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Central Extensions of Coverings of Symplectomorphism Groups

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Abstract. Each even dimensional submanifold of a symplectic manifold defines a Lie algebra 2-cocycle on the Lie algebra of symplectic vector fields. We study its integrability to the group of symplectic diffeomorphisms. When the submanifold is symplectic, we describe a coadjoint orbit of the corresponding extension.
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1. Introduction

In this paper we study central extensions of the group of symplectic diffeomorphisms $\mathrm{Symp}(M,\omega)$ of a symplectic manifold (M,ω), that are defined by a $2k$-dimensional submanifold N of M. The corresponding Lie algebra extension is described by the 2-cocycle σ_N on the Lie algebra of symplectic vector fields of (M,ω) by

$$\sigma_N(X,Y)=\int_N i_X i_Y \omega^{k+1}.$$

The main result is that for $H^{2k+1}(M,\mathbb{R})=0$ and when the cohomology class $[\omega]^{k+1}\in H^{2k+2}(M,\mathbb{R})$ is integral, such a central extension of $\mathrm{Symp}(M,\omega)$ exists. Moreover, when the submanifold N is symplectic, then a coadjoint orbit of this extension is the connected component containing N of the space of $2k$-dimensional symplectic submanifolds of M, the non-linear symplectic Grassmannian $\mathrm{SGr}_{2k}(M)$.

For $k=0$ we recover some results from [5], where a group 2-cocycle on $\mathrm{Symp}(M,\omega)$, integrating the Lie algebra 2-cocycle $(X,Y)\mapsto\omega(X,Y)(x)$, $x\in M$, is given under the assumption that the symplectic form ω is exact. A similar 2-cocycle defined by a $2k$-dimensional submanifold N of M is given in Section 3. Other types of group 2-cocycles on $\mathrm{Symp}(M,\omega)$ were considered in [12] and [13]. In [12] Neretin constructs a group 2-cocycle on the group of compactly supported symplectic diffeomorphisms with the help of a group 2-cocycle on $\mathrm{Sp}(2n,\mathbb{R})$.

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In general it defines a non-trivial cohomology class, while its restriction to the connected component of the identity is a coboundary. In [13] Reznikov uses the closed $\mathrm{Sp}(2n,\mathbb{R})$-invariant closed 2-form on $\mathrm{Sp}(2n,\mathbb{R})/\mathrm{U}(n)$ due to Borel to define a group 2-cocycle on $\mathrm{Symp}(M,\omega)$.

This work is a natural continuation of [2], since it uses the same objects: the non-linear Grassmannian $\mathrm{Gr}_n(M)$ of n-dimensional submanifolds of M with forms induced by the tilda map from forms on M, and same tools: Kostant's central extension associated to a prequantizable presymplectic manifold (presymplectic manifold means a manifold with a closed 2-form).

A 2-cocycle similar to σ_N is the Lichnerowicz 2-cocycle λ_N on the Lie algebra of divergence free vector fields of an m-dimensional manifold M with volume form ν [8] [14]:

$$\lambda_N(X,Y)=\int_N i_X i_Y \nu.$$

In [4] Section 25 it is shown that when ν is an integral volume form, the 2-cocycle λ_N is integrable to the group of exact volume preserving diffeomorphisms of M. The geometric construction uses Kostant's extension associated to the codimension two non-linear Grassmannian $(\mathrm{Gr}_{m-2}(M),\tilde{\nu})$, which is a prequantizable symplectic manifold with a hamiltonian action of the group of exact volume preserving diffeomorphisms. The connected component containing N is a coadjoint orbit of the central extended lie group [2]. The integrability of λ_N to the group of all volume preserving diffeomorphisms is studied in [10] and [15].

Unlike λ_N, whose cohomology class vanishes only for 0-homologous N, the restriction of σ_N to the Lie algebra of hamiltonian vector fields is a coboundary, hence the integrability to the group of hamiltonian diffeomorphisms of (M,ω) is obvious. When the cohomology class $[\omega]^{k+1}\in H^{2k+2}(M,\mathbb{R})$ is integral, the presymplectic manifold $(\mathrm{Gr}_{2k}(M),\widetilde{\omega^{k+1}})$ is prequantizable. The $\mathrm{Symp}(M,\omega)$-action on it is hamiltonian if and only if $H^{2k+1}(M,\mathbb{R})=0$. Pulling back Kostant's central extension by this action, we get a central Lie group extension integrating σ_N. When $H^{2k+1}(M,\mathbb{R})\neq 0$, we pass to covering spaces of $\mathrm{Symp}(M,\omega)$ and $\mathrm{Gr}_{2k}(M)$. The minimal covering group $\overline{\mathrm{Symp}}(M,\omega)$ of $\mathrm{Symp}(M,\omega)$ on which σ_N can be integrated is also determined.

In [2] it is shown that all connected components of the non-linear symplectic Grassmannian $\mathrm{SGr}_{2k}(M)$ appear as coadjoint orbits of the group of Hamiltonian diffeomorphisms. For $H^{2k+1}(M,\mathbb{R})=0$, the connected component containing N is a coadjoint orbit of the central extension of $\mathrm{Symp}(M,\omega)$ integrating σ_N. For $H^{2k+1}(M,\mathbb{R})\neq 0$, a certain covering space of it is a coadjoint orbit of the central extensions of $\widetilde{\mathrm{Symp}}(M,\omega)$ and $\overline{\mathrm{Symp}}(M,\omega)$ integrating σ_N.

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2. Non-linear Grassmannians

We start by collecting some facts from [2] about the non-linear Grassmannian and the non-linear symplectic Grassmannian.

Let $\mathrm{Gr}_n(M)$ be the non-linear Grassmannian of oriented compact n–dimensional submanifolds without boundary of a smooth manifold M. It is a Fréchet manifold in a natural way, see [7] Section 44. Suppose $N \in \mathrm{Gr}_n(M)$. Then the tangent space of $\mathrm{Gr}_n(M)$ at N can naturally be identified with the space of smooth sections of the normal bundle $TN^\perp := (TM|_N)/TN$.

To any k-form α the tilda map associates a $(k-n)$-form $\tilde{\alpha}$ on $\mathrm{Gr}_n(M)$ by:

$$(\tilde{\alpha})_N(Y_1, \ldots, Y_{k-n}) := \int_N i_{Y_{k-n}} \cdots i_{Y_1}\alpha.$$

Here, all Y_j are tangent vectors at $N \in \mathrm{Gr}_n(M)$, i.e. sections of $TN^\perp$. There is a natural action of the group Diff(M) on $\mathrm{Gr}_n(M)$ by $\varphi \cdot N = \varphi(N)$. For every vector field $X \in \mathfrak{X}(M)$ on M, the fundamental vector field ζ_X on $\mathrm{Gr}_n(M)$ is $\zeta_X(N) = X|_N$, viewed as a section of $TN^\perp$. One can verify that

$$\widetilde{d\alpha} = d\tilde{\alpha} \qquad i_{\zeta_X}\tilde{\alpha} = \widetilde{i_X\alpha}$$
$$L_{\zeta_X}\tilde{\alpha} = \widetilde{L_X\alpha} \qquad \varphi^*\tilde{\alpha} = \widetilde{\varphi^*\alpha}.$$

Theorem 1 from [2] shows that, if $[\alpha] \in H^k(M,\mathbb{Z})$, then $(\mathrm{Gr}_{k-2}(M), \tilde{\alpha})$ is prequantizable, i.e. there exist a principal S^1–bundle $\mathcal{P} \to \mathrm{Gr}_{k-2}(M)$ and a principal connection 1-form $\eta \in \Omega^1(\mathcal{P})$ whose curvature form is $\tilde{\alpha}$.

Let $p : \widetilde{\mathrm{Gr}}_n(M) \to \mathrm{Gr}_n(M)$ denote the universal covering projection. The elements in $\widetilde{\mathrm{Gr}}_n(M)$ are seen as homotopy classes $[N_t]$ of curves $t \mapsto N_t$ of n-dimensional submanifolds of M, starting at a fixed point $N_0 \in \mathrm{Gr}_n(M)$. Any closed form $\alpha \in \Omega^{n+1}(M)$ gives rise to a closed 1-form $\tilde{\alpha}$ on $\mathrm{Gr}_n(M)$, hence to a smooth function $\bar{\alpha}$ on $\widetilde{\mathrm{Gr}}_n(M)$, uniquely defined by the conditions

$$p^*\tilde{\alpha} = d\bar{\alpha} \quad \bar{\alpha}([N_0]) = 0,$$

$[N_0]$ denoting the homotopy class of the constant curve.

The value of $\bar{\alpha}$ at $[N_t]$ can be obtained by integrating the $(n+1)$-form α over an $(n+1)$-chain c in M obtained from the class $[N_t]$. This chain is not unique, it depends on a representative curve N_t and on a curve f_t of embeddings $N_0 \hookrightarrow M$ with $f_t(N_0) = N_t$, namely $c : (t,x) \in I \times N_0 \mapsto f_t(x) \in M$. But the integral $\int_c \alpha$ does not depend on these choices and equals $\bar{\alpha}([N_t])$. In the sequel we will use also the notation $\int_{[N_t]}\alpha$.

Suppose (M,ω) is a connected $2n$–dimensional symplectic manifold. We denote by $\mathrm{SGr}_{2k}(M) \subseteq \mathrm{Gr}_{2k}(M)$ the open subset of symplectic submanifolds of M and we call it the non-linear symplectic Grassmannian. The 2-form $\widetilde{\omega^{k+1}}$ is a symplectic form on $\mathrm{SGr}_{2k}(M)$. This symplectic manifold is prequantizable if $[\omega]^{k+1} \in H^{2k+2}(M,\mathbb{Z})$.

Theorem 2.1. *[2] All connected components of the non-linear symplectic Grassmannian* $(\mathrm{SGr}_{2k}(M), \widetilde{\omega^{k+1}})$ *are coadjoint orbits of the group of Hamiltonian diffeomorphisms.*

3. A 2-cocycle on the symplectomorphism group

In this section we generalize the 2-cocycle defined in [5] on the symplectomorphism group of an exact symplectic manifold M with $H^1(M,\mathbb{R})=0$.

The space of normalized locally smooth p-cochains on a Lie group G is the space $C_s^p(G)$ of all maps $c : G^k \to \mathbb{R}$, locally smooth in a neighborhood of the identity, which satisfy $c(g_1,\dots,g_p)=0$ whenever some $g_j=e$. It is a differential complex with the differential

$$(d_G c)(g_0,\dots,g_p)=c(g_1,\dots,g_p)+\sum_{j=1}^{p}(-1)^j c(g_0,\dots,g_{j-1}g_j,\dots,g_p) + (-1)^{p+1}c(g_0,\dots,g_{p-1}).$$

The second locally smooth group cohomology space $H_s^2(G)$ is in bijection with equivalence classes of central Lie group extensions of G by $\mathbb{R}$ possessing a smooth local section.

Let M be a non-compact connected manifold with an exact symplectic form $\omega = d\alpha$. For $k\neq 0$, every oriented compact $2k$-dimensional submanifold without boundary N_0 of M (i.e. $N_0 \in \mathrm{Gr}_{2k}(M)$) defines a normalized group 2-cocycle c_{N_0} on $\mathrm{Symp}(M,\omega)$, the connected component of the identity of the symplectomorphism group, by

$$c_{N_0}(\varphi_1,\varphi_2)=\int_{N_0}^{\varphi_2(N_0)}(\alpha-\varphi_1^*\alpha)\wedge\omega^k. \tag{1}$$

The integral is taken over any bordism with boundary $\varphi_2(N_0)-N_0$, which exists since φ_2 can be connected to the identity by a diffeotopy. This integral does not depend on the chosen bordism since the $(2k+1)$-form $(\alpha-\varphi_1^*\alpha)\wedge\omega^k$ is exact.

Proposition 3.1. *The cohomology class of c_{N_0} on $\mathrm{Symp}(M,\omega)$ does not depend on the choice of the 1-form α satisfying $\omega=d\alpha$ and on the choice of the submanifold N_0 in a connected component of $\mathrm{Gr}_{2k}(M)$, if $k\neq 0$.*

Proof. Let $\omega=d\bar\alpha$ and let $\bar c_{N_0}$ be the group 2-cocycle defined with $\bar\alpha$. The $(2k+1)$-form $(\bar\alpha-\alpha)\wedge\omega^k$ is exact. A $2k$-form β such that $(\bar\alpha-\alpha)\wedge\omega^k=d\beta$ defines a group 1-cochain $a(\varphi)=\int_{N_0}(\varphi^*\beta-\beta)$. Then $\bar c_{N_0}-c_{N_0}=da$.

Given N_0 and N_1 in the same connected component of $\mathrm{Gr}_{2k}(M)$, there is a bordism τ in M with boundary N_1-N_0. Then $c_{N_0}-c_{N_1}=d_G b$ for the group 1-cochain $b(\varphi)=\int_\tau(\alpha-\varphi^*\alpha)\wedge\omega^k$, $\varphi\in\mathrm{Symp}(M,\omega)$. ∎

Remark 3.2. There is a geometric interpretation of the central Lie group extension of $\mathrm{Symp}(M,\omega)$ given by the 2-cocycle c_{N_0}. Let $\mathcal{M}$ be the connected component of N_0 in $\mathrm{Gr}_{2k}(M)$ and the exact 2-form $\Omega=\widetilde{\omega^{k+1}}=d\widetilde{\alpha\wedge\omega^k}$ on $\mathcal{M}$. On the trivial bundle $\mathcal{M}\times\mathbb{R}\to\mathcal{M}$ we consider the connection 1-form $\theta=\widetilde{\alpha\wedge\omega^k}+dt$ with curvature Ω. Then the central extension defined by c_{N_0}, denoted $\mathrm{Symp}(M,\omega)\times_{c_{N_0}}\mathbb{R}$, acts on $\mathcal{M}\times\mathbb{R}$ preserving θ. The action is

$$(\varphi,a)\cdot(N,t)=\Big(\varphi(N),\int_{N_0}^{N}(\varphi^*\alpha-\alpha)\wedge\omega^k+t+a\Big),$$

where the integral is taken over any bordism in M with boundary $N - N_0$. This central extension can be identified with the group of connection preserving automorpisms of $\mathcal{M} \times \mathbb{R}$ projectable to diffeomorphisms of $\mathcal{M}$ coming from the canonical $\mathrm{Symp}(M,\omega)$-action.

Remark 3.3. For $k = 0$ we obtain the construction from [5], but in this case the condition $H^1(M,\mathbb{R}) = 0$ has to be imposed to assure that the 1-form $\alpha - \varphi^*\alpha$ is exact: N_0 is a point $x_0 \in M$, $c_{x_0}(\varphi_1,\varphi_2) = \int_{x_0}^{\varphi_2(x_0)}(\alpha - \varphi_1^*\alpha)$, $\mathcal{M} = M$ and $\Omega = \omega$.

Remark 3.4. Let $M = G/K$ be a non-compact symmetric space admitting a G-invariant complex structure and ω the symplectic form defined by the Hermitian metric. It is shown in [5] that the restriction of c_{x_0} to G is cohomologous to the Guichardet-Wigner 2-cocycle on G. The latter is known to define a non-trivial cohomology class. We define a generalized Guichardet-Wigner 2-cocycle on G corresponding to a $2k$-dimensional submanifold N_0 of M.

Given $g_1, g_2 \in G$, we denote by $(x, g_1(x), g_1g_2(x))$ the oriented geodesic cone having as vertex the point x and as base the geodesic segment from $g_1(x)$ to $g_1g_2(x)$, and by $(N_0, g_1(N_0), g_1g_2(N_0))$ the $(2k+2)$-chain $\cup_{x\in N_0}(x, g_1(x), g_1g_2(x))$. Generalizing the Guichardet-Wigner construction we define

$$w_{N_0}(g_1,g_2) = \int_{(N_0,g_1(N_0),g_1g_2(N_0))} \omega^{k+1}, \tag{2}$$

which is a group 2-cocycle on G.

The restriction of c_{N_0} to G is cohomologous to w_{N_0}. Indeed, given $g \in G$, we denote by $(x, g(x))$ the geodesic segment from x to $g(x)$ and by $(N_0, g(N_0)) = \cup_{x\in N_0}(x, g(x))$ the geodesic cylinder with boundary $g(N_0) - N_0$. The group 1-cochain $a_{N_0}(g) = \int_{(N_0,g(N_0))} \alpha \wedge \omega^k$ on G has the property $da_{N_0} = w_{N_0} - c_{N_0}|_{G\times G}$.

4. 2-cocycles on the Lie algebra of symplectic vector fields

The space of continuous Lie algebra cochains on the topological Lie algebra $\mathfrak{g}$ is $C_c^p(\mathfrak{g}) = \{\sigma : \mathfrak{g}^p \to \mathbb{R} | \sigma \text{ continuous alternating}\}$. With the differential

$$(d_{\mathfrak{g}}\sigma)(X_0,\dots,\mathfrak{X}_p) = \sum_{i\le j}(-1)^{i+j}\sigma([X_i,X_j],X_0,\dots,\hat{X}_i,\dots,\hat{X}_j,\dots,X_p)$$

it becomes a differential complex. Its cohomology is the continuous Lie algebra cohomology $H_c^p(\mathfrak{g})$. The second continuous cohomology space $H_c^2(\mathfrak{g})$ is in bijection with equivalence classes of central topological Lie algebra extensions of $\mathfrak{g}$ by $\mathbb{R}$.

The natural homomorphism from the complex of $\mathbb{R}$-valued locally smooth group cochains to the complex of $\mathbb{R}$-valued continuous Lie algebra cochains in degree two has the following form:

$$c'(X,Y) = \frac{\partial^2}{\partial t\partial s}\Big|_{(0,0)}(c(\exp tX, \exp sY) - c(\exp sY, \exp tX)).$$

For the exact symplectic manifold $(M,\omega=d\alpha)$ we compute the 2-cocycle on the Lie algebra $\mathfrak{symp}(M,\omega)$ of symplectic vector fields corresponding to the 2-cocycle c_{N_0} on $\mathrm{Symp}(M,\omega)$:

$$\begin{aligned} c'_{N_0}(X,Y) &= -\frac{\partial}{\partial s}\Big|_0 \int_{N_0}^{\mathrm{Fl}_s^Y(N_0)} L_X\alpha\wedge\omega^k + \frac{\partial}{\partial t}\Big|_0 \int_{N_0}^{\mathrm{Fl}_t^X(N_0)} L_Y\alpha\wedge\omega^k \\ &= \int_{N_0}(i_XL_Y - i_YL_X)\alpha\wedge\omega^k = \int_{N_0}(i_{[X,Y]}+i_Xi_Yd-di_Xi_Y)\alpha\wedge\omega^k \\ &= \int_{N_0} i_Xi_Y\omega^{k+1} + \int_{N_0} i_{[X,Y]}(\alpha\wedge\omega^k). \end{aligned}$$

This proves that:

Proposition 4.1. *The Lie algebra 2-cocycle associated to the group 2-cocycle c_{N_0} is cohomologous to the Lie algebra 2-cocycle*

$$\sigma_{N_0}(X,Y) = \int_{N_0} i_Xi_Y\omega^{k+1}. \tag{3}$$

Remark 4.2. For a non-exact symplectic form ω on M, σ_{N_0} is stil a continuous Lie algebra 2-cocycle on $\mathfrak{symp}(M,\omega)$.

Remark 4.3. When $k=0$, so $N_0=x_0$ is a point of M, we get $\sigma_{\sigma_0}(X,Y)=\omega(X,Y)(x_0)$. The Lie algebra 2-cocycle σ_{x_0} defines a non-trivial cohomology class in $H^2_c(\mathfrak{symp}(M,\omega)$ if either M is non-compact, or M is compact with non-zero linear map $a\in H^1(M,\mathbb{R})\mapsto a\wedge\omega^{n-1}\in H^{2n-1}(M,\mathbb{R})$ [5].

For the rest of this section M is assumed to be compact. In the exact sequence of Lie algebra homomorphisms

$$0\to\mathfrak{ham}(M,\omega)\to\mathfrak{symp}(M,\omega)\xrightarrow{s_\omega}H^1(M,\mathbb{R})\to 0,$$

for $s_\omega(X)=[i_X\omega]$, the ideal $\mathfrak{ham}(M,\omega)$ of Hamiltonian vector fields is perfect [1]. Therefore the pull-back by s_ω is an injective homomorphism in continuous Lie algebra cohomology: $s_\omega^*: H^2_c(H^1(M,\mathbb{R}))=\Lambda^2H^1(M,\mathbb{R})^*\to H^2_c(\mathfrak{symp}(M,\omega))$.

Remark 4.4. The continuous Lie algebra cohomology space $H^2_c(\mathfrak{ham}(M,\omega))$ is isomorphic to $H^1(M,\mathbb{R})$ [14]. For a surface of genus greater than 2, the Hamiltonian group is contractible and its universal central extension by $H^1(M,\mathbb{R})$ is constructed in [4] as an inductive limit of groups.

Remark 4.5. The continuous cohomology space $H^2_c(\mathfrak{symp}(M,\omega))$ is isomorphic to $\mathrm{Ker}(t)\oplus\mathrm{Im}(s_\omega^*)$ [16] for the transgression map

$$\begin{aligned} t: H^2_c(\mathfrak{ham}(M,\omega)) = H^1(M,\mathbb{R}) &\to H^3_c(H^1(M,\mathbb{R})) = \Lambda^3H^1(M,\mathbb{R})^* \\ t(a)(b_1,b_2,b_3) &= n(n-1)\int_M a\wedge b_1\wedge b_2\wedge b_3\wedge\omega^{n-2} \\ &\quad - \frac{n^2}{\mathrm{Vol}(M)}\sum_{cycl}\langle a,b_1\rangle_M\langle b_2,b_3\rangle_M. \end{aligned}$$

Here $\langle a,b\rangle_M = \int_M a\wedge b\wedge\omega^{n-1}$ denotes the symplectic pairing in $H^1(M,\mathbb{R})$ and $\mathrm{Vol}(M) = \int_M \omega^n$ the symplectic volume.

One can easily see that the restriction of the Lie algebra 2-cocycle σ_{N_0} to the ideal $\mathfrak{ham}(M,\omega)$ of Hamiltonian vector fields is a coboundary. This follows also from the lemma below.

Lemma 4.6. *The cohomology class of the 2-cocycle* $\sigma_{N_0}(X,Y) = \int_{N_0} i_X i_Y \omega^{k+1}$ *on* $\mathfrak{symp}(M,\omega)$ *is in* $\mathrm{Im}(s_\omega^*)$. *More precisely* $[\sigma_{N_0}] = s_\omega^*[\rho]$ *where* $\rho \in \Lambda^2 H^1(M,\mathbb{R})^*$ *is given by*

$$\rho(a,b) = (k+1)\Big(k\langle a,b\rangle_{N_0} - n\frac{\mathrm{Vol}(N_0)}{\mathrm{Vol}(M)}\langle a,b\rangle_M\Big), \quad a,b\in H^1(M,\mathbb{R}),$$

Vol *denoting the symplectic volume and* $\langle a,b\rangle_{N_0} = \int_{N_0} a\wedge b\wedge\omega^{k-1}$.

Proof. The Lie bracket of two symplectic vector fields X and Y is a Hamiltonian vector field with Hamiltonian function $-\omega(X,Y)$. Therefore the unique Hamiltonian function with zero integral on M for $[X,Y]$ is

$$h_{[X,Y]} = -\omega(X,Y) + \frac{n}{\mathrm{Vol}(M)}\langle [i_X\omega],[i_Y\omega]\rangle_M.$$

Then

$$\begin{aligned}\sigma_{N_0}(X,Y) &= \int_{N_0} i_X i_Y \omega^{k+1}\\ &= k(k+1)\int_{N_0} i_X\omega\wedge i_Y\omega\wedge\omega^{k-1} - (k+1)\int_{N_0}\omega(X,Y)\omega^k\\ &= k(k+1)\langle [i_X\omega],[i_Y\omega]\rangle_{N_0} - (k+1)\int_{N_0}(-h_{[X,Y]} + \frac{n}{\mathrm{Vol}(M)}\langle [i_X\omega],[i_Y\omega]\rangle_M)\omega^k\\ &= \rho(s_\omega(X),s_\omega(Y)) + (k+1)\int_{N_0} h_{[X,Y]}\omega^k.\end{aligned}$$

With the help of a continuous linear retraction $r:\mathfrak{symp}(M,\omega)\to\mathfrak{ham}(M,\omega)$, we write $\sigma_{N_0} = s_\omega^*\rho + d\tau$ for the Lie algebra 1-cochain $\tau(X) = -(k+1)\int_{N_0} h_{r(X)}\omega^k$ on $\mathfrak{symp}(M,\omega)$. Hence $[\sigma_{N_0}] = s_\omega^*[\rho]$. ∎

All the elements in $\mathrm{Im}(s_\omega^*)$ can be integrated to $\widetilde{\mathrm{Symp}}(M,\omega)$. The corresponding central extensions are the pull-backs of central extensions of $H^1(M,\mathbb{R})$ by the flux homomorphism $S_\omega : \widetilde{\mathrm{Symp}}(M,\omega)\to H^1(M,\mathbb{R})$, $S_\omega([\varphi_t]) = \int_0^1 [i_{\delta^r\varphi_t}\omega]dt$. Here the right logarithmic derivative is $\delta^r\varphi_t = \frac{d}{dt}\varphi_t\circ\varphi_t^{-1}$.

5. Geometric constructions of central extensions

There is a geometric construction of central Lie group extensions using Kostant's extension. The ingredients are a connected Lie group G, a prequantizable presymplectic manifold $(\mathcal{M},\Omega)$ and a Hamiltonian action of G on $\mathcal{M}$. This means that for all fundametal vector fields ζ_X, $X \in \mathfrak{g}$, the 1-forms $i_{\zeta_X}\Omega$ are exact. Let $\mathcal{P} \to \mathcal{M}$ be the principal S^1-bundle with connection 1-form η and curvature Ω. We denote by $\mathrm{Aut}(\mathcal{P},\eta)$ the group of quantomorphisms, i.e. the connected component of the group of equivariant connection preserving diffeomorphisms of $\mathcal{P}$, and by $\mathrm{Ham}(\mathcal{M},\Omega)$ the group of Hamiltonian diffeomorphisms of $\mathcal{M}$. Kostant's central extension [6] associated to $(\mathcal{M},\Omega)$ is

$$1 \to S^1 \to \mathrm{Aut}(\mathcal{P},\eta) \to \mathrm{Ham}(\mathcal{M},\Omega) \to 1. \tag{4}$$

Its pull-back to G by the Hamiltonian action leads to a 1-dimensional central Lie group extension of G, even if $\mathcal{M}$ is infinite dimensional [11].

Let (M,ω) be a compact connected $2n$-dimensional symplectic manifold with integral $[\omega^{k+1}] \in H^{2k+2}(M,\mathbb{R})$. We fix a closed $2k$-dimensional submanifold N_0 of M and denote by $\mathcal{M}$ the connected component of $\mathrm{Gr}_{2k}(M)$ containing N_0. The integrality of $[\omega^{k+1}]$ assures (see Section 2) the existence of a principal circle bundle $\mathcal{P} \to \mathcal{M}$ and a principal connection 1-form η having curvature Ω. To this data one can associate Kostant's central group extension (4). The canonical action of $\mathrm{Symp}(M,\omega)$ on $\mathcal{M}$ preserves Ω, hence it is a symplectic action.

Theorem 5.1. *If* $H^{2k+1}(M,\mathbb{R}) = 0$*, then* $\mathrm{Symp}(M,\omega)$ *acts in a Hamiltonian way on* $(\mathcal{M},\Omega)$*. The pull-back of Konstant's extension (4) is a central Lie group extension of* $\mathrm{Symp}(M,\omega)$ *integrating* σ_{N_0}*.*

Proof. For $X \in \mathfrak{symp}(M,\omega)$, the $(2k+1)$-form $i_X\omega^{k+1}$ is closed, hence exact. Let $\gamma \in \Omega^{2k}(M)$ such that $i_X\omega^{k+1} = d\gamma$. Then $i_{\zeta_X}\Omega = \widetilde{i_X\omega^{k+1}} = \widetilde{d\gamma} = d\tilde{\gamma}$ is exact too, ensuring that the $\mathrm{Symp}(M,\omega)$-action is Hamiltonian.

The pull-back of Kostant's extension is a Lie group extension by Proposition 3.4 in [11]. Its corresponding Lie algebra 2-cocycle is $(X,Y) \mapsto -\Omega(\zeta_X,\zeta_Y)(N_0) = -\int_{N_0} i_Y i_X \omega^{k+1} = \sigma_{N_0}(X,Y)$ (see Section 3 in [2]). ∎

A momentum map $\mu : \mathcal{M} \to \mathfrak{symp}(M,\omega)^*$ for the Hamiltonian action of $\mathrm{Symp}(M,\omega)$ on $\mathcal{M}$ is $\mu(N)(X) = \int_{N_0}^{N} i_X\omega^{k+1}$. The group 1-cocycle measuring its non-equivariance is

$$\kappa : \mathrm{Symp}(M,\omega) \to \mathfrak{symp}(M,\omega)^*$$
$$\kappa(\varphi)(X) = \mu(\varphi(N_0))(X) = \int_{N_0}^{\varphi(N_0)} i_X\omega^{k+1}.$$

When $k = 0$, then $\mathcal{M} = M$, $\Omega = \omega$, $H^1(M,\mathbb{R}) = 0$ and $\mathrm{Ham}(M,\omega) = \mathrm{Symp}(M,\omega)$, so Theorem 5.1 gives just Kostant's central group extension. Its corresponding Lie algebra extension is trivial.

When $H^{2k+1}(M,\mathbb{R}) \neq 0$, the $\mathrm{Symp}(M,\omega)$-action is no longer Hamiltonian. Passing to universal covering spaces, we get a Hamiltonian action of $\widetilde{\mathrm{Symp}}(M,\omega)$ on $(\tilde{\mathcal{M}}, p^*\Omega)$, where $p : \tilde{\mathcal{M}} \to \mathcal{M}$ denotes the universal covering projection. Given $X \in \mathfrak{symp}(M,\omega)$, the fundamental vector field $\tilde{\zeta}_X$ on $\tilde{\mathcal{M}}$ satisfies $Tp.\tilde{\zeta}_X = \zeta_X$, so

$$i_{\tilde{\zeta}_X} p^*\Omega = p^* i_{\zeta_X}\Omega = p^*(\widetilde{i_X\omega^{k+1}}) = d(\overline{i_X\omega^{k+1}}),$$

where $\overline{i_X\omega^{k+1}}$ is a smooth map on $\tilde{\mathcal{M}}$ defined as in Section 2. The momentum map is in this case

$$\tilde{\mu} : \tilde{\mathcal{M}} \to \mathfrak{symp}(M,\omega)^*, \quad \tilde{\mu}([N_t])(X) = \overline{i_X\omega^{k+1}}([N_t]) = \int_{[N_t]} i_X\omega^{k+1}.$$

The 1-cocycle $\tilde{\kappa} : \widetilde{\mathrm{Symp}}(M,\omega) \to \mathfrak{symp}(M,\omega)^*$ which measures the failure of $\tilde{\mu}$ to be equivariant is

$$\tilde{\kappa}([\varphi_t])(X) = \tilde{\mu}([\varphi_t(N_0)])(X) = \int_{[\varphi_t(N_0)]} i_X\omega^{k+1}.$$

Proposition 5.2. *The pull-back of Kostant's central extension (4) associated to the prequantizable presymplectic manifold $(\tilde{\mathcal{M}}, p^*\Omega)$ by the canonical Hamiltonian action of $\widetilde{\mathrm{Symp}}(M,\omega)$ is a central Lie group extension integrating the Lie algebra 2-cocycle σ_{N_0}.*

Proof. Knowing the $\widetilde{\mathrm{Symp}}(M,\omega)$-action on $\tilde{\mathcal{M}}$ is Hamiltonian, we have just to compute:

$$-p^*\Omega(\tilde{\zeta}_X, \tilde{\zeta}_Y)([N_0]) = -\Omega(\zeta_X,\zeta_Y)(N_0) = -\int_{N_0} i_Y i_X\omega^{k+1} = \sigma_{N_0}(X,Y).$$

Theorem 3.4 from [11] ensures that we indeed get a Lie group extension. ■

6. Minimal covering groups for Lie algebra 2-cocycles

Let $\mathfrak{g}$ be a Lie algebra, $\mathfrak{z}$ a Mackey complete locally convex space and ω a continuous $\mathfrak{z}$-valued Lie algebra 2-cocycle on $\mathfrak{g}$. Then the infinitesimal flux cocycle $f_\omega : X \in \mathfrak{g} \mapsto i_X\omega \in C^1_c(\mathfrak{g},\mathfrak{z})$ is a Lie algebra 1-cocycle on $\mathfrak{g}$ with values in the $\mathfrak{g}$-module $C^1_c(\mathfrak{g},\mathfrak{z})$ of continuous linear maps from $\mathfrak{g}$ to $\mathfrak{z}$.

Let G be a connected Lie group with Lie algebra $\mathfrak{g}$ and $\tilde{G}$ its universal covering group. We denote by X^r the right invariant vector field on G defined by $X \in \mathfrak{g}$ and by ω^l the (closed) left invariant 2-form on G defined by $\omega \in Z^2_c(\mathfrak{g},\mathfrak{z})$. The abstract flux 1-cocycle $\tilde{F}_\omega : \tilde{G} \to C^1(\mathfrak{g},\mathfrak{z})$ associated to ω is defined by $\tilde{F}_\omega([\gamma])(X) = -\int_\gamma i_{X^r}\omega^l$ [9]. Here $[\gamma] \in \tilde{G}$ denotes the homotopy class of a path γ in G starting at the identity. Another expression for the flux 1-cocycle is [10]

$$\tilde{F}_\omega([\gamma])(X) = \int_0^1 \omega(\gamma(t)^{-1}\gamma'(t), \mathrm{Ad}(\gamma(t))^{-1}X)dt. \tag{5}$$

By restricting $\tilde{F}_\omega$ to $\pi_1(G)$ we get the flux homomorphism $F_\omega : \pi_1(G) \to H^1(\mathfrak{g},\mathfrak{z})$. Let Γ_ω be the period group of ω, i.e. the image of the period map $[\beta] \in \pi_2(G) \mapsto \int_{S^2} \beta^*\omega^l \in \mathfrak{z}$.

Theorem 6.1. *[9] Assuming that the period group Γ_ω is discrete, the central Lie algebra extension $\hat{\mathfrak{g}} = \mathfrak{z} \times_\omega \mathfrak{g}$ integrates to a Lie group extension of G by the abelian Lie group $Z = \mathfrak{z}/\Gamma_\omega$ if and only if the flux homomorphism $F_\omega : \pi_1(G) \to H^1(\mathfrak{g},\mathfrak{z})$ vanishes. In particular $\hat{\mathfrak{g}}$ always integrates to a Lie group extension of $\tilde{G}$ by Z.*

Proposition 6.2. *Let Π be the kernel of the flux homomorphism $F_\omega : \pi_1(G) \to H^1(\mathfrak{g}, \mathfrak{z})$ and let $\bar{G}$ be the covering group $\tilde{G}/\Pi$ of G. Then the central Lie algebra extension $\hat{\mathfrak{g}} = \mathfrak{z} \times_\omega \mathfrak{g}$ integrates to a Lie group extension of $\bar{G}$ by $\mathfrak{z}/\Gamma_\omega$. Moreover the covering group $\bar{G}$ of G is minimal with this property.*

Proof. The flux homomorphism for $\bar{G}$ vanishes since it is the restriction of the flux homomorphism F_ω to $\pi_1(\bar{G}) = \Pi = \mathrm{Ker} F_\omega$. Knowing that $\pi_2(\bar{G}) = \pi_2(\tilde{G}) = \pi_2(G)$, the result follows from the previous theorem. ∎

We apply this proposition to the cocycle $\sigma = \sigma_{N_0}$ on the Lie algebra of symplectic vector fields.

Given $V \subset H^{2k+1}(M, \mathbb{R})$, we denote by $V^o \subset H_{2k+1}(M, \mathbb{R})$ the annihilator of V with respect to the canonical pairing between homology and cohomology.

Corollary 6.3. *Let N_0 be a fixed $2k$-dimensional submanifold of M and let σ be the Lie algebra 2-cocycle $\sigma(X,Y) = \int_{N_0} i_X i_Y \omega^{k+1}$ on the Lie algebra of symplectic vector fields on M. We consider the subgroup Π_σ of the fundamental group of the group of symplectic diffeomorphisms defined by*

$$\Pi_\sigma = \{[\varphi_t] \in \pi_1(\mathrm{Symp}(M,\omega)) | [\hat{\varphi}|_{N_0}] \in (H^1(M,\mathbb{R}) \wedge [\omega]^k)^o \subset H_{2k+1}(M,\mathbb{R})\}.$$

where $\hat{\varphi}|_{N_0}$ is the $(2k+1)$-cycle $(t,x) \in [0,1] \times N_0 \mapsto \varphi_t(x) \in M$. Then the minimal covering group of $\mathrm{Symp}(M,\omega)$ *on which σ can be integrated is* $\overline{\mathrm{Symp}}(M,\omega) = \widetilde{\mathrm{Symp}}(M,\omega)/\Pi_\sigma$.

Proof. The flux homomorphism associated to the cocycle σ is

$$F_\sigma : \pi_1(\mathrm{Symp}(M,\omega)) \to H^1_c(\mathfrak{symp}(M,\omega)), \quad F_\sigma([\varphi_t])(X) = \int_{[\varphi_t(N_0)]} i_X \omega^{k+1}.$$

Indeed, the adjoint action in $\mathrm{Diff}(M)$ is $\mathrm{Ad}(\varphi)X = (\varphi^{-1})^* X$ and the relation between the left logarithmic derivative $\delta^l \varphi_t = T\varphi_t^{-1} . \frac{d}{dt}\varphi_t$ and the right logarithmic derivative is $\delta^r \varphi_t = \mathrm{Ad}(\varphi_t)\delta^l \varphi_t = (\varphi_t^{-1})^* \delta^l \varphi_t$. Hence

$$\begin{aligned} F_\sigma([\varphi_t])(X) &= -\int_{\varphi_t} i_{X^r}\sigma^l \overset{(5)}{=} -\int_0^1 \sigma(\mathrm{Ad}(\varphi_t^{-1})X, \delta^l \varphi_t) dt \\ &= -\int_0^1 \int_{N_0} i_{\varphi_t^* X} i_{\delta^l \varphi_t} \omega^{k+1} dt = \int_0^1 \int_{N_0} \varphi_t^* i_{\delta^r \varphi_t} i_X \omega^{k+1} dt = \int_{[\varphi_t(N_0)]} i_X \omega^{k+1}. \end{aligned}$$

The commutator Lie algebra of $\mathfrak{symp}(M,\omega)$ is $\mathfrak{ham}(M,\omega)$ [1], so its first cohomology space is $H^1_c(\mathfrak{symp}(M,\omega)) = H^1(M,\mathbb{R})^*$. Under this identification the flux homomorphism becomes $F_\sigma([\varphi_t])(a) = (k+1)\langle [\hat{\varphi}|_{N_0}], a \wedge [\omega]^k \rangle$ for any $a \in H^1(M,\mathbb{R})$. Then $\Pi_\sigma = \mathrm{Ker} F_\sigma$, so $\overline{\mathrm{Symp}}(M,\omega)$ is the minimal covering group of $\mathrm{Symp}(M,\omega)$ on which σ can be integrated (by Proposition 6.2). ∎

The flux homomorphism F_σ vanishes when $H^{2k+1}(M,\mathbb{R}) = 0$. In this case $\overline{\mathrm{Symp}}(M,\omega) = \mathrm{Symp}(M,\omega)$, fact already known from Theorem 5.1.

A geometric construction of the central extension of $\overline{\mathrm{Symp}}(M,\omega)$ can be done using the covering space $q:\bar{\mathcal{M}}\to\mathcal{M}$ of the connected component $\mathcal{M}$ of $\mathrm{Gr}_{2k}(M)$ containing N_0, defined by $\bar{\mathcal{M}}=\tilde{\mathcal{M}}/\Pi_{\mathcal{M}}$ for

$$\Pi_{\mathcal{M}}=\{[N_t]\in\pi_1(\mathcal{M}):\int_{[N_t]}i_X\omega^{k+1}=0,\text{for all } X\in\mathfrak{symp}(M,\omega)\}. \tag{6}$$

Lemma 6.4. *The groups $\widetilde{\mathrm{Symp}}(M,\omega)$ and $\overline{\mathrm{Symp}}(M,\omega)$ act on $(\bar{\mathcal{M}},q^*\Omega)$ in a Hamiltonian way with momentum map*

$$\bar\mu:\bar{\mathcal{M}}\to\mathfrak{symp}(M,\omega)^*,\quad \bar\mu([N_t])(X)=\int_{[N_t]}i_X\omega^{k+1}. \tag{7}$$

Proof. The group $\widetilde{\mathrm{Symp}}(M,\omega)$ acts on $\bar{\mathcal{M}}$ because for any two representing paths N_t and N_t' of the same element $[N_t]=[N_t']\in\bar{\mathcal{M}}$ and any $[\varphi_t]\in\widetilde{\mathrm{Symp}}(M,\omega)$, the paths $\varphi_t(N_t)$ and $\varphi_t(N_t')$ represent the same element in $\bar{\mathcal{M}}$. Indeed, $N_1=N_1'$ and for all $X\in\mathfrak{symp}(M,\omega)$

$$\begin{aligned}\int_{[\varphi_t(N_t)]}i_X\omega^{k+1}&=\int_{[N_t]}i_X\omega^{k+1}+\int_{[\varphi_t(N_1)]}i_X\omega^{k+1}\\&=\int_{[N_t']}i_X\omega^{k+1}+\int_{[\varphi_t(N_1')]}i_X\omega^{k+1}=\int_{[\varphi_t(N_t')]}i_X\omega^{k+1}.\end{aligned}$$

The action of $\Pi_\sigma\subset\widetilde{\mathrm{Symp}}(M,\omega)$ on $\bar{\mathcal{M}}$ is trivial. Indeed, let $[\varphi_t]\in\Pi_\sigma$ and $[N_t]\in\bar{\mathcal{M}}$. Since $i_X\omega^{k+1}$ is closed and N_0, N_1 cobordant,

$$\int_{[\varphi_t(N_t)]}i_X\omega^{k+1}-\int_{[N_t]}i_X\omega^{k+1}=\int_{[\varphi_t(N_1)]}i_X\omega^{k+1}=\int_{[\varphi_t(N_0)]}i_X\omega^{k+1}=0,$$

so $[\varphi_t(N_t)]=[\varphi_t]\in\bar{\mathcal{M}}$. Hence the $\widetilde{\mathrm{Symp}}(M,\omega)$-action projects to a $\overline{\mathrm{Symp}}(M,\omega)$-action on $\bar{\mathcal{M}}$. Given $X\in\mathfrak{symp}(M,\omega)$, the fundamental vector field $\bar\zeta_X$ on $\bar{\mathcal{M}}$ satisfies $Tq.\bar\zeta_X=\zeta_X$, so the action is Hamiltonian:

$$i_{\bar\zeta_X}q^*\Omega=q^*i_{\zeta_X}\Omega=q^*\widetilde{i_X\omega^{k+1}}=d\overline{i_X\omega^{k+1}},$$

where $\overline{i_X\omega^{k+1}}:[N_t]\mapsto\int_{[N_t]}i_X\omega^{k+1}$ is a well defined function on $\bar{\mathcal{M}}$. Hence a momentum map is $\bar\mu([N_t])(X)=\int_{[N_t]}i_X\omega^{k+1}$. ∎

Observing that $-q^*\Omega(\bar\zeta_X,\bar\zeta_Y)=\sigma_{N_0}(X,Y)$, and using Lemma 2 together with Proposition 3.4 in [11], we get the following proposition:

Proposition 6.5. *By pulling back Kostant's central extension for $(\bar{\mathcal{M}},q^*\Omega)$, we obtain a geometric construction of a central Lie group extension of $\overline{\mathrm{Symp}}(M,\omega)$, integrating the Lie algebra cocycle σ_{N_0}, $\overline{\mathrm{Symp}}(M,\omega)$ being the minimal covering group of $\mathrm{Symp}(M,\omega)$ on which σ_{N_0} can be integrated.*

7. Coadjoint orbits of $\mathrm{Symp}(M, \omega)$

Let (M, ω) be a compact symplectic manifold and N_0 a compact $2k$-dimensional symplectic submanifold without boundary of M. The group $\mathrm{Ham}(M, \omega)$ of Hamiltonian diffeomorphisms acts transitively on every connected component of the non-linear symplectic Grassmannian $\mathrm{SGr}_{2k}(M)$, in particular on the connected component $\mathcal{S}$ containing N_0.

$\mathcal{S}$ is an open submanifold of $\mathcal{M}$, the connected component of N_0 in $\mathrm{Gr}_{2k}(M)$, and the 2-form $\Omega = \widetilde{\omega^{k+1}}$ restricts to a symplectic form on $\mathcal{S}$. The symplectic manifold $(\mathcal{S}, \Omega)$ is a coadjoint orbit of $\mathrm{Ham}(M, \omega)$ by Theorem 2.1. Let $q : \bar{\mathcal{M}} \to \mathcal{M}$ be the covering from Section 6 and $\bar{\mathcal{S}} \subset \bar{\mathcal{M}}$ the connected component of $q^{-1}(\mathcal{S})$ containing $[N_0]$.

Proposition 7.1. *The symplectic manifold $(\bar{\mathcal{S}}, q^*\Omega)$ is a coadjoint orbit of the central extensions of $\widetilde{\mathrm{Symp}}(M, \omega)$ and $\overline{\mathrm{Symp}}(M, \omega)$ integrating σ_{N_0}.*

Proof. The actions of $\widetilde{\mathrm{Symp}}(M, \omega)$ and $\overline{\mathrm{Symp}}(M, \omega)$ on $\bar{\mathcal{S}}$ are Hamiltonian. They lift the transitive action of $\mathrm{Symp}(M, \omega)$ on $\mathcal{S}$ [2], hence it is transitive.

A momentum map $\bar{\mu} : \bar{\mathcal{S}} \to \mathfrak{symp}(M, \omega)^*$ is given by (7) and it is injective. Indeed, if $\bar{\mu}([N_t]) = \bar{\mu}([N_t'])$, then $\int_{N_1} f\omega^k = \int_{N_1'} f\omega^k$ for any smooth function f on M (consider $i_X\omega = df$). It follows that $N_1 = N_1'$ and since $\int_{[N_t]} i_X\omega^{k+1} = \int_{[N_t']} i_X\omega^{k+1}$, in $\bar{\mathcal{S}}$ the classes of $[N_t]$ and $[N_t']$ coincide.

Applying Proposition 1 from [2] to this transitive hamiltonian action, we get the result. ∎

Corollary 7.2. *For $H^{2k+1}(M, \mathbb{R}) = 0$, the symplectic manifold $(\mathcal{S}, \Omega)$ is a coadjoint orbit of the central extension of $\mathrm{Symp}(M, \omega)$ obtained in Theorem 5.1.*

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