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Beyond Euclid: A Mixed-Methods Analysis of Geometry in IB and Cambridge Curricula and a Projective Framework for Reform

Duc Nghia Vu[1], Thanh Son Do[2], Trang Van Vu[2]

[1] *VNU University of Economics and Business, Hanoi, Vietnam*
vuducnghia@vnu.edu.vn

[2] *VNU University of Science, High School for Gifted Students, Hanoi, Vietnam*
thanhsondo240@gmail.com, vutrangvan2010@gmail.com

Abstract. International secondary mathematics curricula, exemplified by the International Baccalaureate (IB) and Cambridge International pathways, remain dominated by a static Euclidean view of geometry. The Projective Framework draws on Klein's Erlangen Program to recompose geometry education around transformation groups and invariants, with projective geometry as the unifying foundation. Critics argue that existing curricula lack a structural narrative, treat non-Euclidean ideas as peripheral, and fragment geometric topics (Jones, 2000). This paper first provides a qualitative comparative analysis of the IB and Cambridge geometry curricula, highlighting structural gaps and the absence of a unifying theory. It then conducts a systematic quantitative content analysis of 321 geometry standards from IB MYP, DP, Cambridge IGCSE, and AS/A Level syllabi, categorizing each into static Euclidean, transformational isometries, coordinate/analytic, group-theoretic/unifying, and non-Euclidean codes. The coding reveals that 69%–84% of all standards are devoted to static Euclidean properties; transformation-based standards never exceed 12% and are completely isolated from group concepts; no mandatory component addresses geometric unification or non-Euclidean geometry. These findings quantitatively confirm the fragmentation and theoretical void that the Projective Framework aims to fill. The paper concludes by integrating the empirical evidence with the framework's design, proposing research-informed design principles for an infusion-based implementation strategy mapped to existing assessment objectives. The research provides both a diagnostic critique and a pragmatically grounded implementation model for moving toward a coherent, modern geometry curriculum.

Key Words: geometry curriculum, Erlangen Program, content analysis, IB, Cambridge International, projective geometry, transformational geometry, invariants

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1 Introduction

Geometry education faces a dual challenge: a persistent decline in coherence and a growing misalignment with the structural reasoning demanded by modern mathematics and STEM fields. While Felix Klein's (1872/1893) Erlangen Program [13] unified geometry through transformation groups and invariants over a century ago, school curricula worldwide remain largely anchored in a static Euclidean paradigm. This fragmentation is not merely a mathematical concern; it has pedagogical consequences, limiting students' opportunities to develop the abstraction, generalization, and spatial reasoning skills that underpin advanced mathematical thinking. Research in geometry education emphasizes that transformational reasoning and attention to invariants support progression through van Hiele levels, foster flexible geometric schemas, and strengthen spatial competencies critical for STEM success [6, 9, 15, 24]. Yet, despite calls for reform, empirical evidence on the extent of this misalignment in influential international curricula remains scarce, and few frameworks bridge Erlangen theory with evidence-based principles of school geometry learning.

This study addresses this gap by analyzing the geometry content of the International Baccalaureate (IB) and Cambridge International pathways. While these programmes serve specific student populations, their influence extends far beyond direct enrollment. As leading providers of international qualifications, IB and Cambridge function as transnational policy benchmarks that shape textbook development, assessment design, and teacher preparation in diverse national systems, particularly in post-colonial and globalized educational contexts where they are often adopted as models of rigor and modernity [3, 20]. Moreover, the structural patterns identified in these curricula reflect broader global trends in secondary mathematics, where high-stakes assessment pressures frequently privilege procedural fluency and isolated theorems over structural coherence and transformational reasoning [16, 21]. By examining these influential pathways, this study offers diagnostic insights transferable to other jurisdictions grappling with similar fragmentation. The analytical framework and coding scheme developed here are designed as adaptable tools, enabling researchers and curriculum designers in varied contexts to audit their own geometry standards and identify structural gaps.

The paper advances the field through three interconnected contributions. First, it introduces the *Projective Framework,* a theoretically grounded model that reorganizes school geometry around Klein's hierarchy while aligning with evidence-based principles of geometric cognition and learning. Second, it provides a methodological contribution by developing a replicable coding scheme based on Erlangen categories, offering the field a systematic tool for analyzing the structural emphasis and unifying content of geometry curricula. Third, it presents empirical findings from a mixed-methods analysis of 321 standards across IB and Cambridge secondary programmes, quantifying the dominance of static Euclidean content and the complete absence of group-theoretic or non-Euclidean perspectives. These findings reveal systemic gaps that hinder the development of transformational reasoning and suggest critical inflection points for reform. Finally, the paper proposes research-informed design principles for an infusion-based implementation strategy that demonstrates how projective and group-theoretic ideas can be integrated into existing curricula without compromising assessment readiness, offering a pragmatic pathway for curriculum renewal that respects high-stakes examination constraints. The research questions guiding this study are:

Qualitative: What are the structural and pedagogical characteristics of geometry in the IB and Cambridge pathways, and what critical gaps emerge when analysed through the

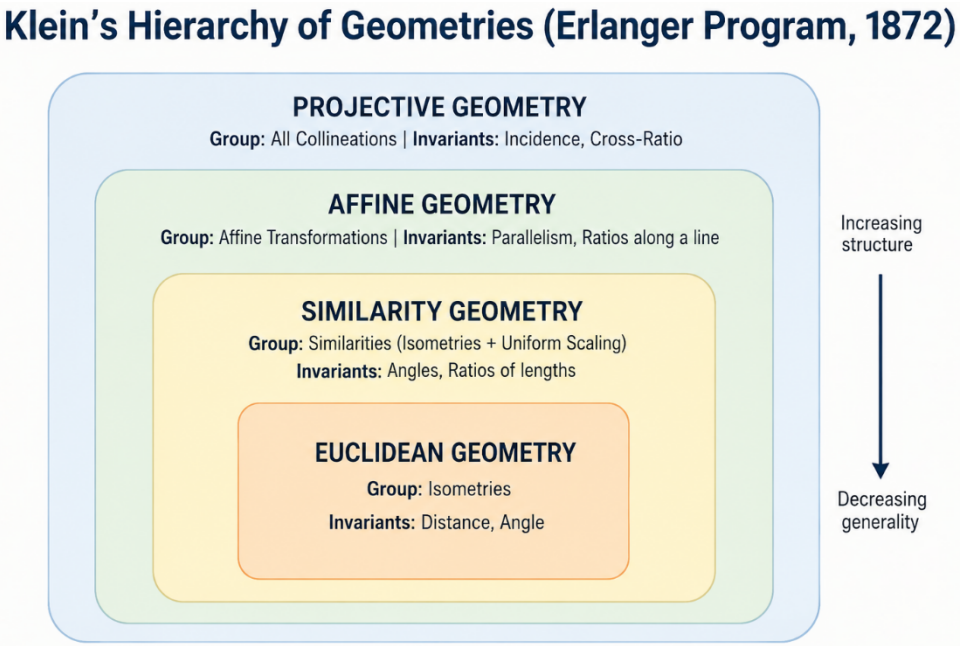


Figure 1: Klein's hierarchy of geometries (Erlangen Program, 1872/1893). Each geometry studies the properties invariant under its transformation group; the nesting Projective ⊃ Affine ⊃ Similarity ⊃ Euclidean expresses that every Euclidean theorem is a special case of a more general projective one. Reading from the outer band inward corresponds to increasing structure / decreasing generality.

lens of Klein's Erlangen Program?

Quantitative: What proportions of geometry curriculum standards in these programmes fall into static Euclidean, transformational isometric, coordinate/analytic, group-theoretic/unifying, and non-Euclidean categories?

Integrative: How can the identified gaps and quantitative patterns inform feasible, research-informed design principles for an assessment-compliant infusion of the Projective Framework that is transferable to diverse educational contexts?

2 Theoretical Framework

Klein's (1872/1893) Erlangen Program [13] famously redefined geometry as the study of invariants under a group of transformations. Euclidean geometry studies properties invariant under the group of isometries (distance, angle); similarity geometry under a larger group that allows uniform scaling; affine geometry under even more general affine transformations (preserving parallelism and ratios along a line); and projective geometry under all collineations, which preserve only incidence and cross-ratio. This hierarchical nesting—projective ⊃ affine ⊃ similarity ⊃ Euclidean—means that every Euclidean theorem is a special case of a more general projective one (Fig. 1), and the property-by-group ladder of preserved invariants that this nesting implies is summarised in Fig. 2. Educational researchers have long argued that such a perspective could provide a powerful unifying narrative for school geometry [6, 8].

Research in geometry education provides a robust pedagogical foundation for reorganizing school geometry around transformation groups and invariants. The van Hiele model of geometric thought [24] describes a developmental progression from visual recognition to formal deduction, emphasizing that advancement through these levels depends on instructional expe-

Property	Projective	Affine	Similarity	Euclidean
Incidence / collinearity	✓	✓	✓	✓
Cross-ratio	✓	✓	✓	✓
Parallelism	X	✓	✓	✓
Ratio of segments on one line	X	✓	✓	✓
Angle measure	X	X	✓	✓
Ratio of arbitrary lengths	X	X	✓	✓
Circle remains a circle	X	X	✓	✓
Distance (length)	X	X	X	✓
Absolute area	X	X	X	✓

Figure 2: Layered Invariants Chart. A property is marked ✓ when preserved by every transformation in the corresponding group, and X otherwise. Read left-to-right, the shrinking group preserves progressively more structure (column totals $2 \to 4 \to 7 \to 9$), making the hierarchical nesting of Fig. 1 explicit row by row; this is the cognitive scaffold operationalised in Design Principle 2 (Section 7.2). *Note:* the *ratio* of two areas is an affine invariant (both scale by $|\det A|$) although *absolute* area is not, and the ratio of arbitrary lengths first becomes invariant at the similarity level; the chart records absolute, not relative, quantities.

riences that engage students with properties and relationships rather than static prototypical images [2, 24]. Curricula dominated by static Euclidean facts risk stalling this progression, whereas transformational reasoning has been shown to facilitate the abstraction and generalization characteristic of higher van Hiele levels by directing attention to structural invariants [6, 8]. Empirical studies indicate that focusing on invariants helps students distinguish between essential and incidental properties, thereby fostering more flexible and coherent geometric schemas [10, 25]. Research on dynamic geometry environments further demonstrates that interactive manipulation of transformations supports students' internalization of invariance and group-like structures, bridging intuitive exploration and formal reasoning [14, 22]. Moreover, spatial reasoning—a critical predictor of STEM success—is strengthened through activities involving mental rotation, perspective-taking, and visualization, all of which are intrinsic to a transformation-based approach [15, 23]. By organizing geometry around Klein's hierarchy of transformation groups, the Projective Framework aligns with these evidence-based principles, offering a coherent structure that supports cognitive development, reduces curricular fragmentation, and connects geometric reasoning to broader mathematical practices.

Beyond its mathematical elegance, the Erlangen Program offers distinct pedagogical advantages that align with core principles of learning and cognitive development. The framework's emphasis on invariants provides a powerful cognitive scaffold, directing students' attention to essential properties that persist under transformation while filtering out incidental

features—a critical skill in geometric reasoning that is often undermined by reliance on static prototypical images [9, 25]. By organizing geometry as a nested hierarchy of transformation groups, the approach fosters abstraction and generalization; students learn to view Euclidean theorems not as isolated facts but as special cases within a broader structural system, thereby developing more coherent and flexible conceptual schemas [6, 8]. Moreover, the explicit linkage between geometric transformations and algebraic group structures bridges the traditional compartmentalization of geometry and algebra, promoting integrated mathematical thinking and reinforcing the concept of structure across domains. The hierarchy also supports developmental progression: intuitive projective phenomena such as perspective, shadows, and collinearity can be explored concretely in early grades, providing an accessible "low-floor" entry point that gradually formalizes into affine and metric reasoning. This trajectory respects students' cognitive development while building toward advanced mathematical maturity, ensuring that the framework is not only theoretically robust but also pedagogically appropriate for school mathematics.

Yet most curricula have remained at best at the level of transformational isometries (reflections, rotations, translations) used merely as tools for proving congruence, without ever naming the underlying group structure. The Projective Framework takes Klein's hierarchy as its spine, proposing that students from Grades 6–12 should encounter geometry as a "family of geometries" generated by varying the transformation group. Projective geometry, introduced through perspective drawing and shadow phenomena, becomes the most general starting point; the Euclidean metric is derived later via the Cayley-Klein construction. The framework explicitly maps activities onto IB and Cambridge assessment objectives, arguing that such an approach fosters deeper conceptual understanding, spatial reasoning, and epistemological reflection (e.g., on the nature of axioms as "variable rules of a game").

3 Research Design and Methodology

A sequential mixed-methods design was employed [7]. Phase 1 comprised a qualitative comparative analysis of the IB and Cambridge geometry curricula. Phase 2 built on Phase 1 with a directed quantitative content analysis of all geometry-related standards. The two phases are integrated in the discussion to produce a diagnosis and an implementation blueprint.

3.1 Qualitative Comparative Analysis

The qualitative phase examined the official curriculum documents for:

1. IB Primary Years Programme (PYP), Middle Years Programme (MYP), and Diploma Programme (DP) Mathematics: Analysis and Approaches (AA) and Applications and Interpretation (AI) guides.
2. Cambridge Primary, Lower Secondary, IGCSE Mathematics (0580), and AS & A Level Mathematics (9709) syllabi.

While IB and Cambridge pathways serve specific student populations, their influence extends far beyond direct enrollment, functioning as transnational policy benchmarks that shape geometry instruction worldwide. As leading providers of international qualifications, these curricula exert substantial normative power over textbook development, assessment design, and teacher preparation in diverse national systems, particularly in post-colonial and globalized educational contexts where they are often adopted as models of rigor and modernity [3, 20]. Moreover, the structural dominance of static Euclidean geometry identified

in this analysis reflects a broader global trend in secondary mathematics, where high-stakes assessment pressures frequently privilege procedural fluency and isolated theorems over structural coherence and transformational reasoning [16, 21]. By examining these influential programmes, this study offers diagnostic insights transferable to other jurisdictions grappling with similar fragmentation. The Projective Framework and coding scheme are designed as adaptable analytical tools, enabling researchers and curriculum designers in varied contexts to audit their own geometry standards and identify structural gaps. Thus, the choice of IB and Cambridge provides not only a critical case of international curriculum design but also a lens through which to examine systemic challenges in geometry education globally, ensuring the findings hold informative power for a broad international audience.

To ensure accessibility for an international audience, key assessment structures are briefly explained. The IB Middle Years Programme (MYP) assesses student work against four criteria: Criterion A (Knowing and Understanding), Criterion B (Investigating Patterns), Criterion C (Communicating), and Criterion D (Applying Mathematics in Real-Life Contexts). The Diploma Programme (DP) includes an Internal Assessment (IA), a student-directed exploration, and Theory of Knowledge (TOK), a core component encouraging reflection on the nature of knowledge. Cambridge International syllabi specify Assessment Objectives (AOs): AO1 (Knowledge and Understanding), AO2 (Reasoning and Communication), and AO3 (Problem Solving). These criteria and objectives define the skills and competencies evaluated in each programme and provide the alignment points for the infusion strategy discussed in Section 7.

Documents were analysed thematically [1] for: (a) the definition and scope of geometry, (b) the role of transformations, (c) integration of non-Euclidean content, (d) presence of a unifying narrative, and (e) alignment with the Erlangen Program. The analysis produced the comparative matrix presented in Section 4.

3.2 Quantitative Content Analysis

3.2.1 Corpus and Unit of Analysis

Four secondary curriculum documents were selected for detailed quantitative mapping:

- IB PYP: From Principles into Practice (2018) [17]
- IB MYP Mathematics guide (2020) [19]
- IB Mathematics: Analysis and Approaches Guide (2019, updated 2021) [18]
- IB DP Mathematics: Analysis and Approaches guide (SL/HL) (2019, updated 2021)
- Cambridge IGCSE Mathematics 0580/0980 syllabus (2023) [5]
- Cambridge AS & A Level Mathematics 9709 syllabus (2023) [4]

The unit of analysis was each discrete content standard, objective, or bullet point involving spatial/geometric reasoning. This included topics under "Geometry," "Trigonometry," "Transformations," "Vectors," and, in the IB DP, relevant portions of the "Geometry and Trigonometry" topic. Standards that were purely arithmetic or algebraic (e.g., solving non-geometric equations) were excluded. A total of 321 standards were identified across the four documents.

Unit Decomposition ("Atomic split"). Several official syllabus statements combined multiple discrete expectations or skills within a single bullet point (e.g., "identify properties of regular polygons and calculate interior angles"). To prevent conflating distinct pedagogical demands and to ensure mutually exclusive coding, compound statements were decomposed

Table 1: Coding Scheme for Geometry Standards

Code	Category	Description	Example
SE	Static Euclidean	Properties of shapes, angle facts, congruence/similarity proved by static criteria (SSS, SAS, etc.), metric calculations, circle theorems, constructions	"Know and use angle properties of parallel lines and polygons"
TI	Transformational Isometries	Reflections, rotations, translations, enlargements, combined transformations, symmetry expressed via transformations	"Describe fully the single transformation that maps shape A onto shape B"
CA	Coordinate/Analytic Geometry	Coordinates, equations of lines and circles, vectors as translation arrows or algebraic objects, matrix representation of transformations (without group language)	"Find the vector equation of a line"
GU	Group-Theoretic/Unifying	Explicit mention of transformation groups, invariants, hierarchy of geometries, projective geometry, axioms treated as variable, Cayley-Klein model	(Hypothetical: "Understand that Euclidean geometry is the study of properties invariant under isometries")
NE	Non-Euclidean	Hyperbolic/elliptic geometry, alternative parallel postulates, non-Euclidean models	(Hypothetical: "Explore the Poincaré disk model")

Note. SE = Static Euclidean; TI = Transformational Isometries; CA = Coordinate/Analytic Geometry; GU = Group-Theoretic/Unifying; NE = Non-Euclidean.

into indivisible, single-concept units prior to analysis. Each decomposed unit was treated as an independent standard for frequency counting and category assignment. This decomposition procedure (labelled "Atomic split" in the Appendix) was conducted by the first author, independently verified by the second author, and completed before inter-rater reliability testing. The final corpus of 321 units reflects these atomic standards rather than raw syllabus line counts.

3.2.2 Coding Scheme

A directed coding framework was developed based on the Erlangen Program categories and the Projective Framework's critique. Each standard was assigned to one of five mutually exclusive primary categories (see Table 1).

The coding categories were designed to align with established distinctions in geometry education research and curriculum analysis methodology. The separation of Static Euclidean (SE) and Transformational Isometries (TI) reflects the well-documented dichotomy between static property-based reasoning and dynamic transformational thinking, a distinction central to research on geometric cognition and van Hiele level progression [6, 24]. The Group-Theoretic/Unifying (GU) category captures standards that explicitly address invariants, hierarchy, or algebraic structure, constructs identified in mathematics education research as critical for fostering abstraction and generalization but rarely present in school curricula [8, 9]. The inclusion of Non-Euclidean (NE) acknowledges the epistemological role of alternative geometries in developing flexibility and reflection on axiomatic systems, a dimension emphasized in reform-oriented frameworks. Methodologically, the directed content analysis approach follows established practices in qualitative research [11] and international curriculum comparison, where standards are decomposed into atomic units and coded for cognitive and

structural emphasis to enable replicable quantification of curricular priorities [16, 21]. This scheme thus provides a theoretically grounded and empirically validated tool for diagnosing the structural alignment of geometry curricula with modern mathematical and pedagogical principles.

3.2.3 Procedure and Reliability

Two coders independently assigned codes to a random 30% subset of the standards. Inter-rater reliability was calculated using Cohen's kappa, yielding $\kappa = 0.87$, indicating strong agreement. Discrepancies were resolved through consensus discussion. The final full coding was then conducted by the first author and spot-checked by the second.

3.2.4 Analysis

Frequencies and percentages of standards in each category were calculated for each curriculum. The presence of any explicit linking statements across topics (e.g., "congruence can be understood through transformations") was also noted.

4 Qualitative Findings: The Geometry Landscape in IB and Cambridge Curricula

4.1 Structural Overview

IB Continuum [17–19]. In the PYP (ages 3–12), mathematics is embedded in transdisciplinary Units of Inquiry, with geometry appearing under "Shape and Space," focusing on spatial awareness and basic transformations through play. The MYP (ages 11–16) organises mathematics around key concepts (Form, Relationships, Space) and five content branches, one of which is Geometry. Assessment is criterion-based (A–D). The DP (ages 16–19) offers four mathematics courses; in Analysis and Approaches, geometry is concentrated in Topic 3: Geometry and Trigonometry, covering vectors, trigonometry, and matrix transformations. Applications and Interpretation also includes a geometry topic but with a greater emphasis on technology and real-world contexts. Non-Euclidean ideas are permitted only within the Internal Assessment (IA), a student-directed exploration.

Cambridge Pathway [4, 5]. Cambridge Primary and Lower Secondary (Stages 1–9) feature a spiral curriculum with geometry topics such as shapes, symmetry, constructions, and transformations. At IGCSE (ages 14–16), geometry appears as Topic 4 (Geometry), Topic 6 (Trigonometry), and Topic 7 (Transformations and Vectors). Content includes circle theorems, symmetry, vector geometry, and trigonometrical problem-solving. AS & A Level Mathematics (9709) continues with coordinate geometry, vectors, and trigonometric functions in Pure Mathematics. Further Mathematics adds modules on matrices and further transformations, but still within a Euclidean, non-group-theoretic frame.

4.2 Comparative Analysis

Table 2 summarises the fundamental contrast between the Projective Framework and the two international programmes.

Table 2: Comparative Analysis of Geometry Definitions and Approaches

Dimension	Projective Framework	IB	Cambridge
Core definition	Study of invariants under transformation groups	Study of shapes, space, and relationships through inquiry	Study of properties, measurements, and problem-solving in Euclidean space
Unifying theory	Erlangen Program; hierarchy of geometries	Geometries taught as separate topics	Geometries taught as separate topics
Starting point	Projective geometry (most general)	Euclidean geometry as default	Euclidean geometry as default
Non-Euclidean	Core component (Grades 9–10)	Optional (IA/extension only)	Minimal/absent
Axiomatic approach	Axioms as "variable rules"	Mostly fixed; some TOK reflection	Fixed Euclidean axioms

Note. IA = Internal Assessment; TOK = Theory of Knowledge.

4.3 Critical Gaps Identified

From the thematic analysis, four interlocking gaps emerge in both IB and Cambridge curricula:

Dominance and Isolation of Euclidean Geometry. The overwhelming majority of content standards assume Euclidean space as the only geometry. Non-Euclidean concepts, when present, are relegated to optional explorations and never form part of the examined core.

Absence of a Unifying Narrative. Congruence, similarity, symmetry, and transformations are taught as disconnected chapters. There is no consistent thread showing that congruence comprises invariance under isometries, or that a dilation changes length but preserves angle, and so on.

Fragmentation Across Topics. Geometry, trigonometry, transformations, and vectors appear in separate syllabus sections with minimal cross-linking. This compartmentalisation reinforces a view of mathematics as a set of isolated procedures.

Static Axiomatic Stance Axioms are treated as fixed, immutable truths. The epistemological question "What if the parallel postulate were different?" is never raised in required content, despite its profound importance.

These gaps align precisely with the theoretical deficiencies the Projective Framework seeks to address. However, the qualitative reading alone cannot tell us the magnitude of the problem. This motivates the quantitative phase.

5 Quantitative Findings: Content Analysis of Geometry Standards

A total of 321 standards were coded. Table 3 presents the distribution by category across the four curricula.

5.1 Key Findings

The quantitative analysis reveals a structural profile consistent with broader trends identified in international curriculum research but markedly misaligned with evidence-based recommen-

Table 3: Distribution of Geometry Standards by Category (%)

Curriculum	SE	TI	CA	GU	NE	Total units
IB MYP	57 (78%)	9 (12%)	7 (10%)	0 (0%)	0 (0%)	73
IB DP AA (SL+HL)	64 (72%)	4 (4%)	21 (24%)	0 (0%)	0 (0%)	89
Cambridge IGCSE 0580	77 (84%)	7 (8%)	8 (9%)	0 (0%)	0 (0%)	92
Cambridge AS/A Level 9709	**46 (69%)**	1 (1%)	**20 (30%)**	0 (0%)	0 (0%)	67
Overall	**244 (76%)**	**21 (7%)**	**56 (17%)**	0 (0%)	0 (0%)	**321**

Note. SE = Static Euclidean; TI = Transformational Isometries; CA = Coordinate/Analytic Geometry; GU = Group-Theoretic/Unifying; NE = Non-Euclidean. Percentages may not total 100% due to rounding.

dations for geometry education.

SE Dominance and Fragmentation. Between 69% and 84% of standards are coded as Static Euclidean, emphasizing properties, metric calculations, and isolated theorems. This predominance mirrors findings from large-scale comparative studies indicating that geometry instruction worldwide often prioritizes procedural fluency and memorization of facts over structural reasoning and coherence [16, 21]. Transformational standards (TI) constitute only 1%–12% of content and are consistently siloed in separate syllabus sections, confirming the fragmentation critique. Research warns that such compartmentalization reinforces a view of mathematics as disconnected procedures and impedes students' ability to perceive underlying unifying structures [21].

Absence of Unifying and Non-Euclidean Content. Zero standards address group-theoretic or unifying concepts (GU), and non-Euclidean geometry (NE) is entirely absent from mandatory syllabi. This finding empirically validates prior qualitative observations that school geometry rarely incorporates algebraic structures or alternative axiomatic systems, despite mathematics education research evidence that exposure to invariants and hierarchy supports higher-order abstraction and epistemological reflection [8, 9]. The complete lack of GU content indicates that students are not provided with the conceptual tools to understand geometry as a coherent family of ideas, a gap that limits opportunities for developing the flexible reasoning demanded by advanced mathematics and STEM fields.

5.2 Qualitative Illustrations from Syllabi

IGCSE 0580: Standard E7.1 "Describe a translation by using a vector" is coded CA. Standard E4.9 "Understand the meaning of congruence" is SE. They are separated by entire topic sections.

DP AA Guide: Under "Geometry and Trigonometry" (Topic 3), students are asked to "represent transformations by matrices." This is coded TI/CA. Nowhere does the guide say, "These matrices form a group; the invariants define the geometry."

MYP Mathematics Guide: The criterion "Investigating Patterns" invites students to "identify patterns in transformations." While this could accommodate group exploration, the content examples provided remain at the level of performing successive transformations rather than analysing the algebraic structure.

These findings provide strong empirical support for the qualitative critique: international geometry curricula are not slightly incomplete, they are structurally devoid of the conceptual

framework that modern mathematics uses to organise geometry.

6 Discussion

The convergence of qualitative and quantitative evidence demonstrates that IB and Cambridge geometry curricula are structurally entrenched in a static Euclidean paradigm, with profound implications for student learning when viewed through the lens of geometry education research.

Implications for Geometric Cognition and Van Hiele Progression. The overwhelming emphasis on static properties and isolated theorems suggests a curricular focus on van Hiele Levels 1 (Analysis) and 2 (Informal Deduction), with limited scaffolding toward Level 3 (Deduction) or Level 4 (Rigor) through structural unification [2, 24]. Research indicates that advancement to higher levels requires instructional experiences that engage students with relationships, invariants, and hierarchical structures rather than prototypical images and disjointed facts [6]. By omitting transformation groups and invariants, the analysed curricula risk stalling this progression, as students are not systematically guided to abstract general principles from specific cases or to reason about properties that persist under transformation.

Invariants, Abstraction, and Spatial Reasoning. The absence of GU content deprives students of opportunities to develop invariant-based reasoning, a cognitive skill central to geometric abstraction and generalization [9, 25]. Focusing on invariants helps learners distinguish essential properties from incidental features, fostering more flexible and coherent schemas. Moreover, the lack of projective and transformational perspectives limits the development of spatial reasoning competencies, such as mental rotation, perspective-taking, and visualization, which are strengthened through activities involving dynamic transformations and are critical predictors of success in STEM disciplines [15, 23]. The curricula thus miss a key opportunity to leverage geometry as a domain for cultivating spatial intelligence.

Bridging the Gap: Klein's Framework as a Pedagogical Tool. Klein's Erlangen Program offers more than mathematical elegance; it provides a pedagogical structure that directly addresses these cognitive gaps. By organizing geometry around transformation groups and invariants, the framework aligns with evidence-based principles: it supports van Hiele progression by making structural relationships explicit; it fosters abstraction through hierarchical nesting; and it enhances spatial reasoning via projective and transformational activities. The Projective Framework's infusion strategy demonstrates how these benefits can be realized within existing high-stakes constraints, offering a pragmatic pathway to align curriculum structure with the developmental needs of learners.

7 Implications and Research-Informed Design Principles

The empirical evidence of structural fragmentation and the complete absence of unifying content in IB and Cambridge curricula points to the need for curriculum renewal grounded in geometry education research. Rather than proposing a prescriptive syllabus, this study offers three research-informed design principles that can guide the infusion of projective and group-theoretic ideas into existing frameworks. These principles align with established findings on

geometric cognition, spatial reasoning, and van Hiele progression, ensuring that reform efforts are pedagogically sound and developmentally appropriate.

7.1 Principle 1: Leverage Intuitive Projective Phenomena to Develop Early Spatial Reasoning

Research indicates that spatial reasoning skills, such as mental rotation and perspective-taking, are malleable and critical predictors of success in STEM disciplines [15, 23]. Moreover, the van Hiele model emphasizes that geometric learning progresses from visual and intuitive experiences toward formal deduction, suggesting that early instruction should engage students with concrete, perceptual activities before introducing abstract properties [2, 24]. This principle recommends capitalizing on intuitive projective phenomena—such as shadows, perspective drawing, and collinearity—as accessible "low-floor" entry points that foster spatial visualization while implicitly introducing the concept of invariance. For example, activities where students investigate how shadows preserve incidence but distort length can help them distinguish between essential and incidental properties, a foundational skill in geometric reasoning that is often undermined by reliance on static prototypical images [9]. Such experiences lay the cognitive groundwork for later formalization of projective and affine concepts.

7.2 Principle 2: Scaffold Invariant-Based Reasoning to Support Abstraction and Van Hiele Progression

Advancement through van Hiele levels requires instructional focus on relationships, hierarchies, and invariants rather than isolated facts and static shapes [6, 24]. Empirical studies demonstrate that directing students' attention to properties preserved under transformation helps them develop flexible geometric schemas and supports the transition to higher levels of abstraction and generalization [9, 25]. This principle advocates for organizing geometry content around transformation groups and explicitly scaffolding invariant-based reasoning. Curriculum materials should highlight which properties persist as the transformation group expands, making the hierarchical nesting of geometries visible to learners. Tools such as *the Layered Invariants Chart (Fig. 2)* can serve as cognitive scaffolds, helping students see Euclidean theorems as special cases within a broader structural system and reducing fragmentation by providing a coherent narrative that links congruence, similarity, and affine properties through their respective invariants [8]. This approach fosters the abstraction and generalization skills that characterize advanced geometric thinking.

7.3 Principle 3: Integrate Group Structures to Bridge Algebra and Geometry

The compartmentalization of geometry and algebra in school curricula limits students' ability to perceive structural connections across mathematical domains. Research on transformational geometry emphasizes the value of linking geometric transformations to algebraic representations, such as matrices, to reinforce the concept of structure and promote integrated mathematical thinking [8, 22]. This principle recommends introducing transformation groups as algebraic objects that act on geometric figures, helping students unify these domains by viewing matrices not merely as computational tools but as generators of geometric symmetries. Engaging with group structures allows students to manipulate algebraic expressions

to explore geometric invariants, thereby deepening conceptual understanding in both areas and supporting the development of abstract reasoning. Such integration aligns with evidence-based calls for curricula that emphasize structural coherence and cross-domain connections [21].

7.4 Illustrative Infusion Strategies and Implementation Considerations

Table 4 presents illustrative examples of how these design principles can be operationalized within existing IB and Cambridge assessment frameworks. These examples are not prescriptive requirements but demonstrate feasible infusion points where projective and group-theoretic concepts can be layered as depth-enriching extensions without compromising examination readiness. Each activity is aligned with specific curriculum standards and assessment criteria, ensuring that enrichment remains subordinate to high-stakes demands while creating structured space for deeper reasoning. The "Research Rationale" column explicitly links each example to the design principles and supporting literature, ensuring that implementation is guided by evidence rather than arbitrary grade-level prescriptions.

7.5 Implications for Teacher Education

The quantitative finding that group-theoretic and unifying content is entirely absent from mandatory curricula (Section 5) has direct implications for teacher preparation. It is likely that most in-service teachers have not encountered the Erlangen Program, projective geometry, or non-Euclidean models in their own education, creating a significant barrier to implementation. Curriculum reform grounded in these design principles therefore requires targeted professional development focused on building teachers' conceptual knowledge of transformation groups, invariants, and the hierarchy of geometries. Effective support models should be differentiated by teaching level, providing primary teachers with experiential familiarity with intuitive projective activities and equipping secondary specialists with the formal background needed to facilitate invariant-based reasoning and algebraic-geometric connections. Future research should investigate the specific professional development structures and resources needed to empower teachers to implement these principles effectively, as well as measure the impact of infused activities on student learning outcomes and spatial reasoning.

8 Limitations

This study has several limitations that should be considered when interpreting the findings. First, the quantitative content analysis assigns equal weight to each coded standard. In practice, standards may vary significantly in the instructional time, cognitive demand, or assessment weight they command. For example, a single standard on vector equations may encompass more classroom hours than multiple standards on basic angle properties. While standard counts provide a robust and replicable measure of structural emphasis and curricular priorities, they do not fully capture the intensity of instructional focus. Future research could incorporate weighting schemes based on examination mark distributions or teacher time-allocation surveys to refine these proportions.

Second, the analysis is confined to official curriculum documents and does not capture the enacted curriculum. Textbooks, teacher interpretations, and classroom practices may

Table 4: Illustrative Infusion Examples Aligned with Research-Informed Design Principles

Design Principle	Illustrative Activity	Curriculum Link	Research Rationale
Principle 1: Intuitive projective phenomena for spatial reasoning	**Perspective & Shadows Investigation:** Students explore how viewpoint changes affect shape perception; identify invariant incidence.	PYP "Shape and Space"; Cambridge Primary Position & Direction	Develops spatial visualization and perspective-taking; leverages intuitive visual reasoning as foundation for formal geometry [15, 24].
Principle 2: Invariant-based reasoning for abstraction	**Cross-Ratio Discovery:** Students use dynamic geometry to explore cross-ratio as an invariant under projection; connect to ratio and similarity.	MYP Criterion B; IGCSE Ratio & Proportion	Focuses attention on invariants to foster flexible schemas; bridges proportional reasoning with projective invariance [9, 25].
Principle 2: Invariant-based reasoning for abstraction	**Layered Invariants Chart:** Students categorize properties preserved under each transformation group; visualize hierarchy.	MYP Criterion A; IGCSE Transformations	Scaffolds abstraction by making hierarchical structure explicit; supports van Hiele progression through structural relationships [6, 8].
Principle 3: Group structures to bridge domains	**Matrix Transformations as Groups:** Students analyze matrices as elements of transformation groups; explore composition and invariants.	DP AA Matrix Transformations; IGCSE Matrices (Ext)	Integrates algebra and geometry; reinforces structural thinking and group properties [8, 22].
Principle 2/3: Epistemological reflection	**Axioms as Variable Rules:** Students explore alternative parallel postulates using dynamic models; reflect on nature of axiomatic systems.	DP TOK; IGCSE Geometry	Promotes epistemological reflection and flexibility; engages students with axiomatic variation as supported by reform frameworks [22].

Note. MYP Criteria A–D and Cambridge Assessment Objectives (AO1–AO3) are explained in Section 3.1. TOK = Theory of Knowledge; IA = Internal Assessment.

introduce unifying ideas or further fragment content beyond what is specified in the syllabi. However, in high-stakes international pathways, official syllabi define the mandatory scope of instruction and assessment; thus, they serve as the primary driver of curricular emphasis and provide the most defensible basis for comparative analysis.

Third, the focus on IB and Cambridge pathways, while globally influential, limits generalizability to other educational systems. Finally, the proposed infusion strategy and professional development model require empirical validation through classroom implementation to assess their impact on student learning and teacher efficacy.

9 Conclusion

This study set out to empirically validate the claim that international geometry curricula are entrenched in a static Euclidean paradigm and to advance a coherent, pedagogically grounded alternative. Through a qualitative comparative analysis and a systematic quantitative content analysis of 321 standards across IB and Cambridge pathways, we have demonstrated that geometry instruction in these influential programmes is 69%–84% static Euclidean, entirely

lacks group-theoretic or unifying narrative, and omits non-Euclidean thought from mandatory syllabi. Transformations, where present, are treated as isolated skills rather than as the foundational grammar of geometric thinking. These findings provide robust evidence of a structural lacuna that limits students' exposure to the invariants, hierarchy, and flexibility that characterize modern geometric reasoning, with implications for the development of abstraction, spatial reasoning, and epistemological reflection.

The contributions of this work extend beyond a critique of specific curricula. The Projective Framework offers a theoretically and pedagogically justified model for reorganizing geometry education, bridging Klein's Erlangen Program with research on geometric cognition, van Hiele progression, and spatial reasoning. The coding scheme developed for this analysis serves as a transferable methodological tool, enabling educators and researchers in varied contexts to audit their own standards and identify structural gaps, thereby supporting comparative curriculum research globally. The infusion strategy demonstrates that reform need not require wholesale replacement; instead, projective and group-theoretic concepts can be layered as depth-enriching extensions at precise inflection points, respecting existing assessment frameworks while fostering deeper conceptual understanding. This approach offers a pragmatic model for curriculum renewal that balances innovation with the realities of high-stakes examination systems.

For the international mathematics education community, this study highlights systemic challenges that transcend individual programmes. The dominance of static Euclidean content and the fragmentation of geometric topics [12] reflect broader tensions between procedural assessment demands and the goal of developing coherent mathematical knowledge. By providing a replicable analytical framework and an evidence-informed implementation model, this work invites curriculum designers, teacher educators, and researchers to reimagine geometry as an interconnected family of ideas rooted in transformation and invariance. Future research should pilot the proposed activities in diverse classrooms, measure their impact on student learning outcomes and spatial reasoning, and investigate the professional development models required to equip teachers with the conceptual background needed to facilitate this shift. In doing so, we can move toward a geometry curriculum that not only reflects the structural elegance of modern mathematics but also equips students with the reasoning tools essential for navigating an increasingly complex world.

In an era where artificial intelligence and computational geometry increasingly demand flexible spatial reasoning, the persistence of a curriculum largely reflecting 19th-century conventions is an anachronism. By reintroducing the Erlangen Program to school mathematics, we can transform geometry from a static collection of isolated theorems into an interconnected family of ideas.

Declaration of Generative AI and AI-assisted technologies in the writing process During the preparation of this manuscript the authors used ChatGPT to generate the schematic figures (Figs. 1–2). The authors reviewed and edited the resulting images, verified their accuracy against the source material, and take full responsibility for the content of this publication.

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Appendix

Syllabus	Section	Standard_ID	Description	Code	Notes
IB MYP Mathematics	Geometric Properties & Classification	MYP_01	Identify and classify 2D polygons by side/angle properties	SE	Atomic split
IB MYP Mathematics	Geometric Properties & Classification	MYP_02	Identify and classify 3D solids by faces/edges/vertices	SE	Atomic split
IB MYP Mathematics	Geometric Properties & Classification	MYP_03	Recognise nets and cross-sections of common solids	SE	Atomic split

IB MYP Mathematics	Geometric Properties & Classification	MYP_04	Use correct geometric notation for points/lines/angles	SE	Atomic split
IB MYP Mathematics	Geometric Properties & Classification	MYP_05	Distinguish between convex and concave polygons	SE	Atomic split
IB MYP Mathematics	Geometric Properties & Classification	MYP_06	Identify properties of regular polygons	SE	Atomic split
IB MYP Mathematics	Geometric Properties & Classification	MYP_07	Classify triangles by sides and angles	SE	Atomic split
IB MYP Mathematics	Geometric Properties & Classification	MYP_08	Classify quadrilaterals by diagonal/symmetry properties	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_09	Apply angle sum properties of triangles	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_10	Apply angle sum properties of quadrilaterals	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_11	Calculate interior angles of regular polygons	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_12	Calculate exterior angles of polygons	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_13	Identify and apply alternate angles with parallel lines	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_14	Identify and apply corresponding angles with parallel lines	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_15	Identify and apply co-interior angles with parallel lines	SE	Atomic split
IB MYP Mathematics	Angle Relationships & Parallel Lines	MYP_16	Solve multi-step angle problems using parallel line facts	SE	Atomic split
IB MYP Mathematics	Congruence & Similarity	MYP_17	Apply SSS criterion to prove triangle congruence	SE	Atomic split
IB MYP Mathematics	Congruence & Similarity	MYP_18	Apply SAS criterion to prove triangle congruence	SE	Atomic split
IB MYP Mathematics	Congruence & Similarity	MYP_19	Apply ASA criterion to prove triangle congruence	SE	Atomic split

IB MYP Mathematics	Congruence & Similarity	MYP_20	Apply RHS criterion to prove right-triangle congruence	SE	Atomic split
IB MYP Mathematics	Congruence & Similarity	MYP_21	Identify similar figures using proportional sides/equal angles	SE	Atomic split
IB MYP Mathematics	Congruence & Similarity	MYP_22	Calculate missing lengths using similarity scale factors	SE	Atomic split
IB MYP Mathematics	Congruence & Similarity	MYP_23	Apply area scale factor relationships in similar figures	SE	Atomic split
IB MYP Mathematics	Congruence & Similarity	MYP_24	Apply volume scale factor relationships in similar solids	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_25	Calculate perimeter of composite 2D shapes	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_26	Calculate area of triangles and quadrilaterals	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_27	Calculate circumference and area of circles	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_28	Calculate arc length and sector area	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_29	Calculate volume of prisms and cylinders	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_30	Calculate volume of pyramids and cones	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_31	Calculate surface area of prisms and cylinders	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_32	Calculate volume and surface area of spheres	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_33	Solve composite solid mensuration problems	SE	Atomic split
IB MYP Mathematics	Mensuration 2D & 3D	MYP_34	Convert between metric units in geometric contexts	SE	Atomic split
IB MYP Mathematics	Right-Angled Trigonometry	MYP_35	Apply sine ratio to find missing sides	SE	Atomic split
IB MYP Mathematics	Right-Angled Trigonometry	MYP_36	Apply cosine ratio to find missing sides	SE	Atomic split
IB MYP Mathematics	Right-Angled Trigonometry	MYP_37	Apply tangent ratio to find missing sides	SE	Atomic split
IB MYP Mathematics	Right-Angled Trigonometry	MYP_38	Use inverse trig ratios to find missing angles	SE	Atomic split
IB MYP Mathematics	Right-Angled Trigonometry	MYP_39	Apply Pythagoras theorem in 2D contexts	SE	Atomic split

IB MYP Mathematics	Right-Angled Trigonometry	MYP_40	Apply Pythagoras theorem in 3D contexts	SE	Atomic split
IB MYP Mathematics	Right-Angled Trigonometry	MYP_41	Solve contextual problems using right-triangle trig	SE	Atomic split
IB MYP Mathematics	Non-Right Trigonometry	MYP_42	Apply sine rule to find missing sides/angles	SE	Atomic split
IB MYP Mathematics	Non-Right Trigonometry	MYP_43	Apply cosine rule to find missing sides/angles	SE	Atomic split
IB MYP Mathematics	Non-Right Trigonometry	MYP_44	Calculate triangle area using 1/2 ab sinC	SE	Atomic split
IB MYP Mathematics	Non-Right Trigonometry	MYP_45	Solve multi-step trigonometry problems in 2D/3D	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_46	Construct logical arguments for geometric properties	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_47	Justify congruence/similarity conclusions formally	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_48	Derive angle relationships from diagrammatic evidence	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_49	Test geometric conjectures using counterexamples	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_50	Communicate geometric reasoning using correct terminology	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_51	Link algebraic expressions to geometric relationships	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_52	Interpret scale drawings and diagrams accurately	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_53	Apply geometric properties to real-world modelling tasks	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_54	Evaluate reasonableness of geometric solutions	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_55	Synthesise multiple geometric facts in extended problems	SE	Atomic split

IB MYP Mathematics	Geometric Reasoning & Proof	MYP_56	Reflect on geometric problem-solving strategies (Criterion D)	SE	Atomic split
IB MYP Mathematics	Geometric Reasoning & Proof	MYP_57	Investigate patterns in geometric sequences (Criterion B)	SE	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_58	Reflect 2D shapes in horizontal/vertical/diagonal lines	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_59	Rotate shapes about given centres and angles	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_60	Translate shapes using coordinate or vector descriptions	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_61	Enlarge shapes with positive integer scale factors	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_62	Identify lines of symmetry in regular/irregular polygons	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_63	Determine order of rotational symmetry for 2D shapes	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_64	Describe fully a single transformation mapping A to B	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_65	Combine two transformations and identify net effect	TI	Atomic split
IB MYP Mathematics	Transformations & Symmetry	MYP_66	Identify invariant points/lines under basic transformations	TI	Atomic split
IB MYP Mathematics	Coordinate & Vector Methods	MYP_67	Plot and interpret points in the Cartesian plane	CA	Atomic split
IB MYP Mathematics	Coordinate & Vector Methods	MYP_68	Calculate gradient of a straight line from coordinates	CA	Atomic split
IB MYP Mathematics	Coordinate & Vector Methods	MYP_69	Find midpoint and distance between two points	CA	Atomic split
IB MYP Mathematics	Coordinate & Vector Methods	MYP_70	Derive equation of a line in $y = mx + c$ form	CA	Atomic split
IB MYP Mathematics	Coordinate & Vector Methods	MYP_71	Identify parallel/perpendicular lines using gradients	CA	Atomic split

IB MYP Mathematics	Coordinate & Vector Methods	MYP_72	Represent and add basic column vectors	CA	Atomic split
IB MYP Mathematics	Coordinate & Vector Methods	MYP_73	Use vectors to describe translations and paths	CA	Atomic split
IB DP AA (SL/HL)	Topic 3.1 Coordinate Geometry	DPAA_01	Calculate distance between two points in 2D/3D	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.1 Coordinate Geometry	DPAA_02	Find midpoint of a line segment in 2D/3D	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.1 Coordinate Geometry	DPAA_03	Determine gradient of a line and interpret geometrically	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.1 Coordinate Geometry	DPAA_04	Find equation of a line in slope-intercept/general form	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.1 Coordinate Geometry	DPAA_05	Apply conditions for parallel and perpendicular lines	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.1 Coordinate Geometry	DPAA_06	Find intersection point of two lines algebraically	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.2 Circle Geometry	DPAA_07	Write equation of a circle in standard form	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.2 Circle Geometry	DPAA_08	Find centre and radius from general circle equation	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.2 Circle Geometry	DPAA_09	Determine intersection of line and circle	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.2 Circle Geometry	DPAA_10	Find equation of tangent to a circle at a given point	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.2 Circle Geometry	DPAA_11	Apply geometric properties of circles in coordinate contexts	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.3 Trigonometric Ratios & Identities	DPAA_12	Define sin/cos/tan for all quadrants using unit circle	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.3 Trigonometric Ratios & Identities	DPAA_13	Apply Pythagorean identity $\sin^2 x + \cos^2 x = 1$	SE	Atomic split
IB DP AA (SL/HL)	Topic 3.3 Trigonometric Ratios & Identities	DPAA_14	Apply double angle identities for sin/cos/tan	SE	Atomic split

IB DP (SL/HL)	AA	Topic 3.3 Trigonometric Ratios & Identities	DPAA_15	Apply compound angle formulas (SL/HL)	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.3 Trigonometric Ratios & Identities	DPAA_16	Prove trigonometric identities algebraically	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.3 Trigonometric Ratios & Identities	DPAA_17	Simplify trig expressions using identities	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.4 Trigonometric Functions & Graphs	DPAA_18	Sketch graphs of sin/cos/tan functions	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.4 Trigonometric Functions & Graphs	DPAA_19	Identify amplitude/period/phase shift from equations	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.4 Trigonometric Functions & Graphs	DPAA_20	Transform trig graphs using vertical/horizontal shifts	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.4 Trigonometric Functions & Graphs	DPAA_21	Model periodic phenomena using trig functions	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.4 Trigonometric Functions & Graphs	DPAA_22	Interpret real-world periodic data using trig models	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.5 Trigonometric Equations	DPAA_23	Solve basic trig equations in given intervals	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.5 Trigonometric Equations	DPAA_24	Solve quadratic-form trig equations	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.5 Trigonometric Equations	DPAA_25	Apply identities to solve complex trig equations	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.5 Trigonometric Equations	DPAA_26	Find general solutions for trig equations (HL)	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.5 Trigonometric Equations	DPAA_27	Use inverse trig functions appropriately in solutions	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.6 Non-Right Triangle Trigonometry	DPAA_28	Apply sine rule to solve oblique triangles	SE	Atomic split

IB DP (SL/HL)	AA	Topic 3.6 Non-Right Triangle Trigonometry	DPAA_29	Apply cosine rule to solve oblique triangles	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.6 Non-Right Triangle Trigonometry	DPAA_30	Calculate area using 1/2 ab sinC	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.6 Non-Right Triangle Trigonometry	DPAA_31	Resolve ambiguous case of sine rule	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.6 Non-Right Triangle Trigonometry	DPAA_32	Solve 3D trigonometry problems using multiple triangles	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.6 Non-Right Triangle Trigonometry	DPAA_33	Apply trig to bearings and navigation contexts	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.6 Non-Right Triangle Trigonometry	DPAA_34	Solve multi-step trig application problems	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.7 3D Geometry & Applications	DPAA_35	Identify and sketch 3D solids and cross-sections	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.7 3D Geometry & Applications	DPAA_36	Calculate angles between lines and planes in 3D	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.7 3D Geometry & Applications	DPAA_37	Apply Pythagoras and trig in 3D figures	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.7 3D Geometry & Applications	DPAA_38	Solve volume/surface area problems for composite 3D shapes	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.7 3D Geometry & Applications	DPAA_39	Interpret 3D diagrams and extract geometric relationships	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.7 3D Geometry & Applications	DPAA_40	Model real-world structures using 3D geometry	SE	Atomic split
IB DP (SL/HL)	AA	Topic 3.8 Vectors (2D/3D)	DPAA_41	Represent vectors in component and i/j/k notation	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.8 Vectors (2D/3D)	**DPAA_42**	**Calculate magnitude and direction of vectors**	**SE**	**Corrected: Metric calculation**
IB DP (SL/HL)	AA	Topic 3.8 Vectors (2D/3D)	DPAA_43	Perform vector addition and subtraction	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.8 Vectors (2D/3D)	DPAA_44	Multiply vectors by scalars and interpret geometrically	CA	Atomic split

IB DP (SL/HL)	AA	Topic 3.8 Vectors (2D/3D)	DPAA_45	Find and use unit vectors	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.8 Vectors (2D/3D)	DPAA_46	Apply position vectors to locate points in space	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.8 Vectors (2D/3D)	DPAA_47	Determine collinearity using vector methods	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.9 Scalar & Vector Products	DPAA_48	Calculate scalar product of two vectors	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.9 Scalar & Vector Products	**DPAA_49**	**Find angle between vectors using dot product**	**SE**	**Corrected: Metric calculation**
IB DP (SL/HL)	AA	Topic 3.9 Scalar & Vector Products	DPAA_50	Test for perpendicularity using scalar product	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.9 Scalar & Vector Products	DPAA_51	Calculate vector product (cross product) (HL)	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.9 Scalar & Vector Products	**DPAA_52**	**Interpret cross product magnitude as area of parallelogram (HL)**	**SE**	**Corrected: Metric calculation**
IB DP (SL/HL)	AA	Topic 3.9 Scalar & Vector Products	DPAA_53	Apply vector products to find normal vectors (HL)	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_54	Write vector equation of a line $r = a + \lambda b$	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_55	Convert between vector/Cartesian/parametric line forms	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_56	Determine if lines are parallel/intersecting/skew	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_57	Find intersection point of two lines	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_58	Find angle between two lines using direction vectors	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_59	Write vector and Cartesian equations of planes (HL)	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_60	Find intersection of line and plane (HL)	CA	Atomic split

IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_61	Calculate distance from point to line/plane (HL)	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_62	Find angle between line and plane (HL)	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_63	Find angle between two planes (HL)	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.10 Vector Lines & Planes	DPAA_64	Solve 3D geometric problems using vector methods	CA	Atomic split
IB DP (SL/HL)	AA	Topic 3.11 Matrix Transformations (HL)	DPAA_65	Represent linear transformations using 2x2 matrices	TI	Atomic split
IB DP (SL/HL)	AA	Topic 3.11 Matrix Transformations (HL)	DPAA_66	Apply matrix multiplication to transform points/vectors	TI	Atomic split
IB DP (SL/HL)	AA	Topic 3.11 Matrix Transformations (HL)	DPAA_67	Identify geometric effect of given transformation matrices	TI	Atomic split
IB DP (SL/HL)	AA	Topic 3.11 Matrix Transformations (HL)	DPAA_68	Compose transformations using matrix products	TI	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_69	Prove geometric results using coordinate methods	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_70	Prove trigonometric relationships algebraically	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_71	Justify vector geometry conclusions formally	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_72	Synthesise trig and coordinate methods in extended problems	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_73	Apply geometric reasoning to optimisation contexts	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_74	Link parametric equations to geometric paths	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_75	Intersect algebraic curves and analyse geometric properties	SE	Atomic split

IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_76	Model physical systems using geometric/trig relationships	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_77	Evaluate validity of geometric assumptions in models	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_78	Communicate multi-step geometric solutions clearly	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_79	Reflect on alternative geometric solution pathways	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_80	Generalise geometric patterns to algebraic forms	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_81	Connect geometric transformations to function mappings	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_82	Analyse invariance in geometric constructions	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_83	Apply geometric constraints in problem-solving	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_84	Use technology to verify geometric conjectures	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_85	Construct geometric arguments for IA exploration	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_86	Critique geometric proofs for logical completeness	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_87	Transfer geometric reasoning to novel contexts	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_88	Integrate multiple Topic 3 strands in exam-style problems	SE	Atomic split
IB DP (SL/HL)	AA	Geometric Proof & Synthesis	DPAA_89	Justify geometric conclusions using formal notation	SE	Atomic split
Cambridge IGCSE 0580		3.1 Coordinate Geometry	**IGCSE_01**	**Calculate gradient of a straight line**	**SE**	**Corrected: Metric calculation**
Cambridge IGCSE 0580		3.1 Coordinate Geometry	**IGCSE_02**	**Calculate length of a line segment**	**SE**	**Corrected: Metric calculation**

Cambridge IGCSE 0580	3.1 Coordinate Geometry	**IGCSE_03**	**Find midpoint of a line segment**	**SE**	**Corrected: Metric calculation**
Cambridge IGCSE 0580	3.1 Coordinate Geometry	IGCSE_04	Derive equation $y = mx + c$ from gradient/point	CA	Atomic split
Cambridge IGCSE 0580	3.1 Coordinate Geometry	**IGCSE_05**	**Identify parallel/perpendicular gradients**	**SE**	**Corrected: Property recognition**
Cambridge IGCSE 0580	4.1 Geometrical Vocabulary	IGCSE_06	Identify and name 2D polygons and properties	SE	Atomic split
Cambridge IGCSE 0580	4.1 Geometrical Vocabulary	IGCSE_07	Identify and name 3D solids and properties	SE	Atomic split
Cambridge IGCSE 0580	4.1 Geometrical Vocabulary	IGCSE_08	Recognise nets and cross-sections of solids	SE	Atomic split
Cambridge IGCSE 0580	4.1 Geometrical Vocabulary	IGCSE_09	Use correct notation for points/lines/angles	SE	Atomic split
Cambridge IGCSE 0580	4.1 Geometrical Vocabulary	IGCSE_10	Classify triangles and quadrilaterals by properties	SE	Atomic split
Cambridge IGCSE 0580	4.2 Constructions & Loci	IGCSE_11	Construct triangles given sides/angles	SE	Atomic split
Cambridge IGCSE 0580	4.2 Constructions & Loci	IGCSE_12	Construct perpendicular bisector of a segment	SE	Atomic split
Cambridge IGCSE 0580	4.2 Constructions & Loci	IGCSE_13	Construct angle bisector	SE	Atomic split
Cambridge IGCSE 0580	4.2 Constructions & Loci	IGCSE_14	Draw and interpret loci in 2D	SE	Atomic split
Cambridge IGCSE 0580	4.2 Constructions & Loci	IGCSE_15	Produce accurate scale drawings	SE	Atomic split
Cambridge IGCSE 0580	4.3 Congruence & Similarity	IGCSE_16	Apply SSS congruence criterion	SE	Atomic split
Cambridge IGCSE 0580	4.3 Congruence & Similarity	IGCSE_17	Apply SAS congruence criterion	SE	Atomic split
Cambridge IGCSE 0580	4.3 Congruence & Similarity	IGCSE_18	Apply ASA congruence criterion	SE	Atomic split
Cambridge IGCSE 0580	4.3 Congruence & Similarity	IGCSE_19	Apply RHS congruence criterion	SE	Atomic split
Cambridge IGCSE 0580	4.3 Congruence & Similarity	IGCSE_20	Identify similar figures and calculate scale factors	SE	Atomic split
Cambridge IGCSE 0580	4.4 Symmetry	**IGCSE_21**	**Identify lines of symmetry in 2D shapes**	**SE**	**Corrected: Static property**

Cambridge IGCSE 0580	4.4 Symmetry	**IGCSE_22**	**Determine order of rotational symmetry**	**SE**	**Corrected: Static property**
Cambridge IGCSE 0580	4.4 Symmetry	**IGCSE_23**	**Analyse symmetry properties of regular polygons**	**SE**	**Corrected: Static property**
Cambridge IGCSE 0580	4.4 Symmetry	**IGCSE_24**	**Identify planes of symmetry in 3D solids**	**SE**	**Corrected: Static property**
Cambridge IGCSE 0580	4.4 Symmetry	**IGCSE_25**	**Relate symmetry to transformational mapping**	**SE**	**Corrected: Synthesis/Property**
Cambridge IGCSE 0580	4.5 Angle Properties	IGCSE_26	Apply angle sum of triangles	SE	Atomic split
Cambridge IGCSE 0580	4.5 Angle Properties	IGCSE_27	Apply angle sum of quadrilaterals	SE	Atomic split
Cambridge IGCSE 0580	4.5 Angle Properties	IGCSE_28	Calculate interior angles of polygons	SE	Atomic split
Cambridge IGCSE 0580	4.5 Angle Properties	IGCSE_29	Calculate exterior angles of polygons	SE	Atomic split
Cambridge IGCSE 0580	4.5 Angle Properties	IGCSE_30	Apply alternate angles with parallel lines	SE	Atomic split
Cambridge IGCSE 0580	4.5 Angle Properties	IGCSE_31	Apply corresponding angles with parallel lines	SE	Atomic split
Cambridge IGCSE 0580	4.5 Angle Properties	IGCSE_32	Apply co-interior angles with parallel lines	SE	Atomic split
Cambridge IGCSE 0580	4.6 Circle Theorems	IGCSE_33	Angle at centre is twice angle at circumference	SE	Atomic split
Cambridge IGCSE 0580	4.6 Circle Theorems	IGCSE_34	Angles in same segment are equal	SE	Atomic split
Cambridge IGCSE 0580	4.6 Circle Theorems	IGCSE_35	Angle in semicircle is 90 degrees	SE	Atomic split
Cambridge IGCSE 0580	4.6 Circle Theorems	IGCSE_36	Opposite angles of cyclic quadrilateral sum to 180	SE	Atomic split
Cambridge IGCSE 0580	4.6 Circle Theorems	IGCSE_37	Tangent is perpendicular to radius at point of contact	SE	Atomic split
Cambridge IGCSE 0580	4.6 Circle Theorems	IGCSE_38	Tangents from external point are equal in length	SE	Atomic split
Cambridge IGCSE 0580	5.1 Mensuration 2D	IGCSE_39	Calculate perimeter of composite 2D shapes	SE	Atomic split
Cambridge IGCSE 0580	5.1 Mensuration 2D	IGCSE_40	Calculate area of triangles and quadrilaterals	SE	Atomic split

Cambridge IGCSE 0580	5.1 Mensuration 2D	IGCSE_41	Calculate circumference and area of circles	SE	Atomic split
Cambridge IGCSE 0580	5.1 Mensuration 2D	IGCSE_42	Calculate arc length of a sector	SE	Atomic split
Cambridge IGCSE 0580	5.1 Mensuration 2D	IGCSE_43	Calculate area of a sector	SE	Atomic split
Cambridge IGCSE 0580	5.2 Mensuration 3D	IGCSE_44	Calculate volume of prisms	SE	Atomic split
Cambridge IGCSE 0580	5.2 Mensuration 3D	IGCSE_45	Calculate volume and surface area of cylinders	SE	Atomic split
Cambridge IGCSE 0580	5.2 Mensuration 3D	IGCSE_46	Calculate volume of pyramids and cones	SE	Atomic split
Cambridge IGCSE 0580	5.2 Mensuration 3D	IGCSE_47	Calculate volume and surface area of spheres	SE	Atomic split
Cambridge IGCSE 0580	5.2 Mensuration 3D	IGCSE_48	Solve composite solid mensuration problems	SE	Atomic split
Cambridge IGCSE 0580	6.1 Pythagoras Theorem	IGCSE_49	Apply Pythagoras theorem in 2D right triangles	SE	Atomic split
Cambridge IGCSE 0580	6.1 Pythagoras Theorem	IGCSE_50	Apply Pythagoras theorem in 3D contexts	SE	Atomic split
Cambridge IGCSE 0580	6.1 Pythagoras Theorem	IGCSE_51	Find missing sides using Pythagoras	SE	Atomic split
Cambridge IGCSE 0580	6.1 Pythagoras Theorem	IGCSE_52	Solve contextual problems using Pythagoras	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_53	Apply sine ratio in right-angled triangles	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_54	Apply cosine ratio in right-angled triangles	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_55	Apply tangent ratio in right-angled triangles	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_56	Solve for missing angles using trig ratios	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_57	Apply sine rule in non-right triangles	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_58	Apply cosine rule in non-right triangles	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_59	Calculate area using 1/2 ab sinC	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	IGCSE_60	Solve 3D trigonometry problems	SE	Atomic split
Cambridge IGCSE 0580	6.2 Trigonometry	**IGCSE_61**	**Sketch and interpret basic trig graphs (Ext)**	**SE**	**Corrected: Graph interpretation**

Cambridge IGCSE 0580	6.2 Trigonometry	**IGCSE_62**	**Solve simple trigonometric equations (Ext)**	**SE**	**Corrected: Equation solving**
Cambridge IGCSE 0580	7.1 Vectors	IGCSE_63	Represent vectors in column notation	CA	Atomic split
Cambridge IGCSE 0580	7.1 Vectors	**IGCSE_64**	**Calculate magnitude and direction of vectors**	**SE**	**Corrected: Metric calculation**
Cambridge IGCSE 0580	7.1 Vectors	IGCSE_65	Add and subtract vectors geometrically/algebraically	CA	Atomic split
Cambridge IGCSE 0580	7.1 Vectors	IGCSE_66	Multiply vectors by scalars	CA	Atomic split
Cambridge IGCSE 0580	7.1 Vectors	IGCSE_67	Use position and resultant vectors in geometry	CA	Atomic split
Cambridge IGCSE 0580	7.2 Transformations	IGCSE_68	Reflect shapes in given lines	TI	Atomic split
Cambridge IGCSE 0580	7.2 Transformations	IGCSE_69	Rotate shapes about given centres/angles	TI	Atomic split
Cambridge IGCSE 0580	7.2 Transformations	IGCSE_70	Translate shapes using vectors	TI	Atomic split
Cambridge IGCSE 0580	7.2 Transformations	IGCSE_71	Enlarge shapes with positive scale factors	TI	Atomic split
Cambridge IGCSE 0580	7.2 Transformations	**IGCSE_72**	**Describe fully a single transformation**	**SE**	**Corrected: Property description**
Cambridge IGCSE 0580	7.2 Transformations	**IGCSE_73**	**Identify invariant points/lines under transformations**	**SE**	Corrected: Static property
Cambridge IGCSE 0580	7.2 Transformations	IGCSE_74	Apply negative/fractional enlargements (Ext)	TI	Atomic split
Cambridge IGCSE 0580	7.2 Transformations	IGCSE_75	Combine two transformations and find equivalent	TI	Atomic split
Cambridge IGCSE 0580	7.2 Transformations	IGCSE_76	Represent transformations using 2x2 matrices (Ext)	CA	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_77	Solve multi-step coordinate geometry problems	CA	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_78	Apply trigonometry to bearings and navigation	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	**IGCSE_79**	**Use vectors in geometric proofs**	**SE**	**Corrected: Proof/Synthesis**

Cambridge IGCSE 0580	Extended Applications	IGCSE_80	Solve circle-tangent coordinate problems	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_81	Analyse combined transformation equivalence	TI	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_82	Apply scale factors to area/volume relationships	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_83	Solve composite mensuration with unit conversion	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_84	Interpret and construct scale diagrams	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_85	Apply trig identities in geometric contexts	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	**IGCSE_86**	**Solve coordinate-trigonometry synthesis problems**	**SE**	**Corrected: Synthesis**
Cambridge IGCSE 0580	Extended Applications	IGCSE_87	Use vector geometry to prove parallelism/collinearity	CA	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_88	Apply circle theorems in algebraic contexts	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_89	Solve 3D Pythagoras/trigonometry composite tasks	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	**IGCSE_90**	**Analyse transformation effects on coordinates**	**SE**	**Corrected: Synthesis**
Cambridge IGCSE 0580	Extended Applications	IGCSE_91	Derive geometric relationships from diagrams	SE	Atomic split
Cambridge IGCSE 0580	Extended Applications	IGCSE_92	Justify geometric conclusions using properties	SE	Atomic split
Cambridge AS/A Level 9709	P1 Coordinate Geometry	**AL_01**	**Calculate gradient/length/midpoint in coordinate plane**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	P1 Coordinate Geometry	AL_02	Find equation of straight line in various forms	CA	Atomic split
Cambridge AS/A Level 9709	P1 Coordinate Geometry	**AL_03**	**Apply parallel/perpendicular gradient conditions**	**SE**	**Corrected: Property application**
Cambridge AS/A Level 9709	P1 Coordinate Geometry	AL_04	Find intersection points of two lines	CA	Atomic split

Cambridge AS/A Level 9709	P1 Coordinate Geometry	AL_05	Solve coordinate geometry problems with parameters	CA	Atomic split
Cambridge AS/A Level 9709	P1 Circles	AL_06	Write equation of circle in standard form	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circles	AL_07	Find centre and radius from general equation	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circles	AL_08	Find equation of tangent to a circle	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circles	AL_09	Determine intersection of line and circle	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circles	AL_10	Apply geometric properties of circles algebraically	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circular Measure	AL_11	Convert between degrees and radians	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circular Measure	AL_12	Calculate arc length using radians	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circular Measure	AL_13	Calculate sector area using radians	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circular Measure	AL_14	Find perimeter of sectors/segments	SE	Atomic split
Cambridge AS/A Level 9709	P1 Circular Measure	AL_15	Solve composite circular measure problems	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_16	Define trig ratios for all quadrants	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_17	Apply fundamental trig identities	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_18	Solve basic trigonometric equations	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_19	Sketch graphs of sin/cos/tan functions	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_20	Use inverse trig functions appropriately	SE	Atomic split

Cambridge AS/A Level 9709	P1 Trigonometry	AL_21	Apply trig to solve triangle problems	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_22	Relate trig graphs to periodic phenomena	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_23	Solve trig equations in given intervals	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_24	Apply trig in coordinate geometry contexts	SE	Atomic split
Cambridge AS/A Level 9709	P1 Trigonometry	AL_25	Prove simple trigonometric identities	SE	Atomic split
Cambridge AS/A Level 9709	P3 Vectors	AL_26	Use vector notation in 2D and 3D	CA	Atomic split
Cambridge AS/A Level 9709	P3 Vectors	**AL_27**	**Calculate magnitude and direction of vectors**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	P3 Vectors	AL_28	Perform vector addition and subtraction	CA	Atomic split
Cambridge AS/A Level 9709	P3 Vectors	AL_29	Multiply vectors by scalars	CA	Atomic split
Cambridge AS/A Level 9709	P3 Vectors	AL_30	Find and use unit vectors	CA	Atomic split
Cambridge AS/A Level 9709	P3 Scalar Product	AL_31	Calculate scalar product of two vectors	CA	Atomic split
Cambridge AS/A Level 9709	P3 Scalar Product	**AL_32**	**Find angle between vectors using dot product**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	P3 Scalar Product	**AL_33**	**Test for perpendicularity using scalar product**	**SE**	**Corrected: Property test**
Cambridge AS/A Level 9709	P3 Scalar Product	AL_34	Interpret scalar product geometrically	CA	Atomic split
Cambridge AS/A Level 9709	P3 Scalar Product	AL_35	Apply scalar product in coordinate problems	CA	Atomic split
Cambridge AS/A Level 9709	P3 Vector Lines	AL_36	Write vector equation of a line $r = a + \lambda b$	CA	Atomic split

Cambridge AS/A Level 9709	P3 Vector Lines	AL_37	Convert between vector and Cartesian forms	CA	Atomic split
Cambridge AS/A Level 9709	P3 Vector Lines	AL_38	Determine if lines are parallel/intersecting/skew	CA	Atomic split
Cambridge AS/A Level 9709	P3 Vector Lines	**AL_39**	**Find angle between two lines**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	P3 Vector Lines	AL_40	Find intersection point of two lines	CA	Atomic split
Cambridge AS/A Level 9709	P3 Distances & Projections	**AL_41**	**Calculate distance from point to line**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	P3 Distances & Projections	**AL_42**	**Find shortest distance between skew lines**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	P3 Distances & Projections	AL_43	Calculate vector projections	CA	Atomic split
Cambridge AS/A Level 9709	P3 Distances & Projections	**AL_44**	**Apply distance formulas in 3D geometry**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	P3 Distances & Projections	AL_45	Solve minimisation problems using vectors	CA	Atomic split
Cambridge AS/A Level 9709	P3 Advanced Trigonometry	AL_46	Apply compound angle formulas	SE	Atomic split
Cambridge AS/A Level 9709	P3 Advanced Trigonometry	AL_47	Apply double angle formulas	SE	Atomic split
Cambridge AS/A Level 9709	P3 Advanced Trigonometry	AL_48	Express a sin x + b cos x in R-form	SE	Atomic split
Cambridge AS/A Level 9709	P3 Advanced Trigonometry	AL_49	Solve complex trigonometric equations	SE	Atomic split
Cambridge AS/A Level 9709	P3 Advanced Trigonometry	AL_50	Prove advanced trigonometric identities	SE	Atomic split
Cambridge AS/A Level 9709	P3 Complex Loci	AL_51	Interpret $\|z - a\| = r$ as circle on Argand diagram	SE	Atomic split
Cambridge AS/A Level 9709	P3 Complex Loci	AL_52	Interpret $\arg(z-a) = \theta$ as half-line	SE	Atomic split

Cambridge AS/A Level 9709	P3 Complex Loci	AL_53	Sketch regions defined by complex inequalities	SE	Atomic split
Cambridge AS/A Level 9709	P3 Complex Loci	AL_54	Apply geometric transformations to complex numbers	TI	Atomic split
Cambridge AS/A Level 9709	P3 Complex Loci	AL_55	Solve locus intersection problems	SE	Atomic split
Cambridge AS/A Level 9709	M1 Vector Applications	AL_56	Resolve forces into vector components	CA	Atomic split
Cambridge AS/A Level 9709	M1 Vector Applications	AL_57	Apply vector triangles for equilibrium	CA	Atomic split
Cambridge AS/A Level 9709	M1 Vector Applications	AL_58	Use vectors in kinematic displacement/velocity	CA	Atomic split
Cambridge AS/A Level 9709	M1 Vector Applications	**AL_59**	**Calculate moments using geometric lever arms**	**SE**	**Corrected: Metric calculation**
Cambridge AS/A Level 9709	M1 Vector Applications	AL_60	Interpret vector results in physical contexts	CA	Atomic split
Cambridge AS/A Level 9709	Synthesis & Problem Solving	AL_61	Solve coordinate-trigonometry synthesis problems	SE	Atomic split
Cambridge AS/A Level 9709	Synthesis & Problem Solving	AL_62	Prove geometric results using vector methods	SE	Atomic split
Cambridge AS/A Level 9709	Synthesis & Problem Solving	AL_63	Apply 3D trigonometry in spatial contexts	SE	Atomic split
Cambridge AS/A Level 9709	Synthesis & Problem Solving	AL_64	Solve circle-line algebraic systems	SE	Atomic split
Cambridge AS/A Level 9709	Synthesis & Problem Solving	AL_65	Construct and analyse geometric loci	SE	Atomic split
Cambridge AS/A Level 9709	Synthesis & Problem Solving	AL_66	Link parametric equations to geometric paths	SE	Atomic split
Cambridge AS/A Level 9709	Synthesis & Problem Solving	AL_67	Justify multi-step geometric solutions	SE	

Note. "Atomic split" indicates that compound syllabus statements were decomposed into discrete, codable units prior to analysis (see Section 3.2.1). All 321 entries represent atomic units, not raw syllabus bullet counts.