Journal for Geometry and Graphics
Volume 28 (2024), No. 1, 89–101

Geometric Design Tool for One-Fold, a Curved Origami with a Single Fold

Aida Safary[1], Hamid Shafieasl[2], Jun Mitani[1]

[1] *University of Tsukuba, Tsukuba, Japan*
s2330170@s.tsukuba.ac.jp, mitani@cs.tsukuba.ac.jp

[2] *Azerbaijan Shahid Madani University, Iran*
h.shafieasl@azaruniv.ac.ir

Abstract. In this paper, we propose a digital system for the Grasshopper Rhinoceros environment for designing a specific temporary architectural structure based on shape with a single fold that can be considered a deployable shelter surface for architectural purposes. We use the system to imitate the One-Fold Project by Patkau Architects, which applied metal sheets as the original material in their prototype. In the proposed method, the system creates conic parts of the original shape part by part in such a way that allows the user to experience a similar model and control and change the value of parameters related to the geometric features of the shape.

Key Words: sustainable architecture, developable surface, origami engineering

MSC 2020: 51N05 (primary), 51M04, 51N15

1 Introduction

This research explores the intersection of computer science and architecture in creating temporary origami-inspired structures. Origami is the art and science of paper folding and has been applied in various fields, such as architecture, space structures, and medical sectors. In architecture, origami techniques can be used to produce fluid, dynamic, interactive, sustainable, and environmentally adaptive structures. These structures can be easily modified and deployed, requiring less material and response to their surroundings [2, 7]. One example of an origami-inspired architecture is the One-Fold Project by Patkau Architects [8] inspired by Paul Jackson's abstract origami artwork [4]. Patkau Architects is a Canadian architectural group that specializes in innovative design. The One-Fold Project consists of three 10×24 feet metal sheets folded along a single diagonal line to create structures with an apex point in the middle which represent abstract origami elements that can be used for architectural purposes. The main objective of our research is to define the mathematical properties of a

ISSN 1433-8157/

One-Fold Project and create digital models in a computer-aided design (CAD) environment. Python script in the Grasshopper Rhinoceros plugin environment was used to construct the One-Fold Project's 3D digital module shape that the user can manipulate and explore. In this study, we first review the literature related to curved origami. We then define the geometric features of the shape to examine it through a digital design process. Next, we explain the methodology used to create the structure. Finally, we present results and data. Figure 1 shows the renderings of shapes generated by the proposed system.

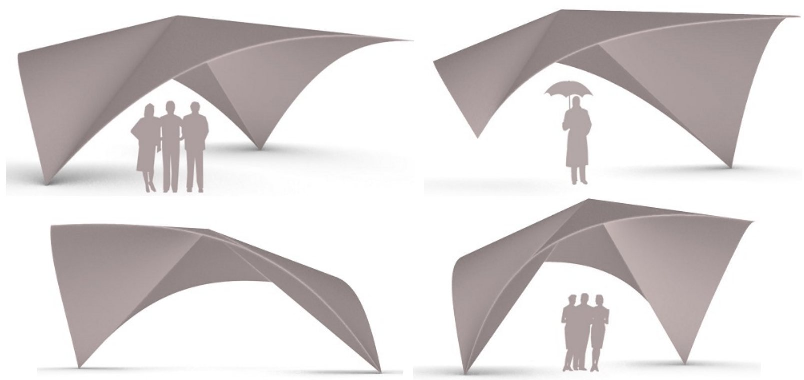


Figure 1: Final 3D renders of the structure designed by our system in various states.

2 Related Works

In this section, we will provide a comprehensive explanation of the One-Fold project, tracing its development from its initial concept as a paper material experiment to the creation of the final shape built with metal sheets in a minimalist expression to show the possibilities of exploration of space and structure designs in the architecture filed that can be triggered by simple origami folds. Additionally, we will discuss manufacturing and finishing the project, providing a complete overview of the entire process.

2.1 One-Fold Project

"One-Fold" is the result of inspiration from Paul Jackson's abstract origami designs, titled "One-Crease"[5] designed with a single fold-line along a square or rectangular paper with one apex along the fold line. Patkau architects initiated the initial steps of their project by folding and experimenting with square papers creating a conventional origami style to understand the potential of this minimalist approach. Although paper can be folded easily to create stable three-dimensional shapes, steel does not share this flexibility. Once folded, the steel sheets become extremely strong and nearly impossible to fold by hand. Their experiment began with two square sheets of 24 feet of galvanized metal. By creating a small hole and folding the metal across it, they can replicate the structure seen on paper. However, this technique proved to be less effective as the size and thickness of steel increased.

Their solution was to invent a machine capable of folding and breaking a structure that was similar to the original shape. This innovation allowed them to achieve with steel, which is typically done sequentially with paper. Via iterative testing with larger sheets, they refined the machine to transform an 18 gauge 5×12 feet steel sheet into a stable, self-supporting structure called a "broken vault". This structure, with its free surfaces and curves, expresses remarkable curves of natural beauty in minimalist architecture. Their final structure was a combination of straight lines and elegant curves assembled as a set of simple reflective landscape shelters using three 10×24 feet stainless-steel sheets [8].

2.2 Developable Surfaces in Origami-Inspired Architecture

Developable surfaces are smooth surfaces with zero Gaussian curvature, which means they can be flattened onto a plane without distortion. The Gaussian curvature of a surface determines its developability as a result of its principal curvatures. Mean curvature, which desc ribes local curvature, is derived from principal curvatures. Developable surfaces have at least one principal curvature value of zero, and straight lines on these surfaces, called rulings, which define the direction along which the surface can be developed. This property makes them ideal for deployable folding architecture due to their ability to transform complex 3D surfaces into paper through bending or folding [11].

Two main approaches appear to be related to deployable architecture. In the first approach, architects and designers imitate foldable origami structures without considering the geometric constraints related to the developability of the structure and its surface. Examples of this approach can be seen in Klein Bottle House by McBride Charles Ryan and Festival Hall in Erl by Delugan Meissl-Associated Architects [6, 10]. In the second approach, ensuring developability is part of the design process. All geometric constraints related to developable surfaces, such as surface length preservation and zero Gaussian curvature, should be applied when architects consider free-form developable surfaces in their design. Examples of the second approach can be found in the designs of architect Frank Gehry for the Guggenheim Museum Bilbao and Walt Disney Concert Hall. Another example is the ARUM installation, which represents a structure with curved folded tessellations on the surface calculated by Zaha Hadid's computational design research group [1, 6, 9, 10].

As mentioned before, developable surfaces are composed of linear elements called rulings. Focusing on the arrangement of these rulings, they can be classified into three types: cylindrical surfaces, where the rulings are parallel; conical surfaces, where the rulings intersect at a point known as the apex; and tangent developable surfaces, where the rulings are tangent to a spatial curve. The shape we aim to create in this study is a conical surface, characterized by the sum of angles around the apex being 360 degrees, ensuring that when unfolded into a plane, no gaps or overlaps occur around the apex.

3 Methodology

To create the structure seen in the One-Fold project with an apex along the fold-line (named *single-vertex crease pattern* in section II of [3]), all we need is a list of 3D vectors, named *direction-vectors*, that indicate the path of the rulings starting from the apex point (see Figure 2). The subsequent steps involve adjusting the lengths of vectors based on the boundary of the target shape.

In this section, we will explain our target surface by providing detailed descriptions and illustrations on how to fold a square paper to craft this project as a practical prototype. Then, we will describe the process of creating a conic structure, named *direction-surface*, which will be the fundamental structure for indicating the orientation of rulings. This structure will give us direction-vectors, and a special point, named *apex*, that will certify the main frame which defines the shape of the surface. To conclude the process, we use the angles between two adjacent direction-vectors to divide the unfolded plane into the parts, then obtain a series of lengths from that division, and finally adjust the lengths of direction-vectors according to the obtained values, which will be described in details in Section 3.3. Via these steps, we achieve the final surface as depicted in Figure 2. Figure 3 expresses two curves and a surface, where curves determine the shape of a surface, such as the tensile structure frame used in architecture involving membrane structures. Moreover, Figure 4 could be helpful in comprehending the impact of the direction-surface on the final surface.

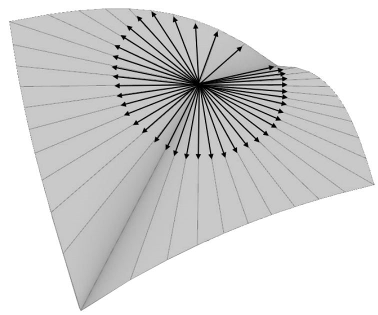


Figure 2: Arrows on the surface represent direction-vectors.

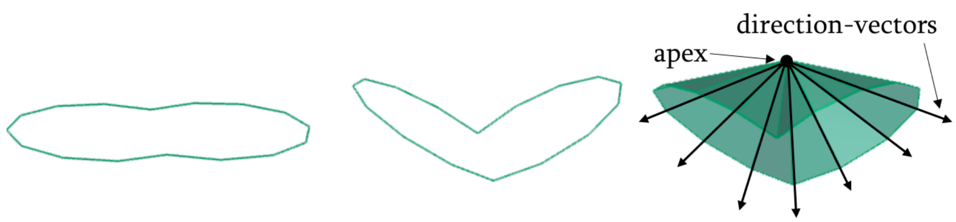


Figure 3: Steps of emerging direction-surface can be seen from left to right. The left shape shows two curves in an unfolded state, and the middle shape shows the frame that emerges upon folding that curves and is the frame of the final surface. The right surface appears when the apex is found and the direction-vectors are the vectors that overlap on the indicated lines.

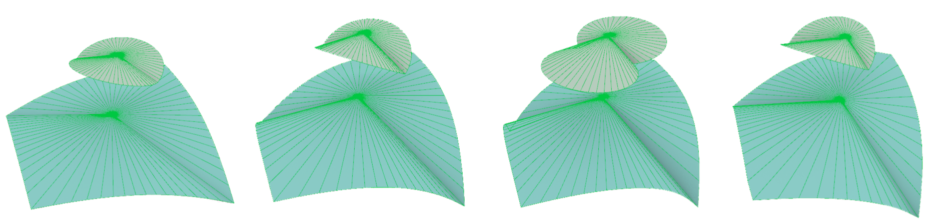


Figure 4: Final surfaces are according to the orientation of direction vectors. Upper surfaces indicate the direction-surface and the lower surfaces are the final surfaces.

3.1 A One-Fold Origami with an Apex Along the Fold-Line

Our shape emerges when a quadrilateral planar sheet is folded along one of its diagonals, as depicted in Figure 5 for a square plane. After folding from one side, the corners are then opened and brought downwards, with a point along that diagonal serving as the apex. Folding the sheet in the opposite direction to the initial fold completes the process, resulting in the final surface. Sample prototypes made with square sheets of paper are shown in Figure 6.

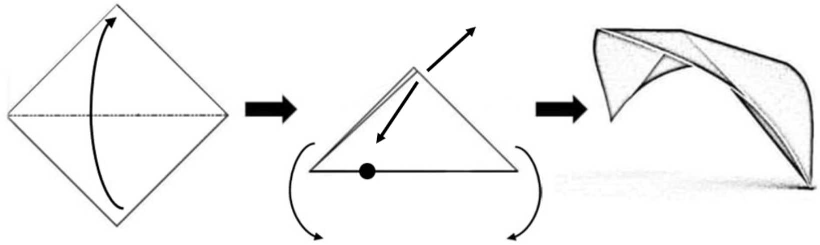


Figure 5: The unfolded and folded state of the paper, the fold line has been reversed.

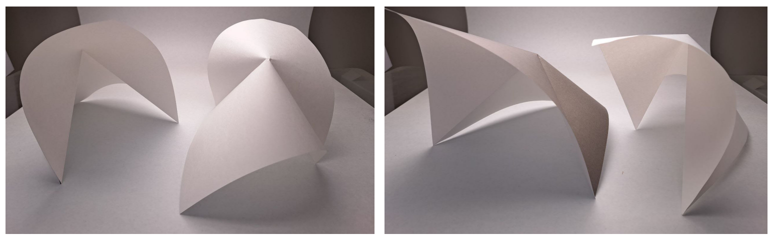


Figure 6: Example of prototypes made by the authors.

3.2 Direction-Surface

One-fold project is made from a square sheet, but here we will explain the more general shape of the sheets (we will explain the process for non-square cases in the section 3.4). We aim to develop a method for folding a 2D shape along a fold-line with a hypothetical point on it serving as the position of the apex on the plane, denoted as H, depicted in Figure 7.

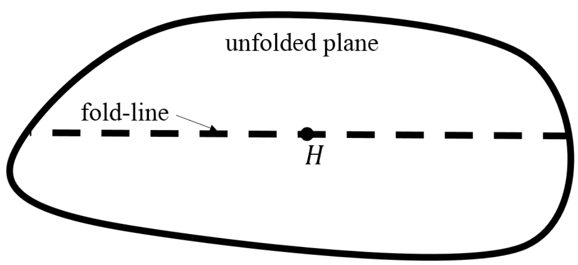


Figure 7: Instance of a planar convex shape with a given fold-line and a position for apex.

The one-fold surface has a kind of concavity along the fold-line (Figure 9), and we start with taking some steps to apply the concavity to the final surface as well. We are going to describe how we encounter concavity and determine some variables to control the intensity of concavity. To accomplish the process, we implement two same curves of a circular arc with an arbitrary, appropriate radius, where they are going to play the main role in achieving the orientation and directions of the rulings of the final surface, (Figure 8). We refer to these curves as “frame-curves” and call the common line between their endpoints the “f-line”.

From now we will try to make a surface using frame-curves, in a way that the sum of the angles around the apex become 360 degrees in order not to create a tear in the development, and the sum of the angles around the apex in one sector becomes 180 degrees. We name these two conditions together the “angle condition around the apex”. As can be seen, our conic surface would be made by a number of triangles, where the apices of triangles overlap the

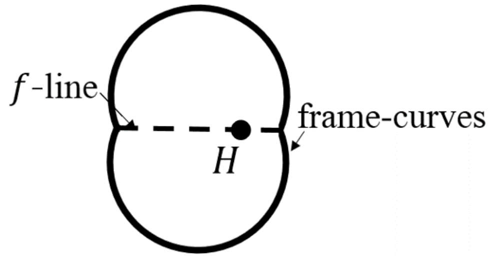


Figure 8: *Frame-curves:* Two same curves of a circular arc connected together over a line segment and the position of a point H on the line segment which could be anywhere on it, where the apex will appear exactly above that.

apex of the surface. We will see that for various positions of H on f-line, the amount of fold angles of direction-surface will be distributed differently. The readers must be careful that the direction-surface is a part of the conical surface that is bounded by frame-curves which hold the angle condition around the apex. This feature of it will guarantee the developability of the final surface. In other words, the direction-surface is a surface that the final surface lies on it.

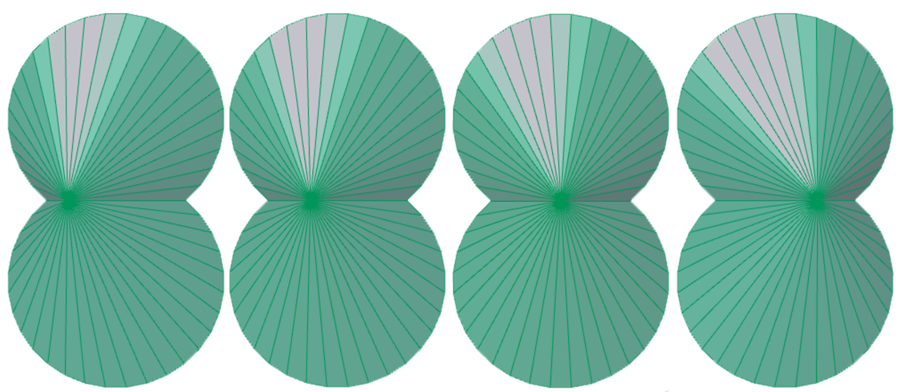


Figure 9: Result of various positions of H on f-line for direction-surface.

Note that we will make the surface on one side of the fold-line, the other side will be made with the same process. To begin with, consider a curve of a circular arc on a plane where its central angle is $180° + 2\alpha$, for some α between 0 and $90°$, as shown in Figure 10.

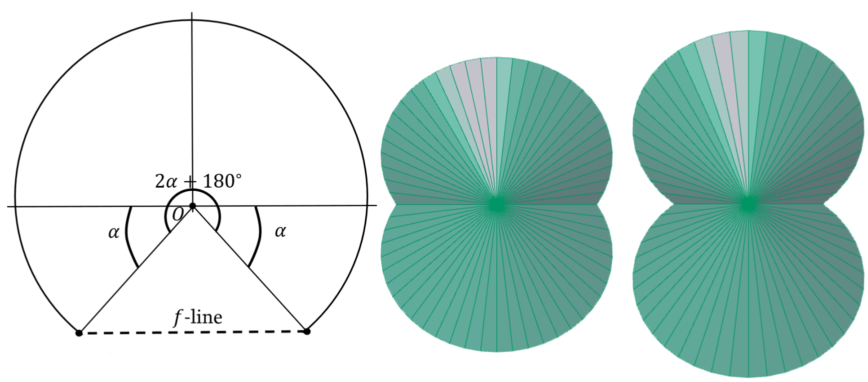


Figure 10: Left: Frame-curve defined by a circular arc with a central angle larger than $180°$. Right: Two render results for $\alpha = 30°$ and $\alpha = 50°$.

Generally, increasing the central angle from 180° to 180° + 2α implies decreasing the length of f-line. We will implement this feature to apply concavity to the surface and we will see that decreasing the length of f-line, increases the intensity of the concavity along the fold-line as shown in Figure 10.

The value of α is the most efficient variable in our process to control the concavity of the fold-line. When $\alpha \to 0°$ the intensity of concavity decreases and for $\alpha \to 90°$ the intensity of concavity increases.

We continue with setting the half of the frame-curves on 3D coordinate system such that it becomes a subset of $\{(x, y, z) \in \mathbb{R}^3 | \ y \geq 0\}$, f-line locates on x-axis and H be the point $(0, 0, 0)$. Moreover, for a given angle β we rotate the frame-curves such that the angle between the plane of frame-curves and y axis becomes β. The angle β is another variable that we use to control concavity and other features of the surface, specifically, it controls the slope of the direction-surface (Figure 12), and consequently that of the final surface. Figure 11 depicts the frame-curves and highlights the angle β. Additionally, in Figure 12, the effects of altering β on direction-surface are depicted.

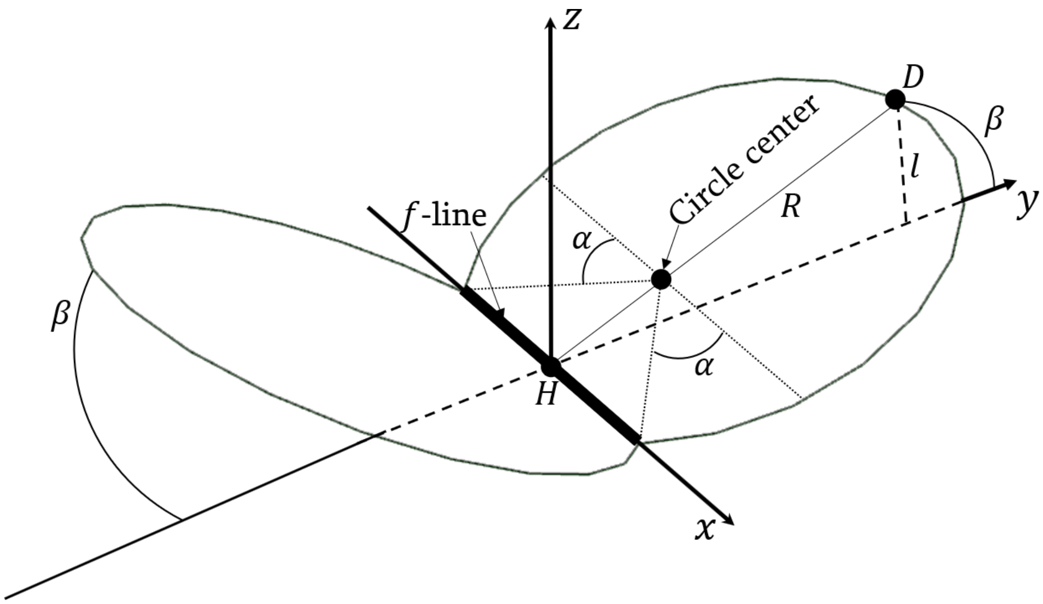
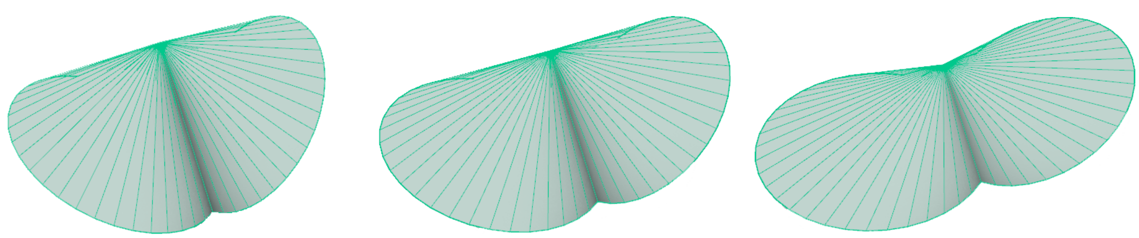


Figure 11: Position and orientation of direction-curves on 3D space.



$\beta = 40°$ $\beta = 30°$ $\beta = 15°$

Figure 12: The impact of β on the frame-curves and direction-surface.

Now we search for a point, called apex $A(0, 0, h)$, along z-axis in the 3D space such that A on the surface satisfies the angle condition around the apex. We utilize the binary search method to determine the value of h. In this process, we take into account the upper bound

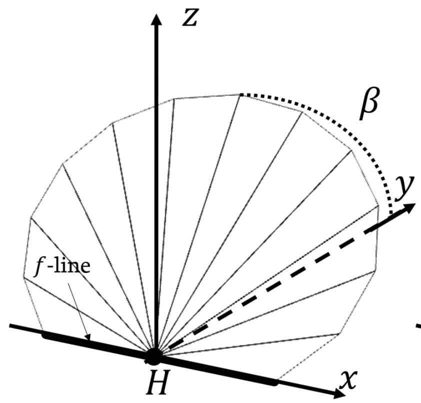


Figure 13: Finding an appropriate position for the apex that satisfies the angle condition around the apex.

as well as the lower bound, which are $2R$ and $l = (R + R\sin\alpha)\sin\beta$, respectively (Figure 11). Figure 13, represents the various surfaces that can be made via changing the value of h, where we use the binary search method to find the best position for A according to the angle condition around the apex.

Consider the surface that we attain via points of divisions of the half of frame-curves and point A. We obtain the second half of the direction-surface through reflection on the xz-plane.

According to previous explanations, direction-surface refers to a surface that the final surface coincides with and it dictates the orientation of the rulings on the final surface. Once the apex is correctly positioned and we are certain about the sum of angles at the apices of triangles, we obtain the direction-vectors that dictate direction to the rulings of the final surface.

3.3 Final Surface

The final surface is the result of the resized and adjusted version of the direction-surface. To get the final surface we need to resize the length of direction-vectors to the desired size which would be achieved from the 2D plane that would be folded along its specified fold-line. In order to maintain the angles of triangles, only the length needs to be changed while keeping the direction of the vector. We refer to the angles of the triangles incident to the apex as γ_i's (Figure 14).

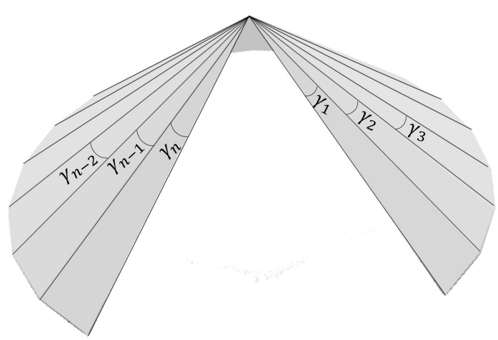


Figure 14: Total sum of γ_i's is 180° degrees due to the angle condition around the apex.

The obtained γ_i's are used for dividing the unfolded plane in any side of the fold-line. We set the unfolded convex shape over xy-plane in such a way that the fold-line overlaps on x-axis and divide it using γ_i's, starting from the positive direction of x-axis.

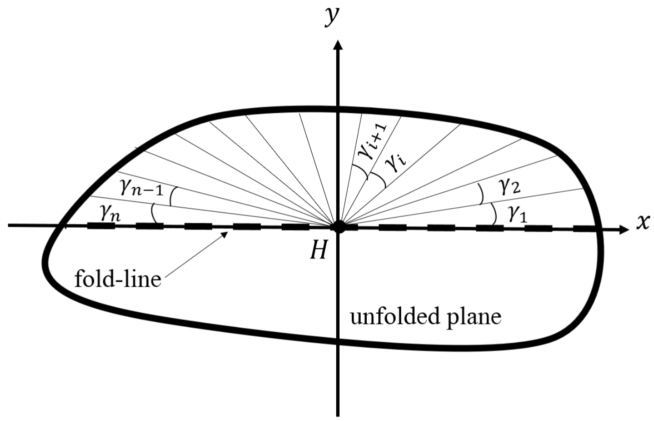


Figure 15: Dividing the flat unfolded plane to attain lengths of final rulings.

Although the shape of the final surface is limited to one that is a rectangle when flattened in the One-Fold project, our method can handle more general shapes. Under the condition that rulings are not fragmented, we can deal with sheets of convex or star-shaped polygons. In Figure 15, a more general shape is used to illustrate this concept.

3.4 One-Fold Surface of Quadrilaterals

We describe the process that works for quadrilateral planes with specific conditions. If r, s be two non-zero values then the equation $\frac{x}{r} + \frac{y}{s} = 1$ indicates a line in 2D space that passes through r of x-axis and s of y-axis. Using the polar coordinate system we would able to specify the coordinates of the points on the line by the corresponding angle with respect to the positive side of x-axis. Setting $f(\gamma) = \frac{rs}{s\cos\gamma + r\sin\gamma}$ we get the equation of the line with variable γ, where γ indicates the angle between the x-axis and the vectors from origin to a point on the line. Every quadrilateral is bounded with four lines and we choose a list of quadrilaterals with the condition that their vertices overlap coordinate axes and we will make the One-Fold surface for them.

Having found γ_i's according to the appropriate direction-surface (Section 3.3), we divide the upper half of the quadrilateral into a number of triangles, see Figure 16.

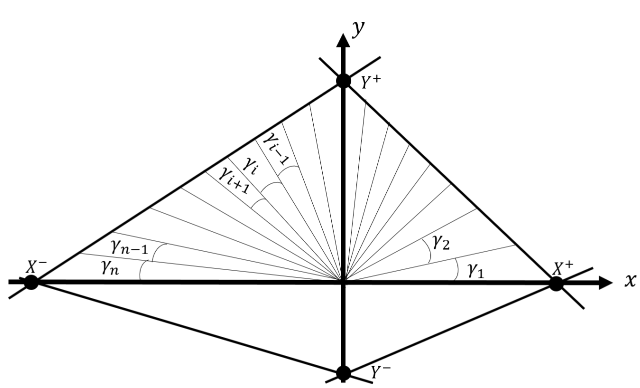


Figure 16: Dividing by γ_i's.

This division gives us the length of the legs of triangles, where they will be the value of the length that we want to apply for the direction-vectors and get the final surface. In Figure 17 we share the result of rendering for various direction-surfaces and various quadrilaterals.

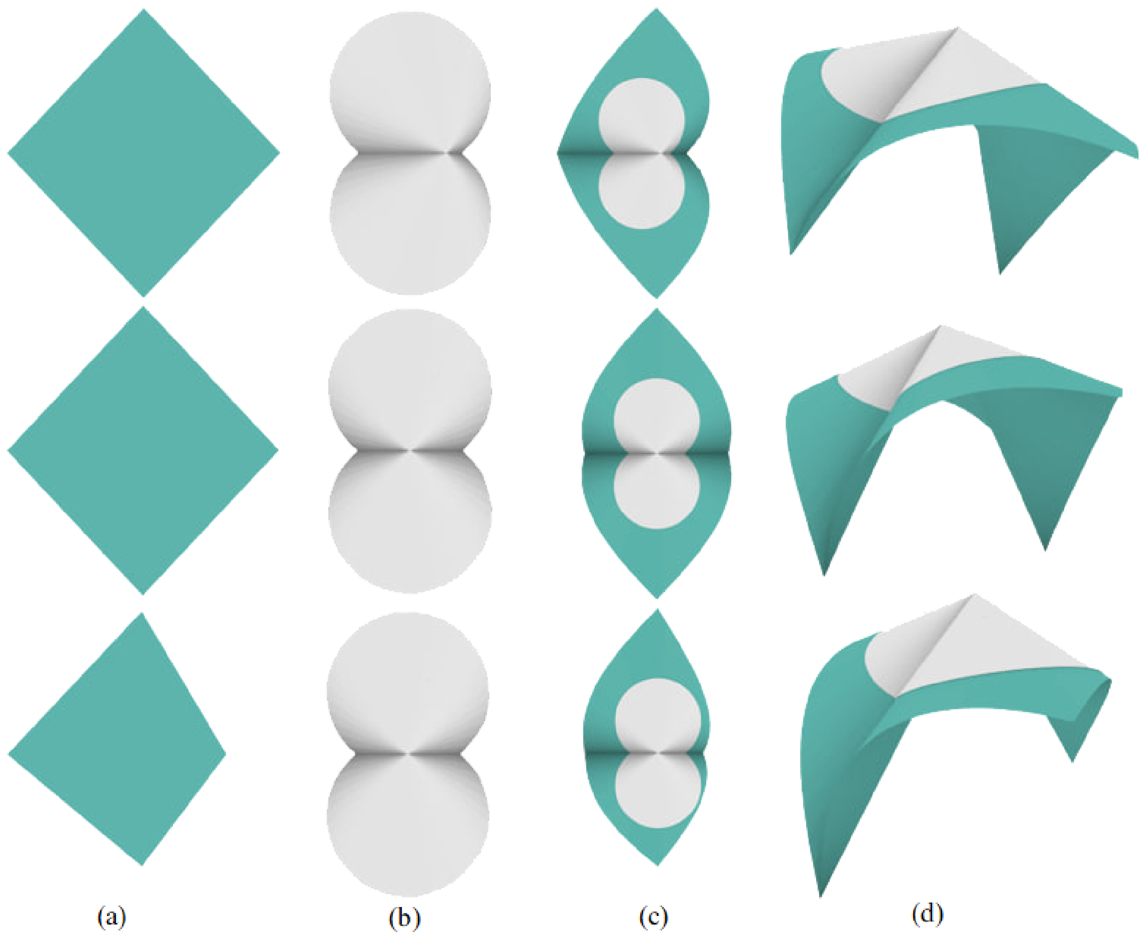


Figure 17: From Left: (a): Unfolded state, (b): Direction-surface, (c),(d): Final surfaces. In the first two rows, quadrilaterals are the same, but direction-surface are not. In the 2nd and 3rd rows direction surfaces are the same, but quadrilaterals are not.

Now we have lengths and direction vectors and we will resize them according to the corresponding angles, lengths, and positions. This process will accomplish one side of the fold line. The same process will make the other side of the surface and the final surface will be accomplished.

Defining new boundaries for planes instead of quadrilaterals and using γ_i's to divide them, we could make various surfaces such as star-shaped polygons. Two examples can be seen in the Figure 18.

4 Numerical Results

We defined several variables in the software to express various models of the surface which we describe here. There are two main scopes of codings in the software, the first one produces the coordinates of direction-vectors, and the second one uses direction-vectors to create the final surface for quadrilaterals. Important input values of the first one are as below:

- α (degree): discussed in Section 3.2 (Figure 10).

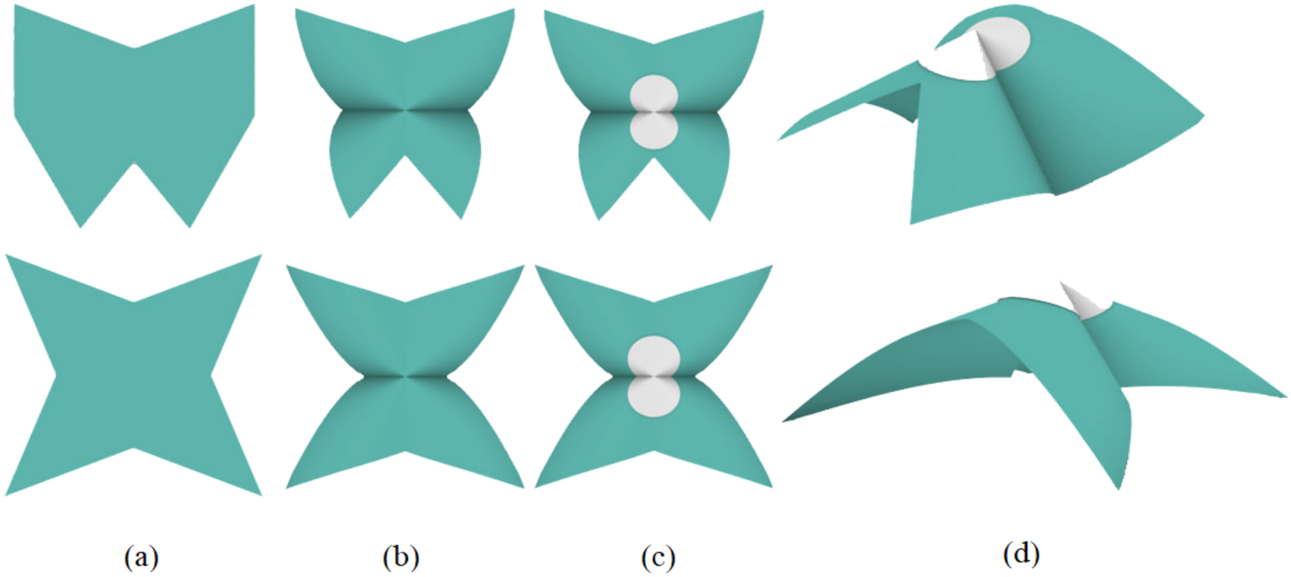


Figure 18: From Left: (a): Unfolded state, (b),(c),(d): Final surfaces. The First column is planes and the rest of them are their (with and without indicated direction-surfaces) final surfaces from different perspectives.

- β (degree): discussed in Section 3.2 (Figure 11).
- H_c: represents the position of point H on the f-line (Figure 9).
- *number_of_division*: represents the number of triangles we assumed for precision.

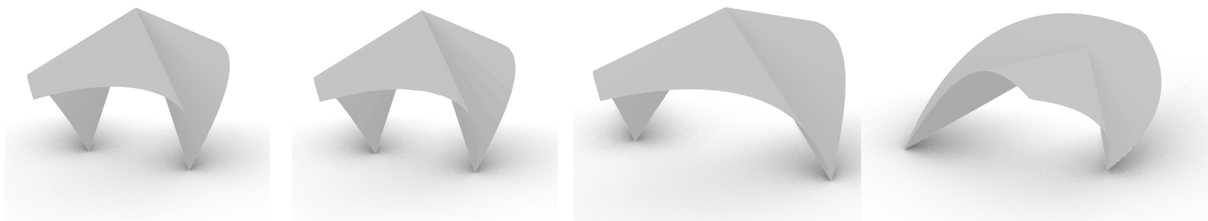
Besides, input values of the second scope of code are as follows where they indicate the position of vertices of the quadrilateral (Figure 16):

- X^+: any point on the positive side of x-axis
- X^-: any point on the negative side of x-axis
- Y^+: any point on the positive side of y-axis
- Y^-: any point on the negative side of y-axis

We evaluate the time of response of the software to get the results for various amount of variables. The PC used for the experiments is a standard laptop PC with the following specifications, Processor: AMD Ryzen 5 5500U, 4 GHz, With Radeon™ Graphics Memory: 8 GB DDR4 RAM, Operating System: Windows 11. The results of the computation time measurements and amount of error for the angle condition around the apex are presented in Table 1 and corresponding Figure 19. In this table, the column of "error" indicates how far the angle condition around the apex is from 360°.

Table 1: Performance based on various settings of values α, β, *number_of_division* and dimensions of quadrilateral plane.

input values (H_c, α, β, # *of division*, Y^+, Y^-, X^+, X^-)	error(deg) 360° condition	time(s) direction-vectors	time(s) final surface
(0, 50, 45, 90, 300, 300, 300, 300)	1.5×10^{-5}	1.1×10^{-2}	1.4×10^{-2}
(0, 50, 45, 20, 300, 300, 300, 300)	1.5×10^{-5}	1.0×10^{-2}	1.2×10^{-2}
(0, 32, 34, 90, 300, 300, 300, 300)	7.0×10^{-5}	4.3×10^{-2}	4.6×10^{-2}
(68, 32, 34, 90, 300, 96, 103, 201)	1.2×10^{-4}	4.2×10^{-2}	4.6×10^{-2}



Row 1 Row 2 Row 3 Row 4

Figure 19: Performance of software about 4 of quadrilaterals as depicted in Table 1.

5 Discussion and Conclusion

A digital module for the creation of a specific temporary and sustainable architectural structure based on origami science is presented in our study. The structure is based on Patkau Architects' One-Fold project, which uses steel sheets as the basic material. Our module uses the Rhino-Grasshopper environment to create a shape with a single fold that can be considered a deployable shelter surface for architectural use. The module allows the user to control and change the value of the angle of the fold line and the apex point and visualize the 3D form of the structure. The angle of the fold line and the apex point, which are our module's primary parameters, can be used to assess the similarity and variance between shapes. The curvature of the shape is determined by the angle of the fold line, while the height and symmetry of the shape are determined by the apex point. The user can view the design area and create new forms and expressions based on that structure by adjusting those parameters.

The results of our study show that it is possible to create a 3D developable origami structure using a single fold and Rhino-Grasshopper system. By incorporating a mathematical model into the CAD environment, we were able to generate a variety of shapes while retaining the origami characteristics of the structure; this method can even be applied to non-symmetrical configurations. The ability to create a 3D developable origami structure using a simple mathematical model and a Rhino-Grasshopper system has exciting implications for the field of architecture. This approach could lead to the development of temporary and sustainable structures easily adapted to various environments. Our research has demonstrated the effectiveness of incorporating origami science into a conceptual architectural design and highlights the potential for future applications in sustainable design practices. Our proposed system has the potential to inspire further exploration into the use of origami science in architecture and encourage innovative thinking in sustainable design. The proposed 3D modular design has some limitations regarding the feasibility and realism of the shape generation. One of the limitations is that the design needs to consider the method of constructing the shape that can be supported by two main bases, which are essential for the stability and balance of the structure. Ano To solve this problem, it is possible to obtain a shape close to the actual model by optimizing the generated model to find a shape that minimizes bending energy, or by applying physical simulation. In addition, we will actually create a physical model and compare it with the digital model obtained using this method.

Acknowledgments

This work was supported by JST CREST Grant Number JPMJCR1911, Japan.

References

[1] S. Z. Bacinoğlu, L. Piskorec, and T. Kotnik: *CURVED. IT: A design tool to integrate making with curved folding into the digital design process.* A/Z ITU Journal of Faculty of Architecture **16**(1), 11–27, 2019.

[2] L. M. Fonseca, G. V. Rodrigues, and M. A. Savi: *An overview of the mechanical description of origami-inspired systems and structures.* International Journal of Mechanical Sciences **223**(107316), 2022.

[3] R. Foschi, T. C. Hull, and S. K. Jason: *Explicit kinematic equations for degree-4 rigid origami vertices, Euclidean and non-Euclidean.* Physical Review E **106**, 055001, 2022.

[4] P. Jackson: *One Crease* `https://www.origami-artist.com/artwork/one-crease/`.

[5] P. Jackson: *Folding Techniques for Designers: From Sheet to Form (How to fold paper and other materials for design projects).* Laurence King Publishing, 2011.

[6] D. Meissl: *Festival Hall in Erl.* `https://www.archdaily.com/306296/festival-hall-in-erl-delugan-meissl-associated-architects`.

[7] M. Meloni, J. Cai, Q. Zhang, D. Sang-Hoon Lee, M. Li, R. Ma, T. E. Parashkevov, and J. Feng: *Engineering origami: A comprehensive review of recent applications, design methods, and tools.* Advanced Science **8(13)**(2000636), 31, 2021.

[8] J. Patkau, P. Patkau, T. Brown, J. E. T. Schroeder, and L. Stern: *One fold.* `https://patkau.ca/projects/one-fold/`.

[9] P. M. Reis, F. L. Jiménez, and J. Marthelot: *Transforming architectures inspired by origami.* Proceedings of the National Academy of Sciences **112**(40), 12234–12235, 2015.

[10] M. C. Ryan: *Klein Bottle house.* `https://www.archdaily.com/7952/`.

[11] C. Yuan, N. Cao, and Y. Shi: *A Survey of Developable Surfaces: From Shape Modeling to Manufacturing.* arXiv:2304.09587v2 2023.

Received February 13, 2024; final form July 12, 2024.