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Geometric Analysis of Architectural Drawings of the 19th Century

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Abstract. The Library of the Department of Architecture of the University of Cagliari houses the drawings of 41 graduation thesis of Architecture that had been debated during the second half of the 19th century. These drawings are the results of a three-year long course of *"Architecture, Drawing and Ornate"* that was taught in Cagliari by Prof. Arch. Gaetano Cima (1805–1878). From the analysis of the drawing it is possible to make some remarks both of historic and methodological aspects. It is very interesting how the students carried out the architectural composition using geometric constructions. The marks of the geometric constructions are yet visible on many of the drawings. In the work presented here, these tiny signs are used as instruments for reconstructing, using the accuracy of CAD, the outlining process. Drawings were sorted according to the rules of construction found for each architectonic composition: golden section, dynamic symmetry (root 2 rectangle), squares and triangles, etc. The classification of the graphic analysis was then related to the note of Prof. Cima and the books he made use of to teach architecture, in order to value affinity and differences. Aim of the work is to regain the Cima's teaching method from the graphic results of his students and to establish a method that could be applied to similar cases where architectural drawings are the only documents to regain the outline of a building.

Key Words: Geometric structures, drawing, proportions, 19th century, architectural design

MSC 2010: 51N05

1. Introduction

To learn from our predecessors is an essential exercise to acquire new skills. The reinterpretation of the drawing methodology, design and project can be a useful stimulus to critically evaluate some passages in the history of architecture, to transform and adapt long forgotten knowledge to current situations. This idea is the basis of the work presented here.

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> *"Composition in architecture starts with elements and their relations. Geometry is able to make a contribution to this process by dealing with geometric figures and forms and elements as well as proportions, angles, and transformations and relations between them"* [6].

An architect is one who thinks the architecture and that is capable of making the transition from idea to form through the mediation of drawing [12]. Starting from these assumptions and the reading of G. Pierluisi [10] and his idea of drawing analysis made for the virtual reconstruction of Giuseppe Terragni's *Danteum*, a similar process of geometric interpretation of drawings is attempted. This attempt though was done without the virtual reconstruction than for the same drawings has been partly addressed long ago, albeit for different reasons that the ones in these communication [3].

The study starts from the analysis of ancient drawings kept in the library of the Department of Architecture at the University of Cagliari. These drawings were made by architecture students during the nineteenth century, precisely in the years from 1843 to 1864. They are the output prepared for the discussion of public examination in Architecture, following the guidance of Professor G. Cima.

2. Historical background

The Sardinian architect Gaetano Cima (1805–1878) began his academic preparation at the School of Bridges and Roads Volunteers in Sardinia, he studied architecture in Turin under the supervision of Ferdinando Bonsignore and Giuseppe Talucchi. He graduated in 1830 then moved to Rome at the Accademia di San Luca following the teachings of Canina and Camporesi. From 1840 until the date of his death he taught *"Architecture, Drawing and Ornate"* at the University of Cagliari. The disciplines of drawing were major topic in the program of lectures by professor Cima.

The structure of the course, the central role of the teaching of geometry and principles of design in the architect training, and the teacher's belief that teaching should be given individually to each student to achieve good results are contained in an autographed document preserved in the historical town archives of Cagliari. This document also shows the structure of the exercises given during the course. One of these practical tests is a compositional exercise "from the ground plan of a building compose his external orthographic view using that order which is best suited to this class of building" [1].

3. Materials

Forty-one theses have come down to us. They are bound in albums that contain the theme of work assigned by the professor and drawings that were executed according to requirements dictated by the subject. These works have been preserved exactly as professor Cima organized them for an exhibition that took place in 1873.

3.1. The drawings

Each thesis contains an average of five tables which consist of ground plans and orthographic views. There are always the plans of the main levels (never more than two) (see Figure 1), one or more sections (usually a longitudinal and transverse) and at least a front (the main one). In some cases there is also the site plan. The internal orthographic views are

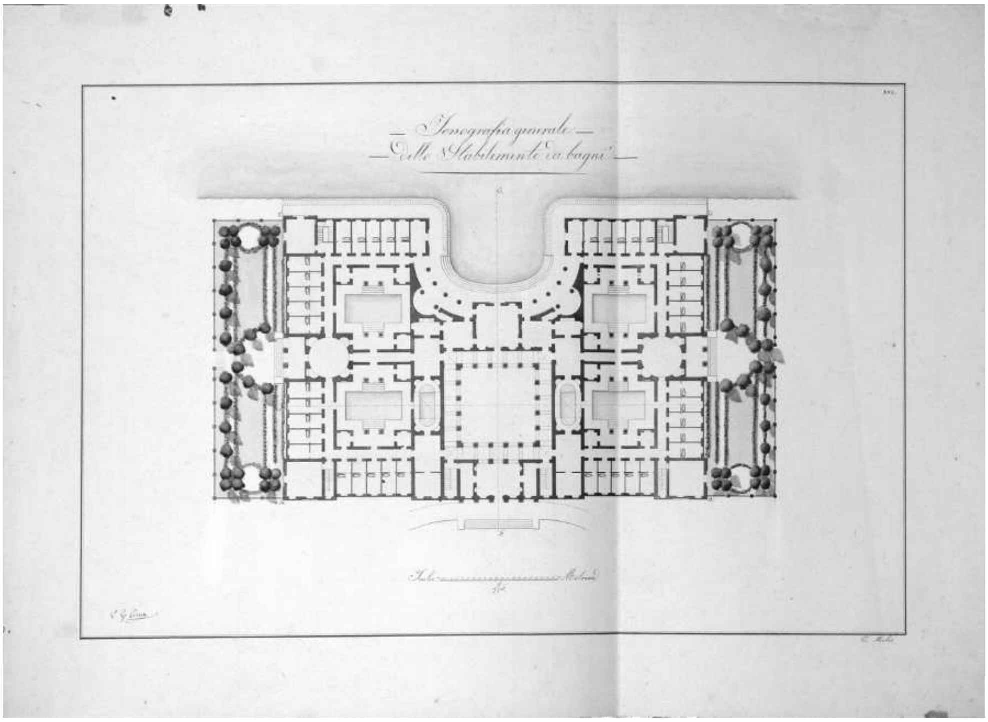

Figure 1: *"icnografia generale dello stabilimento da bagni"*,
General icnography of seaside resort, thesis n. 7, Enrico Melis Romagnino, 1856

more important than external ones. They provide guidance to complete the main plans revealing the existence of mezzanines and attics and by clarifying the types of roofing (roofs and vaults) that is sometimes traced even in plans (cross vault, lunette etc.). The exterior views are designed mostly for the main façade, and only if expressly requested by the theme for the side and rear. The drawings are all made on thick watermarked paper. The size of the sheets does not follow a predetermined convention. Measures range from a minimum size of 415×525 mm to a maximum of 902×584 mm.

3.2. Graphic conventions

The graphic conventions used in the representation of plans and orthographic views are not unique and changes can be detected between one thesis and another. The most common rules are: black ink for lines, solid or watercolour black fills for cut parts in the plans, watercolour red or black fills for cut parts in vertical section, other coloured inks for the representation of particular materials (e.g. brown for wood, blue for water). The convention for the topographic representation of the nineteenth century is often used in the site plan where there are gardens (see Figure 2).

The reduction scales are different for plans and elevations. The fronts and vertical sections are always drawn in a more detailed scale than the plans. This discrepancy between ichnography and orthographic view does not allow the immediate dimensional comparison that is typical of Monge's orthogonal projections. The motivation is probably due to the need for greater detail required by the decorations of the façades.

The project unit is meter. This is relevant for an era in which the variety of units was the rule. The metric standard is related to the fact that Professor Cima was responsible for standardizing the current measurement units in Sardinia to the new system [4].

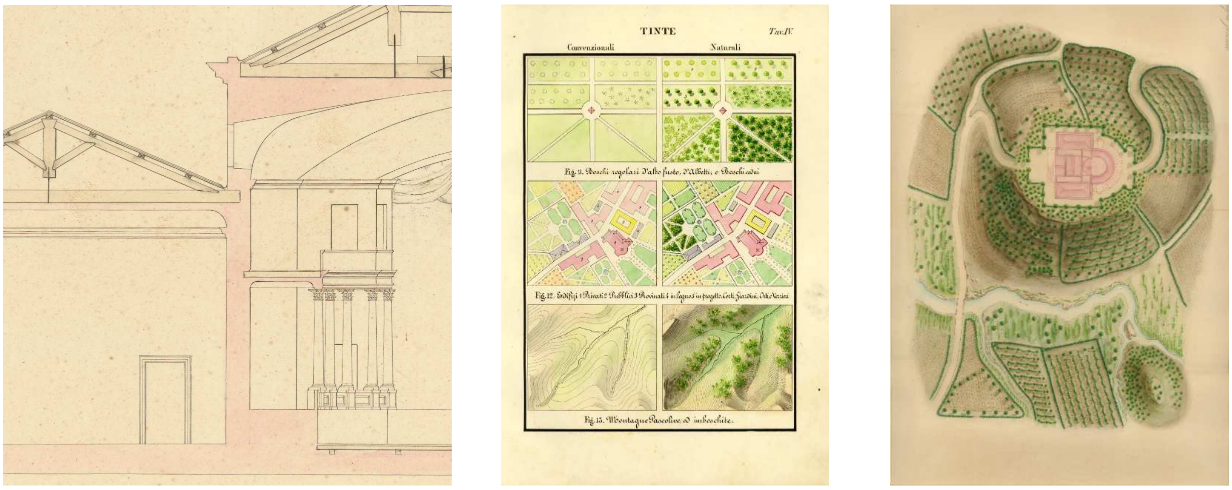

Figure 2: Left: Graphic conventions, thesis n. 33, 1861. Center: Topographic drawing convention (Brignone) [2], Right: General Icnographia, thesis n. 31, 1860.

3.3. The design theme

The themes from which flow the drawing boards includes a description of the project required. It is similar to those that were awarded in the competitions of Academies of Fine Arts. In drafting issues, professor Cima particularly referred to the themes of competitions of the Academies of Florence [9] and Milan [11]. Twenty-seven of forty-one subjects are accompanied by geometric and dimensional indications and especially by ratios between the parts of the architectural composition requested.

Length, width, height, shape of the lot are just some of the indications for the binding of the project geometry. Other constraints are represented by: shape of plans and/or fronts (rectangle, square, circle, and triangle), proportional relationships between the sides of rectangles, and size of area (see Tables 1 and 2).

Other important pieces of information for determining the geometry of the project are those related to the architectural orders. They were closely related to the style to be adopted in the drafting of the façades, and they also affect the height and rhythmic thereof. The relationship between height and geometric dimensions of the plant is suggested in few cases. However, the number of floors of buildings is recommended in many themes, and the presence of the basement often is required. For the latter, the height in meters is always given or the number of steps (see Table 3).

Students drew up the tables and the dissertation based on these themes and knowledge acquired during the studies. The project report contained the justifications of geometric composition and other information describing the project. Although a written copy of this work was requested, unfortunately it cannot be founded. Thus there is no useful contribution to the understanding of the project that is entrusted only to graphic representation.

4. Analysis methodology

4.1. Preliminary operations

The procedure adopted to investigate the composition of geometric designs and test their compliance with the information contained in the themes is described below.

The tables of drawings were digitally scanned at resolutions that allow video to appreciate the details of semi-erased geometric constructions but that are still visible on the original. The image resolution of 300 dpi has proved sufficient for this purpose.

The scanned images were printed in manageable size (A4) by limiting the print to drafted area and not the entire squared paper. Traces of pencil, compass point, and whatever else was found in the originals were noted on the prints to avoid excessive contact with the originals and preserve them from damage.

The scanned images were included in a CAD drawing. The detected traces of elements: axes, points, construction lines, arcs, etc. were drawn using the screen magnification. Paths in pencil, more visible, are found on the iconography and often reveal modular square mesh (see Figure 3 left). The walls are traced on these axes. There are also other detectable traces, especially in the drafting of the fronts and vertical sections. These paths show the geometric constructions of architectural orders (see Figure 3 center) and the second thoughts in the design of elements distribution in faade (see Figure 3 right).

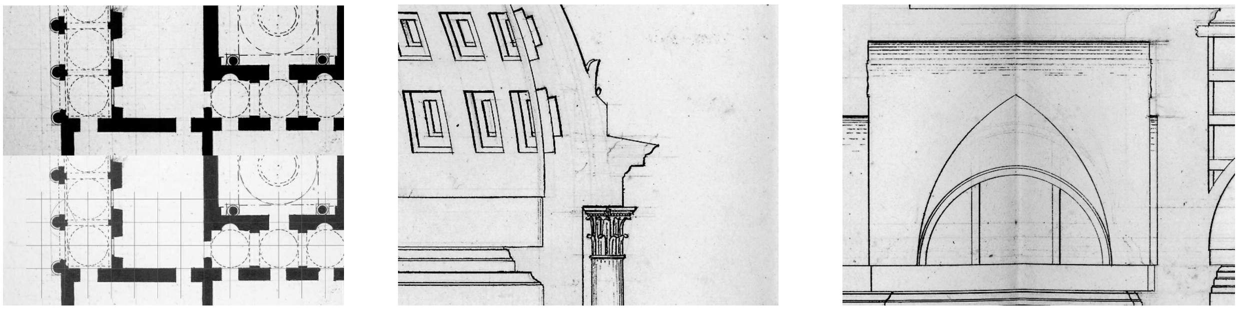

Figure 3: Left: (top) traces of the modular square mesh (bottom) CAD square mesh, thesis n. 9, 1857. Center: Traces of geometric construction of capital and cornice, thesis n. 3, 1845. Right: Traces of second thoughts in the design, thesis n. 3, 1845.

Table 1: Shape constraints

Themes	*Geometric constraints in plans*
22 of 41	rectangle
8 of 41	square
10 of 41	circle or arc
4 of 41	other

Table 2: Dimensional constraints

Themes	*Dimensional constraints in plans*
8 of 41	area
12 of 41	ratio between sides
27 of 41	linear measures
7 of 41	other

Table 3: Dimensional constraints in façade

Themes	*Dimensional constraints in fronts*
15 of 41	architectonic order
18 of 41	number of floor
12 of 41	basement
6 of 41	other

4.2. Determination of the geometry

Based on what we can extrapolate from some of the themes the geometric grid of the compositions is represented by the axes of the walls.

This was indeed the common practice suggested by most architecture manuals of that period or immediately preceding. So, this was the starting point for dimensional verification.

The pencil tracks are lacking by their very nature of removable support. The modular mesh, the sequence of composition, the relations of aggregation between shapes, was reconstructed supplementing the traces and verifying the compliance of these derivatives drawings and indications that are given in the design theme. Operations of reconstruction were often acted on the interpretation by keeping in mind the cultural background of the students.

4.3. Reconstruction of the cultural background of the designer

One operation necessary for the completion of the investigation was the reconstruction of the cultural background of the designer. This reconstruction was made based on:

- Literature on the teaching of architecture in Italy during the nineteenth century (the texts of Gabetti and Marconi [5] and the anthology of Salerno, R. [13] were very helpful);
- Notes the professor took about the course program of *"Architecture, Drawing and Ornate"*;
- Textbooks purchased during his teaching and still preserved in the library of the department of architecture and which he probably referred, in particular Francesco Milizia [7]

In summary it can be stated as follows:

The drawing of geometric figures, the graphic solution of problems of various kinds, descriptive geometry (orthogonal projections) are very important.

Firmitas (strength) *utilitas* (function, purpose); *venustas* (understood as architectural beauty harmony expressed mathematically). Vitruvian rules elaborated in Enlightenment key in the late eighteenth century were followed for architectural composition.

The classic architectural orders were applied according to the proportions dictated by the treatises of the Renaissance (Vignola Scamozzi etc.) and the character of the building typology. Moreover, the knowledge of orders was linked to a deep understanding of the artistic design.

The plans were made from basic geometric shape, often using modular square mesh.

Symmetry, eurhythmy, proportion between the parties played a key role in the composition of the project.

5. Results

The analysis shows that the drawings are related to the theme for what concerns both the dimensions of spaces and the proportions of decorations.

There are some exceptions. The discrepancies are not likely to affect the overall effect required by the theme. Perhaps these variations were introduced under review to improve the composition. The most common cases are those of rectangular buildings or enclosed in a rectangle, they are in full agreement with the neoclassical theories (see Figure 4). These plans are linked rectangular faade thereby rectangular parallelepiped (see Table 4). Buildings with a special function (temple, water tower, lighthouse, theatre) that can influence the shape more are different from previous ones.

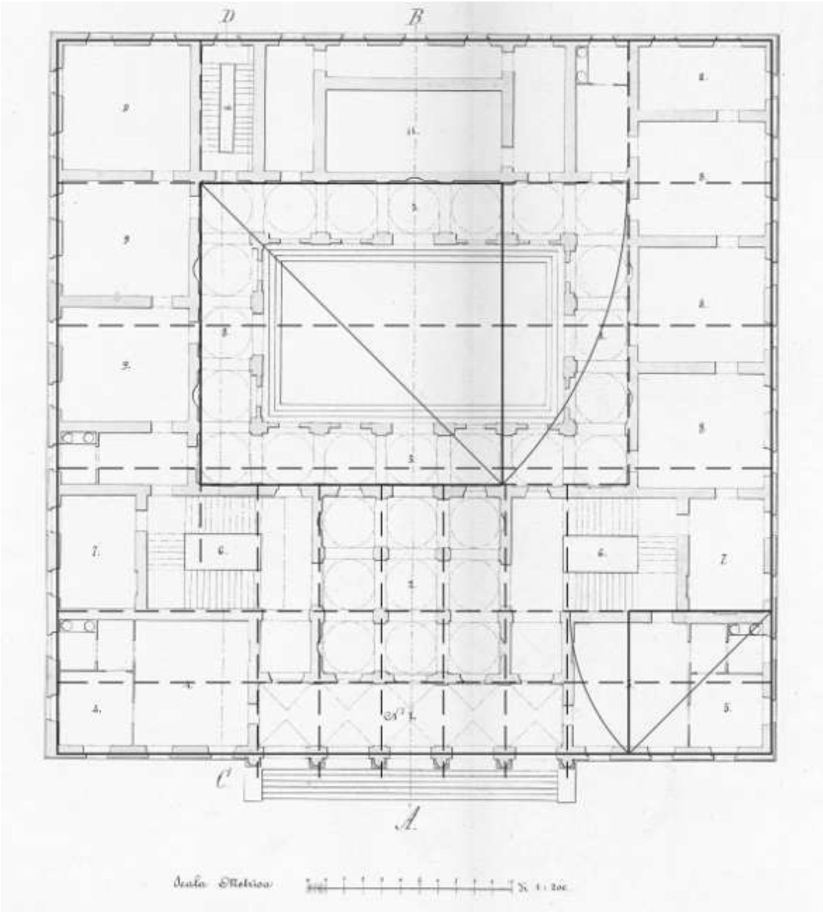

Figure 4: Example of proportion in plans, thesis n. 19, 1859

The proportions observed between the sides of rectangles are 2/3, 2/5, 1/2, 3/5, 6/5. The same ratios were also adopted to determine internal courtyards.

Plans based on a square, those mixtilinear (semicircle + square, rectangle + semicircle) and finally those on regular octagons are less frequent.

The modular square mesh is evident in plan in 50% of cases, but can not be excluded in the rest. It is used as the basis for the tracking of internal divisions. In most cases, the determination of rectangular rooms is made using the half square rectangles with sides in proportion 3/2. In more elaborate projects the operation used is to tip the diagonal of the square to obtain rectangles with sides in proportion $1/\sqrt{2}$. Less frequent are cases in which

Table 4: Shape of plans

Quantity of 41 themes	*Shape of plans*
25	rectangle
2	square
12	composite
2	octagonal

recourse to the golden rectangle. This latter solution is generally linked to the determination of internal walls in mixed-line or square plans.

The module adopted for the allocation of space in plans is directly related to the length of the shaft of the order in faade, or if the column is absent, the height between the base and cornice. The connection appears always like rectangle 2/3, $1/\sqrt{2}$ and the gold standard. However, the modular mesh does not always govern the volumetric composition of the building. Indeed, in some cases the design theme provides the dimensional constraint for elevation. This constraint acts as a hinge for the deduction of the size of the building (see Figure 5).

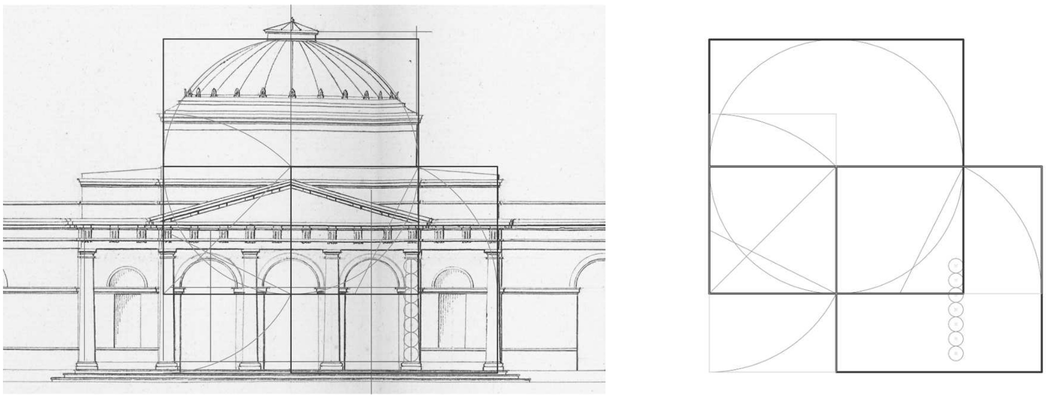

Figure 5: Example of proportion in elevations, thesis n. 20, 1859

Noteworthy are some cases where more than others the professor had left the candidate free in designing the project. One of these is the thesis of Filippo Vivanet who was then professor of descriptive geometry and Superintendent of Fine Arts. The project was made from a central octagonal with two rectangular wings on either side. Everything is contained in a rectangle of ratio 6/5. The main part is designed in a rectangle of ratio 3/1 and the width of the wings is made using golden rectangles (see Figure 6).

6. Conclusions

The work presented here has finally detailed the study about Cima's teaching work hitherto neglected compared to the professional one.

From the findings it is difficult to separate the design made by students from the suggestions made by the professor. The concept of teacher-pupil typical of classic Italian architecture remains clear. However, a new central and disruptive element becomes part of the course: the idea that architecture is not art for a few but an instrument to serve the nation. It is linked

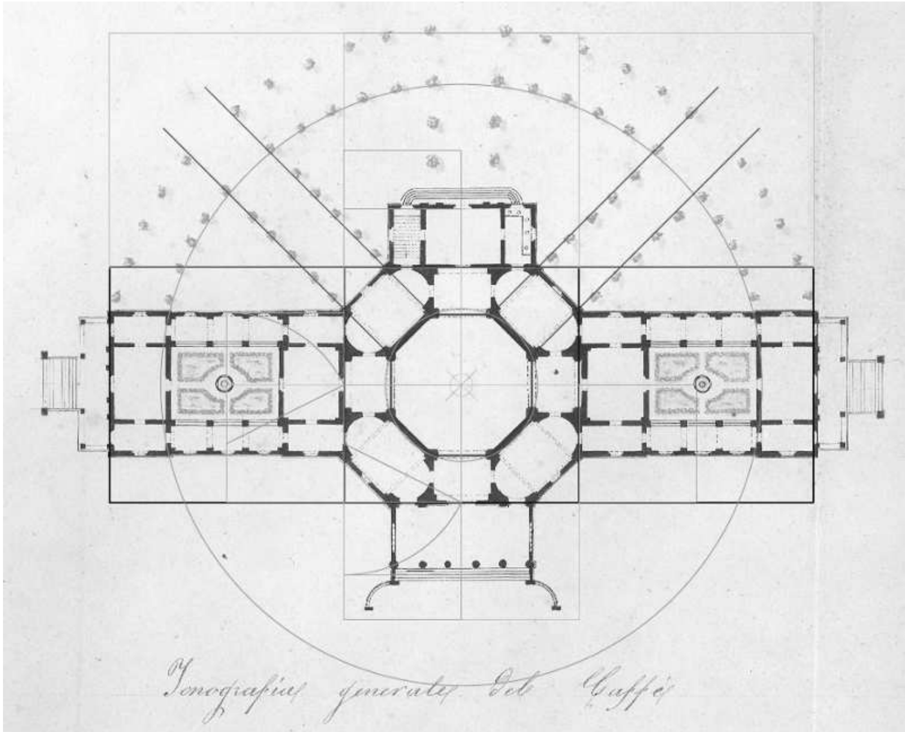

Figure 6: Plan of VIVANET's project, thesis n. 6, 1855

up not only with artistic abstraction, but also with static and functional efficiency. Hinge around which this principle is the geometric drawing, a tool used not only for the composition of plans and views but also and especially for the practical solution. The tracking of the helical curves of staircases, spirals on volute, and graphic solutions for static of arches and vaults and cutting blocks are some examples.

Between 1840 and 1860 Italy is a nascent nation and CIMA endorses the idea of G. MONGE

> *"On contribuera donc donner l'éducation nationale une direction avantageuse, en familiarisant nos jeunes artistes avec l'application de la géométrie descriptive aux costrutions graphiques qui sont necessaries au plus grand nombre des arts ..."* [8].

In my opinion centrality of projective geometry in design is a concept that should never be forgotten, and this research pat is simply a way to bring to light the steps of growth that this discipline has had over time until arriving at what it is today. That is why I tried to draft a protocol for the graphic analysis of drawings that could be helpful in similar research where the only document is a drawn paper, an archaeological view of the mental process of the designer.

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References

[1] A.C.C. Historical town archives of Cagliari: *Carte Cima n. 29.*

[2] G. Brignone: *Cenni e modelli sopra il disegno topografico dell'acquerello: ad uso degli ingegneri civili, misuratori ecc.*. Torino 1845.

[3] P. Casu: *Ricostruzione di architetture ipotizzate*. Proc. of E-Arcom 2004 tecnologie per comunicare l'architettura, Ancona 2004.

[4] A. Del Panta: *Un architetto e la sua città*. Edizioni della torre, Cagliari 1983.

[5] R. Gabetti, P. Marconi: *L'insegnamento dell' architettura nel sistema didattico franco-italiano*. Controspazio **3**, June 1971, Bari.

[6] C. Leopold: *Geometry Concepts in Architectural Design*. Proc. 12th Internat. Conf. on Geometry and Graphics. Salvador/Brasil 2006.

[7] F. Milizia: *Principj di architettura civile*. Milano 1832.

[8] G. Monge: *Géométrie Descriptive. Leçons données aux Écoles Normales par Gaspard Monge (1794)*. Badouin, Paris 1799.

[9] L. Pasqui, C. Lapi, P. Passeri: *Collezione dei progetti d'architettura, premiati nei grandi concorsi triennali dall'I. e R. Accademia delle belle arti in Firenze pubblicati per cura degli architetti Leopoldo Pasqui, Cammillo Lapi, Pietro Passeri ed incisi dall'arch. Angelo Cappiardi*. Firenze 1828.

[10] G. Pierluisi: *Il Danteum di Terragni, note per la costruzione del modello digitale interattivo*. R. Migliari, Prospettiva Dinamica interattiva, Edizioni Kappa, 134–164, Roma 2008.

[11] F. Pizzagalli, G. Aluisetti, P. Comerio: *Opere dei grandi concorsi premiate dall'I. R. Accademia delle Belle Arti in Milano disegnate ed incise per cura degli architetti Felice Pizzagalli, Giulio Aluisetti, e del pittore Agostino Comerio*. Milano 1824.

[12] R. Recht: *Le dessin d'architecture*. Société nouvelle Adam Biro. Paris 1995. Italian edition Editoriale Jaca Book, Milano 2001.

[13] R. Salerno: *La macchina del disegno. Teoria della rappresentazione dell'architettura nel XIX secolo. Antologia critica*. CLUEB, Bologna 2000.

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