Journal for Geometry and Graphics
Volume 5 (2001), No. 1, 101–109.

Evaluation of Mentally Perceived Differences Between the 3D Objects Used in Mental Cutting Tests

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Abstract. We examined the issue of recognizing 3D objects through the measurement of the mentally perceived differences between the 25 solid objects used in the MCT. Subjects were required to arrange actual solid models along a straight line based on their perception of similarities between the solid models. The distances between adjacent solids were transfered into a distance matrix which was analyzed using multidimensional scaling methods. The similarities between the solids were interpreted in a chart of constellation that was constructed using axes of dimension 1 and dimension 2, which were extracted by multidimensional scaling. The results show that subjects seem to pay intense attention to the following details of the solid objects:
(1) impression of the solid shape as a whole which is then compared to some geometrically distinctive fundamental forms, and
(2) characteristic local shapes which symbolize the solid.
In judging mentally perceived differences between solids, high scoring subjects in the paper-pencil MCT were not influenced by the similarity of characteristic local shapes and they were able to classify the objects clearly considering the structural differences. Meanwhile, low scoring subjects in the paper-pencil MCT were influenced by the similarity of characteristic local shapes and had a tendency not to clearly separate the results of classification.

Key Words: spatial ability, Mental Cutting Test
MSC 2000: 51N05

1. Introduction

In courses on graphics at the undergraduate level of study, 3D spatial abilities have received much attention lately. In fact, for the past decade, several tests have been given to evaluate

ISSN 1433-8157/$ 2.50

spatial abilities of students. The MCT (a sub-set of CEEB Special Aptitude Test in Spatial Relations [1] (1939)) was used by Suzuki et al. [5] (1990) for measuring spatial abilities in relations to graphics curricula. Since then the MCT has been widely used for this purpose and a large amount of data has accumulated from tests conducted on various subjects.

The standard MCT consists of 25 problems. In each problem, subjects are given a perspective drawing of a test solid, which is to be cut with an assumed cutting plane. Subjects are asked to choose one correct cross section among 5 alternatives. With perspective drawings like those in the problems of the MCT, there exist various factors concerning shape recognition, which may cause subjects to fail in recognizing the proper objects correctly.

Therefore, if the test solids are shown in the form of stereograms with approximate stereopsis, i.e., a depth sense based solely on stimulation of disparate locations on the retinae, the errors associated with the phase of shape recognition might be reduced. Yao et al. [8] (1996) presented a stereographic MCT system using a time-sharing display controller, glasses with synchronized liquid crystal shutters and a microcomputer. They compared the results of the stereographic MCT (hereafter SMCT) with those of the standard MCT using a microcomputer network that has been developed by Sugai et al. [4] (1994). They showed that for the SMCT

(1) the mean score was a little bit higher although there was no significant difference between the two tests,

(2) correct response rates of pattern problems had a tendency to increase, but those of quantity problems did not change.

However, in their research most of the subjects got extremely high scores. The correct response rates for 16 problems out of the 25 were over 90% and there were only 4 problems whose correct response rates were under 80%. Therefore, it was considered that the results might not reflect the true relation between the two types of tests because of the small number of subjects who could get higher scores in the SMCT.

Tsutsumi et al. [7] (1999) conducted SMCT on female subjects, who are much less likely to get high scores in the standard MCT (N. Tsutsumi [6] (1990), Kashima [2] (1990)). The results have indicated that subjects recognized 3D shapes more easily by the use of stereograms. However, the stereograms did not appear to have any effect on complicated mental image processing tasks such as transformation of a section to a true shape view. Also, low scoring subjects in the study could not recognize the test solid and its cutting plane well, and they were unable to form correct images of objects, even when they used stereograms. Therefore, the question remains: How do subjects recognize the test solids?

Saito and Suzuki [3] (1999) conducted the real solid model MCT and compared the results with standard MCT. The results showed that the real solid models made it easier to construct three-dimensional mental images of the objects. The low spatial ability students, however, made a lot of errors even in the real solid model MCT, indicating that they felt difficulty in constructing correct mental images even when the objects were given by real solid models. Thus, the question how subjects recognize the solid objects of MCT still remains.

In this study we examined the issue of recognizing 3D objects through an experiment, in which the mentally perceived differences between 25 actual solid models of the test solids used in the MCT were measured. Although it might not explain the contents of form perception directly, the items to which the subjects pay attention would be clarified by analyzing features of those solid objects which were judged to have high similarity to each other.

2. Methods

2.1. Actual solid models as stimuli

Actual solid models of 25 test solids used in the MCT were made from chemical wood and were coated with matted light gray paint. Sizes of the models were determined on the basis of a cube whose edge was 65 mm long. Each model was fixed with a black stem of 70 mm in length and 6 mm in diameter (Fig. 1).

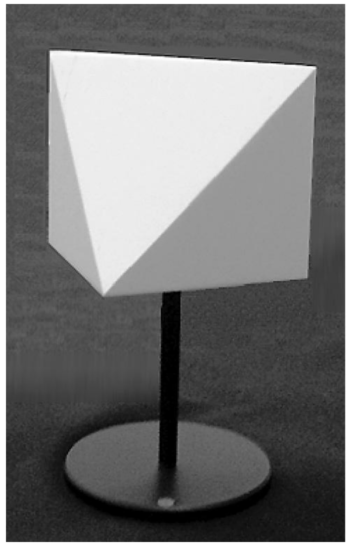

Figure 1: A sample of an actual solid object

2.2. Subjects

Twenty-two subjects participated in the experiments during the period from October to November 1998. All the subjects were female students in the first, second, third and fourth grade at the School of Social Information Studies, Otsuma Women's University.

2.3. Measurement of similarities

The twenty-five actual solid models were used as stimuli in the experiments. Three stimuli (No. 1, No. 10 and No. 16) were selected previously as criteria and used alternately. Subjects were required to arrange 24 stimuli except one criterion along a straight line of 3 m in length based on their perception of the similarity between the stimuli and the criterion (Fig. 2). Two more experiments were carried out after changing the criterion stimulus. In the experiments all the stimuli were positioned so that they were observed from the same direction as in the MCT.

After each experiment the distances between the criterion stimulus and the rest stimuli on the straight line were measured and normalized so that the distance between the criterion and the farthest stimulus became exactly 3 m. Then for each subject all the distances between the simuli were transfered into a distance matrix.

Also the paper-pencil MCT was conducted to all the subjects except two.

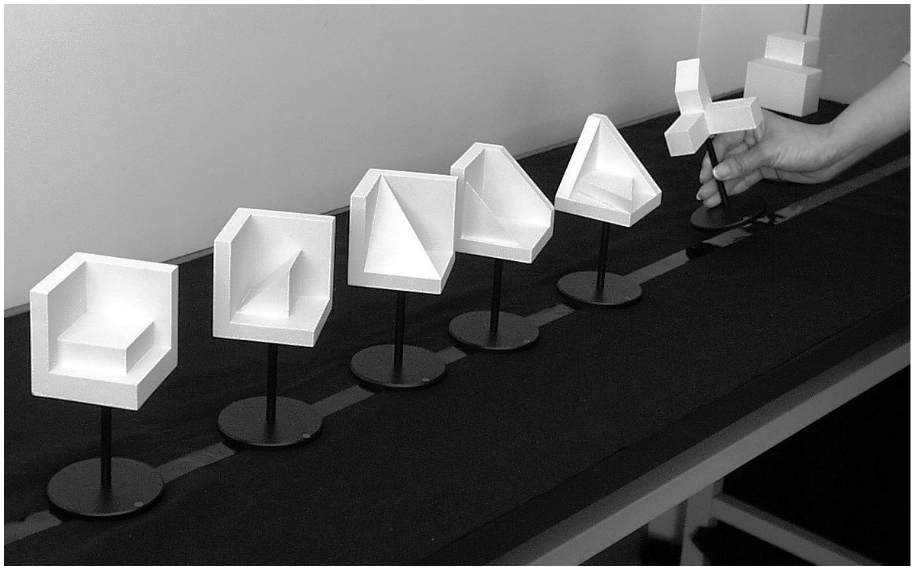

Figure 2: A subject is arranging objects

3. Multidimensional scaling methods

The distance matrices were analyzed applying multidimensional scaling methods (MDS) using the SPSS. In the MDS all the stimuli are constellated in a space so that the distance between any two stimuli reflects the magnitude of similarity. Here an average constellation over all subjects and the constellations for each subject were extracted. In the MDS, Kruskal's stress is used as a measure of fit such that the more the stress decreases the more the results fit. In the analysis, as we couldn't find an improvement on Kruskal's stress against the third dimensionality, we adopted the two-dimensional Euclidean space.

4. Similarity of objects

4.1. Chart of constellation

As shown in Fig. 3, a *chart of constellation* over all subjects was constructed using axes of dimension 1 (horizontal axis) and dimension 2 (vertical axis). The figure shows that the stimuli make several clusters. Dimension 1 and 2 are interpreted as

(1) whether the object remains an image of the cube as a frame of reference or not,

(2) whether the object has convex faces which includes triangular faces,

respectively. As there were individual clusters that include objects with curved surfaces or simple geometrical shape such as prism and pyramid, we examined the constellation in further detail. The results indicated that stimuli were classified into five groups as shown in Table 1.

4.2. Points of observation

Through the analysis of common features of those solid objects that were judged to have high similarity to each other, the items to which the subjects pay attention would be made clear.

It was considered that subjects firstly pay attention to the *"impression of the solid shape as a whole, which is then compared with some geometrically distinctive fundamental forms such as a cube"*. Secondly, subjects pay attention to *"characteristic local shapes, which symbolize*

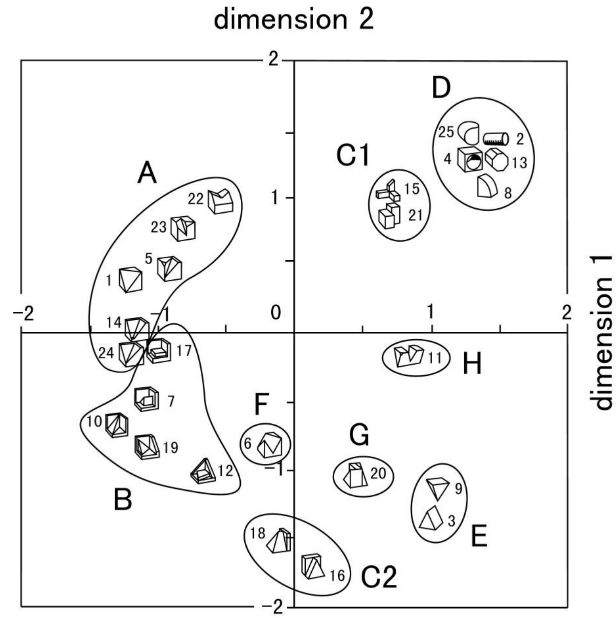

Figure 3: Constellation on dimension 1 and dimension 2

A. *Objects remaining an image of the cube as a frame of reference*	
	A1: a large polyhedron is subtracted from the cube (1,14,24) A2: two small portions are subtracted from the cube (5,23) A2: two large portions are subtracted from the cube (22)
B. *Objects with three panels remaining from the cube*	
	B1: three panels of the cube remain perfect squares (7,10,17) B2: one of the vertical panels is imperfect (19) B3: two vertical panels are imperfect (12)
C. *Combination of geometrical shapes*	
	C1: combination of similar shapes (15,21) C2: combination of a triangular and a quadrangular prism (16,18)
D. *Objects with curved surfaces* (2,4,8,13,25)	
E. *Simple geometrical shape with triangular faces* (9,3)	
Others	
	F: Similar to group B3 and C2 G: Similar to group C2 and E H: Intermediate between C and E

Table 1: Clusters in constellation

the solid", because solid shapes with curved surfaces (D) and shapes with triangular faces (C2, E, D) are separated along the axis of dimension 2. Here the items such as "curved surface or plane surface"or "to be round or angular" are focused on.

Furthermore, following details were observed concerning subjects' items of observation:

- Among the stimuli which remain images of the cube as a frame of reference (group A and group B), there are two types of impression, i.e., the *"mass" type* (group A) and the *"outside frame" type* (group B).

- Among the stimuli which remain images of the cube as a mass (group A), the difference between the shapes which were subtracted from the cube was recognized (A1, A2, A3).
- The stimuli which remain images of the cube as an outside frame (group B) were classified mainly by the shape of the outside panels which covered a larger space compared with the inside bricks (B1, B2, B3). Thus it may be called a combination of outside frames as primary attribute and of bricks as the subordinate.
- Among the combination of geometrical shapes (group C), there are two types of impression, i.e., "a combination of similar shapes" (1) and "a combination of a quadrangular prism as primary attribute and a triangular prism as the subordinate" (C2). Here we judged the quadrangular prism as primary because they were located near group B.
- Stimulus no. 4 is a difference of a cube and a cylinder. Although this stimulus is a cube in shape, it is recognized paying attention mainly to the inside curved surface with negative curvature.

It could be concluded that the details to which the subjects pay attention were as follows:

(1) "Mass" type or "outside frame" type,

(2) result of a Boolean operation, especially of a union or a difference,

(3) comprised curved surface,

(4) comprised angular surface.

4.3. Relation to MCT scores

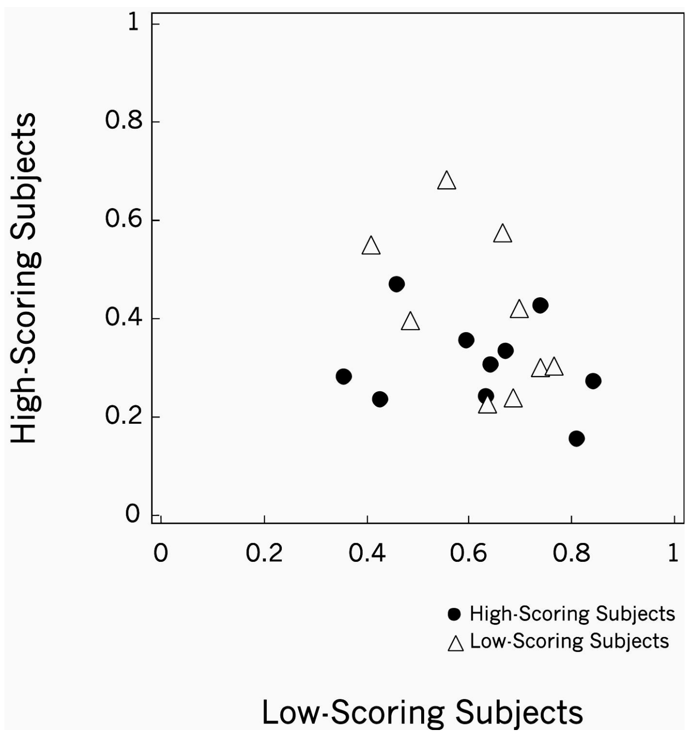

Figure 4: Relation between weights and MCT scores

The relation between the judgement of similarities between the stimuli and the scores of MCT was analyzed comparing the individual constellation, the individual weights for dimension 1 and dimension 2, and the MCT scores.

The subjects of our study were classified into two sub-groups according to their individual scores of MCT. As the average score was 14.5 against the full score 25, subjects whose score was higher than the average were called *high scoring subjects* (score > 14.5, $N = 10$) and the other were called *low scoring subjects* (score < 14.5, $N = 10$).

Fig. 4 shows the relation between the individual weights for dimension 1 and dimension 2, where high-scoring subjects were marked with black dots and low-scoring subjects were marked with triangles. The results were as follows:

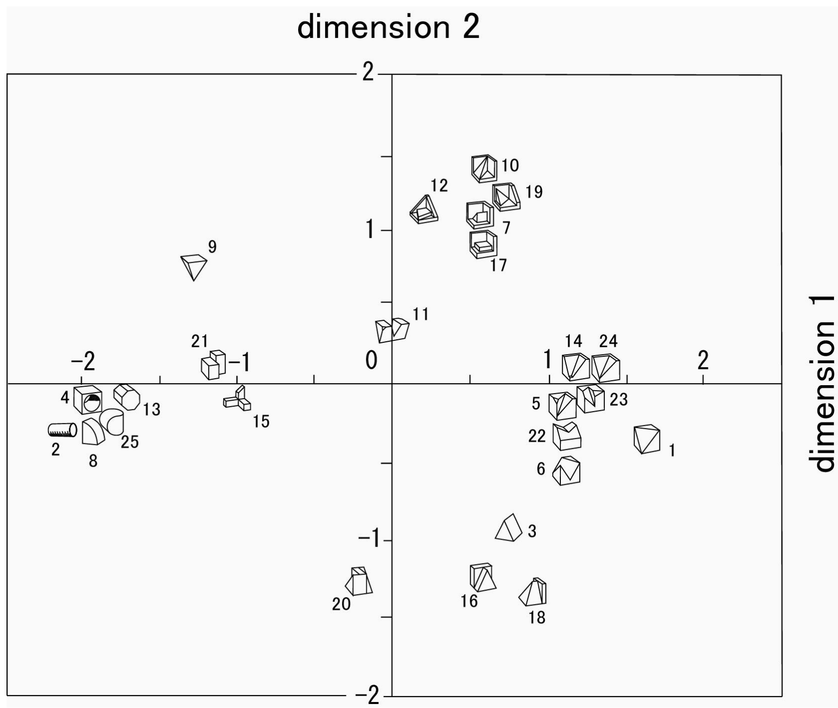

Figure 5: A sample of constellation for a high-scoring subject (subject no. 8, MCT score: 22)

- For high-scoring subjects:
  (1) As there were a few subjects who have highly participated in dimension 2, high-scoring subjects were not influenced by characteristic local shapes such as convex faces.
  (2) They have a tendency to pay attention to structural differences of the objects. In this respect they didn't stick to differences of partial shapes.
  (3) With regard to the constellation, there appeared several distinct clusters individually, as it can be seen in that of subject no. 8 (MCT score: 22) (Fig. 5).
- For low-scoring subjects:
  (1) There were some subjects who have highly participated in dimension 2. The analysis of the individual constellations showed that they were apt to pay attention to local characteristic shapes in the classification.
  (2) The constellations have a tendency not to make clusters but to locate the stimuli continuously, as it can be seen in that of subject no. 12 (MCT score: 8) (Fig. 6).

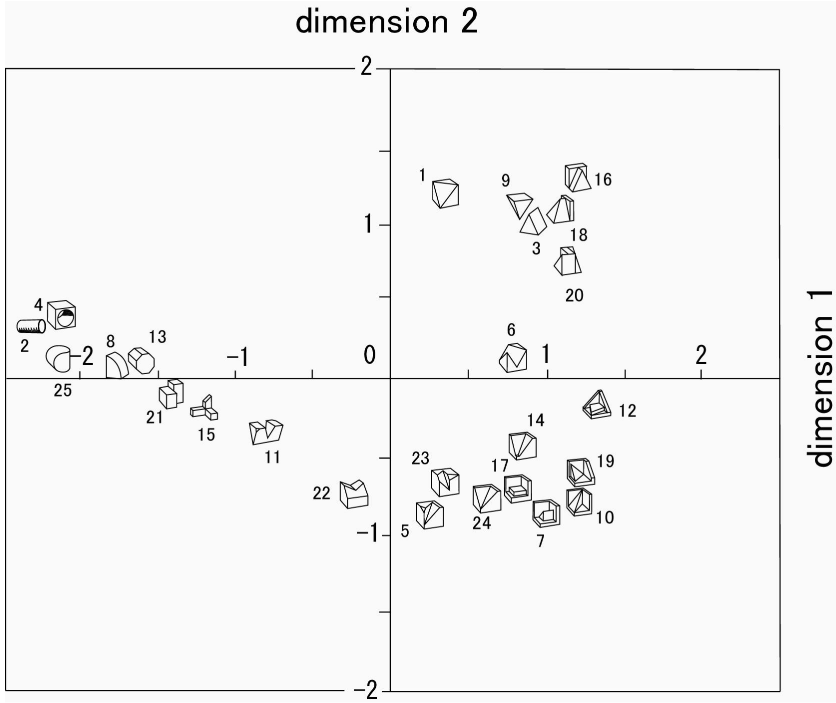

Figure 6: A sample of constellation for a low-scoring subject (subject no. 12, MCT score: 8)

The constellations of subjects who got about the average MCT score have tendencies of both sub-groups.

There was a significant correlation of -0.411 between MCT scores and weights for dimension 2, besides there was no significant correlation between MCT scores and weights for dimension 1.

On the basis of these results, it could be concluded that high-scoring subjects were able to classify the stimuli clearly considering the structural differences between the objects. Meanwhile, low-scoring subjects were influenced by the similarity of the local characteristic shapes and have a tendency not to clearly separate the results of classification.

5. Summary and conclusion

It was examined through measurement of mentally perceived differences between 25 actual solid objects used in the MCT, how students recognize 3D objects. The principal results are as follows:

(1) As regards the criteria for judging mentally perceived differences between solids, subjects seem to pay attention to the following details:

a) Impression of the solid shape as a whole, which is then compared with some geometrically distinctive forms,

b) Characteristic local shapes which symbolize the solid.

(2) It was observed that the weights placed on each dimension varied from subject to subject.

(3) In judging mentally perceived differences between solids, high scoring subjects in the paper-pencil MCT were not influenced by the similarity of characteristic local shapes

and they were able to classify the objects clearly considering the structural differences of the objects. Meanwhile, low scoring subjects in the paper-pencil MCT were influenced by the similarity of the characteristic local shapes and these subjects had a tendency not to clearly separate the results of classification.

(4) The relation between MCT score and items of the observation of solid objects might suggest that it may exert a favorable influence on the recognition of 3D objects when subjects bend their efforts to ascertain the structure of the objects without being wavered by local characteristic shapes.

Acknowledgements

The authors express their thanks to Prof. Kenjiro Suzuki at The University of Tokyo, who gave us helpful discussions and constructive criticisms.

References

[1] *CEEB Special Aptitude Test in Spatial Relations*. Developed by the College Entrance Examination Board, USA, 1939.

[2] S. Kashima: *A Mental Cutting Test and Graphics Education*. Proc. Annual Meeting of Japan Society for Graphic Science XIV (1990) [Japanese].

[3] T. Saito, K. Suzuki: *Error Analysis on a Mental Cutting Test by the Comparison with a Real Solid Model Mental Cutting Test*. Proc. 4th China-Japan Conference on Graphics Education, Dunhuang, China, 1999, pp. 111–116.

[4] Y. Sugai, K. Suzuki, T. Ezaki, K. Irie: *Mental Cutting Test Using a Micro Computer Network*. Proc. 6th ICECGDG Tokyo 1994, pp. 776–780.

[5] K. Suzuki, S. Wakita, S. Nagano: *Improvement of Spatial Ability through Descriptive Geometry Education*. Journal of Graphic Science of Japan **49**, 21–28 (1990) [Japanese].

[6] N. Tsutsumi: *Evaluation of Spatial Abilities by a Mental Cutting Test at Musashino Art University*. Proc. Annual Meeting of Japan Society for Graphic Science XIII (1990) [Japanese].

[7] E. Tsutsumi, Ka. Shiina, A. Suzaki, K. Yamanouchi, T. Saito, K. Suzuki: *A Mental Cutting Test on Female Students Using a Stereographic System*. J. Geometry Graphics **3** (1), 111–119 (1999).

[8] Y. Yao, T. Saito, K. Suzuki: *Analysis of Causes of Errors in a MCT through the Comparison with Stereographic MCT*. Proc. Annual Meeting of Japan Society for Graphic Science 1996, pp. 122–127 [Japanese].

Received August 1, 2000; final form December 19, 2000