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Lie–projective Groups

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1. Introduction

It is often desirable to apply the theory of Lie groups to more general locally compact groups G. This can be done if G can be represented as some kind of limit of Lie groups. In this spirit notions like 'Lie-projective group' or 'group approximated by Lie groups' have been created since the middle of this century. In the present paper, we want to compare some of these notions.
We consider three typical notions of approximation (see [12], [3] and [5]), and characterize them in terms of (projective) limits and of certain families of normal subgroups. Finally, examples show that the class of Lie groups, the class of locally compact Hausdorff groups and the three classes of groups approximated by Lie groups are different. Moreover, we rectify a statement in J. Szenthe's paper [12] on Hilbert's fifth problem.
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2. Three different notions of approximation

In this and the following sections we will use certain (families of) normal subgroups frequently. So, we first of all assign short names to them.

Definition 2.1. For a topological group G a family $\mathcal{N}$ of (compact) normal subgroups with $\bigcap \mathcal{N} = \{1\}$ is called a *(compact) Lie-normal family*, if every member $N \in \mathcal{N}$ is *Lie-normal*, in the sense that the factor group G/N is a Lie group.

Note that only Hausdorff groups can have a Lie-normal family, and that every group with a locally compact Lie-normal subgroup is neccessarily locally compact. Furthermore, we may assume that a (compact) Lie-normal family is not

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empty. Talking about order properties for Lie-normal families, we always refer to the superset order $\supseteq$.
For a start, we recall the usual definition of a Lie-projective group, or a group approximated by Lie groups (see e. g. [3], p. 57 or [11], p. 58). Many authors used this notion (e. g. [10], part 2 p. 101, [2], Lemma 1.7 and [9], p. 153, 175).

Definition 2.2. A locally compact Hausdorff group is called *Lie-projective*, if every neighbourhood of the identity contains a compact Lie-normal subgroup.

Note that we only need one of the two conditions 'the group is locally compact' and 'the Lie-normal subgroup is compact'. A weak sufficient condition for a group to be Lie-projective is the following: the group is a locally compact Hausdorff group, and its factor group modulo the maximal connected subgroup is compact. Therefore each locally compact Hausdorff group has an open, Lie-projective subgroup (see [14], p. 48 together with [2], Lemma 1.4 and 4.5).
The notion of an '(L)-group' in [5] (p. 541) uses a Lie-normal family instead of *small* Lie-normal subgroups (which means that each neighbourhood of the identity contains such a subgroup.) As every Lie-projective group is neccesarily an (L)-group, we call an (L)-group *weakly* Lie-projective here.

Definition 2.3. A locally compact Hausdorff group is called *weakly Lie-projective*, if it has a Lie-normal family.

Remark 2.4. Let G be a locally compact group. If G has a Lie-normal family $\mathcal{N}$, then the canonical maps $G \to G/N$ with $N \in \mathcal{N}$ separate the points of G. Conversely, given Lie groups G_i and continuous homomorphisms $f_i : G \to G_i$ for $i \in I$ separating the points of G, the kernels $\ker f_i$ form a Lie-normal family, as the induced maps $G/\ker f_i \to G_i$ are continuous monomorphisms into Lie groups, and hence $G/\ker f_i$ cannot have small subgroups.

Clearly the Hausdorff condition is redundant. However, there are groups possessing a Lie-normal family which are not locally compact (infinite products of non-compact Lie groups). On the other hand, a locally compact Hausdorff group does not neccessarily have a Lie-normal family (see Example 5.3).
A stronger notion of approximation was given in [12], p. 324. There one only considers *well-ordered compact* Lie-normal families. We can even assume that the family is order isomorphic to the natural numbers (see Section 4), and therefore we use the term 'countably' in the following definition.

In the following we shall call a map $f\colon (I, \leq) \to (X, \preceq)$ *relation preserving*, if $fi \preceq fj$ holds whenever $i \leq j$ is valid. The poset I is called *directed*, if any two elements of I have a common upper bound. It is called *comparing*, if any two elements of I are comparable. A subset $C \subseteq I$ is *cofinal*, if every element of I has an upper bound in C.
Note that a relation preserving image of a well-ordered, directed or comparing set has the same property. The comparing orders are exactly the total orders. A cofinal subset of a directed set is itself directed.

Definition 2.5. A Hausdorff group is called *countably Lie-projective*, if there exists a compact Lie-normal family $(\mathcal{N}, \supseteq)$ which is a relation preserving image of a well-ordered (non-empty) set of indices.

A cofinal subfamily of a Lie-normal family is itself Lie-normal.
It is true, though not obvious from the definition, that a countably Lie-projective group is in fact Lie-projective.

Lemma 2.6. *If $(\mathcal{N}, \supseteq)$ is a non-empty and directed compact Lie-normal family, then every neighbourhood of the identity contains a member of $\mathcal{N}$.*

Proof. Let us assume that there exists a (non-empty) directed compact Lie-normal family $(\mathcal{N}, \supseteq)$ and an open neighbourhood U of the identity such that none of the sets $N \setminus U$ with $N \in \mathcal{N}$ is empty. Thus we have a filter base of compact (and hence closed) sets. Therefore their intersection $(\bigcap \mathcal{N}) \setminus U$ is not empty — which is impossible. ∎

Corollary 2.7. *For the three notions of approximation defined in this section, the following implication chain is valid:*

'countably Lie-projective' $\Longrightarrow$ *'Lie-projective'* $\Longrightarrow$ *'weakly Lie-projective'*

3. Limits of Lie groups

A good motivation for the notion 'Lie-projective' is given in [2], Lemma 1.7 (using results of [13]): Lie-projective groups are exactly the locally compact projective limits of Lie groups. Therefore we recall in this section the categorial definition of a (projective) limit and some basic properties (see also [1], p. 206–208 for a short introduction).

Definition 3.1. For an arbitrary category $\mathbf{C}$ a *diagram* is a functor $D : \mathbf{I} \to \mathbf{C}$. A *cone* (X, f_i) *over* the diagram D consists of an object $X \in \mathbf{C}$, the *vertex*, together with arrows $f_i : X \to D(i)$ for each object $i \in \mathbf{I}$, such that $D(a) \circ f_i = f_j$ holds for every arrow $a : i \to j$ of $\mathbf{I}$.
A *limit* cone (X, f_i) over D is one with the following universal property: for each cone (Y, g_i) over D there is exactly one arrow $g : Y \to X$ factorizing $g_i = f_i \circ g$ for each object $i \in \mathbf{I}$. The vertex X is then called a *limit* of D.

A partially ordered set $\mathbf{I} = (I, \leq)$ may be viewed as a category such that one and only one arrow $i \to j$ exists if and only if $j \leq i$.

Definition 3.2. A diagram from a partially ordered set $\mathbf{I}$ is called a *projective system*, if $\mathbf{I}$ is directed. A limit (cone) of a projective system is called a *projective* limit (cone).

The projective systems are simply the systems $(I, X_i, f_{ji}, j \leq i)$ with a directed order $\leq$ on I, objects X_i for each index $i \in I$, and arrows $f_{ji} : X_i \to X_j$ for each pair $j \leq i$ of indices, where $f_{kk} = \mathrm{id}_{X_k}$ and $f_{kj} \circ f_{ji} = f_{ki}$ holds for all triples $k \leq j \leq i$ (see also [1], p. 208).
It seems to be a strong restriction if we only consider projective limits. However, there is a canonical way to represent a limit as a projective limit of sublimits. To keep things simple, we demand that a sublimit is a limit of the diagram restricted to a *full* subcategory. Thus this subcategory can be regarded as a set of objects of the index category.
Moreover, for projective limits it is sometimes very easy to get more than a directed order by using cofinal subsets of indices.

Lemma 3.3. *In any category the following holds:*

(a) *If they exist, limit cones are unique up to isomorphism. Moreover, every limit of a projective system which is defined on a cofinal subset of a directed index set is a projective limit of the whole system.*

(b) *For a given diagram on a category with a* set I *of objects, let* $(\mathcal{F}, \subseteq)$ *be a directed family of subsets of* I *which covers* I*, and let* X_F *be a sublimit with index category* F *for each subset* $F \in \mathcal{F} \cup \{I\}$*. Then there are, for members* $G \subseteq F$ *of* $\mathcal{F} \cup \{I\}$*, canonical arrows* $f_{GF} : X_F \to X_G$*, such that* (X_I, f_{FI}) *is a projective limit cone over the projective system* $(\mathcal{F}, X_F, f_{GF}, G \subseteq F)$*.*

Proof. Limits are unique by III.4 in [8]. Dualize XI.3.1 in [8] to see that a sublimit over a cofinal subset approximates the whole projective system. For the rest of the lemma we argue as follows:
For a subset G let (X_G, f_i^G) be a limit cone over the diagram restricted to G. Using its universal property we get the canonical arrow f_{GF} for supersets $F \supseteq G$. Thus the system constructed above is projective and (X_I, f_{FI}) is a cone over it. Given another cone (Y, g_F), the morphisms $g_i := f_i^F \circ g_F$ with $i \in F \in \mathcal{F}$ are well-defined and form a cone with vertex Y over the old diagram. Therefore we can show the universal property of (X_I, f_{FI}) using that of (X_I, f_i^I).

■

Now we focus on the category **TG** of topological groups and look at limits of Lie groups in particular. For a given diagram $D : \mathbf{I} \to \mathbf{TG}$ we let $\lim D$ be the topological subgroup of the direct product $\prod D(i)$ (where i ranges over the objects of $\mathbf{I}$) consisting of the points (x_i) with $D(a)x_i = x_j$ for all arrows $a : i \to j$ of $\mathbf{I}$. If we write p_i for the i-th product projection restricted to $\lim D$, then we obtain a limit cone $(\lim D, p_i)$ of D (see [8], V.2.2).
Hence every limit of Lie groups is a closed subgroup of their product, and every sublimit over a finite set of index objects is a Lie group. Applying Lemma 3.3 to the family of all finite sets of index objects we therefore conclude:

Corollary 3.4. *Every limit of Lie groups with a set* I *of index objects is a projective limit of Lie groups with a directed order whose cardinality does not exceed the cardinality of* I*.*

4. Characterizations

This section presents two theorems. Each of them states a list of conditions equivalent to a notion of approximation, involving other notions of approximation, (projective) limits and Lie-normal families.
We start with a characterization of projective limits of Lie groups with index set I by Lie-normal families $\mathcal{N}$, showing the interplay between the order on I and the order on $\mathcal{N}$ — see also [13] (p. 23–26). Remember the remarks accompanying Definition 3.2 concerning the canonical relation.

Lemma 4.1. *For a locally compact group G the following implications hold:*

(a) *Assume that (G, f_i) is a limit cone over a diagram of Lie groups with a directed index category $\mathbf{I}$.*
Then the image $(\mathcal{N}, \supseteq)$ of the relation preserving map $\mathbf{I} \ni i \mapsto \ker f_i$ is a directed Lie-normal family having a cofinal compact Lie-normal subfamily.

(b) *Given a directed and ordered index set I, assume that the image $(\mathcal{N}, \supseteq)$ of a relation preserving map $I \ni i \mapsto N_i$ is a Lie-normal family which has a cofinal compact Lie-normal subfamily.*
Then (G, f_i) is a projective limit cone over the projective system $(I, G/N_i, f_{ji})$ of Lie groups, if we let the maps $f_i : G \to G/N_i$ and $f_{ji} : G/N_i \to G/N_j$ be the canonical ones.

Proof. (a) We may assume $\mathbf{I} \neq \emptyset$ and $(G, f_i) = (\lim D, p_i)$, where D denotes the given diagram of Lie groups (see the end of Section 3). If $j \leq i$ holds for two index objects, we find an arrow $a : i \to j$ of $\mathbf{I}$, and $D(a) \circ f_i = f_j$ is valid. Hence the map $i \mapsto \ker f_i$ is relation preserving, and its image $(\mathcal{N}, \supseteq)$ is directed.
Therefore each neighbourhood of the identity of $\lim D$, being a superset of a finite intersection of members of $\mathcal{N}$, is a superset of a single member of $\mathcal{N}$. Hence the intersection $\bigcap \mathcal{N}$, being a subset of the intersection of all neighbourhoods of the identity, is trivial, as $\lim D$ is a Hausdorff group. Furthermore the subfamily of all $N \in \mathcal{N}$ which are contained in a fixed compact neighbourhood of the identity is cofinal.
It remains to show that $G/\ker f_i$ is a Lie group. But there is a continuous monomorphism from $G/\ker f_i$ into a Lie group, whence it cannot have small subgroups.
(b) As we may assume that $\mathcal{N}$ is not empty, every neighbourhood of the identity contains a member of $\mathcal{N}$ (by Lemma 2.6). Now we can use [13] (p. 25).

∎

The characterization of Lemma 4.1 is the main tool for the proof of the two theorems. However, in order to use it we need directed and compact Lie-normal families. The following lemma will help to find them (see also [2], Lemma 1.6).

Lemma 4.2. *For a locally compact group, every finite intersection of Lie-normal subgroups is itself Lie-normal.*

Proof. For two Lie-normal subgroups N_1 and N_2 of G, the canonical map from $G/(N_1 \cap N_2)$ into $G/N_1 \times G/N_2$ is a continuous monomorphism into a Lie group, and therefore $G/(N_1 \cap N_2)$ cannot have small subgroups.

■

For a limit of Lie groups we may assume the index set to be directed (see Corollary 3.4). Moreover, we may assume a totally ordered index set to be well-ordered, as every totally ordered index set contains a cofinal, well-ordered subset (see [7], Theorem IV.3.1). These results are not restricted to limits of Lie groups. For a limit of Lie groups with a totally ordered index set we can even assume the indices to be natural numbers, and hence the limit is metric (see Theorem 4.4). Here we call a group metric, if its underlying topological space is metric.

Theorem 4.3. *Conditions* (a) *to* (e) *are equivalent for a topological group* G*:*

(a) *G is Lie-projective.*

(b) *G is weakly Lie-projective and has a compact Lie-normal subgroup.*

(c) *G has a Lie-normal family and a compact Lie-normal subgroup.*

(d) *G has a directed compact Lie-normal family.*

(e) *G is a locally compact (projective) limit of Lie groups.*

Proof. Clearly (a) $\implies$ (b) $\implies$ (c), the last implication being merely tautological. If a Lie-normal family $\mathcal{N}$ and a compact Lie-normal subgroup M are given as in condition (c), we use Lemma 4.2 as follows: Intersecting all elements of $\mathcal{N}$ with M we obtain a compact Lie-normal family $\mathcal{C}$. Adding all finite intersections to this family we even get a directed compact Lie-normal family. Thus (c) implies (d). Finally, by Lemma 2.6, (d) implies (a).
Therefore (a) to (d) are equivalent. Corollary 3.4 shows that the condition 'projective' is redundant in (e). By Lemma 4.1 we have (e) $\iff$ (d). ■

Theorem 4.4. *Conditions* (a) *to* (f) *are equivalent for a topological group* G*:*

(a) *G is countably Lie-projective.*

(b) *G is Lie-projective and metric.*

(c) *G has a totally ordered, or equivalently, countable Lie-normal family and a compact Lie-normal subgroup.*

(d) *G is a Lie group, or has a compact Lie-normal family which is order isomorphic to the natural numbers.*

(e) *G is a locally compact limit of Lie groups with a comparing, or equivalently, countable set of index objects.*

(f) *G is a locally compact projective limit of Lie groups with the natural numbers as indices.*

Proof. We show the following cycle: (d) $\Longrightarrow$ (f) $\Longrightarrow$ (e countable) $\Longrightarrow$ (b) $\Longrightarrow$ (c countable) $\Longrightarrow$ (a) $\Longrightarrow$ (e comparing) $\Longrightarrow$ (c totally) $\Longrightarrow$ (d).
By Lemma 4.1 condition (d) implies (f), which obviously implies (e countable). In this case the limit $\lim D$ is a subspace of a countable product of metric spaces and thus is itself metric. By Theorem 4.3 therefore (b) follows from (e countable). If (b) holds, we choose a Lie-normal subgroup in each element of a countable base of compact neighbourhoods of the identity, and infer (c countable).
Let (c countable) be valid. Thus, as in the proof of Theorem 4.3, we have a compact Lie-normal family $\mathcal{C} = \{N_1, N_2, \ldots\}$. The family of all intersections $\bigcap_{\nu=1}^{n} N_\nu$ is then a relation preserving image of the natural numbers, and by Lemma 4.2 it is Lie-normal. Hence condition (c countable) implies (a). Using Lemma 4.1 we see (a) $\Longrightarrow$ (e comparing) $\Longrightarrow$ (c totally).
It remains to show that (c totally) implies (d), which is the interesting case. As in the proof of Theorem 4.3 we may assume that a non-empty and totally ordered compact Lie-normal family $\mathcal{C}$ is given. If the maximum $\max(\mathcal{C}, \supseteq)$ exists, we conclude $\max(\mathcal{C}, \supseteq) = \bigcap \mathcal{C} = \{1\}$, and $G = G/\{1\}$ is a Lie group. Otherwise we construct a cofinal subfamily $\mathcal{S}$ consisting of different elements $N_n \in \mathcal{C}$ with $N_n \supseteq N_{n+1}$ by induction over the natural number n:
(1) If the dimensions $\dim G/N$ are unbounded for $N \in \mathcal{C}$, we fix an arbitrary element $N_0 \in \mathcal{C}$ and choose members $N_{n+1} \in \mathcal{C}$ with $\dim G/N_{n+1} > \dim G/N_n$. Hence N_{n+1} is a proper subset of N_n, as $\mathcal{C}$ is totally ordered and a canonical map $G/N \to G/M$ is never dimension increasing.
Assume that $\mathcal{S}$ is not cofinal in $\mathcal{C}$. As $\mathcal{C}$ is totally ordered we find an upper bound $N \in \mathcal{C}$ of $(\mathcal{S}, \supseteq)$. This is impossible, as the dimension $\dim G/N \geq \dim G/N_n \geq n$ would be infinite in this case.
(2) If m is the maximal dimension $\dim G/N$ of factor groups for $N \in \mathcal{C}$, we choose $N_0 \in \mathcal{C}$ with $\dim G/N_0 = m$. As the totally ordered family $(\mathcal{C}, \supseteq)$ has no maximum, we can choose a proper subset $N_{n+1} \in \mathcal{C}$ of N_n.
Assume that $\mathcal{S}$ is not cofinal in $\mathcal{C}$. Similar to case (a) we get an upper bound $N \in \mathcal{C}$ of $(\mathcal{S}, \supseteq)$, and thus $m \geq \dim G/N \geq \dim G/N_n \geq \dim G/N_0 = m$ holds. Therefore the kernels N_n/N of the canonical maps $G/N \to G/N_n$ are zero-dimensional compact Lie groups and hence finite. Now the sets N_{n+1}/N are proper subsets of N_n/N. Therefore the sets N_n/N are infinite, which is a contradiction. ∎

What we have seen implies that the groups 'approximated by Lie groups' in the paper [12] are in fact the *metric* groups approximated by Lie groups in the usual sense. There are many compact Hausdorff (and therefore Lie-projective) groups which are not metric (see the next section) — and thus they cannot contain an open metric subgroup.
In [12] it is stated (above Theorem 2 and at the beginning of the proof of Theorem 3) that a locally compact Hausdorff group contains an open countably Lie-projective subgroup. By our results, this is correct only under the hypothesis that the group is metric (or equivalently, has a countable base of neighbourhoods of the identity). It therefore appears that this hypothesis should be added to the assumptions made in Theorem 3, Theorem 4 and the final corollary of [12].

5. Examples

In this section we present examples showing that none of the implications in Corollary 2.7 can be reversed, including the additional implications 'Lie group $\Longrightarrow$ ' at the beginning and ' $\Longrightarrow$ locally compact Hausdorff' at the end of the chain. The deeper we go down the implication chain the more general are the conditions, and more complicated examples are required.

Example 5.1. We consider a product $G = \prod_{i \in I} G_i$ of non-trivial Lie groups G_i, such that almost all of them are compact. Then G is a locally compact group, and even compact if all its factors G_i are compact. Furthermore, G is a limit of its factors (when I is viewed as a category with only identical arrows).
If the index set I is infinite, the product G has small subgroups, and thus it cannot be a Lie group. If the index set is moreover uncountable, the product has no countable base of neighbourhoods of the identity, and hence it cannot be metric. Using Theorem 4.3 and 4.4 we conclude: the product G is a Lie group exactly when the index set I is finite, it is countably Lie-projective exactly when I is countable, and it is Lie-projective in general.

The following two examples use semidirect products $G = A \ltimes N$ of topological groups A and N. The multiplication is given by $(a, n) \cdot (b, m) = (ab, n^b m)$ where $N \times A \ni (n, a) \mapsto n^a \in N$ is a given topological action of A on N from the right. Then we can identify A and N with the corresponding subgroups in G, and the multiplication is determined by $n \cdot a = a \cdot n^a$.
In both of the examples, the groups A and N (and hence G) are totally disconnected, locally compact Hausdorff and with a countable base for the open sets (and thus metric). Therefore, we have $\dim G = 0$ and hence $\dim G/K = 0$ for every Lie-normal subgroup K, which means that K is open in G. So the Lie-normal subgroups are exactly the open normal subgroups.

Example 5.2. This example of a weakly Lie-projective group which is not Lie-projective is due to Ta-Sun Wu (see [4], Example 3.4.ii):
Assume that F is a finite group, and T is a group of automorphisms of F with at least two elements. We write 1 and id for the identities in F and T, respectively. For non-negative integers $i \geq 0$ let $N \subseteq \prod_{i \geq 0} F$ be the group of all finite sequences in F, endowed with the discrete topology. Then the group $A := \prod_{i \geq 0} T$ acts on N componentwise, $(n_i)^{(a_i)} = (n_i^{a_i})$, and we obtain a semidirect product $G := A \ltimes N$. Then the subgroups $K_k := (\{\mathrm{id}\}^k \times A) \ltimes (\{1\}^k \times N)$ with $k \geq 0$ are kernels of projections to discrete groups. Thus they are open and normal and form a countable Lie-normal family.
We now assume that there exists a *compact* open normal subgroup K of G. We choose elements $a \in T$ and $m \in F$ with $n := m^a m^{-1} \neq 1$. Let $[m]_l \in N$ denote the sequence in F with m at the coordinate $l \geq 0$ and 1 at all other coordinates (analogously for $[a]_l \in A$). As K is open it contains an open subgroup $B = \{\mathrm{id}\}^k \times A \subseteq A$. For $l > k$ we therefore get

$$K = K \cdot [m]_l \cdot K \cdot [m^{-1}]_l \ni [a^{-1}]_l \cdot [m]_l \cdot [a]_l \cdot [m^{-1}]_l = ([m]_l)^{[a]_l} \cdot [m^{-1}]_l = [n]_l$$

as K is normal. Hence K contains the set $C := \bigcup_{l>k} B \cdot [n]_l$, which is closed (as B is compact) and thus compact. This is impossible, as a finite number of the open sets $B \cdot [n]_l$ never covers C. Hence G is not Lie-projective.
Analogously we can also use different groups F_i and T_i for each coordinate $i \geq 0$.

Example 5.3. The final example of a locally compact Hausdorff group which is not weakly Lie-projective is a slightly modified version of [9], p. 57:
Let F be a finite group with at least two elements and let A denote the group of integers. Then A acts on $P := \prod_{i \in A} F$ via shifts, $(f_i)^a = (f_{i+a})$. Let the normal subgroup U of P consist of all sequences (f_i) such that f_i is the identity for negative integers i. We endow the subgroup $N := \bigcup_{a \in A} U^a$ of P with the topology for which the sets U^a form a base of open neighbourhoods of the identity. This is possible because the sets U^a form a filter base of normal subgroups. Then all the open subgroups U^a of N are isomorphic to $\prod_{i \geq 0} F$. Furthermore, N is continuously embedded into P, and hence is a totally disconnected, locally compact Hausdorff group with a countable base for the open sets. The action of A on N remains topological, and we obtain a semidirect product $G := A \ltimes N$.
We finally show that every open normal subgroup K of G contains N: For a neighbourhood $U^a \subseteq K$ we conclude that $K = (-b) \cdot K \cdot b \supseteq (-b) \cdot U^a \cdot b = U^{a+b}$ for every integer $b \in A$.

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