$, $*** = p < 0.001$.

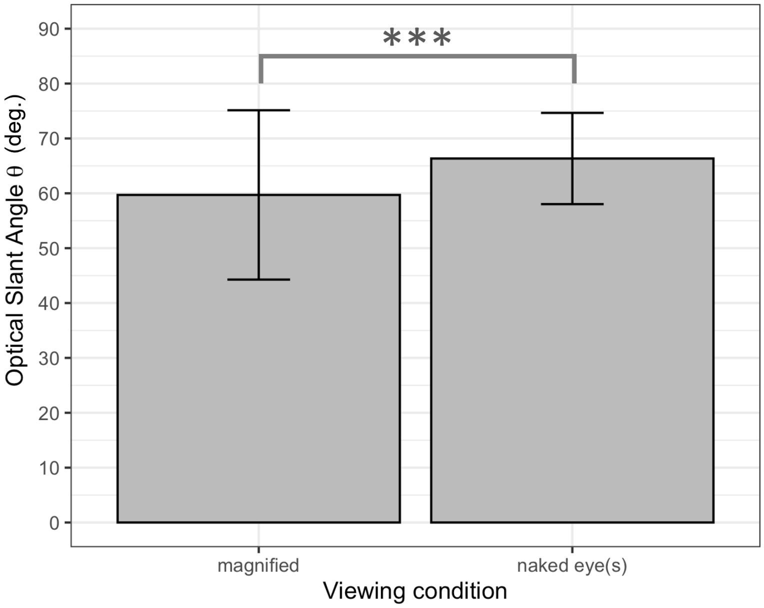


Figure 14: The comparison of $\theta$ between magnification and naked eye conditions regardless of stimulus and number of eyes used.

$\beta_G$, which are $-1°$, $0°$, $1.4°$, and $3.75°$ for stimulus 1 to 4, respectively. It is to be noted that Figure 10 and Figure 11 are zoomed in for clarity. However, there still exist some individual data points that overlap with each other, while some outlier data points are located beyond the graph. Therefore all 23 data points may not be visible in each graph.

From the scatter plots showing $\beta$ and $\theta$ under monocular vision in Figure 10, we observe a consistent trend that $\theta$ is smaller when magnified while $\beta$ is also smaller except for stimulus 4. For binocular vision in Figure 11, a similar pattern is observed, with both $\beta$ and $\theta$ becoming

Table 2: Average value of the perspective angle $\beta$ collected from all participants in the experiment, for four stimuli, monocular or binocular vision, magnified or naked eye(s) condition. The rightmost 3 columns show the statistical comparison result of $\beta$ for magnification and naked eyes viewing conditions.

| Measure | | Magnified | | Naked eye(s) | | Pairwise Comparisons | | |
|---|---|---|---|---|---|---|---|---|
| Stimulus | Number of eyes | Mean (deg.) | Std.Dev | Mean (deg.) | Std.Dev | Mean Difference (Magnified - Naked eyes) | Std. Error | $p$-value |
| 1 | Monocular | -2.318 | 1.980 | 0.402 | 2.545 | -2.815* | 0.694 | **0.003** |
| 2 | | -0.514 | 1.923 | 1.150 | 3.054 | -1.591 | 0.750 | 0.272 |
| 3 | | 1.710 | 2.432 | 3.385 | 3.550 | -1.424 | 0.816 | 0.569 |
| 4 | | 5.161 | 2.788 | 3.819 | 3.463 | 1.097 | 0.629 | 0.572 |
| 1 | Binocular | -3.606 | 2.874 | -0.478 | 0.679 | -2.991* | 0.619 | **0.000** |
| 2 | | -0.780 | 1.595 | 0.191 | 0.739 | -.928* | 0.298 | **0.030** |
| 3 | | 0.809 | 2.335 | 3.624 | 2.215 | -2.708* | 0.577 | **0.001** |
| 4 | | 3.738 | 3.098 | 6.399 | 2.807 | -2.600* | 0.635 | **0.003** |

smaller under magnification compared to the naked eyes condition for all stimuli. To further investigate these changes, a three-way repeated measure ANOVA was run on a sample of 23 participants to examine the effect of the stimulus, magnification, and binocular/monocular vision on the perspective angle, $\beta$. There was a significant three-way interaction, $F(3, 66) = 5.295$, $p = .002$. The comparison of $\beta$ between magnification and naked eye viewing conditions are shown with statistical difference notation using solid lines in Figures 12 and 13 for monocular vision and binocular vision, respectively. The statistical detail for the pairwise comparison is shown in Table 2. From the comparison, however, the change of $\beta$ is not statistically significant except the stimulus 1 ($p = 0.003$) under monocular vision (see Figure 12 for illustration and Table 2 for statistical details). Meanwhile, all changes of $\beta$ under the binocular vision are statistically significant, stimulus 1: $p < 0.0001$, stimulus 2: $p < 0.05$, stimulus 3: $p < 0.01$, and stimulus 4: $p < 0.05$ (see Figure 13 for illustration and Table 2 for statistical details). Because magnification illusion is defined as the change of the interpretation when viewed using the telescope, the shift may confirm the illusion mainly when viewed using two eyes.

To further investigate these findings, we conducted the paired t-test comparisons between the naked eye condition of stimulus $i$ and the magnified condition of stimulus $i+1$ as represented by the dashed lines in Figure 12 and 13. The statistical detail for the pairwise comparison is shown in Table 3. We did not find statistically significant differences under binocular vision. As mentioned in Section 5.1, we examined the physical changes in $\beta$ and conducted statistical comparisons within each viewing condition for neighboring stimuli, $i$ and $i + 1$. It indicates that participants were capable of discriminating between neighboring stimuli when observed with either naked eyes or magnification. However, when stimulus $i + 1$ was magnified while the stimulus $i$ remained with naked eyes, participants could not discriminate between those stimuli. This implies that the physical changes in $\beta$ became indistinguishable to participants when stimulus $i + 1$ was magnified. In other words, magnification may have reduced $\beta$ of stimulus $i + 1$ to that of stimulus $i$.

For the changes of the optical slant angle $\theta$, we found a significant three-way interaction, $F(3, 66) = .637$, $p < .05$, but not for two-way interaction between stimuli and number of eyes used in the observation for both magnified ($F(1.457, 32.058) = 1.217, p = .297$) and naked eye viewing conditions ($F(2.254, 49, 58) = .907$, $p = .421$). Therefore, we performed paired t-test between magnification and naked eye conditions, regardless of stimulus and number of eyes used for the observation. The participants interpreted optical slant as being closer

Table 3: The statistical detail of the paired t-test comparison between the naked eye condition of stimulus $i$ and the magnified condition of stimulus $i+1$.

| Stimulus | | Number of eyes | Paired differences | | Statistic | |
|---|---|---|---|---|---|---|
| Naked eye | Magnified | | Mean difference (deg.) | Std. Dev | t(22) | *p*-value |
| 1 | 2 | Monocular | 1.089 | 2.825 | 1.849 | 0.078 |
| 2 | 3 | | -0.714 | 3.527 | -0.971 | 0.342 |
| 3 | 4 | | -1.836 | 4.015 | -2.193 | **0.039** |
| 1 | 2 | Binocular | 0.288 | 1.363 | 1.014 | 0.322 |
| 2 | 3 | | -0.767 | 2.288 | -1.609 | 0.122 |
| 3 | 4 | | -0.117 | 2.893 | -0.195 | 0.847 |

to the frontoparallel plane in the magnification viewing condition ($M = 59.70$, $SD = 15.43$) as opposed to the naked eye(s) viewing condition ($M = 66.33$, $SD = 8.32$), a statistically significant mean decreased of 6.64, 95% CI $[-8.50, -4.77]$, $t(183) = -7.018$, $p < 0.001$, $d = -.517$. The difference is illustrated in Figure 14.

## 5.3 Comparison with the Actual $\beta$ and $\theta$

We are also interested to analyze how deviates the perceived perspective angle $\beta$ and the perceived optical slant $\theta$ is from the actual perspective and optical slant angles, $\beta_G, \theta_G$ using the following equations;

$$\Delta\beta = \beta - \beta_G \tag{1}$$

$$\Delta\theta = \theta - \theta_G. \tag{2}$$

We show the average and standard deviation of $\Delta\beta$ and $\Delta\theta$ in Tables 4 and 5, respectively.

To visualize the difference between the perceived angles and the actual angles, bar graphs are shown for the average, and standard deviation of $\Delta\beta$ and $\Delta\theta$ in Figures 15 left and 16, respectively. For $\beta$, we also show the average of absolute difference, $|\Delta\beta|$ in the bar graphs (Figure 15 right) for extended discussion.

### 5.3.1 Changes in the Perspective Angles

Figure 15 left shows the average difference of $\beta$ from the ground truth values. It can be seen from the figure that $\beta$ are larger than the ground truth, when seeing all stimuli using naked eye(s), both in binocular and monocular vision. Meanwhile, when magnified, the observed $\beta$ may be larger or smaller than the ground truths. Especially for the magnification condition, the difference seems to increase with the stimulus, from a negative difference to a positive difference. In summary, the differences are always positive when observed with the naked eye. On the contrary, the differences, when magnified may be positive or negative, and sometimes even closer to zero than that of the naked eye condition. Again, our definition of magnification illusion does not refer to incorrect perception according to magnification. When we compare all the standard deviations of $\Delta\beta$, it seems that the standard deviations of stimuli 1 and 2 under binocular vision with naked eyes are smaller than those of the other stimuli. This suggests that for these particular stimuli and viewing conditions, there was a relatively higher consistency in the perceived changes in $\beta$ among participants.

Table 4: Average and standard deviation of perspective angles and the actual perspective angles, $\beta - \beta_G$ for each stimulus, grouped by monocular and binocular vision, magnified or naked eyes conditions.

| Measure | | Magnified | | Naked eyes | |
|---|---|---|---|---|---|
| Stimulus | Number of eyes | mean (deg.) | sd | mean (deg.) | sd |
| 1 | Monocular | -1.217 | 1.994 | 1.598 | 2.658 |
| 2 | | -0.491 | 1.882 | 1.100 | 2.993 |
| 3 | | 0.414 | 2.428 | 1.838 | 3.539 |
| 4 | | 1.324 | 2.756 | 0.227 | 3.467 |
| 1 | Binocular | -2.449 | 2.907 | 0.542 | 0.671 |
| 2 | | -0.746 | 1.567 | 0.183 | 0.723 |
| 3 | | -0.450 | 2.379 | 2.258 | 2.170 |
| 4 | | 0.025 | 3.033 | 2.625 | 2.744 |

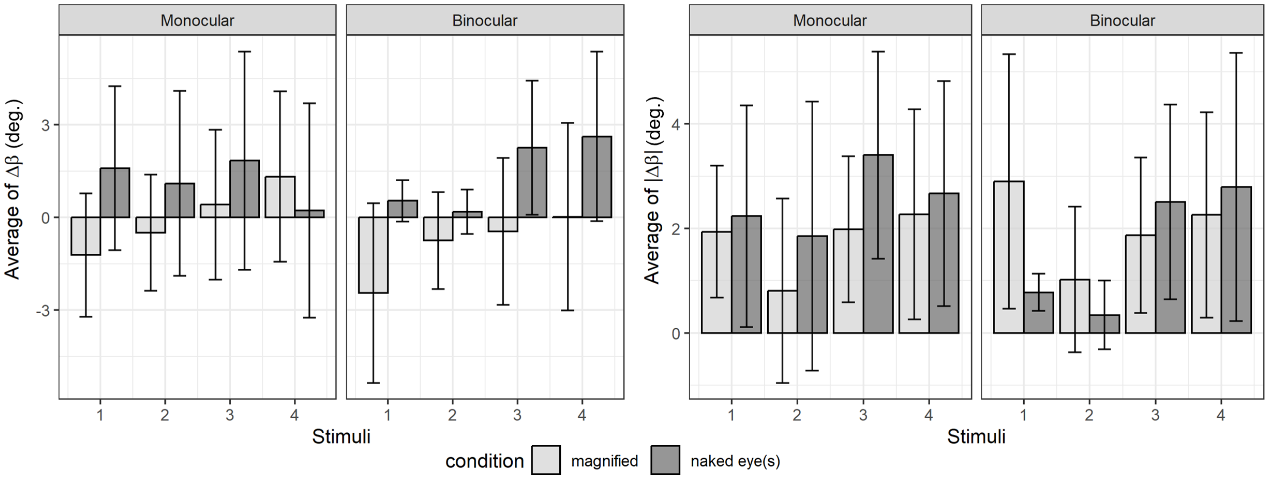


Figure 15: Average values of $\Delta\beta$ and $|\Delta\beta|$ for each stimulus exposure, separated by magnified and naked eye(s) conditions, monocular vision or binocular vision

To investigate only the amount of difference, we further analyzed by calculating the absolute difference $|\Delta\beta|$, as shown on the right of Figure 15. Even after taking the absolute difference, it is obvious that stimuli 1 and 2 for binocular vision with the naked eyes are special. In most cases, absolute differences are 1 degree or more. The exceptions are stimuli 1 and 2 for binocular vision with the naked eyes, which could be implied that everyone perceives similarly under these specific conditions. In Figure 15 left, however, the difference of $\beta$ for stimulus 1 is slightly larger than that for stimulus 2. That is the perception of stimulus 1 is on average about 1 degree larger. This indicates that stimulus 1 (diverged lines) and stimulus 2 (parallel lines) are both perceived to be parallel when observed with naked eyes, in binocular vision. It may be implied that human seems to have special perception when seeing the parallel stimulus under binocular vision with the naked eyes.

Table 5: Average and standard deviation of perceived optical slant angles and the actual optical slant angles, $\theta - \theta_G$ for each stimulus, grouped by monocular and binocular vision, magnified or naked eyes conditions.

| Measure | | Magnified | | Naked eyes | |
|---|---|---|---|---|---|
| Stimulus | Number of eyes | mean (deg.) | sd | mean (deg.) | sd |
| 1 | Monocular | -15.465 | 16.150 | -7.428 | 9.946 |
| 2 | | -8.911 | 28.666 | -5.759 | 7.702 |
| 3 | | -13.311 | 12.766 | -5.459 | 8.105 |
| 4 | | -9.737 | 10.253 | -5.705 | 9.226 |
| 1 | Binocular | -12.963 | 14.225 | -5.287 | 7.410 |
| 2 | | -13.855 | 12.416 | -5.474 | 7.007 |
| 3 | | -11.307 | 11.615 | -3.037 | 7.057 |
| 4 | | -9.622 | 9.898 | -3.935 | 9.998 |

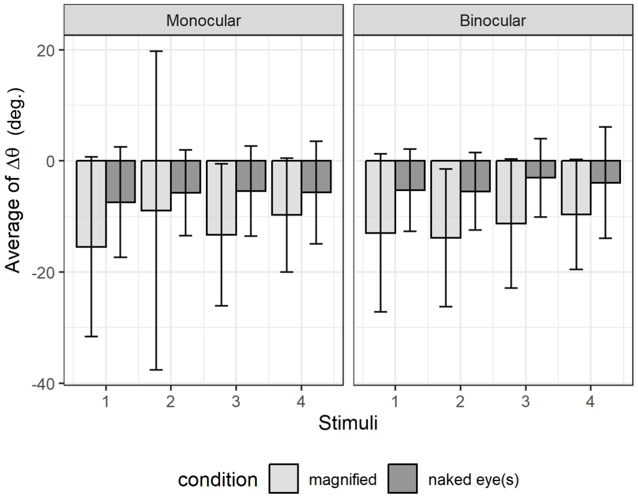


Figure 16: Average values of $\delta\theta$ for each stimulus exposure, separated by magnified and naked eye(s) conditions, monocular vision or binocular vision

### 5.3.2 Changes in the Optical Slant Angles

The result from Table 5 shows that the optical slant could not be accurately perceived for all types of stimuli, under any conditions. In general, all stimuli are perceived as being placed on a more slanted plane even when observed with naked eye(s), as a negative difference indicates that the stimulus is perceived as being slanted more toward fronto-parallel plane. The underestimation of slant is observed from our results, which is similar to previous works [15, 22, 32]. Moreover, the perceived optical slants are much more underestimated when the scene is magnified. These deviations in optical slant are observed for both monocular and binocular vision.

## 6 Discussion

We describe the discussion of the experiment result with regard to each of the hypotheses as follows.

### Hypotheses 1 and 2: Change in $\beta$ and $\theta$ when Magnified and the Effect of Monocular/Binocular Vision

We hypothesized that when the scene is magnified (by looking through a telescope), $\theta$ and $\beta$ will change. From the visualization of $\theta$ and $\beta$ in scatterplots shown in Figures 10 and 11, when comparing the magnified condition to the naked eye condition, both $\theta$ and $\beta$ seem to decrease across all stimuli. The result of the statistical tests shows that the significant changes in $\beta$ when magnified are observed for all stimuli, only in binocular vision, as indicated by the significant three-way interaction and the pairwise comparison shown in Table 2. Meanwhile, the underestimation of $\theta$ when magnified is observed regardless of the number of eyes used or the stimulus, as no significant two-way interaction is observed. Therefore, it can be concluded that the interpretation of $\theta$ is underestimated in the magnification condition under both monocular vision and binocular vision. However, the underestimation of $\beta$ is only observed when viewed using binocular vision.

When compared with ground truth, we found that $\Delta\theta$ is negative even for the naked eyes condition, and $\Delta\theta$ decreases (becomes more negative) when the scene is magnified. The negative value for $\Delta\theta$ indicates that the same plane is interpreted as being more slanted toward the fronto-parallel, and it seems to slant more in the magnified condition. From the finding, the slant perception under monocular and binocular vision seems to be similar, which is different from distance estimation, as also mentioned in [5, 7–9].

On the contrary, the monocular/binocular conditions affect the perspective angle interpretations. It may indicate that the perspective angle perception could be somehow related to the distance perception, as discussed in [6] that the perspective angle is related to the estimated distance of the vanishing point. The mechanism for optical slant angle perception may differ from that of perspective angle perception which is not so affected by the difference of monocular/binocular conditions. We have to further investigate the relationship among interpretations of distance, perspective angle, and slant angle.

### Hypothesis 3: Change in Stimuli Other than the Parallel Lines

The perspective angle $\beta$ is interpreted as smaller when magnified even if the lines are not parallel only under binocular vision. On the contrary, the illusion reported by Eltenton and Tsuinashi [4, 30] focused only on looking at the parallel lines using binocular vision through a pair of binoculars. The illusion was reported that the further ends of the parallel lines appear to converge when seen through a pair of binoculars. When viewing the parallel lines using binocular vision with the naked eye, the interpretation of $\beta$ is considerably accurate. Accurate perception is observed only when the three conditions are met: parallel lines, binocular vision, and the naked eye. The interpretation is less accurate when one of the three conditions is violated, which is different from $\theta$. Previous studies [4, 30] reported perceptions of parallel lines as diverged lines when magnified using a pair of binoculars. These reports seemed to be due to the fact that we can perceive the parallel lines almost accurately with the naked eyes, so that people were sensitive to the change of perception when magnified.

## 7 Conclusion

In this work, we investigated the change in the interpretation when looking at the two lines receding into the distance through a telescope, namely magnification illusion. We hypothesized that the perspective angle and the optical slant will change when magnified, and monocular or binocular vision may affect such changes and that the change does not limit only to the parallel lines as reported in the previous literature. We designed the experiment according to the hypothesis by collecting perspective angles and optical slant angles for four different types of stimuli, in various viewing conditions. From the experiment, we found that the perspective angle is underestimated for all types of stimuli when magnification only when viewed with binocular vision. We also found that the interpretation of the perspective angle is accurate only when the lines are parallel lines, viewed using naked eyes, under binocular vision. Meanwhile, the optical slant perception does not rely on such conditions, and regardless of the viewing condition, the optical slant is always underestimated. These findings suggest that the illusion reported in the previous work seems to be due to the fact that we can perceive the parallel lines almost accurately with the naked eye so that people are sensitive to the change of perception when magnified. However, we found that the change in the interpretation of perspective angles when magnified does occur in all stimuli under binocular vision. This finding could be useful when designing an environment that limits the field of view such as a virtual reality scene.

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