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# A note on the Bruhat decomposition of semisimple Lie groups

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**Abstract.** Let a split element of a connected semisimple Lie group act on one of its flag manifolds. We prove that each connected set of fixed points of this action is itself a flag manifold. With this we can obtain a generalized Bruhat decomposition of a semisimple Lie group by entirely dynamical arguments.
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## 0. Introduction

Let a split element $h$ of a connected semisimple Lie group $G$ act on one of its flag manifolds $\mathbb{F}_\Theta$ (notation of semisimple Lie groups and its flag manifolds is recalled in Section 1). We prove that each connected set of fixed points of this action is itself a flag manifold, but a flag manifold of a semisimple Lie subgroup of $G$. This generalizes directly the fact that each connected fixed point set of a diagonalizable matrix acting on a projective space is given by a projective subspace. Apart from being interesting in itself, this result also allows us to obtain generalized Bruhat decomposition of a semisimple Lie group by dynamical arguments, as we explain below.

Standard textbooks on semisimple Lie groups [2, 5] prove by algebraic arguments what we will call the regular Bruhat decomposition of a connected semisimple Lie group $G$, namely

$$G = \coprod_{w \in W} PwP = \coprod_{w \in W} N^+ wP,$$

where $P$ is the minimal parabolic and $W$ the Weyl group of $G$. This decomposition is equivalent to the regular Bruhat decomposition of the maximal flag manifold $\mathbb{F} = \mathrm{Ad}(G)\mathfrak{p}$ of $G$, given by

$$\mathbb{F} = \coprod_{w \in W} Pw\mathfrak{p} = \coprod_{w \in W} N^+ w\mathfrak{p},$$

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which can be seen as the decomposition of $\mathbb{F}$ into unstable manifolds of the action of a split-regular element $h \in A^+$ (cf. Section 3 of [1] for a proof of this by dynamical arguments). From this regular Bruhat decomposition on the maximal flag manifold $\mathbb{F}$ one readily obtains the regular Bruhat decomposition on the partial flag manifolds $\mathbb{F}_\Theta = \mathrm{Ad}(G)\mathfrak{p}_\Theta$ given by

$$\mathbb{F}_\Theta = \coprod_{w \in W/W_\Theta} Pw\mathfrak{p}_\Theta = \coprod_{w \in W/W_\Theta} N^+ w\mathfrak{p}_\Theta, \tag{1}$$

where the argument goes as follows. Projecting the regular Bruhat decomposition of $\mathbb{F}$ onto $\mathbb{F}_\Theta$ one needs only to show the disjointedness of the decomposition in (1). If the unstable manifolds $N^+ w\mathfrak{p}_\Theta$ and $N^+ s\mathfrak{p}_\Theta$ meet, for $s, w \in W$, then there exists $n \in N^+$ such that $w\mathfrak{p}_\Theta = ns\mathfrak{p}_\Theta$. Taking the regular element $h \in A^+$ we have for $k \in \mathbb{Z}$ that $w\mathfrak{p}_\Theta$ is a fixed point so that

$$w\mathfrak{p}_\Theta = h^{-k} w\mathfrak{p}_\Theta = h^{-k} ns\mathfrak{p}_\Theta \to s\mathfrak{p}_\Theta,$$

when $k \to +\infty$. It follows that $w\mathfrak{p}_\Theta = s\mathfrak{p}_\Theta$, so that $s^{-1}w\mathfrak{p}_\Theta = \mathfrak{p}_\Theta$ which, by the Langlands decomposition $P_\Theta = K_\Theta AN^+$, implies that $s^{-1}w \in K_\Theta \cap M^*$ so that $s^{-1}w \in W_\Theta$, that is, $w \in sW_\Theta$, as claimed. The corresponding decomposition in $G$ is the regular Bruhat decomposition

$$G = \coprod_{w \in W/W_\Theta} PwP_\Theta = \coprod_{w \in W/W_\Theta} N^+ wP_\Theta.$$

Usually much harder to obtain is what we will call a generalized Bruhat decomposition of $G$, given by

$$G = \coprod_{w \in W_\Delta \backslash W/W_\Theta} P_\Delta w P_\Theta,$$

where $P_\Delta$ and $P_\Theta$ are standard parabolic subgroups of $G$. This is proved in [5] by using Tits Systems (see Section 1.2 of [5]). This decomposition is equivalent to the generalized Bruhat decomposition of the partial flag manifold $\mathbb{F}_\Theta$ given by

$$\mathbb{F}_\Theta = \coprod_{w \in W_\Delta \backslash W/W_\Theta} P_\Delta w\mathfrak{p}_\Theta = \coprod_{w \in W_\Delta \backslash W/W_\Theta} N_\Delta^+ Z_\Delta w\mathfrak{p}_\Theta. \tag{2}$$

Dynamically, this can be seen as the decomposition of $\mathbb{F}_\Theta$ into unstable manifolds of the action of an non-regular split-element $h \in \mathrm{cl}A^+$, $h = \exp(H)$, where $\Delta$ is the set of simple roots which annihilate $H$. In this case, the fixed points of $h$ in $\mathbb{F}_\Theta$ degenerate into the fixed point manifolds given by (see Proposition 1.2 of [1])

$$\mathrm{fix}(H, w)_\Theta = Z_\Delta w\mathfrak{p}_\Theta. \tag{3}$$

Equation (1) already imply that the orbits $P_\Delta w\mathfrak{p}_\Theta$ in equation (2) exhaust $\mathbb{F}_\Theta$. To show that these orbits are disjoint we can argue as in the previous paragraph to get rid of the unstable part $N_\Delta^+$ of $P_\Delta = N_\Delta^+ Z_\Delta$ so that the only difficulty is to show that fixed point manifolds are disjoint when we take $w \in W_\Delta \backslash W/W_\Theta$. At this point of the argument [1] appeals to a general theorem of Borel-Tits (see Proposition 1.3 of [1]).

In this note we show the disjointedness of the above fixed point manifolds (Corollary 2.3) as a byproduct of showing that each of these fixed point manifolds is itself equivariantly diffeomorphic to a flag manifold (Theorem 2.2). With this, one can obtain a generalized Bruhat decomposition of a semisimple Lie group by entirely dynamical arguments: one follows Section 3 of [1] to prove the regular Bruhat decomposition and then uses the result of this article to prove the generalized Bruhat decomposition.

In the first section we recall notation and preliminary results on semisimple Lie theory and in the second section we prove the main results.

## 1. Preliminaries on Semi-simple Lie Theory

For the theory of semi-simple Lie groups and their flag manifolds we refer to Duistermat-Kolk-Varadarajan [1], Helgason [2] and Warner [5]. To set notation let $G$ be a connected noncompact semi-simple Lie group with Lie algebra $\mathfrak{g}$. We assume throughout that $G$ has finite center. Fix a Cartan involution $\theta$ of $\mathfrak{g}$ with Cartan decomposition $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{s}$. The form $B_\theta(X, Y) = -\langle X, \theta Y\rangle$, where $\langle \cdot, \cdot \rangle$ is the Cartan-Killing form of $\mathfrak{g}$, is an inner product.

Fix a maximal abelian subspace $\mathfrak{a} \subset \mathfrak{s}$ and a Weyl chamber $\mathfrak{a}^+ \subset \mathfrak{a}$. We let $\Pi$ be the set of roots of $\mathfrak{a}$, $\Pi^+$ the positive roots corresponding to $\mathfrak{a}^+$ and $\Sigma$ the set of simple roots in $\Pi^+$. The Iwasawa decomposition of the Lie algebra $\mathfrak{g}$ reads $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{a} \oplus \mathfrak{n}^+$ with $\mathfrak{n}^+ = \oplus_{\alpha \in \Pi^+} \mathfrak{g}_\alpha$ where $\mathfrak{g}_\alpha$ is the root space associated to $\alpha$. As to the global decompositions of the group we write $G = KAN^+$, where $K$, $A$, $N^+$ are the connected subgroups with Lie algebra $\mathfrak{k}$, $\mathfrak{a}$, $\mathfrak{n}^+$ respectively. The Weyl group $W$ associated to $\mathfrak{a}$ is the finite group generated by the reflections over the root hyperplanes $\alpha = 0$ in $\mathfrak{a}$, $\alpha \in \Pi$. $W$ acts on $\mathfrak{a}$ by isometries and can be alternatively be given as $W = M^*/M$ where $M^*$ and $M$ are the normalizer and the centralizer of $A$ in $K$, respectively. We write $\mathfrak{m}$ for the Lie algebra of $M$.

Associated to a subset of simple roots $\Theta \subset \Sigma$ there are several Lie algebras and groups (cf. Section 1.2.4 of [5]): We write $\mathfrak{g}(\Theta)$ for the (semi-simple) Lie subalgebra generated by $\mathfrak{g}_\alpha$, $\alpha \in \Theta$, and put $\mathfrak{k}(\Theta) = \mathfrak{g}(\Theta) \cap \mathfrak{k}$, $\mathfrak{a}(\Theta) = \mathfrak{g}(\Theta) \cap \mathfrak{a}$, and $\mathfrak{n}^\pm(\Theta) = \mathfrak{g}(\Theta) \cap \mathfrak{n}^\pm$. The simple roots of $\mathfrak{g}(\Theta)$ are given by $\Theta$, more precisely, by restricting the functionals of $\Theta$ to $\mathfrak{a}(\Theta)$. Let $G(\Theta)$, $K(\Theta)$, $A(\Theta)$, $N^+(\Theta)$ be the connected groups with Lie algebra $\mathfrak{g}(\Theta)$ $\mathfrak{k}(\Theta)$, $\mathfrak{a}(\Theta)$, $\mathfrak{n}^+(\Theta)$ respectively. Then $G(\Theta)$ is a connected semisimple Lie group with finite center and we have the Iwasawa decomposition $G(\Theta) = K(\Theta)A(\Theta)N^+(\Theta)$. Let $\mathfrak{a}_\Theta = \{H \in \mathfrak{a} : \alpha(H) = 0,\ \alpha \in \Theta\}$ be the orthocomplement of $\mathfrak{a}(\Theta)$ in $\mathfrak{a}$ with respect to the $B_\theta$-inner product and put $A_\Theta = \exp \mathfrak{a}_\Theta$. The subset $\Theta$ singles out the subgroup $W_\Theta$ of the Weyl group which acts trivially on $\mathfrak{a}_\Theta$. Alternatively $W_\Theta$ can be given as the subgroup generated by the reflections with respect to the roots $\alpha \in \Theta$. The restriction of $w \in W_\Theta$ to $\mathfrak{a}(\Theta)$ furnishes an isomorphism between $W_\Theta$ and the Weyl groyp $W(\Theta)$ of $G(\Theta)$

Denote by $Z_\Theta$ the centralizer of $\mathfrak{a}_\Theta$ in $G$ and $K_\Theta = Z_\Theta \cap K$. We have that $K_\Theta$ decomposes as $K_\Theta = MK(\Theta)$ and that $Z_\Theta$ decomposes as

$$Z_\Theta = G(\Theta) M A_\Theta. \tag{4}$$

For $H \in \mathfrak{a}$ we denote by $Z_H$, $K_H$, $W_H$ the centralizer of $H$ in $G$, $K$, $W$

respectively. When $H \in \mathrm{cl}\mathfrak{a}^+$ we put

$$\Theta(H) = \{\alpha \in \Sigma : \alpha(H) = 0\},$$

and we have that $Z_H = Z_{\Theta(H)}$, $K_H = K_{\Theta(H)}$ and $W_H = W_{\Theta(H)}$.

The standard parabolic subalgebra of type $\Theta$ is defined by

$$\mathfrak{p}_\Theta = \mathfrak{m} \oplus \mathfrak{a} \oplus \mathfrak{n}^+ \oplus \mathfrak{n}^- (\Theta),$$

and the corresponding standard parabolic subgroup $P_\Theta$ is the normalizer of $\mathfrak{p}_\Theta$ in $G$. It has the Langlands decomposition $P_\Theta = K_\Theta A N^+$. The empty set $\Theta = \emptyset$ gives the minimal parabolic subalgebra $\mathfrak{p} = \mathfrak{m} \oplus \mathfrak{a} \oplus \mathfrak{n}^+$ whose minimal parabolic subgroup $P = P_\emptyset$ has Langlands decomposition $P = MAN^+$. Let $\mathfrak{n}_\Theta^+ = \oplus_{\alpha \in \Pi^\pm - \langle\Theta\rangle} \mathfrak{g}_\alpha$ and $N_\Theta^+ = \exp(\mathfrak{n}_\Theta^+)$. We have that $P_\Theta$ decomposes as $P_\Theta = Z_\Theta N_\Theta^+$. Take $H \in \mathrm{cl}\mathfrak{a}^+$, $\Theta = \Theta(H)$ and $h = \exp(H)$. Then for $n \in N_\Theta^+$ we have

$$h^{-k} n h^k \to 1, \quad t \to +\infty.$$

Define the flag manifold of type $\Theta$ by the orbit

$$\mathbb{F}_\Theta = \mathrm{Ad}(G)\mathfrak{p}_\Theta,$$

which identifies with the homogeneous space $G/P_\Theta$. Since the center of $G$ normalizes $\mathfrak{p}_\Theta$, the flag manifold depends only on the Lie algebra $\mathfrak{g}$ of $G$. The empty set $\Theta = \emptyset$ gives the maximal flag manifold $\mathbb{F} = \mathbb{F}_\emptyset$. The flag manifolds of $\mathfrak{g}$ can be defined alternatively by the choice of an element $H \in \mathfrak{a}$ as follows. The parabolic subalgebra of type $H$ is defined by

$$\mathfrak{p}_H = \oplus\{\mathfrak{g}_\alpha : \alpha(H) \geq 0\},$$

where $\alpha$ runs through all the weights $\Pi \cup 0$, and the corresponding parabolic subgroup $P_H$ is the normalizer of $\mathfrak{p}_H$. Define the flag manifold of type $H$ by the orbit

$$\mathbb{F}_H = \mathrm{Ad}(G)\mathfrak{p}_H.$$

Now choose a chamber $\mathfrak{a}^+$ of $\mathfrak{a}$ which contains $H$ in its closure, consider the simple roots $\Sigma$ associated to $\mathfrak{a}^+$ and take $\Theta(H) \subset \Sigma$. We have that $\mathfrak{p}_H = \mathfrak{p}_{\Theta(H)}$, so it follows that

$$\mathbb{F}_H = \mathbb{F}_{\Theta(H)},$$

and that $P_H = P_\Theta$, so it decomposes as $P_H = K_{\Theta(H)} A N^+ = K_H A N^+$. We can proceed reciprocally. That is, if $\mathfrak{a}^+$ and $\Theta$ are given, we can choose an $H \in \mathrm{cl}\mathfrak{a}^+$ such that $\Theta(H) = \Theta$ and describe the objects that depend on $\mathfrak{a}^+$ and $\Theta$ by $H$ (clearly, such an $H$ is not unique.) Note that we have

$$w\mathfrak{p}_{\Theta(H)} = w\mathfrak{p}_H = \mathfrak{p}_{wH}. \tag{5}$$

Finally, let $H \in \mathrm{cl}\mathfrak{a}^+$ and put $\Delta = \Theta(H)$. From equation (3) and from the decomposition (4) applied to $Z_\Delta$ we have

$$\mathrm{fix}(H, w) = Z_\Delta w \mathfrak{p}_\Theta = G(\Delta) w \mathfrak{p}_\Theta, \tag{6}$$

since $w$ normalizes $MA$ which fixes $\mathfrak{p}_\Theta$.

## 2. Fixed points as flag manifolds

Let $\pi_\Theta : \mathfrak{a} \to \mathfrak{a}(\Theta)$ be the orthogonal projection parallel to $\mathfrak{a}_\Theta$.

**Lemma 2.1.** *The following assertions are true.*

1. *The projection by $\pi_\Theta$ of a regular element of $\mathfrak{a}$ is a regular element of $\mathfrak{a}(\Theta)$.*
2. *The projection by $\pi_\Theta$ of a chamber in $\mathfrak{a}$ is contained inside a chamber of $\mathfrak{a}(\Theta)$.*
3. *For $w \in W$ denote by $\mathfrak{a}(\Theta)^w$ the chamber of $\mathfrak{a}(\Theta)$ which contains the projection $\pi_\Theta(w\mathfrak{a}^+)$. Then the nilpotent subalgebras $\mathfrak{n}^+$ and $\mathfrak{n}(\Theta)^w$ w.r.t. the chambers $\mathfrak{a}^+$ and $\mathfrak{a}(\Theta)^w$, respectively, satisfy*

$$\mathfrak{n}(\Theta)^w \subset w\mathfrak{n}^+.$$

**Proof.** We first observe that for $\alpha \in \Theta$ we have $\alpha|_{\mathfrak{a}_\Theta} = 0$ so that for $H \in \mathfrak{a}$ we have $\alpha(\pi_\Theta(H)) = \alpha(H)$. Since $\Theta$ is the set of simple roots of $\mathfrak{a}(\Theta)$, it follows that $\pi_\Theta(H)$ is regular in $\mathfrak{a}(\Theta)$ if $H$ is regular in $\mathfrak{a}$. This proves the first item. For the second item we observe that the projection of a chamber of $\mathfrak{a}$ is a convex set of $\mathfrak{a}(\Theta)$ which, by the first item, consists of regular elements of $\mathfrak{a}(\Theta)$, hence it is contained in a chamber of $\mathfrak{a}(\Theta)$. For the third item let $\alpha \in \Theta$. If $\alpha > 0$ in the chamber $\mathfrak{a}(\Theta)^w$ then $\alpha > 0$ in $\pi_\Theta(w\mathfrak{a}^+)$ hence, by the first remark of the proof, we have that $\alpha > 0$ in $w\mathfrak{a}^+$. It follows that

$$\mathfrak{n}(\Theta)^w = \oplus\{\mathfrak{g}_\alpha : \alpha|_{\mathfrak{a}(\Theta)^w} > 0, \alpha \in \prod \Theta\} \subset \oplus\{\mathfrak{g}_\alpha : \alpha|_{w\mathfrak{a}^+} > 0, \alpha \in \Pi\} = w\mathfrak{n}^+,$$

as desired. ■

In what follows fix $H \in \mathrm{cl}\mathfrak{a}^+$ and $\Theta \subset \Sigma$.

**Theorem 2.2.** *Let $\Delta = \Theta(H)$ and take $H_\Theta \in \mathrm{cl}\mathfrak{a}^+$ such that $\Theta(H_\Theta) = \Theta$. Then, for $w \in W$, the map*

$$\psi : \mathrm{fix}(H, w)_\Theta \to \mathbb{F}_{\pi_\Delta(wH_\Theta)}(\mathfrak{g}(\Delta)), \quad g\mathfrak{p}_{wH_\Theta} \mapsto g\mathfrak{p}_{\pi_\Delta(wH_\Theta)}, \quad g \in G(\Delta),$$

*is a well defined $G(\Delta)$-equivariant diffeomorphism.*

**Proof.** From equations (5) and (6) we have $\mathrm{fix}(H, w)_\Theta = G(\Delta)\mathfrak{p}_{wH_\Theta}$. We prove that $\psi$ is well defined and injective. Both facts will follow if we show that the isotropy of $\mathfrak{p}_{wH_\Theta}$ in $G(\Delta)$ coincides with the isotropy of $\mathfrak{p}_{\pi_\Delta(wH_\Theta)}$ of the $G(\Delta)$-action. For this, let $\mathfrak{a}(\Delta)^w$ be the chamber of $\mathfrak{a}(\Delta)$ which contains the projection $\pi_\Delta(w\mathfrak{a}^+)$. Consider the Iwasawa decomposition of $G$ and $P_{wH_\Theta}$ w.r.t. the chamber $w\mathfrak{a}^+$

$$G = KAwN^+w^{-1}, \quad P_{wH_\Theta} = K_{wH_\Theta}AwN^+w^{-1}.$$

Consider also the Iwasawa decomposition of $G(\Delta)$ w.r.t. the chamber $\mathfrak{a}(\Delta)^w$

$$G(\Delta) = K(\Delta)A(\Delta)N(\Delta)^w,$$

where $N(\Delta)^w = \exp(\mathfrak{n}(\Delta)^w)$. By item (3) of the previous Lemma, we have that

$$N(\Delta)^w \subset wN^+w^{-1}.$$

Thus, by the uniqueness of the Iwasawa decomposition of $G$, it follows that the isotropy of $\mathfrak{p}_{wH_\Theta}$ in $G(\Delta)$ is given by

$$G(\Delta) \cap P_{wH_\Theta} = (K(\Delta) \cap K_{wH_\Theta})A(\Delta)N(\Delta)^w.$$

The first term in the right hand side can be written as

$$K(\Delta) \cap K_{wH_\Theta} = K(\Delta)_{wH_\Theta} = K(\Delta)_{\pi_\Delta(wH_\Theta)},$$

where in the last equality we used that $K(\Delta)$ already centralizes $\mathfrak{a}_\Delta$. It follows that

$$G(\Delta) \cap P_{wH_\Theta} = K(\Delta)_{\pi_\Delta(wH_\Theta)}A(\Delta)N(\Delta)^w = P(\Delta)_{\pi_\Delta(wH_\Theta)},$$

which is precisely the isotropy of $\mathfrak{p}_{\pi_\Delta(wH_\Theta)}$ in $G(\Delta)$. It is immediate that $\psi$ is equivariant and that the inverse of $\psi$ is given by $g\mathfrak{p}_{\pi_\Delta(wH_\Theta)} \mapsto g\mathfrak{p}_{wH_\Theta}$, $g \in G(\Delta)$. This shows that $\psi$ is an $G(\Delta)$-equivariant diffeomorphism. ■

**Corollary 2.3.** *Let $w, w' \in W$. If $\mathrm{fix}(H, w')_\Theta \cap \mathrm{fix}(H, w)_\Theta \neq \emptyset$ then $w' \in W_\Delta w W_\Theta$, where $\Delta = \Theta(H)$.*

**Proof.** If $\mathrm{fix}(H, w')_\Theta \cap \mathrm{fix}(H, w)_\Theta \neq \emptyset$ then there exists $g \in G(\Delta)$ such that $w'\mathfrak{p}_\Theta = gw\mathfrak{p}_\Theta$. Take a regular $h \in A(\Delta)$, using the $G(\Delta)$-equivariance of $\psi$ we have for $k \in \mathbb{Z}$

$$w'\mathfrak{p}_\Theta = h^k w'\mathfrak{p}_\Theta = h^k gw\mathfrak{p}_\Theta = h^k g\mathfrak{p}_{wH_\Theta} = \psi^{-1}(h^k g\mathfrak{p}_{\pi_\Delta(wH_\Theta)}) = (*).$$

By the regular Bruhat decomposition of the flag manifold $\mathbb{F}_{\pi_\Delta(wH_\Theta)}(\mathfrak{g}(\Delta))$ (cf. equation (1)), letting $k \to \infty$ we have that there exists $s \in W(\Delta) = W_\Delta$ such that

$$(*) \to \psi^{-1}(s\mathfrak{p}_{\pi_\Delta(wH_\Theta)}) = s\mathfrak{p}_{wH_\Theta} = sw\mathfrak{p}_\Theta.$$

It follows that $w^{-1}s^{-1}w'\mathfrak{p}_\Theta = \mathfrak{p}_\Theta$, so that $w^{-1}s^{-1}w' \in M^* \cap K_\Theta$, which implies that $w^{-1}s^{-1}w \in W_\Theta$. Hence $w \in swW_\Theta \subset W_\Delta wW_\Theta$, as desired. ■

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