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ON YJ-INJECTIVITY AND ANNIHILATORS

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Abstract. This note contains the following results for a ring A: (1) A is a quasi-Frobenius ring iff A is a left and right YJ-injective, left Noetherian ring whose prime factor rings are right YJ-injective iff every non-zero one-sided ideal of A is the annihilator of a finite subset of elements of A; (2) if A is a right YJ-injective ring such that any finitely generated right ideal is either a maximal right annihilator or a projective right annihilator, then A is either quasi-Frobenius or a right p.p. ring such that every non-zero left ideal of A contains a non-zero idempotent; (3) a commutative YJ-injective Goldie ring is quasi-Frobenius; (4) if the Jacobson radical of A is reduced, every simple left A-module is either YJ-injective or flat and every maximal left ideal of A is either injective or a two-sided ideal of A, then A is either strongly regular or left self-injective regular with non-zero socle.

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Introduction

Quasi-Frobenius rings introduced by T. Nakayama (1939) have since been extensively studied (cf. [8], [10], [11], [17], [20], [22]). We here consider quasi-Frobenius rings in terms of certain special annihilators. In [25] p-injective modules were introduced to study von Neumann regular rings, V-rings, self-injective rings and associated rings, have drawn the attention of various authors (cf. for example [2], [3], [9], [11], [15], [16], [21], [24]).

Throughout the paper, A denotes an associative ring with identity and A-modules are unital. J, Y will always stand respectively for the Jacobson radical and the right singular ideal of A. Recall that a right A-module M is p-injective if, for every principal right ideal P of A, any right A-homomorphism of P into M extends to one of A into M (cf. [11, p. 122], [20, p. 340], [21], [24], [25]). It is well known that A is von Neumann regular iff every right (left) A-module is flat (M. Harada (1956); M. Auslander (1957)). This remains true if "flat" is replaced by "p-injective" (cf. [2], [15], [21], [24], [25]). The generalization of p-injectivity to YJ-injectivity is performed as follows: a right A-module M is called YJ-injective if, for every $0 \neq a \in A$, there exist a positive integer n such that $a^n \neq 0$ and any right A-homomorphism of a^nA into M extends to one of A into M ([7], [21], [28], [29]). A is called a right p-injective (resp. YJ-injective) ring if A_A is p-injective (resp. YJ-injective). Similarly, p-injectivity and YJ-injectivity are defined on the left side.

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An ideal of A will always mean a two-sided ideal of A. As usual, a right (resp. left) ideal I of A is called a right (resp. left) annihilator if $I = r(S)$ (resp. $1(S)$) for some subset S of elements of A. In [29, Lemma 3], it is proved that A is a right YJ-injective ring iff for every $0 \neq a \in A$, there exist a positive integer n such that Aa^n is a non-zero left annihilator. Also, if A is right YJ-injective, then $J = Y$ (cf. [27, p. 222] and [28, p. 103]) (where this notation was introduced).

It is well known that quasi-Frobenius rings are left and right self-injective, left and right Artinian rings whose one-sided ideals are annihilators. Note that certain special annihilators will impose the chain conditions on rings (cf. [29], [33]).

Rings whose one-sided ideals are annihilators of an element are studied by C. R. Yohe [23]. The next result is motivated by the remark at the end of Yohe's paper. The proof of [33, Theorem 16] shows that if every non-zero one-sided ideal of A is the annihilator of an element of A, then A is a principal ideal ring. By a theorem of M. Ikeda and T. Nakayama, A is left and right self-injective. Consequently, [10, Theorem 24.20] yields

Theorem 1. *The following conditions are equivalent:*

(1) *Every factor ring of A is quasi-Frobenius;*
(2) *A is a principal left and right ideal ring which is quasi-Frobenius;*
(3) *Every non-zero one-sided ideal of A is the annihilator of an element of A.*

We here give an example of a commutative principal ideal quasi-Frobenius ring.

Example (Q). Set $A = \mathbb{Z}/4\mathbb{Z}$. Then $M = \{0, 2\}$ is the unique non-trivial ideal of A. Every non-zero ideal of A is the annihilator of an element of A (M is the annihilator of 2). A is a principal ideal ring (M is generated by 2). A is therefore Artinian, self-injective, and $M^2 = 0$. Consequently, M is not an injective A-module and A is not semi-prime. Note that every factor ring of A is quasi-Frobenius (A/M being a field).

Following [11], we say that "A is VNR" if A is a von Neumann regular ring. Since 1979 K. R. Goodearl's classic [14] has motivated numerous papers on VNR and associated rings. Following C. Faith, A is called a right (resp. left) V-ring if every simple right (resp. left) A-module is injective. A theorem of I. Kaplansky asserts that a commutative ring is a V-ring iff it is VNR. In the non-commutative case, there is no implication between these two classes of rings. A vast amount of work on injective modules over non semi-simple Artinian rings and on flat modules over non-VNR rings motivate the study of p-injectivity and YJ-injectivity over rings which are not necessarily VNR (cf. for example, [3], [11], [15], [16], [22]).

A result of P. Menal and P. Vamos asserts that any arbitrary ring can be embedded in a FP-injective ring [11, p. 308]. (This does not hold if "FP-injective" is replaced by "self-injective".) Consequently, any ring can be embedded in a p-injective or YJ-injective ring. We are thus motivated to study YJ-injective rings. A is called a right CF-ring if every cyclic right A-module embeds in a free module (cf. [21]). A semi-perfect, right CF, right YJ-injective ring is quasi-Frobenius [21, Corollary 8].

Theorem 2. *The following conditions are equivalent:*

(1) *A is quasi-Frobenius;*
(2) *A is a right CF, right YJ-injective ring satisfying the maximum condition on left annihilators of elements of A;*
(3) *A is a left and right YJ-injective, left Noetherian ring whose prime factor rings are right YJ-injective;*
(4) *Every non-zero one-sided ideal of A is the annihilator of a finite subset of elements of A.*

Proof. (1) Clearly implies (2), (3) and (4).

Assume (2). Since A is right CF, then it is left p-injective and therefore every principal right ideal of A is a right annihilator. Now A satisfies the descending chain condition on principal right ideals which means that A is left perfect. By [17, Corollary 11.6.2], and [21, Corollary 8], A is quasi-Frobenius and therefore, (2) implies (1).

Assume (3). Let B be a prime factor ring of A. Then B is left Noetherian and since B is right YJ-injective, every non-zero-divisor of B is invertible in B which implies that B coincides with a classical left (and right) quotient ring. By Goldie's theorem, B is simple Artinian. If A is prime, then A is Artinian as just seen. If not, then every proper prime factor ring of A is simple Artinian and by [10, Lemma 18.34B], A is left Artinian. In any case, A is left Artinian. Since A is right YJ-injective, for any minimal left ideal U of A, $U = Au$, $u \in A$, there exist a positive integer n such that Au^n is a non-zero left annihilator [28, Lemma 3]. If $U^2 = 0$, then $u^2 = 0$ which implies that Au is a left annihilator. If $U = Ae$, $e = e^2 \in A$, then U is again a left annihilator. Similarly, every minimal right ideal of A is a right annihilator. A is therefore quasi-Frobenius by a result of H. H. Storrer [19]. Thus (3) implies (1).

Finally, assume (4). Let L be a non-zero proper left ideal of A. If $L = l(F)$, where $F = \{x_1, \ldots, x_m\}$, $x_i \in A$, with $R = \sum_{i=1}^{m} x_i A$, $L = l(R)$ and by hypothesis, $R = r(G)$, where $G = \{y_1, \ldots, y_n\}$ is a finite subset of elements of A. With $K = \sum_{i=1}^{n} Ay_i$, $R = r(K)$ and K is also a left annihilator which implies that $L = l(R) = l(r(K)) = K$ is a finitely generated left ideal of A. Therefore A is left Noetherian. Since every principal one-sided ideal of A is an annihilator, then A is quasi-Frobenius (a left Noetherian, left or right perfect ring is left Artinian ([10, Proposition 18.12])). Thus (4) implies (1). □

Remark 1. It follows from Theorem 2 that if every factor ring of A is a left and right YJ-injective, left Noetherian ring, then A is quasi-Frobenius.

This remark, together with [28, Proposition 3.1(3)] motivate the following

Question 1. Is a left and right YJ-injective, left Noetherian ring quasi-Frobenius? (It is known that if A is a left Noetherian ring whose minimal one-sided ideals are annihilators, then A needs not be left Artinian.)

Remark 2. If A is left p-injective with maximum condition on left annihilators of elements of A and A contains no nilpotent minimal right ideal, then A is semi-simple Artinian.

The singular submodule of a module is a fundamental concept in ring theory (a standard reference is K.R. Goodearl [13]). Recall that for a right A-module M, the right singular submodule of M is $Z(M) = \{y \in M \setminus r(y)$ is an essential right ideal of $A\}$ and M is called right non-singular if $Z(M) = 0$. In this note, we write $Y = Z(A_A)$ and Y is called the right singular ideal of A.

A well-known result asserts that A is right non-singular (i.e. $Y = 0$) iff A has VNR maximal right quotient ring Q. In that case, Q_A is the injective hull of A_A and Q is a right self-injective VNR ring. Another result on non-singular rings is due to Y. Utumi [13, Theorem 2.38]: If A is right and left non-singular, then the maximal right and left quotient rings of A coincide iff every complement one-sided ideal of A is an annihilator (the terms “complement” and “annihilator” in [11, p. 181] should be permuted).

The next result is motivated by Example (Q) and depends mainly on the right singular ideal of A.

Proposition 3. *Let A be a right YJ-injective ring such that any finitely generated right ideal is either a maximal right annihilator or a projective right annihilator. Then A is either quasi-Frobenius or a right p.p. ring such that every non-zero left ideal contains a non-zero idempotent.*

Proof. Suppose that Y, the right singular ideal of A, is non-zero. For any $0 \neq y \in Y$, yA_A is not projective which implies that yA is a maximal right annihilator. If $w \neq yA$, then $yA + wA = A$. This proves that $Y = yA$. Therefore Y is a minimal right ideal of A. But Y is also a maximal right ideal of A. Since Y contains no non-zero idempotent, then Y is an essential right ideal of A. For any proper non-zero right ideal R of A, $R \cap Y \neq 0$, which implies that $R \cap Y = Y$ by the minimality of Y. Then $Y \subseteq R$ which yields $Y = R$ by the maximality of Y. We have proved that Y is the unique non-zero proper right ideal of A. A is therefore right Artinian local ring. Since A is right YJ-injective, then every minimal left ideal of A is a left annihilator as in the proof of Theorem 2. Also, every minimal right ideal of A is a right annihilator by hypothesis, which proves that A is quasi-Frobenius. Now suppose that $Y = 0$. If $0 \neq a \in A$ such that aA is a maximal right annihilator, then aA is not an essential right ideal (in as much as $Y = 0$). If $0 \neq b \in A$ such that $aA \cap bA = 0$, then $A = aA + bA$ since aA is a maximal right annihilator. This proves that every principal right ideal of A is projective and A is therefore a right p.p. ring. For any $0 \neq c \in A$, by [28, Lemma 3], there exists a positive integer n such that Ac^n is a non-zero left annihilator. Since $r(c^n)$ is a direct summand of A_A, then $Ac^n = l(r(Ac^n))$ is a direct summand of ${}_AA$. Therefore Ac contains a non-zero idempotent, which completes the proof. □

The next proposition is motivated by [12, Proposition 3.3(ii)].

Proposition 4. *The following conditions are equivalent for a left YJ-injective ring A:*

(1) *A is right Artinian;*
(2) *A is a semi-perfect ring with maximum condition on left annihilators and finite right Goldie dimension.*

Proof. It is clear that (1) implies (2).

Assume (2). Since A satisfies the maximum conditions on left annihilators, then Z, the left singular of A, is nilpotent [13, Proposition 3.31]. From [28, p. 103], $J = Z$ is nilpotent. Since A is semi-perfect, A/J is semi-simple Artinian which implies that A is semi-primary. Since A is left YJ-injective, every minimal right ideal of A is a right annihilator (cf. the proof of Theorem 2). Since A has finite right Goldie dimension, $\mathrm{Soc}(A_A)$, the right socle of A, is a finitely generated right ideal. Also $\mathrm{Soc}(A_A)$ is an essential right ideal of A (because A is left perfect). Then, $\mathrm{Soc}(A_A)$ coincides with $\mathrm{Soc}({}_AA)$, the left socle of A, by [4, Theorem 3.1]. Now A, being semi-primary with maximum condition on left annihilators such that $\mathrm{Soc}({}_AA) = \mathrm{Soc}(A_A)$ is finite-dimensional as a right A-module must imply that A is right Artinian by [5, Lemma 6]. Thus (2) implies (1). □

Since there exist left and right Artinian rings whose right ideals are annihilators which are not quasi-Frobenius, the ring considered in Proposition 4 needs not be quasi-Frobenius (not even right YJ-injective).

Corollary 5. *A is quasi-Frobenius iff A is a semi-perfect, left and right YJ-injective, left and right Goldie ring.*

Corollary 6. *If A is a left and right YJ-injective, left and right Noetherian ring, then A is quasi-Frobenius iff A/J is a right YJ-injective ring.*

Theorem 2(3) motivates the next remark.

Remark 3. The following conditions are equivalent: (a) Every factor ring of A is quasi-Frobenius, (b) Every factor ring of A is left and right YJ-injective, left Noetherian.

Remark 4. If A is a left YJ-injective ring with maximum condition on left annihilators, then idempotents can be lifted mod J.

Remark 5. A commutative YJ-injective ring with maximum condition on annihilators is semi-primary (cf. [11, Theorem 16.31] and [28, Remark 11]).

A is called a right (resp. left) FPF ring (finite pseudo-Frobenius) if every finitely generated faithful right (resp. left) A-module generates mod-A (resp. A-mod). Such rings are closely connected with self-injective rings.

Remark 6. A commutative YJ-injective FPF ring is self-injective (cf. [11, Theorem 5.42]).

Remark 7. If A is left Noetherian, then A is left Artinian iff every prime factor ring of A is left-injective iff every prime factor ring of A is right YJ-injective.

Applying [11, Theorem 12.4D] and [31, Corollary 7], we get

Remark 8. If A is right Noetherian with $J^2 = 0$ and every essential right ideal of A is an idempotent ideal of A, then A is right Artinian. (Such rings need not be right duo.)

A theorem of S. Page [11, Theorem 5.49] yields the following characterization.

Remark 9. A is right and left self-injective regular ring of a bounded index iff A is a right YJ-injective, right non-singular, right FPF ring.

Remark 10. A is a right pseudo-Frobeniusean ring iff A is a semi-perfect, right YJ-injective, right FPF ring with essential right socle.

We now characterize commutative quasi-Frobenius rings in terms of Goldie rings ([28, Corollary 3.2] is improved).

Theorem 7. *The following conditions are equivalent for a commutative ring A:*

(1) *A is quasi-Frobenius;*
(2) *A is YJ-injective Goldie ring.*

Proof. It is obvious that (1) implies (2).

Assume (2). Then Y, the singular ideal of A, is nilpotent. Since A is YJ-injective, $Y = J$, the Jacobson radical of A [28, p. 103]. Therefore J is nilpotent. Again, since A is YJ-injective, A coincides with its classical quotient ring. By [11, Theorem 9.4], A is Artinian. Every minimal ideal of A is an annihilator (in as much as A is YJ-injective). A is therefore quasi-Frobenius and (2) implies (1). □

The study of rings whose simple modules are injective or projective is initiated in [1]. Such rings are called GV-rings by V. S. Ramamurthy and K. M. Rangaswamy (cf. [2], [18]).

The next example motivates our last propositions.

Example (GV). If A denotes the 2×2 upper triangular matrix ring over a field, A is a P.I. left and right Artinian, left and right hereditary, left and right quasi-duo ring whose singular left and right modules are injective but A is not semi-prime (indeed, the Jacobson radical J is non-zero with $J^2 = 0$). Also, all non-singular left and right modules are projective and the maximal left and right quotient rings of A coincide (cf. [13, Theorems 5.21 and 5.23] and [34]). Note that A is neither left nor right p-injective.

As usual, a left (right) ideal of A is called reduced if it contains no non-zero nilpotent element. Kaplansky's theorem on commutative V-rings has motivated several generalizations of non-commutative V-rings. In [7, Lemma 1], it is proved that if every simple left A-module is YJ-injective, then the Jacobson radical J is reduced.

Proposition 8. *Let A be a ring with reduced Jacobson radical J such that every simple left A-module is either YJ-injective or flat. Then $J = 0$.*

Proof. Suppose there exist $0 \neq u \in J$. For any positive integer m, $l(u^m) = l(u) = r(u) = r(u^m)$ (because J is reduced). If $AuA + l(u) \neq A$, let M be a maximal left ideal of A containing $AuA + l(u)$. Then ${}_AA/M$ is simple and therefore either YJ-injective or flat. First suppose that ${}_AA/M$ is YJ-injective. There exist a positive integer n such that any left A-homomorphism of $Au^n \to {}_AA/M$ extends to one of ${}_AA$ into ${}_AA/M$. Define a left A-homomorphism $f : Au^n \to A/M$ by $f(au^n) = a + M$ for all $a \in A$ (f is well-defined because $l(u^n) = l(u)$). Then $1 + M = f(u^n) = u^n y + M$ for some $y \in A$. Now $1 - u^n y \in M$ and since $u^n y \in AuA \subseteq M$, then $1 \in M$, which contradicts $M \neq A$. Now suppose that ${}_AA/M$ is flat. Then $u \in M$ implies that $u = uw$ for some $w \in M$ (cf. [6, p. 458]). Therefore $1 - w \in r(u) = l(u) \subseteq M$, which implies that $1 \in M$, again a contradiction! We thus have $AuA + l(u) = A$. If $1 = b + c$, $b \in AuA$, $c \in l(u)$, then $u = bu + cu = bu$ and $(1 - b)u = 0$. Since $b \in AuA \subseteq J$, $1 - b$ is left invertible in A which yield $u = 0$, contradicting $u \neq 0$. We have proved that $J = 0$. □

Corollary 9. *If every simple left A-module is YJ-injective, then $J = 0$.* (*Apply* [7, Lemma 1].)

Corollary 10. *The following conditions are equivalent for a right self-injective ring A:* (1) *A is VNR;* (2) *Every simple right A-module is either YJ-injective or projective;* (3) *Every simple left A-module is YJ-injective.*

Question 2. If A is a right self-injective ring whose simple left modules are either YJ-injective or projective, is A VNR?

The next proposition is again motivated by Example (GV).

Proposition 11. *Let A be a ring with reduced Jacobson radical J such that every simple left A-module is either YJ-injective or flat and every maximal left ideal of A is either injective or an ideal of A. Then A is either strongly regular or left self-injective regular with non-zero socle.*

Proof. By proposition 8, $J = 0$.

First suppose that each maximal left ideal of A is an ideal of A. Then A is semi-primitive, left quasi-duo which is therefore reduced (cf. the proof of "(2) implies (3)" in [26, Theorem 2.1]). Following the proof of Proposition 8, we see that for any $0 \neq a \in A$, any positive integer m, $l(a^m) = l(a) = r(a) = r(a^m)$. Since every simple left A-module is either YJ-injective or flat, we must have $AaA + l(a) = AaA + r(a) = A$ which implies that A is fully left and right idempotent. Since A is left quasi-duo, by [3, Proposition 9], A is VNR and is therefore strongly regular. Now suppose that there exist a maximal left ideal M of A which is not an ideal of A. Then ${}_AM$ is injective by [3, Lemma 4], A is left self-injective and since $J = 0$, A is VNR with non-zero socle. □

Note that Proposition 11 remains valid if we replace "every simple left A-module" by "every simple right A-module".

Our last result is motivated by a question raised in [32] (cf. U.Q.1(c)).

Proposition 12. *If every principal left ideal of A is the flat left annihilator of an element of A, then A is VNR.*

Proof. Let $0 \neq a \in A$. Then $Aa = l(c)$ for some $c \in A$. Since $A/l(c) \approx Ac$ is a flat left A-module and $l(c) = Aa$, A/Aa is a finitely related flat left A-module which implies that ${}_AA/Aa$ is projective. It follows that Aa is a direct summand of ${}_AA$, which proves that A is VNR. □

Question 3. Is A VNR if every principal left ideal of A is a flat complement left ideal of A? (The answer is positive if A is commutative.)

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