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An Analysis of Vermeer's Perspective in Composition*

Noriko Sato

Joshibi University of Art and Design, 1900 Asamizodai, Sagamihara, Kanagawa, Japan
sato14004@venus.joshibi.jp

Abstract. In this study, the relationship between the works of Vermeer, camera obscura light images and frame composition is considered. It has been suggested that Vermeer may have traced camera obscura images to create his paintings. Although it is unlikely that he could have traced the entire image at once, similar to scanning, it is possible that Vermeer may have taken models, motifs, or relative sizes of different objects from these light images and incorporated them directly into his compositions.

One of his works during the early stages of his career is called "Milk Maid." The results of the analysis of two works from the same period, "Woman in Blue" and "Woman with a Water Jug," show that multiple horizon lines can be derived, indicating a geometric inaccuracy. This shows that the works of Vermeer were not geometrically accurate during the start of his career. He created stable compositions with geometric accuracy after a certain point in his career.

The analysis of "Milk Maid" has created doubts regarding the possibility that Vermeer used a camera obscura to trace the surface of the floor. However, because of factors such as the necessity for focus adjustment and limitations of the light image, he may have faced problems in constructing a geometrically accurate space. This report describes the boundary lines drawn on floors, which are essential for creating a space with a sense of depth in paintings.

Key Words: modeling theory, 17th century Dutch painting, perspective painting
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1 Introduction

Johannes Vermeer (1632–1675) was one of the most prominent painters of the 17th century. His chamber scenes have fascinated numerous audiences. His works portray a calm

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atmosphere, and the use of gentle light filtering through the window adds to their appeal. Additionally, Vermeer uses a geometrically accurate and detailed style that is reminiscent of photography.

Out of a desire to determine how Vermeer composed the frame in his works, we geometrically evaluated the spaces in the paintings [11, 14]. However, it is unclear whether the painters during Vermeer's time were aware of these theories. As mathematicians had only recently found the distance point at this stage in history, application of this concept in artwork may have been difficult, c. f. Booker [3, pp. 30–33][1] and Ruurs [10, p. 30].[2] However, the paintings of Vermeer appear to show the correct perspective technique, because of which it has been suggested that he may have used a camera obscura to trace them. The reason his paintings give the impression of a photograph is not only limited to their precise geometrical accuracy. Also worth mentioning are his expressive aspects such as his use of light, which evokes the halation created by optical instruments.

Professor P. Steadman (1942–), an architecture specialist, conducted a graphical analysis on the paintings of Vermeer and investigated them for traces of the use of a camera obscura [15]. The methods and precision of his study yielded good results. It was stated that Vermeer created the composition by linking the elements of the background in his paintings while adjusting the focus for each element, and we agree with this assessment. Accordingly, the objective of this study is to determine how the scope of the scene affected the overall structure.

In this study, we consider Vermeer's artistic compositions as a combination of partial images. First, we conduct a graphical analysis on "Milk Maid," a work from the early period of Vermeerss career. Next, we consider the light image scope of camera obscura. Finally, we express our opinion regarding the relationship between the frame composition and classical theories of perspective techniques.

2 Analysis of "Milk Maid"

"Milk Maid," which is kept at the National Museum in Netherlands, depicts a woman preparing pap, a common meal at that time. Within the body of work of Vermeer, this piece is the only one that depicts food preparations. The maid is standing next to a table by the window and pouring milk into a pot. The faint light filtering in through the window hits the ingredients on the table.

In this piece, the table and window frame play important roles in the geometric construction of the painting. We analyzed these geometric motifs based on perspective principles. To find point s_1 (= station point; this is a plan view of the viewpoint) in Figure 1, the table is assumed to be a half octagon. The viewing point is s_2, assuming that the foot warmer has a rectangular shape and the s_1–Vc (= visual center or central vanishing point; this is an elevation view of the viewpoint) view axes of the table are shared. The analysis lines of the foot warmer, which is depicted in the background, converge on V_1V_2 (= vanishing points; these are on the horizon line, and straight lines that are not perpendicular to the screen converge and appear to vanish) on the horizon line, thereby matching the projection height

[1] When existence of the distance point was first proven mathematically, the appearance of drafting lines for correctly identifying the horizon line using the distance point were extremely similar to those of the visual cone section determination method used until that time. For this reason, Booker suggests that medieval painters may have been a bit confused about this property.

[2] According to Ruurs, there was a clear separation between artistic practitioners such as painters and mathematicians in pursuit of hidden line drawing theory in the latter 16th century.

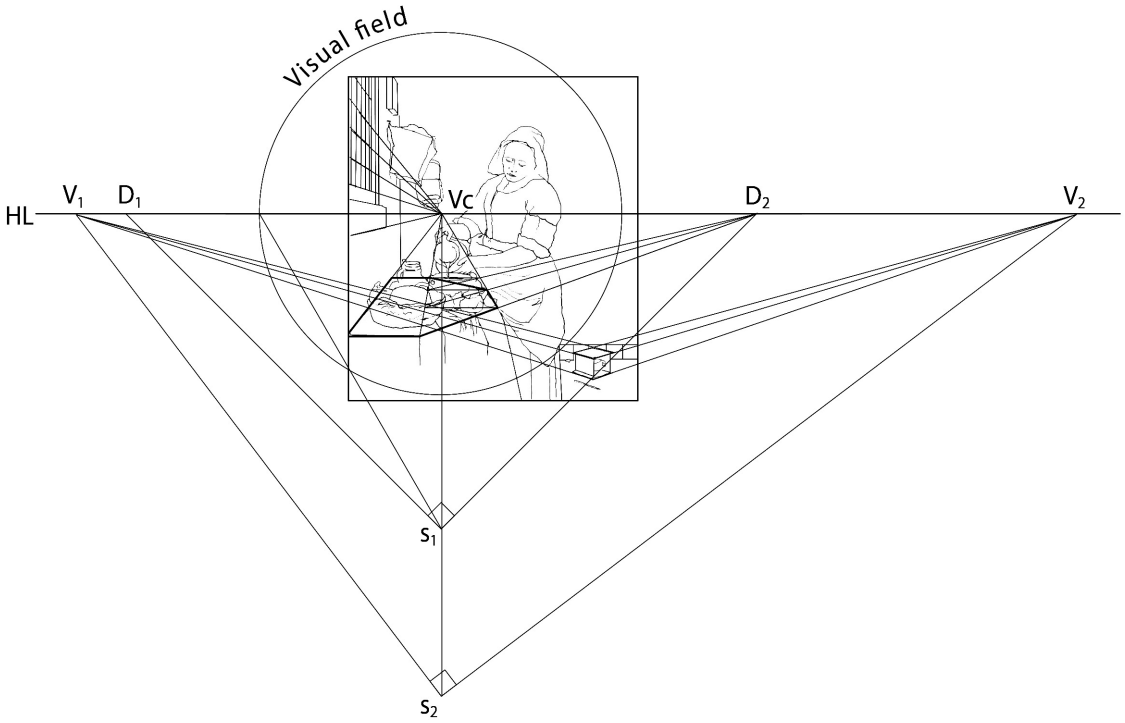

Figure 1: Analysis of "Milk Maid." There is a geometrically different viewpoint in the space depending on where the artist is standing. The shape of the foot warmer when viewed from the viewpoint s_1 is shown in Figure 2. "Milk Maid," 45.5 × 41.0 (1658–59) Rijksmuseum, Amsterdam.

of the window frame and foreground table.

"Milk Maid" is believed to have been painted during the same time period as "Woman in Blue" and "Woman with a Water Jug." When the geometric motifs of these two pieces were analyzed, multiple horizon lines were derived, indicating an inaccurate representation. However, the "Milk Maid" frame composition does not have multiple horizon lines. This change indicates that Vermeer adopted a fixed viewing height and achieved a stable and geometrically accurate frame composition.

2.1 Milk Maid's Plan View and Projection Drawing

Figure 2 shows the plan view and projection drawing for "Milk Maid." As the depth of the side wall has a steep elevation, there may be some slippage in the drafting process.

Additionally, accurate determination of the standing position of the model is difficult. For this reason, the size of the back of the model was set based on the extent that the depiction of the model overlapped with visual height.

This plan view shows the foot warmer when viewed from viewpoint s_1, which also provides a view of the table and window frame. Consequently, the foot warmer appears to be a parallelogram. However, according to the perspective theory, this object appears as rectangle A, as shown in Figure 3, when viewed from point s_2 in Figure 1. Furthermore, when viewed from point s_3 in Figure 3, it appears as rectangle B, a shape that is almost a square.[3]

[3]The shape of the foot warmer can be confirmed in prints from the time, and the top and bottom appear to be nearly square. This kind of shape can be seen in the 1614 print by V. Roemer, "Emblem from Sinnepoppen."

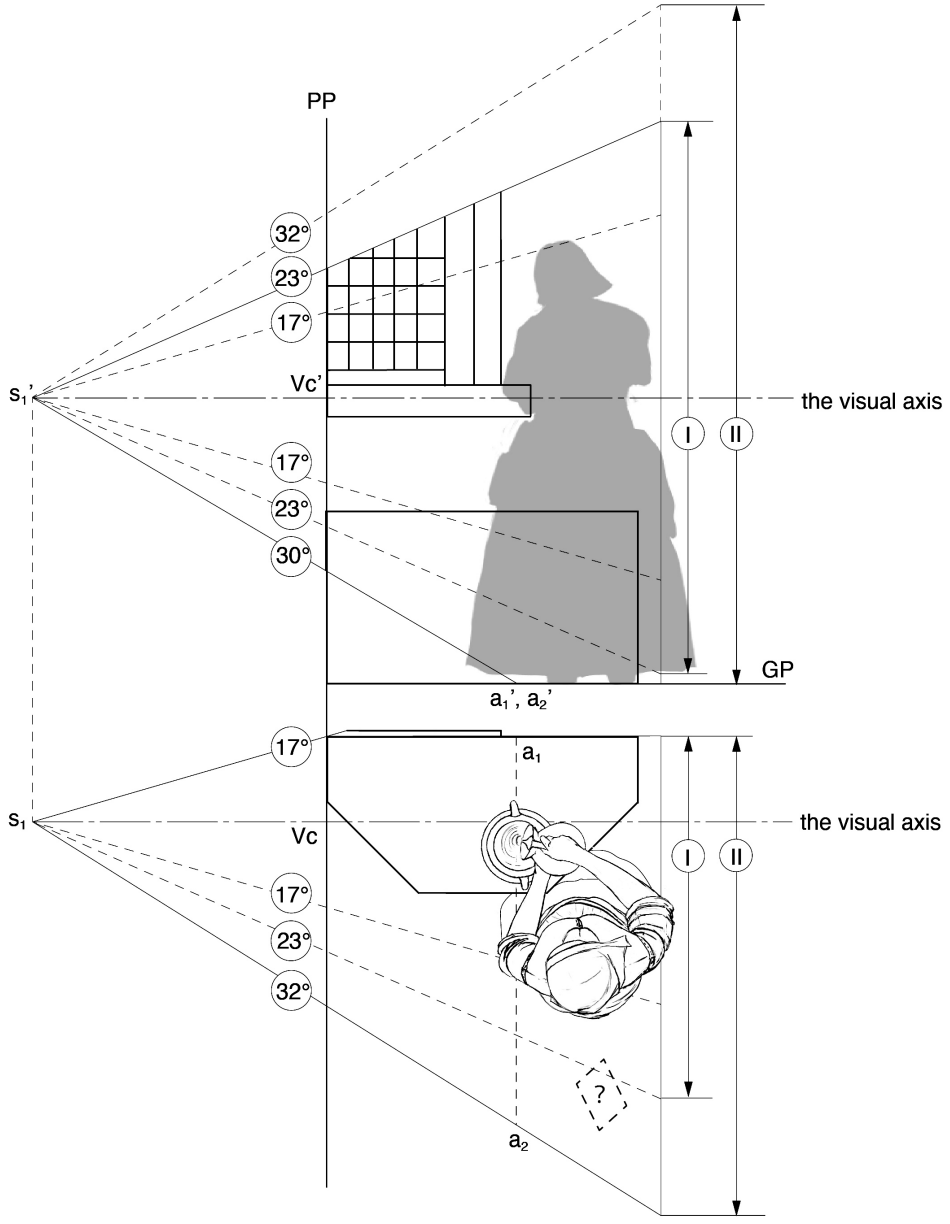

Figure 2: Elevation and plan of "Milk Maid." The right side of the viewing axis is drawn considerably wider than the left side in the plan view. In elevation, the region lower than the viewing axis is drawn wider than the top. When drawing a foot heater from the viewpoint s_1, it becomes a parallelogram.

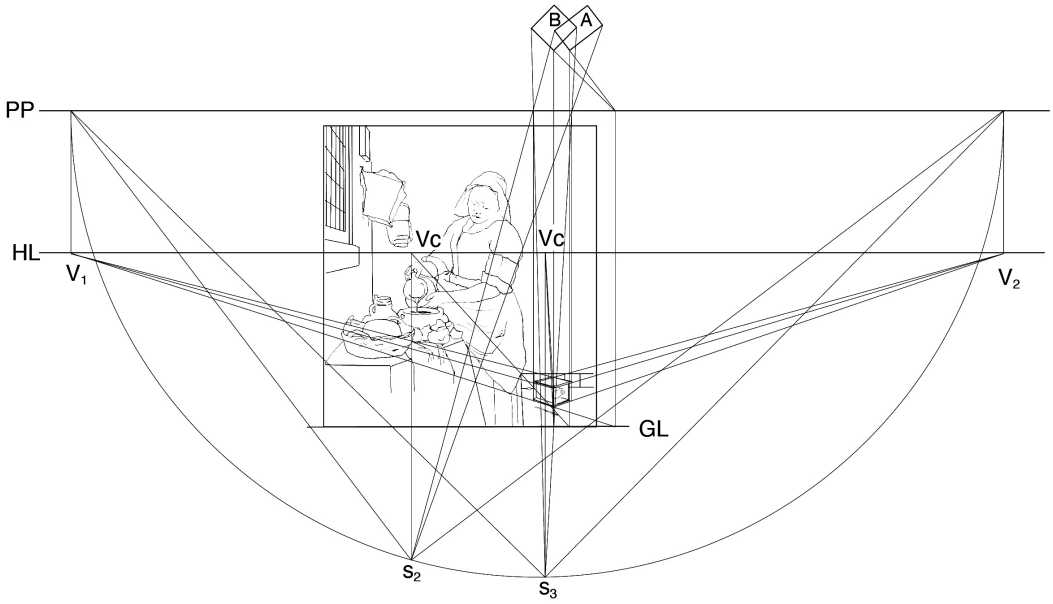

Figure 3: Analysis of foot warmer. When drawing a plan of the foot warmer from the viewpoint s_2, it is a rectangular shape and looks like a rectangle A. However, from the perspective of s_3 the foot warmer appears to get close to a square and looks like B.

The table, window frame, and foot warmer depicted in this work are viewed from different perspectives. These varying views are configured as a single composition in the painting. This phenomenon could have been caused by a lens focus adjustment and lens movement. Moreover, this suggests that the painter fused landscapes from differing view distances into a single space using his knowledge of perspective.

2.2 An Analysis of Milk Maid's Angle of View

Generally, the viewing angle of a camera indicates the range for the image projection. However, if we consider the angle of view in Figure 2 based on this fact, a discrepancy can be observed. In the plan view, with the viewing angle as a vertical axis, the sum of the angles on the left and right sides should be the same value. With the viewing axis at the center, if we consider double the values for up, down, left, and right, the narrowest angle is the left half of the plan view: 17°, or 34° when doubled. The next narrowest angle is the lower half of the projection drawing, and it is 23°, or 46° when doubled (I in Figure 2). In contrast, the widest angle is the right half of the plan view, and it is 32°, or 64° when doubled, exactly twice the size of the narrowest angle (II in Figure 2). It is important to note that these numerical values may have errors resulting from the aspect ratio of the digital image which were available for drawing and analysis by the author.

Figure 4 shows the range of angles I and II, derived above, with a circle. The question is whether this light range can be considered as the circle projected by a camera obscura. Within the actual canvas size, the diameters are approximately 39 cm for angle I and 55 cm for angle II.

Determination of the conditions for the camera obscura lens in the atelier of Vermeer is not the purpose of this study. Professor Steadman has already analyzed the interior scenes in the works during the middle and later stages of Vermeer's career and conducted sufficient experimentation and consideration. In his study, Steadman states that the type of work

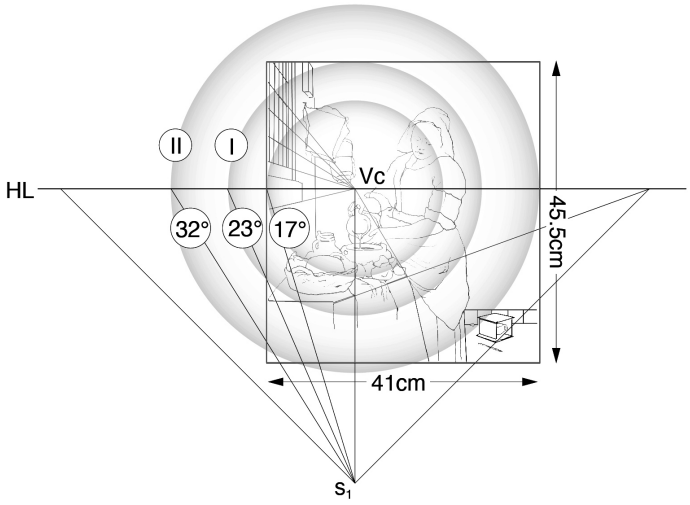

Figure 4: Expected range of light. This circle means the range of images of light in the camera obscura. The size of the circle which is the bottom of the viewing pyramid is determined by the four sides of the canvas. One can observe angle I extend to the top edge of the canvas, and angle II indicates up to the right edge of the canvas.

without focus adjustment "has a lens in a fixed position, resulting in a point in the scene that is the distance with the best focus" [15, p. 55] (based on D. Fink's experiment, see Note 116). In addition, he also notes the following regarding 17th century camera obscura: "focusing on each part of the scene individually is completely possible" [15, 186–187]. Although it is impossible to know what type of lens Vermeer had at his disposal, we do know that the depth of scenes, similar to those depicted in his paintings, cannot be captured in a single projection.

2.3 "Milk Maid" Canvas Angles and Depicted Models

Applying the ranges of angle I and angle II, we investigated the position of the models depicted in other works. Figure 5 shows the result of overlapping the perspective center, Vc, with the center of the angle of sight on a work from the early period. In "Officer and a Laughing Girl," there are two perspective centers, and when the perspective center closer to the soldier's face is centered on II, the entire window frame fits within it, but the girl's chair is outside the center. Although the space between these perspective centers is only a few centimeters, the girl can be fit into the frame by shifting the lens or canvas to the left or right. Naturally, shifting the canvas is the easier option. A majority of people depicted in the works of Vermeer from the middle and later periods fit within this frame. Even when the canvas size differs, the people in the paintings fit within a 30–40 cm diameter circle (angle I). Sometimes the entire bodies of the models are depicted, and sometimes only their upper bodies are included.

3 Camera Obscura and Frame Composition

According to Professor Steadman's hypothesis, if Vermeer did possess a camera obscura, it was a box type camera obscura. He says: "Vermeer shifted the position around 50 cm forward and backward, and a larger distance from side to side. This means that the lens was mobile. The front of the camera was most likely made of a lightweight cover or curtain rather than a

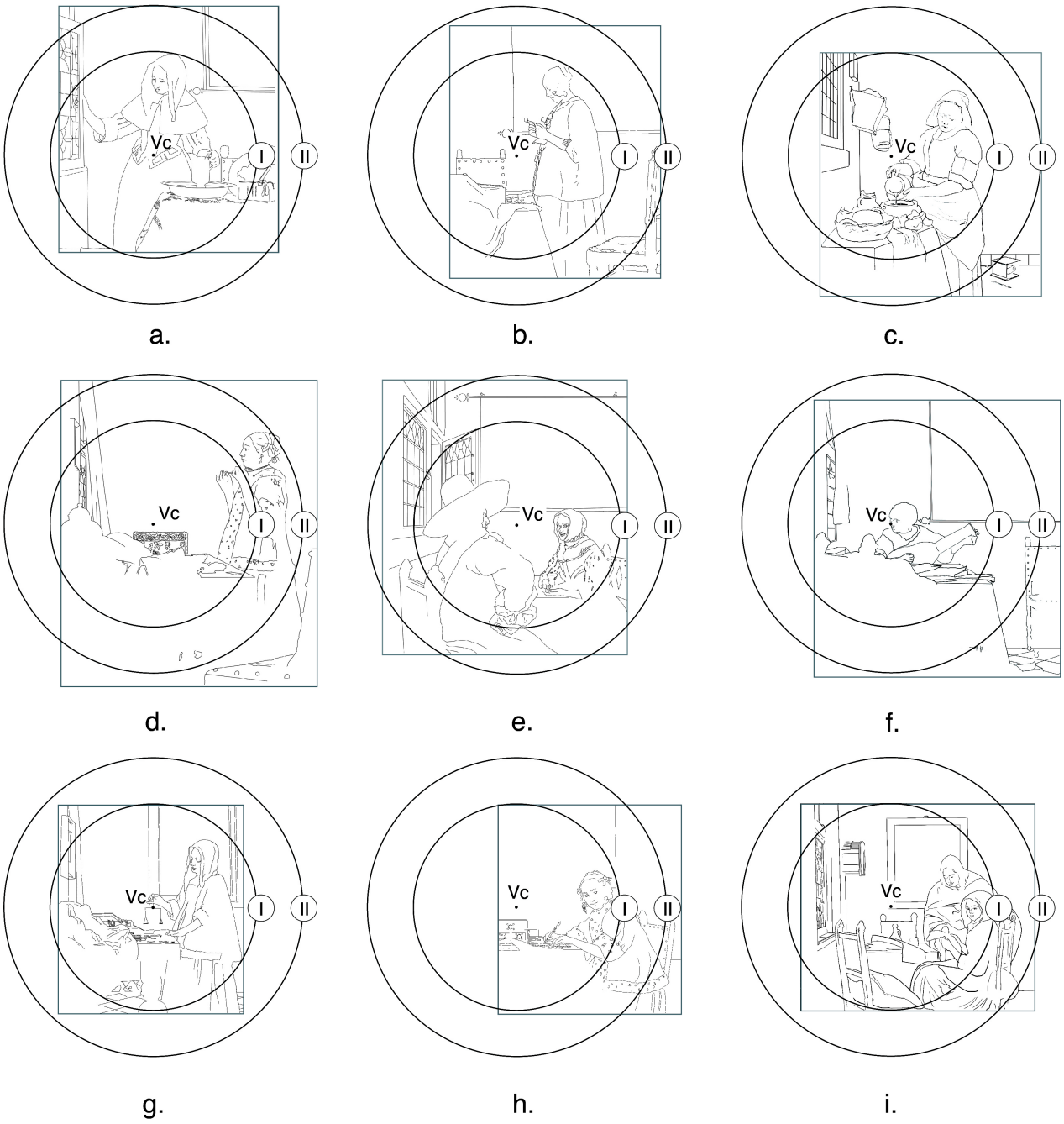

Figure 5: Comparison of angle of view and canvas size. These figures show the state in which the range of light assumed from "Milk Maid" is superimposed to match the canvas size. a. "Young Woman with a Water Pitcher," 45.7×40.6 (1662–1665) Metropolitan Museum of Art, New York. b. "Woman in Blue Reading a Letter," 46.5×39 (1662–1665) Rijksmuseum, Amsterdam. c. "Milk Maid," Ibid. d. "Woman with a Pearl Necklace," 55×45 (1662–1665) Staatliche Museen Preußischer Kulturbesitz, Gemäldegalerie, Berlin. e. "Officer and Laughing Girl," 50.5×46 (1657–1660) Frick Collection, New York. f. "Woman with a Lute," 51.4×45.7 (1662–1665) Metropolitan Museum of Art, New York. g. "Woman Holding a Balance," 42.5×38 (1662–1665) National Gallery of Art, Washington D.C. h. "A Lady Writing," 45×39.9 (1662–1667) National Gallery of Art, Washington. i. "Girl Interrupted at Her Music," 39.3×44.4, (1658–1661) Frick Collection, New York.

solid wall" [15, p. 141].

In works where the models are arranged near the center wall, Vermeer may have been able to capture them in the same light image as the pictures and maps on this wall. In other words, he captured the relative sizes, shapes, and forms of the object he wanted to depict, without adjusting the focus within the frame. However, there are also some works in which the models are separated from the center wall. Accordingly, in these works, the viewing distance differs for the center wall and the models, and focus adjustment is required. Without a doubt, the appearance of the floor tiles would have changed with each adjustment. This means Vermeer would have needed to employ some method to ensure a consistent diminution ratio for the tiles.

In the areas not contained within the presented angle I, the center wall does not contain any geometric elements. Therefore, if these were added, he may have focused on the depiction of the light. As maps and pictures within the frame face the lens, they can be painted anywhere within the composition by moving the canvas. The various motifs on the tables are geometrically accurate and finely-detailed depictions, suggesting that they were painted in focus, along with the models, which are usually positioned close to the table.

The question is how the geometric structure of the floor tiles was achieved. For example, there are hundreds of surviving paintings by another artist of the same period, Pieter de Hooch (1629–1684), and a graphical analysis of these works shows traces of focus adjustment [12]. These traces are the same multiple horizon lines in varying positions seen in the early works of Vermeer [8, pp. 60–64][4]. However, in "Milk Maid" and later works, these traces have disappeared.

3.1 Light Form and Floor Depiction

Some works that depict a single female model do not include the floor in the composition. These are organized in one of two ways. The first features the model standing close to the central wall and captured from a close perspective. The second features motifs depicted in the foreground, with a floor that should be visible, but is not shown. As Vermeer gained experience with painting, he gradually incorporated the floor into his compositions, in addition to the center walls and side walls with windows, depicting a larger interior space.

The boundary line between the center wall and the floor surface is an important element in the depiction of the space. The decision of incorporating or not incorporating this line causes major changes to the structure. As shown in Figure 5, "Woman with a Water Jug" (a) and "Woman in Blue" (b) use almost the same-size canvas as "Milk Maid," and all three paintings feature a standing model. However, unlike "Milk Maid," the other two works do not include the floor in their composition.

3.2 Appearance of the Floor

When the composition of the depicted space includes the floor, a boundary line between the center wall and the floor surface is created. Figure 6 compares the position of this boundary line. When after measuring the distance from the perspective center to the bottom edge of the canvas is the same, the position of the boundary line between the floor surface and the center wall seems to be roughly the same as well. In Vermeer's works, if the distance from the

[4]The position of the vanishing point is higher when focusing on the near view of the scene than when focusing on the background.

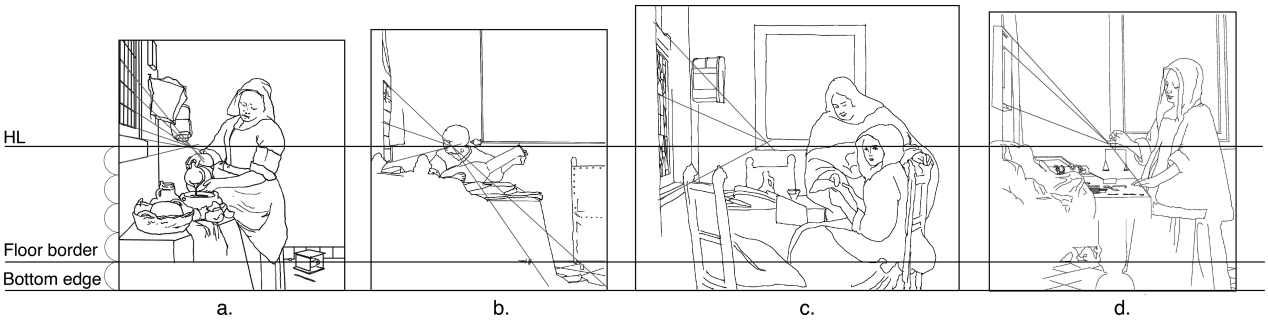

Figure 6: Comparison of floor positions on the screen. Both the HL (horizontal line running parallel to the screen through the visual center) and the floor in the painting as well as the bottom of the canvas are parallel lines, the boundary of the drawn floor is about the same height on the canvas. a. "Milk Maid," b. "Woman with a Lute," c. "Girl Interrupted at Her Music," d. "Woman Holding a Balance."

perspective center to the bottom edge of the canvas is considered to be 1 unit, the position of this boundary line frequently forms a uniform partitioning of the space. In all of the works shown in Figure 6, this division is 1/5 of the space.[5]

At first glance, this even partitioning of the length from the bottom edge of the canvas to the horizon line seems coincidental. Additionally, the interior space, which supports the composition of the works of Vermeer, can be considered to have sprung up out of inevitability from the lighting design. The impact of the aspect ratio of the canvas when painting a space with a sense of depth using perspective should be discussed as well.

3.3 Definition of Euclid

Pietro Accolti (1579–1642), an Italian mathematician, painter, and architect, wrote "Lo Inganno degli Occhi" [1] in 1625 [9, p. 135] (from this book, the author learned about the existence of Accolti), [2, pp. 377–378]. There is no record of this book ever being translated into Dutch. However, as shown in Figure 7, even without needing to set a half way point (1/2D) [6], a floor can easily be depicted with a sense of depth. With this method, using the concept of a square, with the length of the side equal to the shorter side of the canvas, this quadrilateral can be compressed to depict a floor with depth. The boundary line of the floor surface is determined by dividing the distance from the lower edge of the canvas to the horizontal line into two equal parts. Accolti describes it as follows: "This is a distance from the eye which is no longer related to perspective. It is proven in Euclid's Volume 6, No. 4" [1, pp. 25–27]. In addition, the reason for using the short side of the canvas is described as follows: "This is because painters prefer the shorter distance of the canvas" [1, p. 26].

Similar triangles ABC and DBE are formed, as shown in Figure 8, and the distance points unrelated to the viewing distance are defined solely on the canvas. For this reason, a virtual distance point is set, while the original viewing point is ignored.

4 Considerations

Figure 6 shows the works of Vermeer when he started to paint genre paintings. In these four paintings, tiles were drawn on "Woman Holding a Balance" and "Woman with a Lute." The

[5]In other works, the position repeatedly partitions the frame into 1/3 or 1/2 divided spaces.

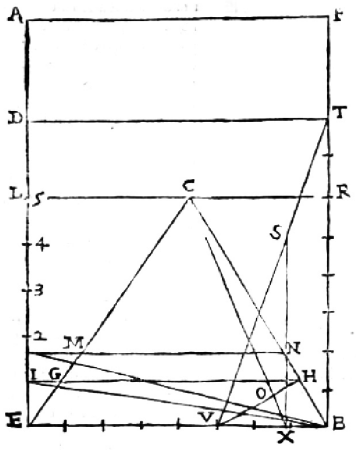

Figure 7: Illustration by Accolti. From the material shown in reference [1], the diagram used in the explanation for obtaining the distance point of Chapter 20 arithmetically. The actual article contains several drawings related to this.

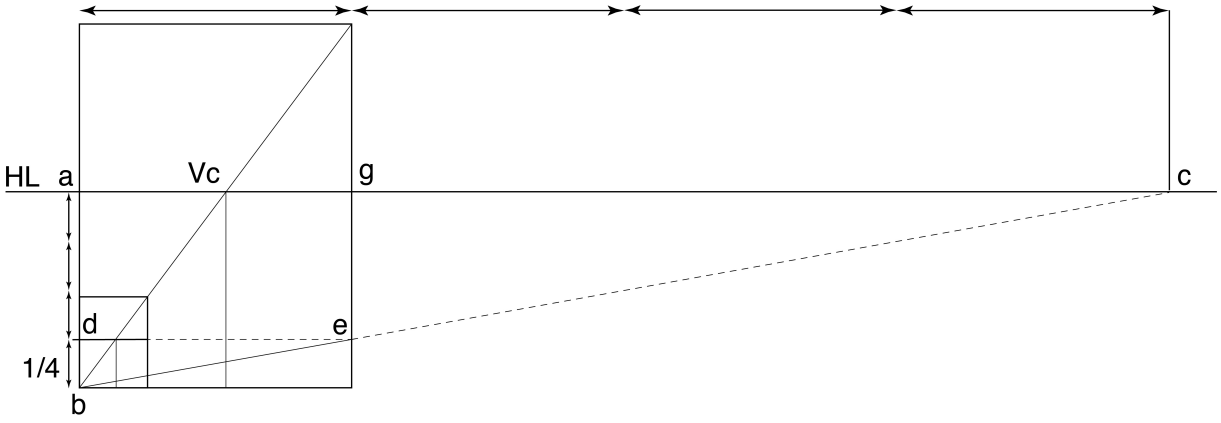

Figure 8: Accolti's theory. This indicates when the floor is set to a height of 1/4 when the bottom edge of the canvas is set to 1 from the bottom edge to the HL. The distance gc is three times the length of the canvas' short side. If you set the floor to 1/5, the distance gc is four times the length of the short side.

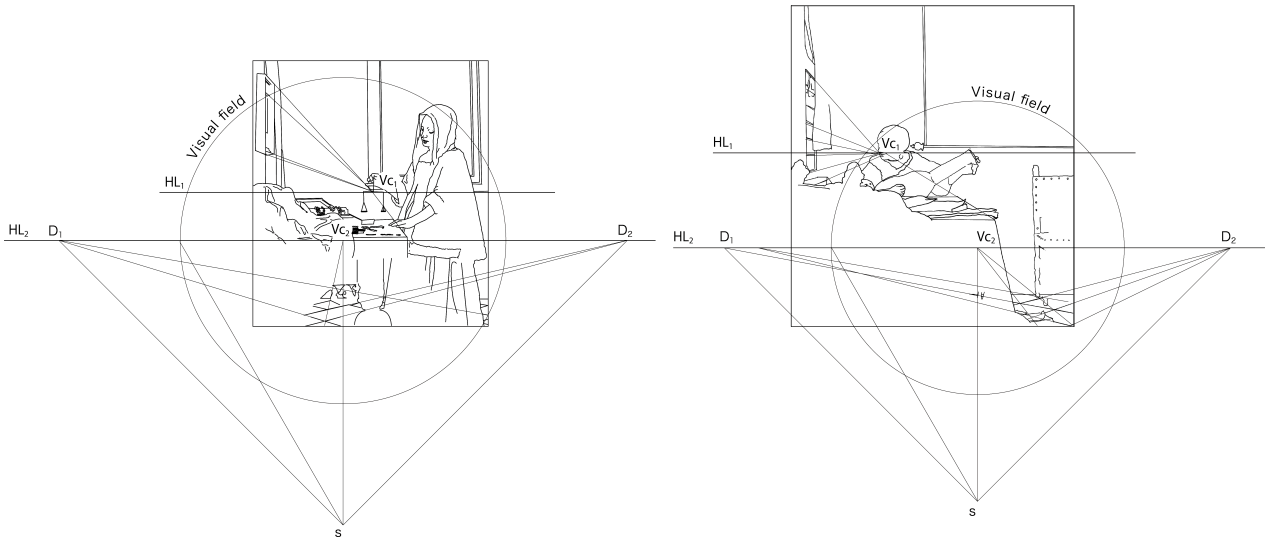

Figure 9: Analysis of drawn tiles. The model and floor tiles were captured from a different perspective. In the analysis, the total perspective at the distance point of the floor did not converge neatly, thereby resulting in a deviation. As an analyst, we state that the diagram was made easy to understand.

analysis diagram of these two paintings based on the aspect ratio of the canvas size is shown in Figure 9. As a result of the analysis, horizon lines were found in the upper and lower halves of the screen.

During Vermeer's period, a painter understood the concept that the height of the horizon lines coincide with the perspective of the observer. This is necessary for perspective drawing. Additionally, it was the time when painters were trying to acquire the concept of distance points. Therefore, it is not known if Vermeer knew about distance points. However, except for the concept of distance points, it had been known then that the vanishing point of the diagonal of a square can be obtained from the horizon in the same manner as for the height of the perspective. Therefore, it can be inferred that Vermeer created paintings with some sense of incongruity about the fact that the diagonal of the floor tile will not reach the height of the horizon line. It has already been clarified in "Painting Art" that the tiles on the floor and other motifs do not match geometrically [7].

Contemplating about the reason why this sense of incongruity is left on the screen, we think that there is something he wanted to prioritize more than the concept of drawing. Although this is just a possibility, perhaps Vermeer managed to draw a tile that could not be captured near the center of the camera obscura image. At that time, the part that was away from the center of that image was distorted and blurred. Therefore, the boundary line between the white and black tiles that could not be captured in the center of the image was unclear, and we assume that he was hesitant about how to express it. In other words, he had to determine whether depictions of the tiles should be geometric or whether they should be drawn as the impression obtained from the image of light.

The canvas size of "Woman Holding a Balance" and "Woman with a Lute" are different. However, finding a circle with a field of view having a 60° apex angle from the constructed floor, one can see that the screen does not fit in this circle. Nevertheless, the screen of "Lady Standing at a Virginal" fits perfectly into the circle with a field of view having a 60° apex angle. Moreover, the floor tiles are arranged in an orderly manner (Figure 10).

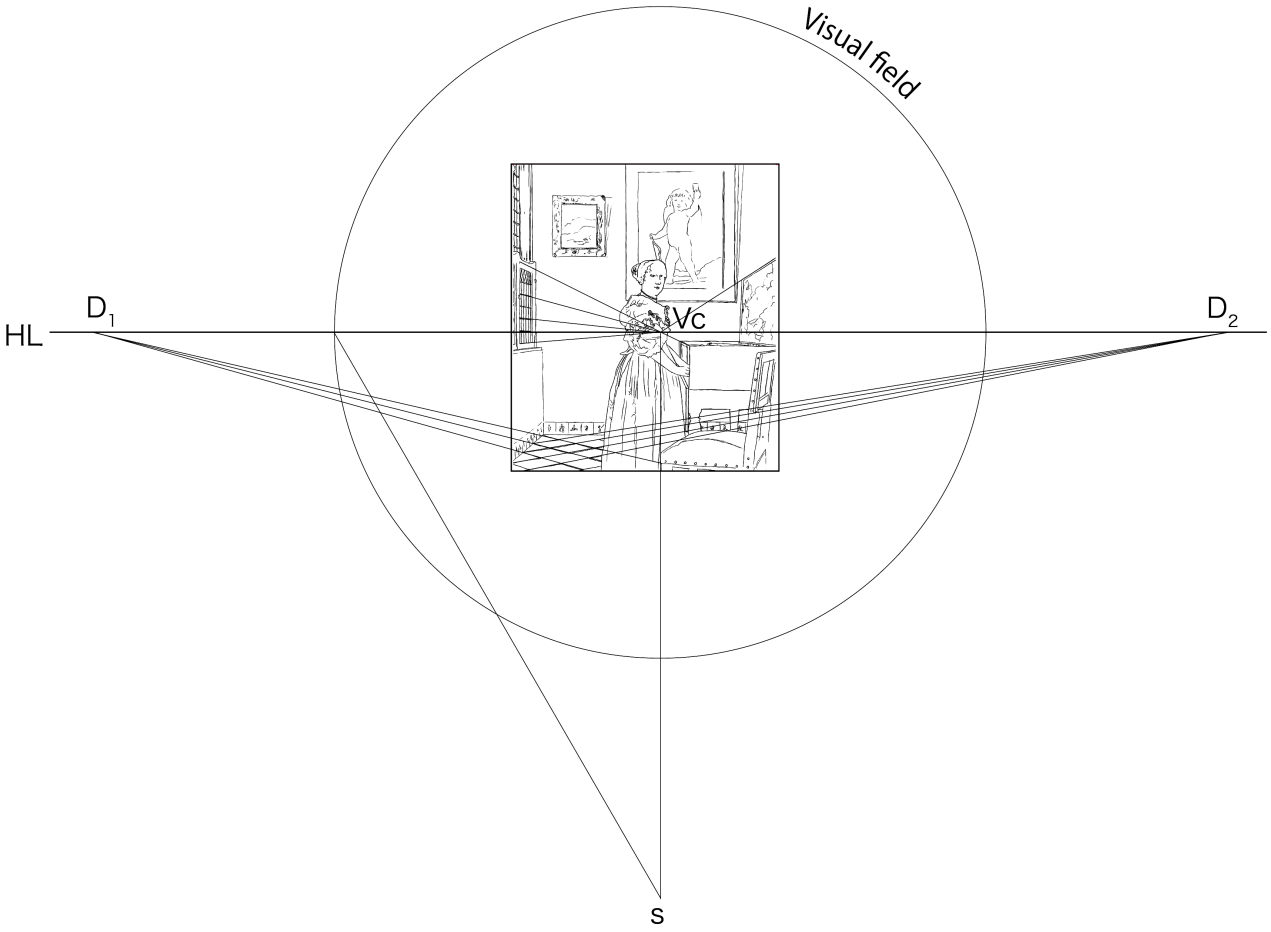

Figure 10: Analysis of distance points. The analysis lines are beautifully focused on one point as shown in the photograph.

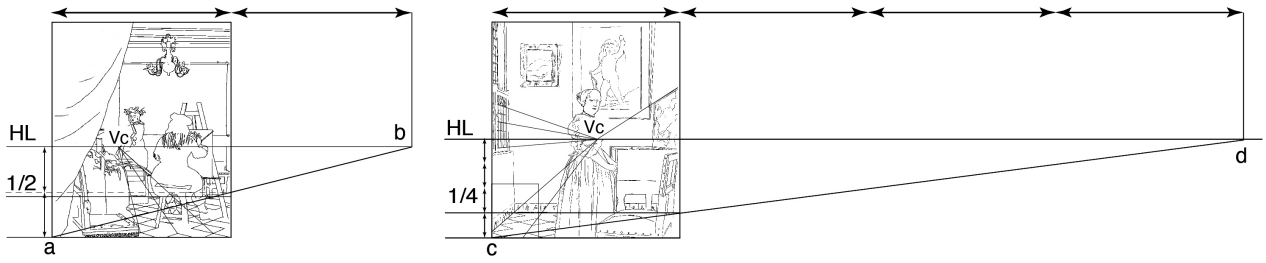

Figure 11: The relationship between the floor and Accolti's theory. The number of tiles drawn on "The Art of Painting" (left) is 5 multiplied by 6 (vertical×horizontal). Because it is close to the square, it may match Accolti's description. The straight line *ab* in the figure is along the reducing rate of the tiles. The number of tiles drawn on "Lady Standing at a Virginal" (right) is 5 multiplied by 3.5 (vertical×horizontal). If the floor is square, there is a shortage of one and a half rows of tiles in the depth direction. It should be considered that the number of tiles can be easily increased or decreased by knowledge of perspective. The line *cd* is not along the reducing rate of the tiles. "The Art of Painting," 120 × 100 (1662–1668) Kunsthistorisches Museum, Vienna. "Lady Standing at a Virginal," 51.7 × 45.28 (1670–1674) National Gallery, London.

However, it should be noted that owing to the limitations of the lens of the camera obscura, it is impossible to capture the model and the background of the room as depicted on the canvas. In other words, Vermeer, as a painter, decided to incorporate the floor into the geometric harmony of the entire screen. Moreover, he knew that not only the floor but also the processing of other geometric motifs must be considered simultaneously.

In the works of Vermeer, in addition to the tiles, the chairs depicted in the foreground are also geometric motifs. For example, when the structure of these foreground chairs in Vermeer's paintings is analyzed, the vanishing point is approximately 170 cm away from the perspective center [13]. For this reason, we doubt that Vermeer used a needle above the horizon with long threads tied to it to construct the floor surface, regardless of the distance point. When considering methods for the accurate depiction of these tiles, without the use of thread, the first that comes to mind is capturing the actual positions using a camera obscura. As a result of the limitations of focus for the model and the room, light formations can be acquired through difference distance points with this method. Geometric disorder will not occur, provided that knowledge of perspective is used to integrate the images. By the time Vermeer noticed this, he had made some experimental works.

Another possibility is the limitations of the scope of light formation considered for "Milk Maid." If Accolti's method was used to structure the geometric floor surface, a square with the length of the sides equal to the shorter side of the canvas and the diminution ratio had to be determined (Figure 11, left). To depict a geometric pattern on the surface, the square is simply divided. However, even though the floor in Vermeer's works is positioned in a way that could indicate the use of the Accolti distance point, there are some cases where it is not a square (Figure 11, right).

To determine if the proportions explained by Accolti could be found in interior scenes painted by Vermeer's contemporaries, we analyzed the "Man Writing a Letter" and "Man and Woman Sitting at the Virginal" by Gabriël Metsu (1629–1667). These two works are painted on different-sized canvases. As shown in Figure 12, the distance from the bottom edge of the canvas to the horizon line is split into three parts, with the boundary line of the floor surface set approximately 1/3 of the way up.

In addition to the floor tiles, there are also other tiles that are common to the works of both

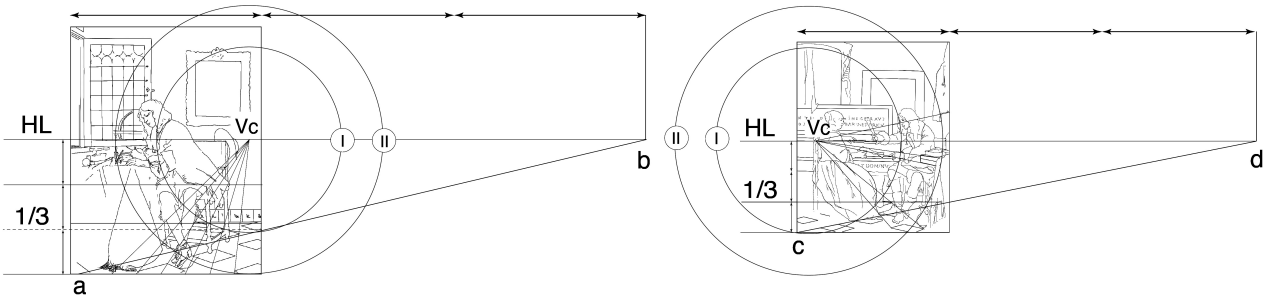

Figure 12: The relationship between Gabriel Metsu's work and Accolti's theory. The range of light assumed from "Milk Maid" is superimposed onto the work of Metsu. And the floor lines and HL shows their relationship. It seems that Euclid's proportionality is used here as well. The straight line ab in the "Man writing a letter" (left) figure is along the reducing rate of the tile. In this work, the drawn tiles is about 4.5 multiplied by 4 (vertical×horizontal). The line ab is along the reducing rate of the tiles. The tiles drawn on "Man and Woman Sitting at the Virginal" (right) is about 4.5 multiplied by 5 (vertical×horizontal). The line cd is along the reducing rate of the tiles partially. "Man writing a letter," 53×40 (1662–1665) National Gallery of Ireland, Dublin. "Man and Woman Sitting at the Virginal," 38×32 (1658–1660) National Gallery, London.

Vermeer and Metsu. These are the drainage tiles that surround the lower edge of the wall, and are squares with a side of 13 cm. If you measure the diagonal length of a checkerboard tiles on the floor with the drainage tiles, this is equivalent to 3 tiles for Vermeer and 2.5 tiles for Metsu, respectively. Perhaps these square drainage tiles might have functioned as a reference for measuring and dividing up the squares of the floor.

5 Conclusions

The theory of perspective originated in Italy, and when it reached the northern parts of the Netherlands, the cultural environment there gave rise to the interior-scene genre. In the 17th century, this area was the most prosperous in all of Europe, and the development of various technologies was underway. However, the means by which Vermeer and his contemporaries learned perspective is not clearly known.

In this era, it is difficult to uncover the truth about how painters used camera obscura, how they structured their works, and other related aspects. In 17th century Netherlands, books on practical perspective techniques were published widely, separate from mathematical proofs. If Vermeer had tried to paint all of his compositions using perspective, the type of geometric inconsistencies seen in his early-period works should not have been present. These inconsistencies disappeared in his later works, and the geometric accuracy increased.

The possibility that Vermeer chose Accolti's "distance point of eye" technique is very interesting as he knew about the geometric accuracy of perspective drawing and the realism provided by optical instruments. By the time his book was published in 1625 in Italy, the technique book written by architect Jacopo Barozzi da Vignola (1507–1573) had already been published and showed that the point of extension of the diagonal of the square was at the same height as that of the perspective [4, p. 6]. Nevertheless, it can be said that Accolti dared to describe in his book how to set "points" that were different from the perspective. On the other hand, the geometric inaccuracy observed in Vermeer's early genre paintings is not due to the lack of knowledge of the painter. The correct location of the vanishing point of the square is shown in the book of Hans Vredeman de Vries (1527–1607), who had a great

influence on architects and artists in the Netherlands during the early 17th century [5]. The fusion of Euclid's theorem and art is different from constructing a virtual space in a painting with perspective drawing. As if against the times, we do not understand Accolti's motive to describe Euclid's method. However, Vermeer's virtual space may not be a world completely dominated by optical instruments and perspective drawing machines. The Dutch painters of the 17th century painted a space of painting that was a mixture of reality and imagination. In these paintings, a glimpse of humanity of each painter can be observed. Therefore, it is not difficult to think that Vermeer imaginatively constructed a space of painting. In the future, we would like to conduct further studies analyzing the later works of Vermeer and the theory of Accolti.

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