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The Inversion of the X-ray Transform on a Compact Symmetric Space

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Abstract. The X-ray transform on a compact symmetric space M is here inverted by means of an explicit inversion formula. The proof uses the conjugacy of the minimal closed geodesics in M and of the maximally curved totally geodesic spheres in M, proved in [3].
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1. Introduction

In his paper [1] Funk showed, using tools from a geometric paper by Minkowski, that an even function f on the sphere $\mathbf{S}^2$ is explicitly determined by its integrals $\widehat{f}(\xi)$ over the great circles ξ on the sphere. The evenness condition is clearly necessary since $\widehat{f} \equiv 0$ if f is odd.

The negative aspect of the result would suggest that Funk's theorem might not extend to geodesic integrals on a compact symmetric space since the concept of an even function is not present. However we shall see that in restated form the theorem generalizes to compact symmetric spaces.

Let $\mathbf{S}^+$ denote the top half $x_3 > 0$ of $\mathbf{S}^2$ and $f \in \mathcal{C}_c^\infty(\mathbf{S}^+)$. Then $g(x) = \frac{1}{2}(f(x) + f(-x))$ is even and $\widehat{f} = \widehat{g}$. The inversion formula for g (Corollary 2.1) thus gives an inversion formula for f on $\mathbf{S}^+$. In this form we extend Funk's injectivity result to compact symmetric spaces, even with an explicit inversion formula. See Corollary 3.3.

Let $M = U/K$ be an irreducible compact simply connected symmetric space, U being a compact semisimple Lie group. For this space we shall use results from our paper [3]; see also [4], VII, §11, whose notation we follow. Let $\mathfrak{u} = \mathfrak{k} + \mathfrak{p}_*$ be the eigenspace decomposition for the involution of the Lie algebra $\mathfrak{u}$ of U. If the metric on M is given by the negative of the Killing form B of $\mathfrak{u}$, the

* This paper is dedicated to François Rouvière whose paper [13] prompted me to reconsider the old problem dealt with here.

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maximal sectional curvature on M equals $\|\bar{\delta}\|^2$ where $\bar{\delta}$ is the highest restricted root. We normalize the metric such that this maximal curvature is 1. We shall then use the following result from [3]. Here $m(\bar{\delta})$ denotes the multiplicity of $\bar{\delta}$.

Theorem 1.1.

(i) *The shortest closed geodesics in M have length 2π and they are permuted transitively by U.*

(ii) *M has totally geodesic spheres of curvature 1. Their maximum dimension is $1+m(\bar{\delta})$. All such spheres $\mathbf{S}^{1+m(\bar{\delta})}$ are conjugate under U.*

We need some further results for the geometry of M. Fix a maximal abelian subspace $\mathfrak{a}_* \subset \mathfrak{p}_*$. Let

$$\mathfrak{p}_\delta = \{X \in \mathfrak{p}_* : (\text{ad } H)^2 X = \delta(H)^2 X \text{ for } H \in \mathfrak{a}_*\}$$

fix $A(\bar{\delta}) \in \mathfrak{a}_*$ such that

$$B(H, A(\bar{\delta})) = \pi i \delta(H) \qquad H \in \mathfrak{a}_*\,,$$

and let $\mathfrak{a}_\delta$ denote the line $\mathbf{R}A(\bar{\delta})$ (notation of [4], VII, §11). If Exp denotes the map from $\mathfrak{p}_*$ to U/K given by Exp $X = (\exp X)K$ we have from [4], p. 343 that the set

$$M_\delta = \text{Exp}\,(\mathfrak{a}_\delta + \mathfrak{p}_\delta)$$

is a sphere, totally geodesic in M, of dimension $1+m(\delta)$ and curvature 1. The point Exp $A(\bar{\delta})$ is the point on M_δ antipodal to the point $o = eK$. Let $S \subset K$ be the subgroup fixing both o and Exp $A(\bar{\delta})$. From [4], p. 343 we have the following result.

Proposition 1.2. *The restriction of $\text{Ad}_U(S)$ to the tangent space $(M_\delta)_0$ contains $\mathbf{SO}((M_\delta)_0)$.*

Definition. Let $x \in M$. The *midpoint locus* A_x associated to x is the set of midpoints $m(\gamma)$ of all the closed minimal geodesics γ starting at x. Let $e_1(\gamma), e_2(\gamma)$ denote the midpoints of the arcs of γ which join x and $m(\gamma)$. Let E_x denote the set of these $e(\gamma)$. We call E_x the *equator* associated to x.

Theorem 1.3. *A_0 and E_0 are K-orbits and A_0 is a totally geodesic submanifold of M. Also*

$$A_0 = K/S\,.$$

Proof. A_0 is a K-orbit because of Theorem 1.1. For a similar statement for E_0 we must verify that the two midpoints $e(\gamma)$ on the same minimal γ are conjugate under K. This is obvious if we take Proposition 1.2 into account. For the rest see [4], VII, §11. ∎

Definition. The Funk transform for $M = U/K$ is the map $f \to \widehat{f}$ where

$$\widehat{f}(\xi) = \int_{\xi} dm(x)\,, \tag{1.1}$$

ξ being a closed geodesic in M of minimal length and dm the arc element.

Because of Theorem 1.1 we have a pair of homogeneous spaces:

$$M = U/K\,, \quad \Xi = \{\text{minimal geodesics}\} = U/H$$

where H is the stabilizer of a specific minimal geodesic ξ in M. We then have the corresponding dual transform $\varphi \to \check{\varphi}$ where

$$\check{\varphi}(gK) = \int_K \varphi(gk\cdot\xi)\,dk\,. \tag{1.2}$$

Notation: In a metric space $B_r(p)$ denotes the open ball with center p and radius r. $S_r(p)$ denotes the corresponding sphere.

2. Inversion on S^n

We consider now the sphere

$$X = \mathbf{S}^n = \mathbf{O}(n+1)/\mathbf{O}(n)\,,$$

where $L = \mathbf{O}(n)$ is the isotropy group of $o = (0,\ldots,0,1)$ and the space

$$\Xi = \{\text{totally geodesic } \mathbf{S}^k \subset \mathbf{S}^n\}$$

for k fixed, $1 \le k \le n-1$. We write

$$\Xi = \mathbf{O}(n+1)/H_p\,,$$

where H_p is the stability group of a k-sphere $\xi_p \subset X$ which has distance p from o. In addition to the Funk transform $f \to \widehat{f}$

$$\widehat{f}(\xi) = \int_{\xi} f(x)\,dm(x) \qquad \xi \in \Xi \tag{2.3}$$

we consider also the dual transform,

$$\check{\varphi}_p(gL) = \int_L \varphi(g\ell\cdot\xi_p)\,d\ell\,, \tag{2.4}$$

the average of φ over the set of $\mathbf{S}^k$ at distance p from $g\cdot o$. We write $\check{\varphi}$ for $\check{\varphi}_0$.

In [2] we inverted the transform $f \to \widehat{f}$ by the formula

$$f = P_k(\Delta)\left((\widehat{f})^{\vee}\right) \tag{2.5}$$

for k even, $P_k(\Delta)$ being an explicit polynomial in the Laplacian Δ of degree $k/2$. In the paper [16] this is augmented by the case $k = n-1$, k odd, and the transform $f \to \widehat{f}$ inverted by an integral which is then suitably regularized.

In [6] I published the inversion formula (2.6) below for $f \to \widehat{f}$, valid for all k and n; in comparison with (2.5) it seemed so unwieldy that I did not publish it in [2]; at that time the case $k=1$ (the X-ray transform) had not gained the later prominence. Unexpectedly, the formula simplifies considerably for $k=1$ and this version is the basis for the extension below to the compact space $M = U/K$. One more inversion of $f \to \widehat{f}$ on $\mathbf{S}^n$ with k arbitrary was given by Rubin [15].

From Theorem 3.2 in [6] we have the following inversion formula for (2.3). For $f \in \mathcal{C}^\infty(\mathbf{S}^n)$ even

$$f(x) = \frac{c}{2}\Big[\Big(\frac{d}{d(u^2)}\Big)^k \int_0^u (\widehat{f})^\vee_{\cos^{-1} v}(x) v^k (u^2 - v^2)^{\frac{k}{2}-1}\, dv\Big]_{u=1} \tag{2.6}$$

where

$$c = \frac{2^{k+1}}{(k-1)!\Omega_{k+1}},$$

and Ω_{k+1} is the area of the unit sphere in $\mathbf{R}^{k+1}$.

Remark 2.1. Let $(M^r f)(x)$ denote the average of f over a sphere in X with center x and radius r. The proof of (2.6) in [6] used the fact that the function $y \to (M^{d(x,y)} f)(x)$ on ξ_p is even, d denoting distance and $d(x, \xi_p) = p$. Indeed, if $g = \mathbf{O}(n+1)$ is such that $g \cdot o = x$ then it was shown that

$$(M^{d(x,y)} f)(x) = \int_L f(g\ell g^{-1} \cdot y)\, d\ell\,,$$

which is indeed even in y because of the linearity of $g\ell g^{-1}$.

For the case $k=1$ we can derive a better version even without the evenness assumption. Note though that because of the integration over E_x formula (2.7) is not an exact inversion. The proof resembles that of Theorem 4.3 in [8].

Corollary 2.2. *The X-ray transform on $\mathbf{S}^n$ is inverted by the formula*

$$\tfrac{1}{2}(f(x) + f(-x)) = \int_{E_x} f(\omega)\, d\omega - \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} \frac{d}{dp} (\widehat{f})^\vee_p(x) \frac{dp}{\sin p} \tag{2.7}$$

for every $f \in \mathcal{C}^\infty(\mathbf{S}^n)$. Here $d\omega$ is the normalized measure on the equator E_x.

Proof. Replacing f by $\frac{1}{2}(f(x)+f(-x))$ has no effect on $\widehat{f}$ so with $\widehat{F}(\cos p) = (\widehat{f})^\vee_p(x)$ we have for the right hand side of (2.6)

$$\begin{aligned}
&\frac{1}{2\pi}\Big\{\frac{d}{du}\int_0^u (u^2 - v^2)^{-\frac{1}{2}} v \widehat{F}(v)\, dv\Big\}_{u=1} \\
&= -\frac{1}{2\pi}\Big\{\frac{d}{du}\int_0^u \frac{d}{dv}(u^2 - v^2)^{\frac{1}{2}} \widehat{F}(v)\, dv\Big\}_{u=1},
\end{aligned}$$

which by integration by parts becomes

$$-\frac{1}{2\pi}\left\{\frac{d}{du}\left[-u\widehat{F}(0)-\int_0^u (u^2-v^2)^{\frac{1}{2}}\frac{d}{dv}\widehat{F}(v)\,dv\right]\right\}_{u=1}$$

$$=\frac{1}{2\pi}\widehat{F}(0)+\frac{1}{2\pi}\int_0^1 (1-v^2)^{-\frac{1}{2}}\frac{d}{dv}\widehat{F}(v)\,dv$$

$$=\frac{1}{2\pi}(\widehat{f})^\vee_{\frac{\pi}{2}}(x)-\frac{1}{2\pi}\int_0^{\frac{\pi}{2}}\frac{d}{dp}(\widehat{f})^\vee_p(x)\frac{dp}{\sin p}\,.$$

The first term is an average of the integrals of f over geodesics at distance $\pi/2$ from x which thus lie in E_x. It represents a rotation–invariant functional on E_x hence a constant multiple of the integral over E_x. Taking $f \equiv 1$ the constant is 1 and the formula is proved. ■

Corollary 2.3. *Suppose $f \in \mathcal{C}^\infty(\mathbf{S}^n)$ has support in the ball $B = \{x \in \mathbf{S}^n : d(o,x) < \frac{\pi}{4}\}$. Then*

$$f(x) = -\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{d}{dp}\Big((\widehat{f})^\vee_p(x)\Big)\frac{dp}{\sin p}\,, \qquad x \in B\,. \tag{2.8}$$

In fact if $x \in B$ then $f(-x) = 0$. If $x \in B$ any $y \in E_x$ then $d(o,y) \geq d(x,y) - d(o,x) \geq \frac{\pi}{2} - \frac{\pi}{4}$ so $f(y) = 0$.

3. The case of a compact symmetric space

Let $M = U/K$ be as in the Introduction. We shall now combine Theorem 1.1 and Corollary 2.2 to study the Funk transform (1.1). Note that this is the X-ray transform restricted to minimal geodesics.

Given $f \in \mathcal{C}^\infty(M)$ we consider its restriction $f|M_\delta$ to the sphere M_δ. For $0 \leq p \leq \frac{\pi}{2}$ we fix a geodesic $\xi_p \subset M_\delta$ at distance p from o. Let f_* denote the Funk transform $(f|M_\delta)\widehat{}$ and φ^*_p the dual transform (2.4). Note that φ^*_p is (for given p) independent of the choice of ξ_p. Then (2.7) implies

$$\tfrac{1}{2}\big(f(o)+f(\mathrm{Exp}\;A(\bar{\delta}))\big) = \int_{E'_0} f(\omega)\,d\omega - \frac{1}{2\pi}\int_0^{\frac{\pi}{2}}\frac{d}{dp}(f_*)^*_p(o)\frac{dp}{\sin p}\,, \tag{3.9}$$

where E'_0 is the equator in M_δ associated to o. By Proposition 1.2, $E'_0 = S\cdot \mathrm{Exp}\,(\frac{1}{2}A(\bar{\delta}))$. We now apply (3.9) to the function

$$f^\natural(x) = \int_K f(k\cdot x)\,dk\,.$$

Since $A_0 = K \cdot \mathrm{Exp}\;A(\bar{\delta})$ the left hand side becomes

$$\tfrac{1}{2}\Big(f(o) + \int_{A_0} f(\omega)\,d\omega\Big)\,,$$

where $d\omega$ stands for average. The first term on the right becomes

$$\int_{K\cdot E'_0} f(k\cdot\omega)\,dk\,d\omega = \int_K f\Big(k\cdot \mathrm{Exp}\;\tfrac{1}{2}A(\bar{\delta})\Big)\,dk = \int_{E_0} f(\omega)\,d\omega\,,$$

where $E_0 = K \cdot \mathrm{Exp}\, \frac{1}{2}A(\bar{\delta})$ which is contained in $S_{\frac{\pi}{2}}(o)$. For the second term on the right note that by the transitivity of the group S (Prop. 1.2)

$$\begin{aligned}
\left((f^\natural)_*\right)_p^*(o) &= \int_S (f^\natural)_*(s\cdot\xi_p)\,ds = \int_S (f^\natural)\widehat{\ }(s\cdot\xi_p)\,ds \\
&= \int_K\int_S \widehat{f}(ks\cdot\xi_p)\,ds\,dk = \int_K \widehat{f}(k\cdot\xi_p)\,dk = (\widehat{f})_p^\vee(o)\,,
\end{aligned}$$

where $\check{\varphi}_p$ is the dual transform (1.2) for $\xi = \xi_p$. Thus we have

$$\tfrac{1}{2}\Big(f(o) + \int_{A_0} f(\omega)\,d\omega\Big) = \int_{E_0} f(\omega)\,d\omega - \frac{1}{2\pi}\int_0^{\frac{\pi}{2}} \frac{d}{dp}\Big((\widehat{f})_p^\vee(o)\Big)\frac{dp}{\sin p}\,.$$

The set $\Xi_p = \{k\cdot\xi_p : k \in K\}$ constitutes the set of all minimal geodesics each lying in some totally geodesic sphere $S^{1+m(\delta)}$ through o having distance p from o. Let ω_p^0 denote the normalized K-invariant measure on this set. Thus

$$(\widehat{f})_p^\vee(o) = \int_{\Xi_p} \widehat{f}(\xi)\,d\omega_p^0(\xi)\,.$$

Let $\Xi_p(x)$ be defined similarly for the point $x \in M$ and let ω_p denote the corresponding measure. Choose $g \in U$ such that $g\cdot o = x$. Then $A_x = gA_0$, $E_x = gE_0$ and $g\cdot\Xi_p = \Xi_p(x)$. Then we obtain the following result.

Theorem 3.1. *Let $f \in \mathcal{C}^\infty(M)$. Then*

$$\tfrac{1}{2}\Big(f(x) + \int_{A_x} f(\omega)\,d\omega\Big) = \int_{E_x} f(\omega)\,d\omega - \frac{1}{2\pi}\int_0^{\frac{\pi}{2}} \frac{d}{dp}\Big(\int_{\Xi_p(x)} \widehat{f}(\xi)\,d\omega_p(\xi)\Big)\frac{dp}{\sin p}\,.$$

Restricting the support of f we obtain the following result.

Corollary 3.2. *Let $f \in \mathcal{C}_c^\infty(B_{\frac{\pi}{2}}(o))$. Then for $x \in B_{\frac{\pi}{2}}(o)$*

$$f(x) = 2\int_{E_x} f(\omega)\,d\omega - \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{d}{dp}\Big(\int_{\Xi_p(x)} \widehat{f}(\xi)\,d\omega_p(\xi)\Big)\frac{dp}{\sin p}\,.$$

For a full inversion formula we restrict the support of f further.

Corollary 3.3. *Let $f \in \mathcal{C}_c^\infty(B_{\frac{\pi}{4}}(o))$. Then*

$$f(x) = -\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{d}{dp}\Big(\int_{\Xi_p(x)} \widehat{f}(\xi)\,d\omega_p(\xi)\Big)\frac{dp}{\sin p}\,, \quad x \in B_{\frac{\pi}{4}}(o)\,.$$

For Corollary 3.2 we must show that

$$A_x \cap B_{\frac{\pi}{2}}(o) = \emptyset \text{ if } x \in B_{\frac{\pi}{2}}(o)\,. \tag{3.10}$$

If $g\cdot o = x$ we have $A_x = g\cdot A_0$ and

$$\begin{aligned}
d(o, gk\cdot\mathrm{Exp}\; A(\bar{\delta})) &= d(g^{-1}\cdot o\,,\, k\cdot\mathrm{Exp}\; A(\bar{\delta})) \\
&\geq d(o, k\cdot\mathrm{Exp}\; A(\bar{\delta})) - d(o, g^{-1}\cdot o) \\
&\geq \pi - \frac{\pi}{2} = \frac{\pi}{2}
\end{aligned}$$

so (3.10) follows. For Cor. 3.3 we must show

$$B_{\frac{\pi}{4}}(o) \cap E_x = \emptyset \quad \text{if} \quad x \in B_{\frac{\pi}{4}}(o)\,.$$

Better still we show that

$$B_{\frac{\pi}{4}}(o) \cap S_{\frac{\pi}{2}}(x) = \emptyset \quad \text{if} \quad x \in B_{\frac{\pi}{4}}(o)\,. \tag{3.11}$$

But if $z \in S_{\frac{\pi}{2}}(x)$ then

$$d(o,z) \geq d(x,z) - d(o,x) \geq \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

proving (3.11).

Using Theorem 1.1 we can derive the following support theorem for the rank one case.

Theorem 3.4. *Assume M has rank one. Let $0 < \delta < \frac{\pi}{2}$. Suppose $f \in \mathcal{C}^\infty(B_{\frac{\pi}{2}}(o))$ satisfies*

(i) $\widehat{f}(\xi) = 0$ *for* $d(o,\xi) > \delta$.

(ii) For each $m > 0$

$$f(x) \cos d(o,x)^{-m} \text{ is bounded.}$$

Then

$$f(x) = 0 \quad \text{for} \quad d(o,x) > \delta\,. \tag{3.12}$$

Proof. Consider the restriction $f|M_\delta$. Because of (ii) $f|M_\delta$ can be extended to a symmetric function on M_δ so by the support theorem for the sphere ([12], [10] or [7]. I, § 3) (3.12) holds for $x \in M_\delta$. Since the rank is one the spheres kM_δ, $(k \in K)$ fill up M so (3.12) holds for $x \in M$. ∎

4. The non-compact case

Here we consider the case of an irreducible symmetric space $X = G/K$ of the non-compact type where G is simple, connected with finite center and K a maximal compact subgroup. As proved in [5] the X-ray transform is here injective. In his elegant paper [14] Rouvière proved an explicit inversion formula by a reduction to the hyperbolic plane. In my paper [9] another inversion formula is given which, however, requires rank $X > 1$.

In this section we present a third formula suggested by our method for the compact case. The proof is much simpler than in the compact case since there is no midpoint locus and no equator. Again we normalize the metric on X such that the maximal negative curvature is -1. A geodesic in X which lies in a totally geodesic hyperbolic space of curvature -1 will be called a *flexed geodesic.* From the duality for symmetric spaces ([4], Ch. V, §2) we know that the tangent spaces to G/K and U/K at o correspond under multiplication by i. This commutes with the action of K. Lie triple systems are mapped into Lie triple systems by this correspondence and sectional curvatures are turned into their negatives. Thus we have the following analog to Theorem 1.1.

Theorem 4.1.

(i) *X has hyperbolic totally geodesic submanifolds of curvature -1. Their maximum dimension is $1+m(\bar{\delta})$ and those $\mathbf{H}^{1+m(\bar{\delta})}$ are all conjugate under G.*

(ii) *The flexed geodesics in X are permuted transitively by G.*

We consider now the hyperbolic analog of M_δ, namely X_δ of curvature -1, dimension $1+m(\bar{\delta})$, passing through $o=eK$. We have then the following analog of Corollary 2.2 proved in [8], for $\mathbf{H}^n$ of all dimensions,

$$f(x) = -\frac{1}{\pi}\int_0^\infty \frac{d}{dp}\Big((\widehat{f})_p^\vee(x)\Big)\frac{dp}{\sinh p}, \quad x \in X_\delta . \tag{4.13}$$

For each $p \geq 0$ let $\Xi_p(x)$ denote the set of all flexed geodesics ξ in X, each lying in a totally geodesic $\mathbf{H}^{1+m(\bar{\delta})}$ passing through x with $d(x,\xi)=p$. Let ω_p denote the normalized measure on Ξ_p invariant under the isotropy group of x. The proof of Theorem 3.4 then yields the following result.

Theorem 4.2. *Let $f \in \mathcal{C}_c^\infty(X)$. Then*

$$f(x) = -\frac{1}{\pi}\int_0^\infty \frac{d}{dp}\bigg(\int_{\Xi_p(x)} \widehat{f}(\xi)\,d\omega_p(\xi)\bigg)\frac{dp}{\sinh p}. \tag{4.14}$$

Remark 4.3. As kindly pointed out by Rouvière, (4.14) agrees with his formula in Theorem 1 in [14] which more generally holds for each root of $(\mathfrak{g},\mathfrak{a})$.

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