

Strengthening Higher-Order Thinking Skills in 3D Geometry: Problem Solving or Direct Instruction?

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Abstract. Evidence shows students' higher-order thinking in solid (3D) geometry remains weak, partly because classes are dominated by lecture and routine exercises. Yet few studies have directly compared problem-solving instruction with traditional direct instruction on 3D-geometry HOTS, while controlling for prior ability. This gap limits clear guidance for classroom practice. In this quasi-experimental study, we compared a problem-solving approach versus traditional direct instruction for enhancing HOTS in high school solid geometry. Two intact eleventh-grade classes ($n = 20$ each) in Indonesia were assigned to one of the two methods. Both groups took identical pre- and post-tests of challenging geometry problems (essay tasks on cube and pyramid sections, angles, and distances) and participated in brief post-unit interviews about their learning. Pre-post gains were analyzed via ANCOVA (controlling pretest scores) and effect sizes. The results showed that the problem-solving group achieved significantly higher gains in the HOTS indicators (argument clarity, multiple representations, cross-disciplinary connections) than the direct-instruction group (adjusted $F(1, 37) = 55.18$, $p < .001$, $\eta^2 \approx 0.60$; Cohen's $d \approx 0.82$). Qualitative feedback echoed this: students in the problem-solving class reported deeper engagement with spatial concepts, while many in the lecture-based class noted passive learning. These findings align with recent research: inquiry-based and active strategies strengthen mathematical thinking more than teacher-centered methods. This study is among the first to quantify the effect of a problem-solving curriculum on HOTS in 3D geometry, showing that replacing procedural lectures with rich problem tasks substantially strengthens students' geometric reasoning. The implications are clear: to prepare students for STEM fields, geometry teaching should shift toward student-centered, problem-based activities that explicitly target spatial visualization and critical analysis.

Key Words: higher-order thinking skills (HOTS), solid geometry, problem-solving instruction, direct instruction, spatial visualization, quasi-experimental design

MSC 2020: 97G40 (primary), 97D50, 97D60

1 Introduction

Geometry is widely recognized as a foundation for STEM education, fostering visualization, intuition, critical thinking, problem solving, deductive reasoning, and logical argumentation [1]. Solid geometry (3D shapes) in particular demands spatial visualization and integration of multiple representations. However, many students perform poorly in geometry: international assessments often show geometry subscale scores trailing other math areas, due to weak spatial skills and conceptual gaps [9]. For example, a global study found 8th-graders across 60+ countries scored lowest on geometry items [5]. In our context, Indonesian studies report students' HOTS in geometry are low; Rasch analysis of HOTS geometry items found an average ability below zero logit [6]. Addressing this requires effective pedagogy.

Traditional geometry instruction often relies on lectures and memorization. Such direct instruction is teacher-centered: the teacher presents concepts and solution steps while students listen and practice routine exercises [2]. Critics note this approach “does not enable meaningful and active learning” and gives few opportunities for student reasoning. In contrast, student-centered methods — such as problem-based learning (PBL) or collaborative problem solving — engage students in grappling with challenging tasks. These active strategies have been shown to improve reasoning: for instance, an inquiry-based (IBL) approach yielded greater gains in mathematical thinking than traditional lectures. Similarly, [3] found that a framework combining inquiry learning and collaboration “significantly enhances higher-order thinking” in math. In geometry education, hands-on or PBL tasks have also shown promise: [8] reported that hands-on activities in geometry increased problem-solving performance and reduced cognitive load, while [2] found that a PBL model significantly improved students' spatial reasoning ability. Guided inquiry methods in solid geometry likewise produced very large achievement gains ($d > 1.3$) compared to traditional teaching.

Despite this evidence in related contexts, few studies have directly compared problem-solving versus direct instruction specifically for 3D geometry HOTS. Many geometry studies focus on 2D topics or basic achievement; systematic research on HOTS (e.g. argumentation, creativity in geometry) is scarce. This study addresses that gap by using a quasi-experimental design to compare two approaches in teaching solid geometry to high-schoolers: one emphasizing non-routine problem solving, group discussion, and exploration, versus the other using conventional direct instruction. We hypothesize that the problem-solving approach will produce larger gains in HOTS (spatial reasoning, critical thinking, integrative reasoning). The novelty lies in quantifying how these pedagogies affect higher-order outcomes specifically in 3D geometry, with rigorous statistical controls.

2 Method

Design and Sample: We conducted a quantitative quasi-experiment with a non-equivalent pretest-posttest design. Two intact eleventh-grade classes (XI SCIENCE 1 and XI SCIENCE 2) at SMA Santa Maria Cirebon (a high school in Indonesia) were selected. Each class had 20 students. One class was assigned as the experimental group (Problem-Solving instruction)

and the other as control (Direct Instruction). Assignment was by administrative convenience rather than random, reflecting typical classroom settings.

Instructional Conditions: Both groups covered identical geometry content (solid geometry: cubes, prisms, pyramids; topics detailed below) over a unit of several weeks. In the Problem-Solving class, lessons centered on open-ended problems. Students worked individually or in small groups to tackle non-routine tasks requiring them to draw diagrams, form conjectures, and apply multiple solution strategies. The teacher facilitated by posing guiding questions and encouraging discussion, but solutions were primarily student-derived. In the Direct Instruction class, teaching was teacher-centered: the instructor presented formulas and solved example problems step-by-step on the board, with students practicing similar problems individually. This mirrors the lecture-style approach described as direct instruction.

Instrument: We developed a structured 10 essay test to measure HOTS in 3D geometry, based on the provided pretest items. The test covered three content areas: (1) Cross-sections: e.g. “Given cube ABCD.EFGH with side 4 cm, what is the shape and area of the cross-section through points P , C , Q (where P and Q are specified midpoints)?” (2) Angles: e.g. “Find the angle between segment AH and segment EG in the cube.” (3) Distances: e.g. “In the same cube, find the distance from point A to plane BDF .” Items required multi-step reasoning, spatial visualization, and connecting different representations (drawing views, using 3D geometry formulas, etc.). We refined wording for clarity but kept contexts equivalent. A sample item: “In cube ABCD.EFGH (edge 4 cm), what is the area of triangle $A-C-H$?” was included.

Student responses were scored for both accuracy and reasoning quality. We used a reasoning rubric to evaluate higher-order aspects, including: clarity of geometric argument (justification of answer), use of multiple representations (e.g. diagrams, algebra), and making cross-disciplinary or real-world connections (e.g. relating 3D problems to planar analogues or contexts). Two raters independently scored a subset of responses to ensure consistency. Pretest (before instruction) and posttest (after instruction) used the same test. In addition, we conducted semi-structured student interviews after the unit. Typical interview questions included: “How did the way geometry was taught in class affect your understanding of these problems?”, “What strategies did you use to visualize or solve the problems?”, and “What was the most challenging or helpful part of learning this geometry unit?” These questions probed students’ perceptions of the learning process under each approach.

Analysis Plan: We computed descriptive statistics for pretest and posttest scores in each group. Pre-post gain was calculated for each student. To compare groups while controlling for any pretest differences, we performed an ANCOVA with posttest score as the dependent variable, teaching method as the fixed factor, and pretest score as the covariate. Effect size was assessed via partial eta-squared from ANCOVA and Cohen’s d on group gains. We also qualitatively analyzed interview transcripts for themes about engagement, strategy, and perceived HOTS development.

3 Results and Discussion

3.1 Results

Descriptive and Gain Scores: Both groups began with similar geometry ability. The mean pretest score (out of 100) was 50.0 ($SD \approx 8.7$) in the problem-solving group and 50.0 ($SD \approx 12.2$) in the direct-instruction group (Table 1). After instruction, the problem-solving

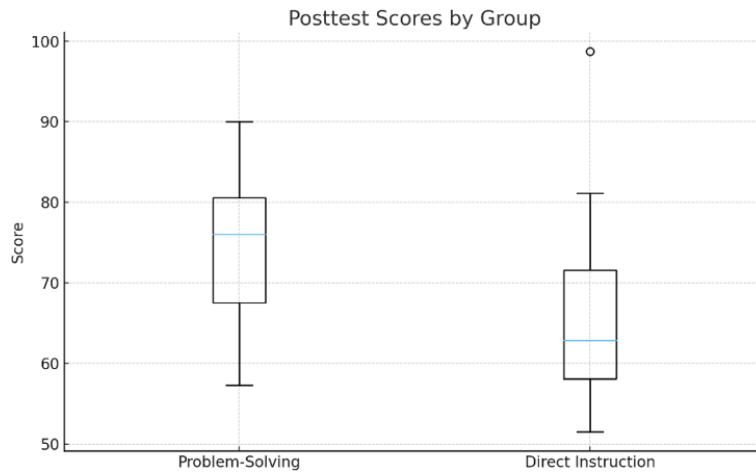


Figure 1: Distribution of posttest scores by instructional method.

class had a higher mean posttest ($M = 73.0$, $SD = 10.6$) than the direct-instruction class ($M = 63.2$, $SD = 13.1$). The mean gain (post-pre) was $+23.0$ ($SD \approx 4.5$) for problem-solving, vs. $+13.2$ ($SD \approx 3.8$) for direct instruction. The difference in gains (≈ 9.8 points) corresponds to a large Cohen's $d \approx 0.82$ favoring the problem-solving group.

Table 1: Descriptive statistics for geometry test scores by group ($N = 20$ each). Gains = (posttest – pretest).

Group	N	Pretest Mean	Pretest SD	Posttest Mean	Posttest SD
Problem-Solving	20	50.0	8.7	73.0	10.6
Direct Instruction	20	50.0	12.2	63.2	13.1

Group	Gain Mean	Gain SD
Problem-Solving	23.0	4.5
Direct Instruction	13.2	3.8

ANCOVA: Controlling for pretest scores, the group effect on posttest was highly significant. ANCOVA showed a significant difference between methods: $F(1, 37) = 55.18$, $p < .001$, partial $\eta^2 = 0.60$. This indicates that, after accounting for initial ability, the problem-solving group scored significantly higher on the posttest. (The pretest covariate itself was also significant, $F(1, 37) = 274.7$, $p < .001$, reflecting expected score correlation.) Table 2 summarizes the ANCOVA.

Table 2: ANCOVA predicting posttest geometry score (controlling pretest).

Source	df	F	p	Partial η^2
Pretest (covariate)	1, 37	274.7	$< .001$	—
Instructional method	1, 37	55.18	$< .001$	0.60

The problem-solving group exhibits a higher median and upper quartile than the direct-instruction group, which shows greater spread and one high outlier (≈ 99). Scores range from 0 to 100, with $N = 20$ per group. As shown in Figure 1, the distribution of posttest scores in the problem-solving class is shifted upward relative to the direct-instruction class, consistent with the ANCOVA result ($F(1, 37) = 55.18$, $p < .001$).

Interview Findings: Analysis of interviews revealed distinct student perspectives in each condition.

Problem-Solving group: Most students described increased engagement and deeper thinking. For example, one student said, “I had to draw the cube and really picture the triangle in it. Solving it on my own helped me understand why the formula works.” Another noted, “We talked a lot in groups and shared different ways to see the shape. That made the answer clearer to me.” Several students explicitly mentioned connecting 3D figures to 2D analogies or real-life objects, indicating the use of multiple representations. This suggests they were actively using spatial visualization and reasoning.

Direct-instruction group: Students often emphasized clarity but also passivity. One commented, “The teacher explained everything step by step, so I knew what to do. But when I got a new problem, I was stuck because I just followed the example.” Another said, “I understood the steps in class, but I felt like I wasn’t really thinking about why things worked.” These responses imply that while explanations were clear, students viewed learning as following procedures rather than engaging critically.

In summary, interview themes mirrored the quantitative results: the problem-solving approach led students to articulate how they reasoned through 3D problems (using diagrams, discussing ideas), whereas direct instruction led to less self-generated reasoning. No student cited feeling overwhelmed in the problem-solving class; rather, they appreciated the challenge as motivating careful thinking.

3.2 Discussion

This study found that a problem-solving-based geometry curriculum produced significantly larger HOTS gains in 3D geometry than a direct instruction approach. Students in the problem-solving class not only earned higher posttest scores, but their mean gains and effect sizes were substantial (Cohen’s $d \approx 0.82$). These findings are consistent with broader research showing the superiority of student-centered methods for developing mathematical thinking. For example, [4] reported that inquiry-based learning yielded greater improvements in secondary students’ mathematical thinking than traditional lecturing. Likewise, in an international (Kazakhstan) context, [9] found that a problem-based geometry course significantly boosted students’ problem-solving skills and creativity compared to a control group. Our results extend these conclusions specifically to the domain of solid geometry HOTS.

The problem-solving method especially enhanced qualitative HOTS indicators: students showed clearer argumentation, used multiple representations (drawings, different solution paths), and connected geometric ideas. [3] similarly noted that inquiry-based frameworks with collaboration “significantly enhance higher-order thinking” in mathematics, highlighting benefits in reasoning and creative problem solving. In contrast, the direct instruction group improved less on these dimensions. Interviews indicated they often lacked deep spatial reasoning; they “understood steps” but “weren’t thinking about why,” echoing observations that lecture-based methods can leave students passive.

From the cognitive perspective, these results align with embodied cognition theories. Hands-on and problem-oriented tasks tend to produce richer mental representations. [8] found that giving students physical tasks in geometry both improved solution performance and reduced cognitive load. In our study, even without special equipment, asking students to actively visualize and construct solutions appears to have had a similar effect: students reported drawing models and working through problems themselves, which likely strengthened

spatial understanding.

Notably, prior Indonesian studies have underscored a general deficit in geometry HOTS: [6] found students' geometry HOTS were substantially below the desired level. Our intervention demonstrates one way to address this gap. [7] also found that problem-based learning significantly improved students' spatial reasoning. Our findings build on this by showing that a structured problem-solving approach not only improves spatial ability but also other HOTS facets (argument clarity, integrative thinking) in 3D geometry.

Comparing effect sizes, our results are notable but somewhat smaller than some intensive interventions. For example, [10] combined guided inquiry with “variation theory” and achieved very large gains in solid geometry ($d \approx 1.5$ vs control). Our study used a more standard classroom implementation, yet still produced a large effect ($\eta^2 \approx 0.60$). This suggests that even without elaborate design research frameworks, meaningful HOTS gains are achievable with problem-solving tasks.

Contributions: This research contributes to geometry education literature by providing controlled evidence on teaching method effects specifically in 3D geometry. While many studies focus on planar geometry or general math, ours directly targets solid geometry concepts — an area crucial for fields like engineering and computer science. We also integrate qualitative and quantitative data, illustrating how students' learning experiences differ under each pedagogy. The study thus highlights the practical importance of method choice for STEM readiness: geometry is well-known to underpin spatial reasoning needed in science and engineering, so strengthening geometry HOTS can have long-term benefits.

3.3 Implication of Research

These findings have clear implications for teachers and curriculum designers. Teachers should incorporate more non-routine problem tasks and collaborative exploration in geometry lessons. Instead of merely demonstrating solutions, teachers can pose challenging puzzles (e.g. finding volumes via cross-sections, or angles in cubes) and guide students to reason through them. This approach scaffolds HOTS such as analysis and representation skills. For example, drawing 3D diagrams or constructing nets for solids can be routinely integrated. The interview feedback suggests students appreciate and benefit from such engagement — they remembered concepts better when they figured them out themselves.

Curriculum: Textbooks and standards should allocate more emphasis to HOTS problems in solid geometry (not just formula application). Algebra and geometry curricula often separate topics; our results support curricula that intertwine conceptual reasoning. Curricula might include explicit objectives for “constructing and evaluating geometric arguments” and “connecting spatial problems to real-world contexts.” Teacher guides can provide problem sequences that gradually build reasoning skills, moving from guided examples to open problems.

STEM Education: The study reinforces the role of geometry in STEM literacy. Geometry tasks train spatial thinking crucial for disciplines like physics (visualizing forces in 3D), chemistry (molecular shapes), and engineering (designing 3D structures). Strengthening geometry HOTS thus prepares students for advanced STEM learning. Educational policy should thus recognize geometry not as an isolated subtopic, but as a lever for developing overall critical thinking and spatial ability. For example, integrating geometry tasks with technology (simulations, 3D printers) could further enhance engagement and HOTS.

3.4 Limitation

Several limitations must be acknowledged. First, the sample size was small ($N = 40$) and drawn from a single school, limiting generalizability. The two classes may have had unmeasured differences (though pretests were matched). Second, teacher effects were not controlled — the problem-solving lessons may have been delivered with different teacher enthusiasm or skill than the direct instruction. Third, our pre-post test only measured immediate gains; we did not assess long-term retention or transfer of HOTS to new problems. Fourth, our qualitative data (interviews) were limited to a few illustrative quotes; a fuller analysis with more interviews would strengthen the conclusions. Finally, scoring HOTS with a rubric involves subjectivity. We mitigated this with multiple raters on a subset, but perfect reliability is hard to achieve. Future studies should include more schools, random assignment if possible, and long-term follow-up.

4 Conclusion

In summary, this study demonstrates that a problem-solving approach in teaching 3D geometry substantially outperforms traditional lecture-based instruction in developing higher-order thinking. Through structured exploratory tasks, students not only learned solid geometry formulas but also practiced reasoning and representation. The strong performance gains and positive student feedback underscore the value of active, inquiry-based pedagogy for geometry. These results suggest educators should shift from procedure-focused lessons to rich problem-based activities. By doing so, teachers can strengthen students' critical thinking and spatial reasoning in geometry, laying a stronger foundation for STEM learning.

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