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Study on Optimization of Curves That Compose The Reflector of A Regular Reflection Light Shelf and Its Application

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Abstract. Recently, it is considered that it is important to reduce primary energy consumption in buildings. One way to do this is to use daylight. To use daylight as lighting, the authors focused on a light shelf, one of the daylighting devices. This study aimed to clarify the cross-sectional shape of a light shelf that can bring in light evenly into a room. In order to derive a cross-sectional shape with high daylighting performance, we focused on a reflector composed of multiple quadratic curves. A numerical program was constructed to constitute the optimal shape of the light shelf, and the performance of the derived light shelf was verified. Moreover, by applying the optimization method, we confirmed that it is possible to derive a reflective surface shape to achieve an any ceiling surface illuminance distribution.

Key Words: daylighting, daylighting system, light shelf, regular reflection, optimization

MSC 2020: 51-08

1 Introduction

In recent years, it is a social issue to reduce the environmental burden, and in the architectural field as well. There is a growing movement toward reducing energy consumption in buildings. One way to saving energy is to use daylight. It is clear that the use of daylight is effective, since lighting fixtures account for approximately 30% of a building's total primary energy consumption [1]. Against this background, a large number of studies have been conducted on system development and performance verification of a wide variety of daylighting devices [2, 5, 6]. Most of them are aimed to reduce energy consumption in the entire rooms. While this is one of the major benefits of daylight utilization, other effects of natural light on the body and mind, such as adjustment of life rhythm and improvement of intellectual productivity, are also attracting attention, and studies on these effects are also being conducted [3, 4]. There

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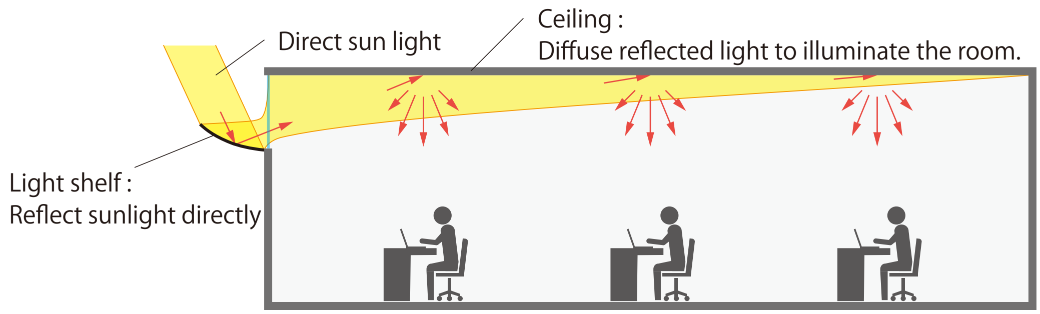


Figure 1: Lighting policy with regular reflection light shelf.

are some studies aimed at achieving the latter effect [7]. However, few studies that target light shelf were found.

Therefore, the purpose of this research was to propose a lighting device that would enable the reduction of energy consumption and also provide the physical and mental benefits of natural light in the entire room by using daylight. Focusing on a light shelf, one of the lighting devices, we devised a method to optimize the shape of the reflective surface that allows light to enter the room evenly. The performance of the light shelf formed by this method was verified, and its usefulness was indicated. We also showed that the application of the optimization method can realize a reflective surface that can control the light distribution of reflected light with a high degree of freedom.

2 Optimization Method

2.1 Conditions for Optimization

The optimization of the cross-sectional shape of the reflective surface of a regular reflection light shelf is to be conducted in a room space with a window in the upper part of the south face. The light shelf is installed outside of this window surface. The reflective surface of the light shelf is a regular reflective material, which reflects direct sunlight up to the ceiling of the room. The incident light on the ceiling surface is then diffused into the room to illuminate (Figure 1). In the optimization, the ceiling surface is assumed to be an even diffuse surface. In addition, to simplify the theory, only the profile angle, the apparent altitude of the sun, is used in this study as a variable to represent the sun's position, instead of the solar orientation angle and solar altitude. In this case, the sun position is on the plane of the Figure 1, which is applicable only for a moment of each day (south central time). If the sun is out of the plane, the illuminated band on the ceiling will rotate up to transition to the corresponding wall and the brightness of the room becomes unbalanced.

2.2 Optimal Light Shelf Reflector Shape

In optimizing the cross-sectional curves that compose the reflective surface of a regular-reflection light shelf, the two factors to be sought are as follows:

1. Daylight can be used as a substitute for artificial lighting to reduce energy consumption in buildings.
2. All areas of the room can receive the physical and mental benefits of exposure to daylight, such as the adjustment of life rhythms and improvement of intellectual productivity.

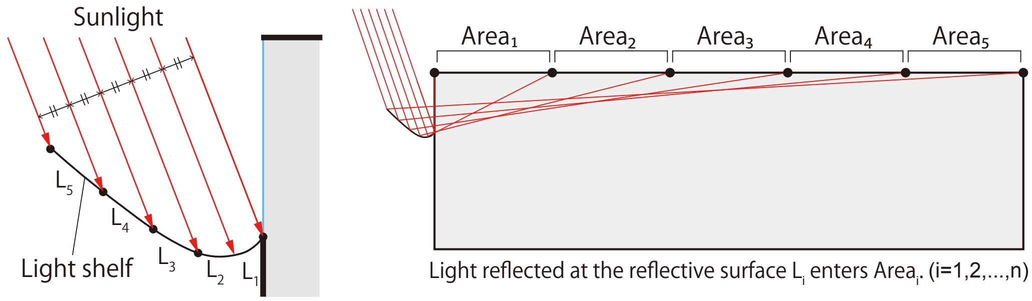


Figure 2: Diagram representing the concept of optimization theory (the case of $n = 5$).

In order to satisfy the above two factors, we considered that a reflective surface shape that can illuminate the entire room evenly and brightly by taking in daylight would be optimal. This not only saves artificial lighting by preventing some parts of the room from being excessively bright, but also provides the effect of natural lighting throughout the entire room. Therefore, the method that leads to "a reflective surface shape capable of illuminating the entire room evenly and brightly" defines the optimization method.

2.3 Optimization Theory

In case the ceiling surface is a even diffuse surface, even if the incident light on the ceiling surface is uniform, the floor surface of the room will not be perfectly uniform. It is because the fact is that the middle area of the floor is illuminated mostly by light irradiated from the ceiling surface, while the floor surface near the walls is illuminated by some proportion of light irradiated from the nearby wall surface. However, light reflected from the ceiling has the largest impact on the illuminance of the floor directly below it, and the further away from it, the smaller the impact. Therefore, in the optimization, the objective was to derive a reflector shape that reflects light uniformly on the ceiling.

For the optimization, we used a reflective surface whose cross-section is a curve composed of multiple quadratic curves whose positive quadratic terms have monotonically increasing slopes connected with them so that their slopes are continuous. The reason for selecting these curves is that since they have monotonically increasing slopes, connecting them yields spline curves whose change in slope is always positive. By doing so, a single cross-sectional curve is uniquely defined by optimization.

As shown in Figure 2, the number of quadratic curves constituting the reflective cross-section is defined as the number of divisions n, and each curve is denoted by L_i $(i = 1, 2, \dots, n)$ in order from the window side. Also, divide the ceiling surface into n equal parts, and let each divided ceiling surface be Area_i $(i = 1, 2, \dots, n)$ in order from the window side. If S_i, the apparent length of the curve L_i viewed from the sun direction, are all equal and the curve shape is such that light reflected at the curve L_i enters Area_i, the amount of light entering each divided ceiling surface Area_i is equal. Under these conditions, the distribution of light incident on the ceiling surface is expected to approach uniformity as n increases.

2.4 Optimization Calculations

In order to clarify the curves that satisfy the conditions described in Section 2.3, optimization was performed by constructing a system of multivariable equations based on the conditions

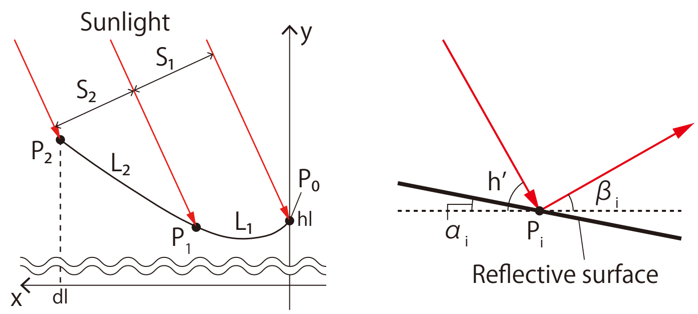


Figure 3: Definition of variables and constants related to optimization calculations. ($n = 2$ is shown here as an example).

and obtaining solutions through numerical calculations.

The definitions of the variables are as follows (Figure 3):

L_i: The i-th curve from the window side among the quadratic curves constituting the composite curve ($i = 1, 2, \ldots, n$)
a_i: Coefficient of the second order term in the equation representing L_i
b_i: Coefficient of the first order term in the equation representing L_i
c_i: Coefficient of the constant term in the equation representing L_i
S_i: Apparent length of L_i when viewed from the sun direction ($i = 1, 2, \ldots, n$)
P_i: Boundary points (x_i, y_i) of each curve constituting the cross-sectional curve ($i = 1, 2, \ldots, n$)
x_i: x-coordinate at $P_i[m]$.
y_i: y-coordinate at $P_i[m]$.
y'_i: slope of the section curve at P_i
α_i: Angle of cross section curve at P_i with horizontal direction.
β_i: Angle of reflected light at P_i horizontal direction.

The definitions of the constants are as follows (Figure 3):

h': profile angle (apparent solar altitude)
hl: Installation height of the light shelf
dl: horizontal length of the light shelf
dr: depth of the room space

The following is a summary of the method used to derive the equations.

1. Equations (1) to (6) are shown for variables that become constants by setting conditions.

$$x_0 = 0 \tag{1}$$

$$y_0 = \mathrm{hl} \tag{2}$$

$$y'_0 = \tan \alpha_0 \tag{3}$$

$$x_n = \mathrm{dl} \tag{4}$$

$$\beta_0 = 90^\circ \tag{5}$$

$$\alpha_0 = \frac{1}{2}(h' - \beta_0) \tag{6}$$

2. Equations for quadratic curves L_i, the quadratic curve that constitutes the reflector, is expressed by the following Equation (7) using the variables a_i, b_i, c_i ($i = 1, 2, \ldots, n$).

$$y = a_i x^2 + b_i x + c_i \quad (x_{i-1} \leq x \leq x_i) \tag{7}$$

Differentiating Equation (7) provides a relational equation for the coordinates and slope of the curve, which is shown in Equation (8).

$$y' = 2a_i x + b_i x \tag{8}$$

Since the curve L_i passes through P_{i-1} (x_{i-1}, y_{i-1}) and P_i (x_i, y_i) and their slopes are continuous, the following Equations (9)–(12) are obtained from Equations (7) and (8).

$$y_i = a_i x_i^2 + b_i x_i + c_i \tag{9}$$

$$y_{i-1} = a_i x_{i-1}^2 + b_i x_{i-1} + c_i \tag{10}$$

$$y_i' = 2a_i x_i + b_i \tag{11}$$

$$y_{i-1}' = 2a_i x_{i-1} + b_i \tag{12}$$

Therefore, a total of $4n$ relational equations can be obtained from Equations (9)–(12).

3. Relational equations for S_i, the apparent length of the curve: Since S_i is the apparent length of the quadratic curve L_i when viewed from the direction of direct sunlight, the following equation (Equation 13) is valid for S_i.

$$S_i = \sqrt{\Delta x^2 + \Delta y^2} \cos 90° - \left(h' - \tan^{-1} \frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right) \tag{13}$$

To reflect light uniformly in a room, the amount of rays of direct sunlight incident on the n curves constituting the reflector should be equal, so Equation (14) is obtained.

$$S_i = S_{i+1} \quad (i = 1, 2, \ldots, n-1) \tag{14}$$

From Equations (13) and (14), there are $(2n - 1)$ relational equations for S_i.

4. Relational equations for slope and angle of reflective surfaces and rays of light The following Equations (15) and (16) are valid for α_i and β_i $(i = 1, 2, \ldots, n)$.

$$\beta_i = \tan^{-1} \frac{\mathrm{hr} - y_i}{\frac{i\mathrm{dr}}{n} + x_i} \tag{15}$$

$$\alpha_i = \frac{1}{2}(h' - \beta_i) \tag{16}$$

The following Equation (17) holds for y_i' $(i = 1, 2, \ldots, n)$

$$y_i' = \tan \alpha_i \tag{17}$$

From Equations (15) to (17), we can obtain $3n$ relational equations related to the slope and angle of the reflector and rays of light.

From 1. to 4., $(9n + 5)$ equations are obtained for $(9n + 5)$ variables. Optimization is achieved by finding solutions to this system of equations. However, the system of equations is complex, and the solution cannot be obtained by direct methods. Therefore, the Newton-Raphson method, one of the convergence calculation methods, was used to obtain the numerical solution to derive the reflection cross-section shape.

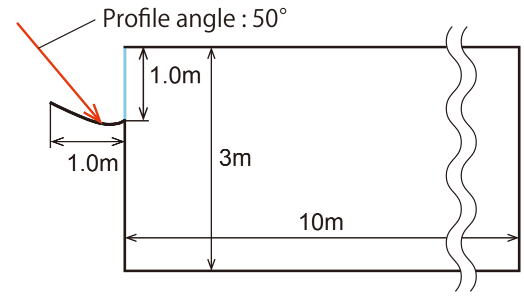


Figure 4: Details of the room conditions.

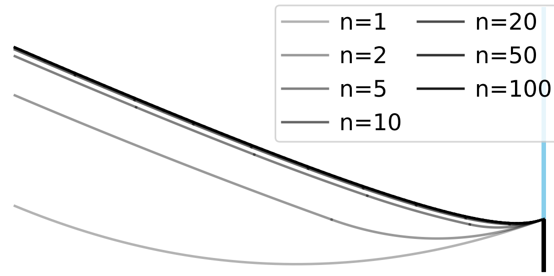


Figure 5: Reflective surfaces generated by the optimization method.

2.5 Results of Optimization Method

Figure 5 shows the reflector shape generated by the optimization method described in above sections under the conditions of Figure 4. The change in the reflector shape becomes smaller as the number of segments increases, and there is almost no difference in the cross-sectional shape when the number of divisions is approximately 50 or more. It can be confirmed that the cross-sectional shape obtained by optimization is a curve with a large curvature near the window surface and a curve close to a straight line near the tip.

Next, ray tracing was performed to visualize the reflection of light on the reflective surface obtained by the optimization method. As the ray tracing results in Figure 6 show, by increasing the number of divisions, it can be confirmed that light is reflected on the ceiling surface of the entire room, from near the window to the back of the room. Also, by increasing the number of divisions, it seems that light can be taken in evenly. Furthermore, in order to confirm the detailed lighting performance, the lighting performance by the number of divisions was quantified by the illuminance distribution and uniformity ratio of illuminance on the ceiling surface.

3 Performance Verification

3.1 Illuminance Distribution on the Ceiling Surface

To obtain the illuminance distribution on the ceiling surface, direct sunlight is considered as a finite number of light rays. In this case, each ray is considered to have a luminous flux of a certain size. Since illuminance is "the luminous flux incident per unit area," the number of rays incident on a unit area can be regarded as the illuminance. Therefore, the ceiling surface illuminance distribution is quantified here by means of this way. The unit area was defined as each ceiling area divided into 500 equal sections. The number of rays of direct sunlight was assumed to be $10,000$ and to be evenly incident on an area of $1.0\,\mathrm{m}$ from the window surface at the height of the light shelf installation.

From Figure 7, it can be seen that when the number of divisions is small, the variation of the ceiling surface illuminance is large, and as the number of divisions increases, the distribution approaches uniformity. For 100 or more divisions, the ceiling illuminance is almost uniform, indicating that optimization can be performed with a high degree of accuracy.

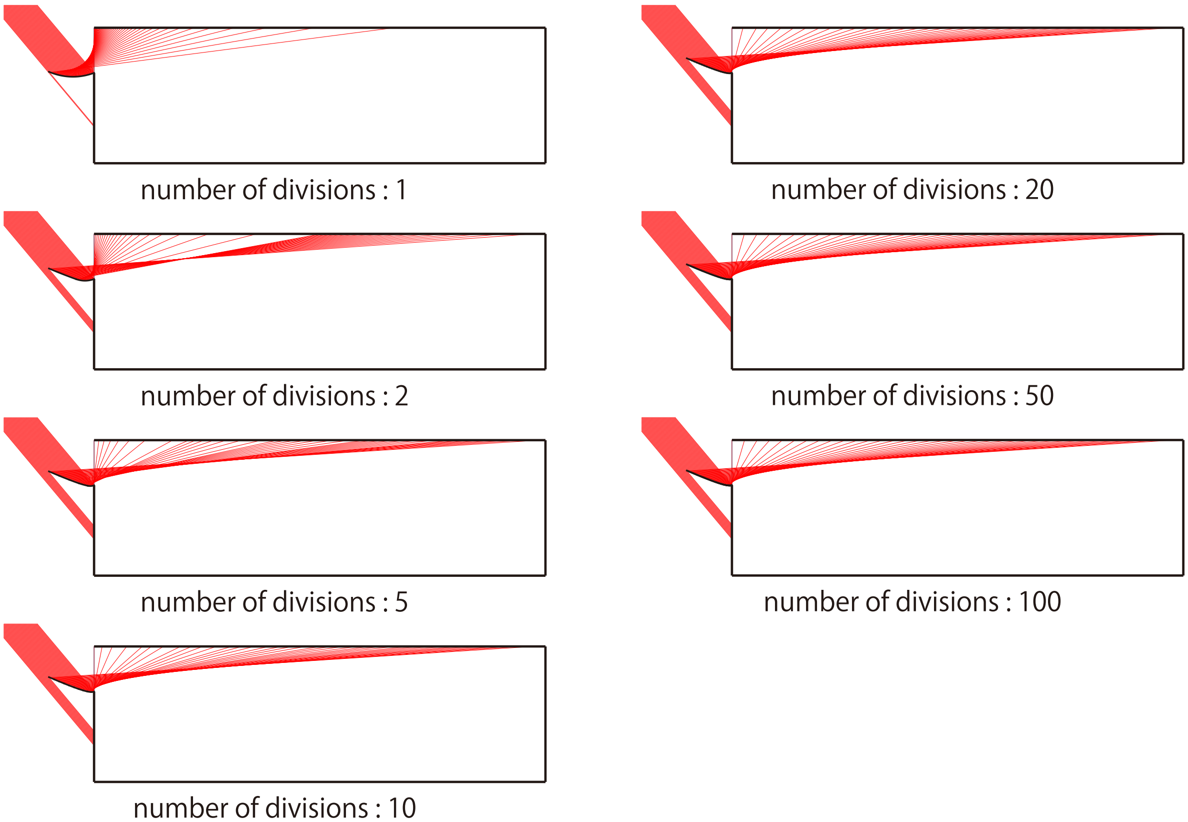


Figure 6: Ray-traced image on a reflective surface generated by the optimization method.

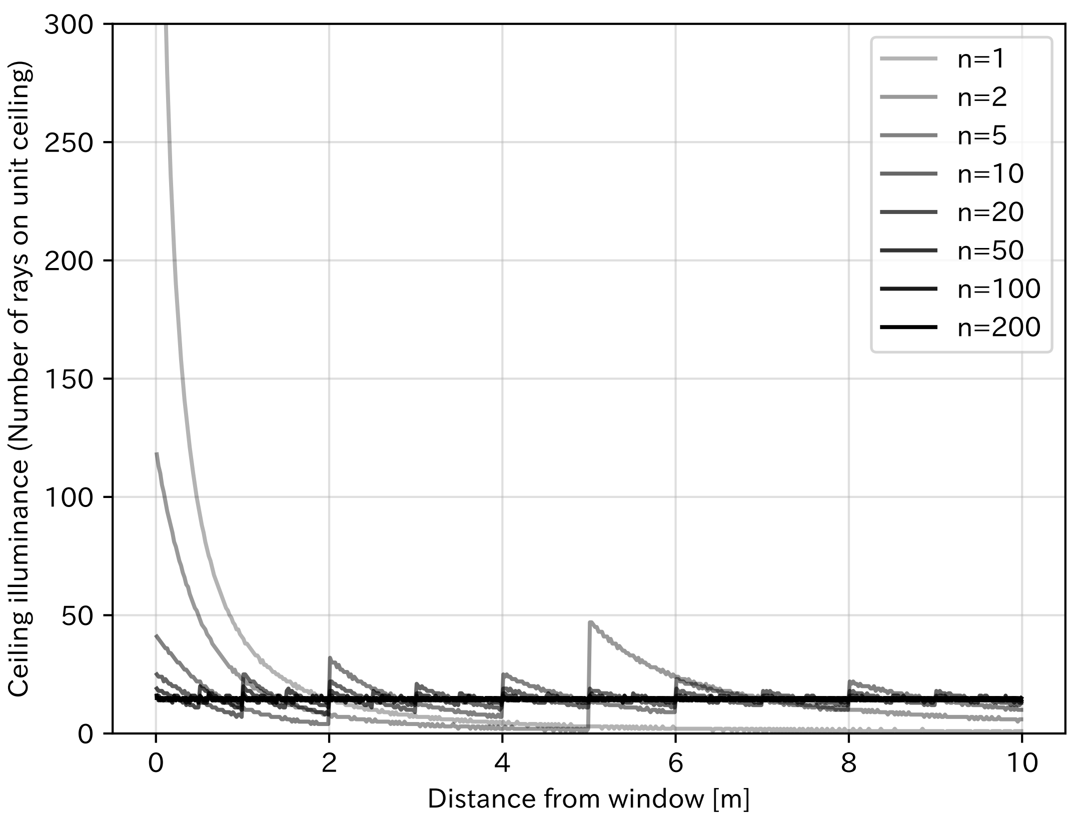


Figure 7: Illuminance distribution on the ceiling surface.

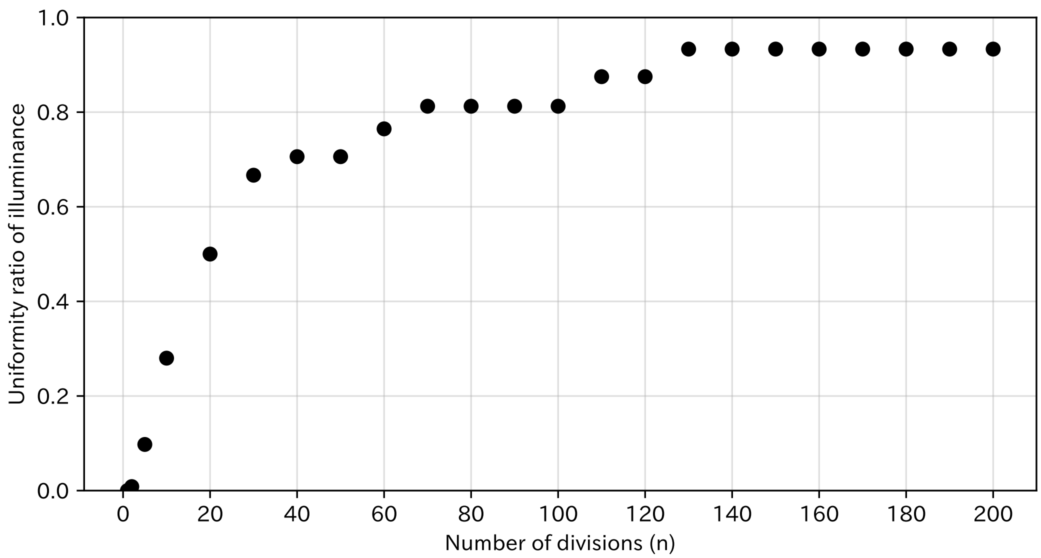


Figure 8: Uniformity ratio of illuminance on the ceiling surface.

3.2 Uniformity Ratio of Illuminance

Next, the uniformity ratio of illuminance on the ceiling surface was used to quantify the degree of uniformity of brightness due to the use of daylight in the entire room. Uniformity ratio is a value obtained by the ratio of the minimum illuminance to the maximum illuminance, and the closer it is to 1, the more uniform the brightness in the room is. Figure 8 shows the change in uniformity ratio of illuminance on the ceiling surface for each number of divisions.

It can be seen that the uniformity ratio increases with the number of divisions. After 40 divisions, the amount of change decreases significantly, and the uniformity ratio exceeds 0.9 at more than 130 divisions. The reason why the change in uniformity ratio is not smooth may be due to the fact that the number of rays was set to $10,000$, and by further increasing the number of rays, it may be possible to obtain a detailed difference in uniformity ratio for each number of divisions.

3.3 Results of Performance Verification

When the reflective surface generated by the proposed optimization method is applied, the performance verification confirmed that it is possible to uniformly brighten a room by using daylight. However, since the increase in uniformity decreases with the increase in the number of divisions, the advantage of using a very large value of n is small, and it can be said that a regular reflection light shelf with sufficient lighting performance can be realized with a certain number of divisions.

4 Application of the Optimization Method

Through Sections 2 and 3, we proposed a method for optimizing the shape of the reflective surface of a daylighting device capable of uniformly reflecting light, and verified its performance. Since this method can be applied to realize a free light environment that is not limited to uniform light intake, its applied use is summarized below.

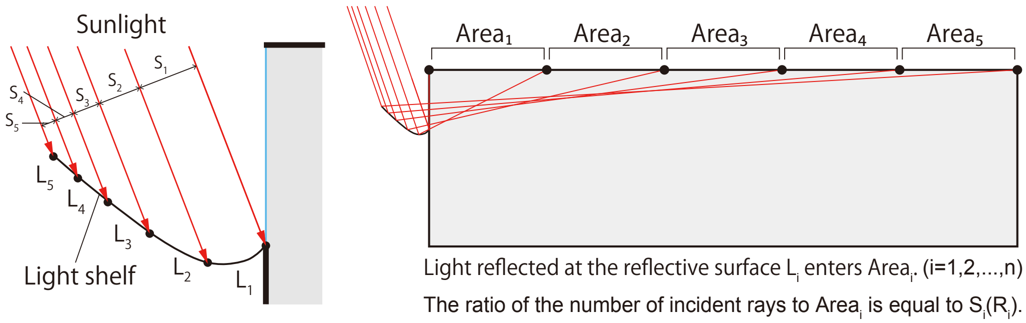


Figure 9: Conceptual diagram of the theory in the application of optimization methods (the case of $n = 5$).

4.1 Theory in Application

In order to equalize the amount of light incident on each of the divided ceiling surfaces (Area$_i$) when performing optimization, restrictions such as "S_i, the apparent lengths of the curves L_i viewed from the sun direction, are all equal and the curve shape is such that light reflected at the curve L_i is incident on Area$_i$" were established. Since the ratio of S_i is equal to the ratio of the number of rays incident on each Area$_i$, as shown in Figure 9, by setting this ratio according to the illuminance distribution to be realized, the shape of the reflecting surface can be optimized to realize a desired daylight illumination environment. Here, the correctness of the above theory is confirmed through two examples of the application of the optimization method.

The equations used in the optimization calculations are the equations in Section 2.4, except for Equation (14), plus Equation (18) shown below, for a total of $(9n + 5)$ equations. The reflection cross section shape was derived by obtaining numerical solutions for these equation systems using the Newton-Raphson method. R_i in Equation (18) represents the relative value of S_i.

$$\frac{S_i}{R_i} = \frac{S_{i+1}}{R_{i+1}} \quad (i = 1, 2, \ldots, n-1) \tag{18}$$

4.2 Results of Application

The reflector geometry that achieves the two ceiling surface illuminance distributions shown below is derived by the applied use of the optimization method under the same conditions as the performance verification in Section 3, with $n = 100$ divisions.

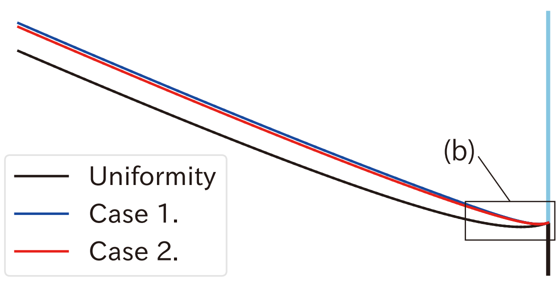
1. When more light is taken into the back half of the room
2. Case where more light is taken in as one moves toward the back of the room

Equations (19) and (20) show the formulas for calculating the value of R_i $(i = 1, 2, \ldots, 100)$ to be used when optimizing Case 1. and 2., respectively.

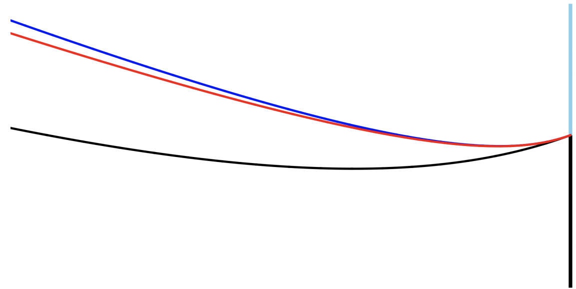
$$\begin{cases} R_i = 1 & (1 \leq x \leq 50) \\ R_i = 5 & (51 \leq x \leq 100) \end{cases} \tag{19}$$

$$R_i = 2\left\{\sin\left(\frac{i-1}{99}\pi - \frac{\pi}{2}\right) + 1\right\} + 1 \tag{20}$$

Then, optimization was performed using these equations. Reflected cross-sectional shapes generated by optimization are shown in Figure 10. In addition, the reflection cross section

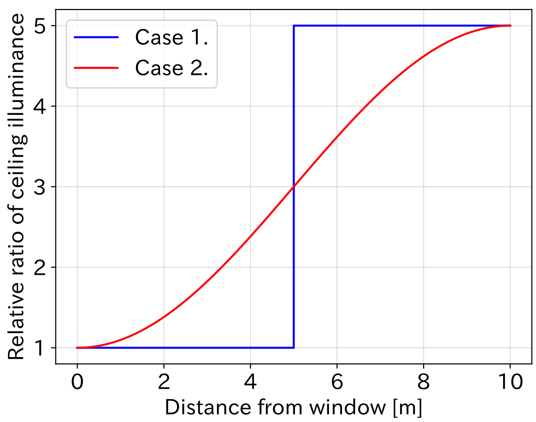


(a) Overall diagram.

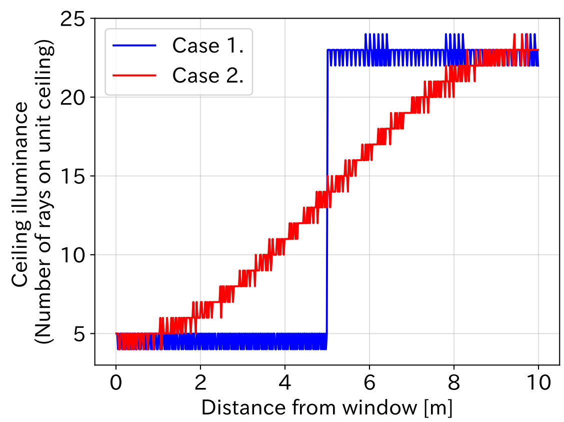


(b) Enlarged diagram.

Figure 10: Cross-sectional shapes of the reflector generated by application of the optimization method.



(a) Reproduction targets.



(b) Result of applying the method.

Figure 11: Illuminance distribution on the ceiling surface.

for $n = 100$, the number of divisions obtained in Section 3, is drawn in black. No significant differences in reflector shapes were observed in Cases 1. and 2. This is thought to be because, although there are differences in the ceiling surface illuminance distribution, they are consistent in terms of taking in more light at the back of the room. When more light is taken in at the back of the room (Case 2.), the ratio of the area with large curvature near the window is smaller, and the shape is closer to a straight line.

Next, the illuminance distribution on the ceiling surface to be realized and obtained by the optical light shelf are shown in Figure 11.

From Figure 11, it can be seen that the illuminance distribution when the optimized method is applied overlaps with the illuminance distribution to be reproduced. The ratio of minimum to maximum values is approximately $1 : 5$, and the relative ratio of illuminance is also reproduced. From this, it can be said that this method can be used to create a reflector shape that can achieve an arbitrary illuminance distribution with high accuracy. As a result, the optimization method can be used to create a regular reflective light shelf that can satisfy a wide variety of user requirements for daylight use in addition to uniform light intake.

5 Conclusion

By using the optimization method, it was possible to derive the shape of the reflective surface of a regular-reflection light shelf that brings light into a room evenly. By verifying the performance of this reflective surface, we confirmed that a uniformity of over 0.9 can be obtained by increasing the number of divisions ($n \geq 130$), showing its usefulness in terms of bringing in light uniformly. We also confirmed that the applied optimization method can be used to derive a reflective surface shape to achieve an desired illuminance distribution on ceiling surface.

Future work includes consideration of how to respond to changes in the position of the sun and how to control the mechanism and machinery for practical use as a daylighting device. In addition, it is necessary to confirm the practicable performance of the device by creating a model or an actual device and evaluating its performance in real space. On the other hand, since the theory proposed in this research can be applied not only to daylighting but also to a wide range of areas related to light reflection, we will also examine the applicability of this theory.

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